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# **SERIE RESEARCH MEMORANDA**

PARAMETRIZATION OF SIMPLICIAL ALGORITHMS  
WITH AN APPLICATION TO AN EMPIRICAL  
GENERAL EQUILIBRIUM MODEL

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**VRIJE UNIVERSITEIT**  
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**A M S T E R D A M**



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WITH AN APPLICATION TO AN EMPIRICAL GENERAL EQUILIBRIUM MODEL

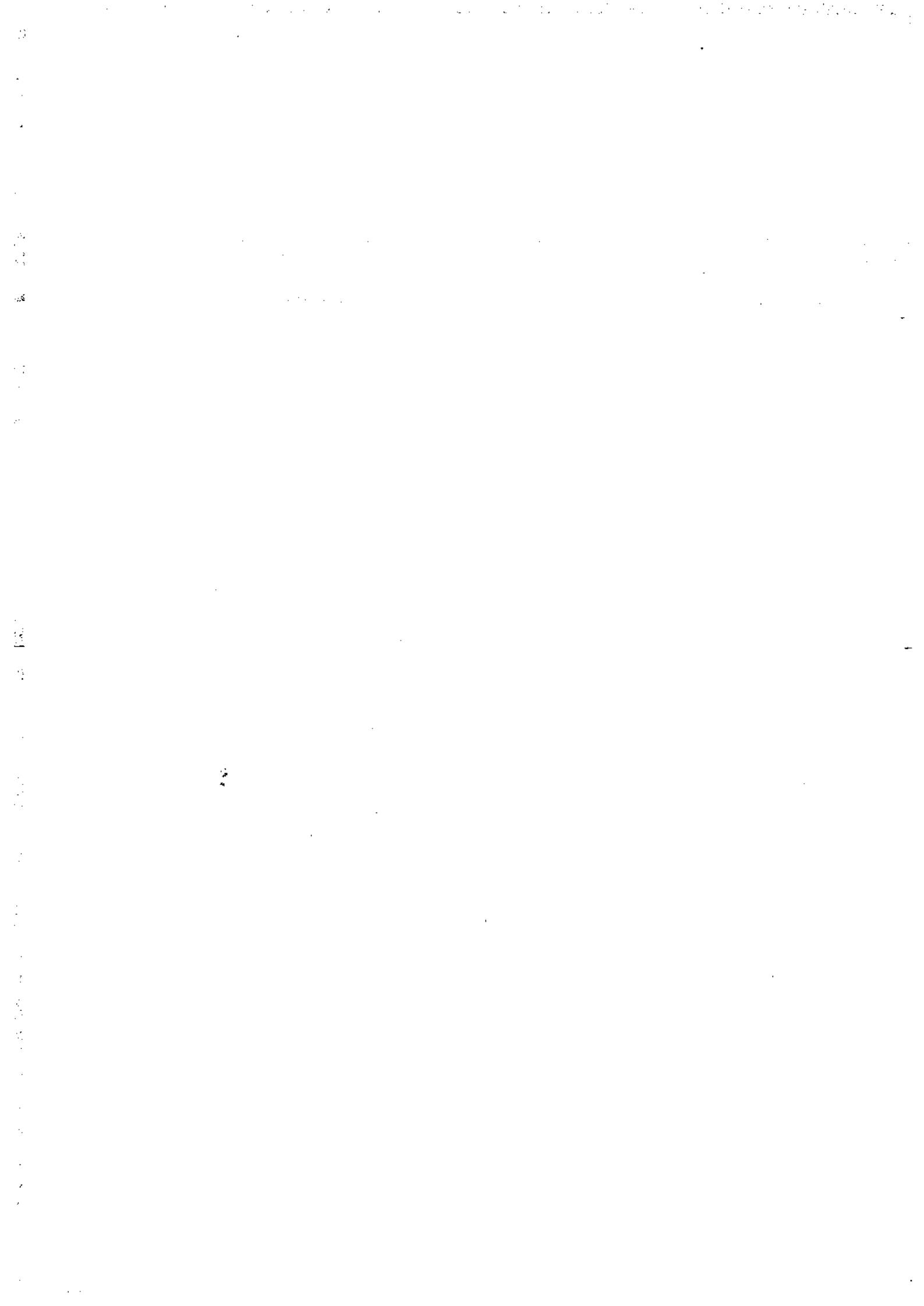
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PARAMETRIZATION OF SIMPLICIAL ALGORITHMS  
WITH AN APPLICATION TO AN EMPIRICAL GENERAL EQUILIBRIUM MODEL

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Abstract

In this paper a technique is developed to follow paths of approximate equilibria of a general equilibrium model under deformation. This technique, which we will call in accordance with John (1981) the homotopy technique, is a parametric version of the simplicial algorithms as developed by van der Laan and Talman (1979) and Doup and Talman (1987). Furthermore, this homotopy technique is applied to an empirical general equilibrium model of the Netherlands and some computational results are given.

Acknowledgements

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N.B. The results in section 4 are preliminary.



## 1. Introduction

During the last three decades a lot of research has been done on the development of several methods to determine equilibria of economic systems. Among these methods are the so-called tatonnement processes and quasi-Newton methods. The main disadvantage of these algorithms is that they do not globally converge.

In the mid-sixties Scarf (1967) introduced a fixed point algorithm on the unit simplex  $S^n$ . Research of, among others, Kuhn (1968), Eaves (1972,1978), Merrill (1972), van der Laan and Talman (1979) eventually led to a class of algorithms which do not cycle and are globally convergent. Furthermore, increasingly accurate solutions can easily be computed by restarting the algorithm in or close to an approximate solution. These simplicial algorithms are used nowadays in many practical applications to compute (static) equilibria.

A next phase in the development of computing equilibria is the computation of paths of equilibria when the economy is gradually changed from the existing one into another, i.e. to "follow" the equilibrium continuously. Not only is it much more efficient to follow the equilibrium continuously under a deformation instead of recomputing the equilibrium for each new situation, but on top of that there is a more fundamental reason. Whenever the model we are dealing with has no unique solution, it is not guaranteed that the new equilibrium we compute will in fact be realