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On "stop = repeat" servicing for non-exponential queueing networks with blocking

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Abstract

The communication blocking protocols of stopping or repeating a service in non-exponential queueing networks with blocking are shown to be equivalent under a partial balance condition. Examples both with a reversible and non-reversible routing are given.

Keywords

Queueing networks * job-local-balance * stop/repeat blocking protocol * product form.

1. Introduction

Blocking is a common feature in queueing network applications such as due to a restricted number of links in communication networks, shared processor units or finite store and forward buffers in computer systems or limited storage pools along assembly lines in manufacturing. Standard blocking protocols are the "stop (service or interrruption)" and the "repeat (or rejection)" communication protocol, under which a service such as a message transmission is instantaneously stopped (interrupted) respectively repeated upon blocking, and the "production (transfer or manufacturing)" protocol under which a job has to wait untill deblocking (cf. [1], [18], [25]).

In the exponential case the "stop" and "repeat" protocol can be argued to be the same based upon the memoryless property, while also equivalencies with the production protocol are established (cf. [18]). In the non-exponential case, however, equivalencies of interrupting or repeating services have not been reported and do not seem to hold generally.

This note will show that the "stop" and "repeat" protocol are effectively the same also for non-exponential services under a notion of partial balance. This notion itself is known to be responsible for insensitivity results and to be directly related to product form expressions (cf. [3], [4], [6], [7], [8], [10], [11], [12], [14], [17], [20], [24]). Roughly speaking, the equivalence of both protocols thus appears to be related to insensitive product form results.

Recently the author has revealed a somewhat similar equivalence result between the "stop" and the so-called "recirculate" protocol for exponential Jacksonian networks with blocked system departures. Under the latter protocol a job which is blocked to leave the system is instantaneously rescheduled as a newly arriving job, e.g. from the end back to the beginning of an assembly line. This protocol has already been introduced in Jackson's classical paper [15] and investigated more extensively in [16] as it retains product form results when total system size constraints are imposed. The present paper is different in a threefold manner:

(i) Non-exponential services are involved.

(ii) Job-local-balance rather than station balance is the key-notion.

(iii) A general framework of stochastic networks with blocking is studied.

Insensitive product form results have been widely reported in the literature for concrete reversible non-exponential queueing networks with blocking under the "repeat (rejection or recirculate)" protocol (cf. [2], [13], [23], [25]). A general insensitivity result under the "repeat" protocol for other stochastic network structures, such as networks with blocking and non-reversible routing, however, is not available. For the "stop" protocol no explicit insensitivity result at all for concrete non-exponential networks with blocking seems to be around.

Only indirectly insensitivity results under the stop protocol can be derived from either of the general frameworks in [12], [20] or [24]. These references, however, do not explicitly deal with blocking phenomena. This paper investigates both protocols in a general framework and proves their equivalence under a partial balance condition. The equivalence result is of both practical and theoretical interest as

1. The stop protocol seems more practical,
2. It formalizes an intuitively appealing result.

For presentational convenience the paper is restricted to a closed framework. A generalization to open systems can be given by standard arguments (e.g. [11], [12], [14]). First, the general equivalence result is established in section 2. Next, in section 3 the result is illustrated for networks with reversible routing and two specific examples with non-reversible routing.

2. Equivalence result

Consider a stochastic network with a fixed number of M jobs. A state $L = (\ell_1, \dots, \ell_M)$ denotes that job i has a current jobmark ℓ_i with $\ell_i \in S$, where S is some countable set of possible jobmarks.

For example, when dealing with a queueing network a jobmark ℓ can be of the form $\ell = (t,s,p)$ representing the type (or number) t of the job, the station s at which it is present and the position (or server) p that it occupies at this station.

The law of motion is determined by the characteristics

(i) F_ℓ : the distribution function of the amount of service that a job with jobmark ℓ requires, for all $\ell \in S$.

(ii) $f(\ell|L)$: the service capacity (or amount of service per unit of time) scheduled for a job with jobmark $\ell \in L$ when the system is in state L.

(iii) $p(\ell,\ell'|L)$: the transition probability for a job with jobmark ℓ to change its jobmark in ℓ' when the system is in state L.

(iv) $b(\ell,\ell'|L)$: the acceptation probability involved with this transition.

and either one of the following protocol descriptions:

P1 (Stop protocol) A job with jobmark ℓ in state L is effectively provided an amount of service per unit of time:

$$f(\ell|L)[\textstyle\sum_{\ell'} p(\ell,\ell'|L)\; b(\ell,\ell'|L)]. \tag{2.1}$$

Upon completion of its service in state L it changes its jobmark in ℓ' with probability

$$\frac{p(\ell,\ell'|L)\; b(\ell,\ell'|L)}{\sum_{\ell'} p(\ell,\ell'|L)\; b(\ell,\ell'|L)}\;. \tag{2.2}$$

P2 (Repeat protocol) A job with jobmark ℓ in state L is always provided an amount of service per unit of time $f(\ell|L)$. Upon completion of its service, however, it changes its jobmark in ℓ' with probability

$$(2.3)\qquad \begin{cases} p(\ell,\ell'|L)\ b(\ell,\ell'|L) & \ell'\neq\ell \\ p(\ell,\ell|L) + \sum_{\ell'\neq\ell} p(\ell,\ell'|L)[1-b(\ell,\ell'|L)] & \ell'=\ell \end{cases}$$

Roughly speaking that is, under the stop protocol a job's servicing is delayed by the probability that it would be blocked upon service completion at that moment, whereas under the repeat protocol the servicing is not delayed but a job can be blocked to change its jobmark in which case it has to redo a complete new service.

Remark Note that (2.1) delays a job's servicing by the blocking probability "averaged" over "all" possible new jobmarks ℓ', while the job will eventually change its jobmark is some specific jobmark ℓ'.

Exponential case First assume that the service requirements are exponential with parameter μ_ℓ for jobmark ℓ. The underlying process under protocol P1 or P2 then constitutes a continuous-time Markov chain. Let $q_1(L,L')$ and $q_2(L,L')$ denote the corresponding transition rate for a transition from state L into L' under P1 and P2 respectively. Also, for a state L let $L_i+\ell_i'$ denote the same state with the job-i jobmark ℓ_i replaced by ℓ_i'. Then one easily verifies that for $\ell_i'\neq\ell_i$:

$$(2.4)\qquad q_1(L_i+\ell_i, L_i+\ell_i') = q_2(L_i+\ell_i, L_i+\ell_i') =$$

$$\mu_{\ell_i}\ f(\ell_i|L_i+\ell_i)\ p(\ell_i,\ell_i'|L_i+\ell_i)\ b(\ell_i,\ell_i'|L_i+\ell_i),$$

while transition rates of any other form, with exception of $\ell_i'=\ell$, are equal to 0. Assuming that these chains are irreducible with unique stationary distribution $\pi_1(.)$ and $\pi_2(.)$ at one and the same set S, for both p=1 and p=2 these distributions are thus determined by the global balance equations:

$$(2.5)\qquad \pi_p(L) \sum_i \sum_{\ell_i'} q_p(L_i+\ell_i, L_i+\ell_i') =$$

$$\sum_i \sum_{\ell_i'} \pi_p(L_i+\ell_i')\ q_p(L_i+\ell_i', L_i+\ell_i) \qquad (L \in S)$$

and normalization as a probability distribution. Now note that transitions with $\ell_i'=\ell_i$ contribute equally to both the left and right hand side of (2.5) and can thus be deleted. As a consequence, from (2.4) it thus follows that $\pi_1 = \pi_2 = \pi$ for some π at S.

Job-local-balance The chains are said to satisfy job-local-balance (JLB) if for some distribution π at S, any i=1,...,M and $L_i+\ell_i \in S$:

$$(2.6) \qquad \pi(L_i+\ell_i)\ \mu_{\ell_i}\ f(\ell_i|L)\ \sum_{\ell_i'} p(\ell_i,\ell_i'|L)\ b(\ell_i,\ell_i'|L) =$$

$$\sum_{\ell_i'} \pi(L_i+\ell_i')\ \mu_{\ell_i'}\ f(\ell_i'|L)\ p(\ell_i',\ell_i|L)\ b(\ell_i',\ell_i|L).$$

By substituting (2.4) in (2.5) and summing over i, one directly concludes that any distribution π satisfying the JLB-equations (2.6) also satisfies (2.5) and thus $\pi = \pi_1 = \pi_2$.

The notion of JLB is directly related to the notion of local-balance as defined for generalized semi-Markov processes in [20] and has been introduced in [11] and [12]. By appropriate substitutions it can be concluded from these references that a distribution π satisfying JLB is insensitive under the "stop" protocol as (2.2) effectively does no longer contain blocking. Below, it will be shown, however, that (2.6) implies insensiti-vity under both the "stop" and "repeat" protocol and that is thus guarantees stationary equivalence also under non-exponential services.

Non-exponential case Assume that the distribution functions F_ℓ are absolute continuous with density functions $q_\ell(.)$. Denote by

$$(L,T) = ((\ell_1,t_1),\ldots,(\ell_M,t_M))$$

that job i currently has a jobmark ℓ_i and a residual service requirement t_i up to completion of its current service requirement. Let $\pi_1(L,T)$ and $\pi_2(L,T)$ be the unique stationary densities of the corresponding Markov processes under the P_1 (stop) and P_2 (repeat) protocol respectively.

Theorem 2.1 (Equivalence result) If JLB is satisfied, i.e. (2.6) for all i and $L_i+\ell_i \in S$, then for all (L,T) with $L \in S$:

$$\pi_1(L,T) = \pi_2(L,T) = \pi(L) \prod_{i=1}^{M} \{[\tau_{\ell_i}]^{-1} \; [1-F_{\ell_i}(t_i)]\}. \tag{2.7}$$

Proof We need to verify the global balance (or stationary forward Kolmogorov) equations under either protocol, where without loss of generality it is assumed that these have a unique solution.
To this end, we introduced the notation:

$$D(\ell|L) = \textstyle\sum_{\ell'} p(\ell,\ell'|L) \; b(\ell,\ell'|L).$$

Also, for a state (L,T) write

$$(L_i, T_i) + (\ell_i', t_i')$$

to denote the same state with the job-i specification (ℓ_i, t_i) replaced by (ℓ_i', t_i'). Further, the symbol 0^+ indicates a right hand limit at 0 and for an event A, let $1_{\{A\}} = 1$ if A is satisfied and 0 otherwise. The global balance equations (GBE) can then be derived in a standard manner by considering a point of time t, conditioning upon time t-Δt, dividing by Δt and letting Δt tend to 0.

GBE under P_1 (stop protocol)

$$\begin{aligned} &\textstyle\sum_i \Big\{ \frac{\partial}{\partial t_i} \pi_1(L,T) \; f(\ell|L) \; D(\ell|L) \; + \\ &\textstyle\sum_{\ell_i'} \pi_1((L_i,T_i) + (\ell_i',0^+)) \; f(\ell_i'|L_i+\ell_i') \; D(\ell_i'|L_i+\ell_i') \\ &1_{(D(\ell_i'|L_i+\ell_i') > 0)} \; p(\ell_i',\ell_i|L_i+\ell_i') \; b(\ell_i',\ell_i|L_i+\ell_i) \\ &/ \; D(\ell_i'|L_i+\ell_i)] q_{\ell_i}(t_i) \Big\} = 0. \end{aligned} \tag{2.8}$$

GBE under P_2 (repeat protocol)

$$(2.9)\qquad \sum_i \Big\{ \frac{\partial}{\partial t_i} \pi_2(L,T)\ f(\ell|L)\ +$$

$$\sum_{\ell_i'} \pi_2((L_i,T_i) + (\ell_i',0^+))\ f(\ell_i'|L_i+\ell_i') \times$$

$$p(\ell_i',\ell_i|L_i+\ell_i')\ b(\ell_i',\ell_i|L_i+\ell_i)\ q_{\ell_i}(t_i) +$$

$$\pi_2((L_i,T_i) + (\ell_i,0^+))\ f(\ell_i|L) \times$$

$$\sum_{\ell_i'} p(\ell_i,\ell_i'|L)[1-b(\ell_i,\ell_i'|L)]q_{\ell_i}(t_i) \Big\} = 0.$$

Expression (2.7) yields for both p=1 and p=2:

$$(2.10)\qquad \begin{aligned} \frac{\partial}{\partial t_i} \pi_p(L,T) &= -q_{\ell_i}(t_i)[\tau_{\ell_i}]^{-1}\ \pi_p(L) \prod_{j\neq i} \{[\tau_{\ell_j}]^{-1}[1-F_{\ell_j}(t_j)]\} \\ \pi_p((L_i,T_i) + (\ell_i',0^+)) &= [\tau_{\ell_i}]^{-1}\ \pi_p(L_i+\ell_i) \prod_{j\neq i} \{[\tau_{\ell_j}]^{-1}[1-F_{\ell_j}(t_j)]\} \end{aligned}$$

By substituting (2.10) in (2.8) and (2.9), recalling (2.7) and writing $\mu_\ell=\tau_\ell^{-1}$, for each fixed job the term within braces {.} in the left hand side of both (2.8) and (2.9) reduces to:

$$(2.11)\qquad \{q_{\ell_i}(t_i) \prod_{j\neq i} \{[\tau_{\ell_j}]^{-1}[1-F_{\ell_j}(t_j)]\} \times$$

$$[-\pi_p(L)\mu_{\ell_i}\ f(\ell_i|L) \sum_{\ell_i} p(\ell_i,\ell_i'|L)\ b(\ell_i,\ell_i'|L) +$$

$$\sum_{\ell_i'} \pi_p(L_i+\ell_i')\mu_{\ell_i'}\ f(\ell_i'|L_i+\ell_i')\ p(\ell_i',\ell_i|L_i+\ell_i')\ b(\ell_i',\ell_i|L_i+\ell_i')]\}.$$

By the assumption of job-local-balance (2.6), we have hereby proven that for each job i separately the term within braces {.} in both (2.8) and (2.9) is equal to 0. This completes the proof. □

Theorem 2.2 (Insensitivity result) Under the assumption of JLB we have

$$(2.12)\qquad \pi_1(L) = \pi_2(L) = \pi(L)$$

Proof Immediately by integrating over all possible residual service requirements t_i for each job i and the identity

$$\int_0^\infty [1-F_\ell(t)]dt = \tau_\ell.$$

Remark 2.3 (General distributions) It is well-known that arbitrary distributions, e.g. for deterministic service requirements, can be approximated arbitrarily closely (in the sense of weak convergence) by absolute continuous distributions (e.g. by mixtures of Erlang distributions, cf. [9]). By standard though technical weak convergence arguments on so-called D-sample path spaces, therefore, (cf. [9]) theorem 2.1 and corollary 2.2 can be proven to remain valid for arbitrary service requirements.

Remark 2.4 (Partial Job-local-balance) The insensitivity and non-exponential equivalence result of theorems 2.1 and 2.2 can be generalized to local results in particular states and for particular jobs only, if the job-local-balance condition (2.6) is not satisfied allover but just locally for specific L_i and ℓ_i. For presentational simplicity this is not included. For example, equivalence and partial insensitivity results can so be concluded for product form networks with infinite server and FCFS-stations (see remark 3.2).

3. Examples

This section provides three queueing network examples with blocking for which the JLB-condition (2.6) and thus also the equivalence and insensitivity results (2.7) and (2.13) hold.

The first example is rather standard. Its insensitive product form result, however, seems to be new under the stop protocol. The other two examples are more specific and have been reported in the literature only for exponential services.

In all three examples let $L = (\ell_1,\ldots\ell_M)$ just denote by ℓ_i the number of the station at which job i is present.

3.1 Jackson networks with finite stations and reversible routing

Consider a closed Jackson network with N service stations, M jobs of one class and state independent routing probabilities p_{ij} from station i to j. All stations i are assumed to be loss stations with N_i servers and no waiting facility, that is they can contain no more than N_i jobs and each job present is assigned one server. The mean service requirement of a job at station i is $\tau_i = \mu_i^{-1}$. The routing probabilities, furthermore, are assumed to be reversible, i.e. for some $\lambda_1, \ldots, \lambda_N$, uniquely determined up to normalization, and all i,j we have

$$(3.1) \qquad \lambda_i p_{ij} = \lambda_j p_{ji}.$$

With $\bar{n} = (n_1, \ldots, n_N)$ denoting the numbers n_i of jobs at station i, i=1,...,N, the "stop" and "repeat protocol" as described in section 2 particularize to:

Stop protocol When the system is in state $\bar{n}$ the servers at station i are all delayed by a factor

$$(3.2) \qquad \textstyle\sum_j p_{ij} 1_{\{n_j < N_j\}}.$$

Particularly, when this factor is equal to zero, a service at station i is stopped completely. When this factor is positive, a job completing service at station i routes to station j with probability:

$$(3.3) \qquad p_{ij} 1_{\{n_j < N_j\}} / \textstyle\sum_j p_{ij} 1_{\{n_j < N_j\}}.$$

Particularly, routing to saturated stations j with $n_j = N_j$ is prohibited and rerouted as according to (3.3).

Repeat protocol A job which completes its service at station i routes to a station j with $n_j < N_j$ with probability p_{ij} and has to restart a new service at station i with probability

(3.4) $\quad p_{ii} + \sum_{j \neq i} p_{ij}\, 1_{\{n_j = N_j\}}$.

As argued before, for the exponential case both protocols are directly seen to be effectively the same and in this case the stationary distribution is well-known (e.g. [2], [10], [13], [14], [19], [22], [23], [25]) to exhibit the product form

(3.5) $$\pi(\bar{n}) = c \prod_{j=1}^{N} \frac{[\lambda_j \tau_j]^{n_j}}{n_j!} \qquad (n_j \leq N_j,\ j=1,\dots,N).$$

For the non-exponential case this expression has been shown to remain valid under the "repeat protocol" (cf. [2], [22], [23], [25]). Under the "stop" protocol, however, an explicit insensitive product form result for this network has not been reported.

Result 3.1.1 Under either protocol and with c a normalizing constant, we have for all $L \in S$:

(3.6) $$\pi(L) = c \prod_i [\lambda_{\ell_i} \tau_{\ell_i}].$$

Proof By virtue of (3.1), the substitutions $\mu_\ell = \tau_\ell^{-1}$, $f(\ell|L) = 1$, $p(\ell,\ell'|L) = p_{\ell\ell'}$ and

(3.7) $$b(\ell,\ell'|L) = 1_{\{n_{\ell'} < N_{\ell'}\}},$$

and noting that $\pi(L+\ell')=0$ if $n_{\ell'}=N_{\ell'}$, one immediately verifies (2.6) in the detailed manner:

(3.8) $$\pi(L_i+\ell_i)\mu_{\ell_i}\, p_{\ell_i\ell_i'}\, 1_{\{n_{\ell_i'} < N_{\ell_i'}\}} = \pi(L_i+\ell_i')\mu_{\ell_i'}\, p_{\ell_i'\ell_i}\, 1_{\{n_{\ell_i} < N_{\ell_i}\}}. \qquad \square$$

Result 3.1.2. Expression (3.5) generally holds under both protocols.

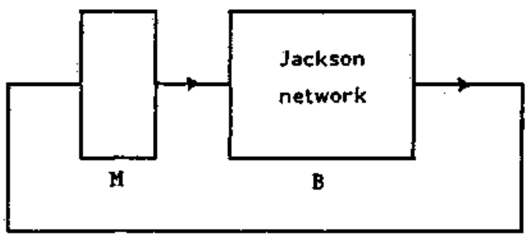
Proof Immediately from (3.6), the substitution $c = \bar{c}\, M!$ and the identity

(3.9) $$\pi(\bar{n}) = \binom{M}{n_1,\dots,n_N} \pi(L). \qquad \square$$

Remark 3.1.3 (Extensions) Rather than infinite server disciplines we could have allowed both non-symmetric (such as FCFS) and other symmetric (such as processor sharing or LCFS) disciplines, provided the service requirements are exponential at the non-symmetric stations (see [14] for definitions). As based upon remark 2.4 a similar product form expression can then be shown to be valid for both protocols again and to be insensitive to service requirements at symmetric stations.

3.2 Restricted Jackson network

Now consider a Jackson network with infinite server stations $1,\ldots,N$ and a finite source input, represented by a station 0, with M sources. The network thus contains a fixed number of M jobs and each station $0,1,\ldots,N$ is to be regarded as an infinite server station, say with mean service times τ_i at station i for $i=0,1,\ldots,N$. Let p_{ij} be the routing probability from station i to j for $i,j=0,1,\ldots,N$.



The main network, i.e. stations $1,\ldots,N$ cannot contain more than B jobs in total, where $B \leq M$. To this end, with $n = n_1 +\ldots+ n_N$, we distinguish the protocols:

Stop protocol When $r=B$, servicing at station 0 is "stopped".

Repeat protocol When $r=B$, a job which completes a service at station 0 has to receive a new service at station 0.

With $\bar{n} = (n_1,\ldots,n_N)$ as before and $\{\lambda_1,\ldots,\lambda_N\}$ uniquely determined, under the standard irreducibility assumption, by the traffic equations:

$$(3.10) \qquad \lambda_j = p_{0j} + \textstyle\sum_{i=1}^{N} \lambda_i p_{ij} \qquad (j=1,\ldots,N)$$

and c a normalizing constant, for the exponential case the following product form can already be concluded from Jackson's classical paper [15]:

$$(3.11) \qquad \pi(\bar{n}) = c \prod_{i=1}^{N} \frac{[\lambda_i \tau_i]^{n_i}}{n_i!} \qquad (\bar{n}|n \le B)$$

For the non-exponential case, however, and most notably with a non-exponential input, i.e. with station 0 non-exponential, no such result has been reported. In contrast, the so-called loss and triggering blocking protocol in [15] and [16] essentially requires a Poissonian input.

Result 3.2.1 Under either protocol and with $\bar{c}$ a normalizing constant

$$(3.12) \qquad \pi(L) = c \prod_{i} [\lambda_{\ell_i} \tau_{\ell_i}] \qquad (\bar{n}|n \le B).$$

Proof With n(L) the number of jobs at stations 1,...,N in state L and

$$(3.13) \qquad b(\ell,\ell'|L) = \begin{cases} 1_{\{n(L)<B\}} & \ell=0 \qquad (\ell'=1,\ldots,N) \\ 1 & \ell\neq 0 \qquad (\ell'=0,\ldots,N) \end{cases}$$

the job-local-balance equations (2.6) are standardly verified by substituting (3.12) and using (3.10) for any i and $L = L_i + \ell_i$ such that either $\ell_i \neq 0$ or $\ell_i = 0$ but $n(L) < B$, while for $\ell_i = 0$ and $n(L) = B$ both the left and right hand side of (2.6) are equal to 0 by virtue of (3.13) and $\pi(\bar{n}) = 0$ for $n > B$ respectively. □

Similarly to result 3.1.2, we now obtain:

Result 3.2.2 Expression (3.11) generally holds under both protocols.

Remark 3.2.3 Again, based upon remark 2.4, the results extend to arbitrary product form networks with a total capacity constraint, such as with FCFS stations and a non-exponential finite source input.

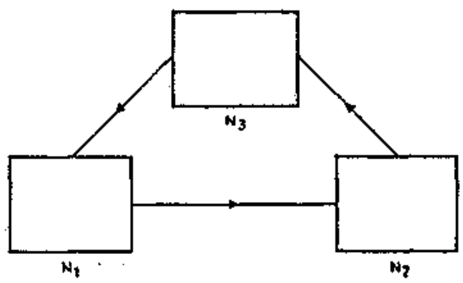
3.3 Jackson networks with finite stations and overall blocking

Reconsider the Jackson network of section 3.1 with finite capacity constraint N_i for station $i, i=1,\ldots,N$, but without the reversible routing condition (3.1). Instead, either one of the following two overall blocking protocols is in order.

Stop protocol As soon and long as one of the stations becomes and is saturated, say station i when $n_i = N_i$, then servicing at all the other stations $j \neq i$ is stopped.

Repeat protocol A job which completes a service at station j has to undergo a new service at station j if any of the other stations $i \neq j$ is saturated, i.e. $n_i = N_i$. Otherwise it routes according to the routing probabilities (p_{ij}).

For example, in the cyclic three station network depicted below, service at not only station 2 but also station 1 is to be stopped respectively repeated upon completion when station 3 is saturated.



Note that under either protocol no two stations can become saturated at the same time, so that the set of admissible states $\bar{n}$ is now restricted to:

$$(3.14) \qquad V = \{ \bar{n} \mid n_i \leq N_i, \ \forall i, \ n_i + n_j < N_i + N_j \ \forall j \neq i \}.$$

With $\{\lambda_1, \ldots, \lambda_N\}$ uniquely determined, up to normalization and under irreducibility assumption, by the traffic equations (3.10) again, with $p_{0j} = 0$ substituted and c a normalizing constant, the following product form is to be expected

$$(3.15) \qquad \pi(\bar{n}) = c \prod_{i=1}^{N} \frac{[\lambda_i \tau_i]^{n_i}}{n_i!} \qquad (\bar{n} \in V).$$

For the exponential case and the specific three station example given above, this product form has been presented in [10]. For the non-exponential case no such result has been given.

Result 3.3.1 Under either protocol and with $\bar{c}$ a normalizing constant:

$$(3.16) \qquad \pi(L) = \bar{c} \prod_i [\lambda_{\ell_i} \tau_{\ell_i}] \qquad (\bar{n} \in V).$$

Proof With

$$(3.17) \qquad b(\ell,\ell' | L) = \begin{cases} 1 & \text{if } n_j < N_j \text{ for all } j \neq \ell \qquad (\ell'=1,\ldots,N) \\ 0 & \text{otherwise} \qquad (\ell'=1,\ldots,N), \end{cases}$$

the job-local-balance equations (2.6) are standardly verified by substituting (3.16) and using (3.10) for any i and $L=L_i+\ell_i$ with $n_j < N_j$ for all $j \neq \ell_i$, while for i and $L=L_i+\ell_i$ such that $n_j=N_j$ for some $j \neq \ell_i$ both the left and right hand side of (2.6) are equal to 0 by virtue of (3.17) and $\pi(\bar{n})=0$ for $\bar{n} \notin V$. □

Again, similarly to result 3.1.2, we so obtain:

Result 3.3.2 Expression (3.15) holds under both protocols.

Remark 3.3.3 Note that a general nón-reversible routing is allowed.

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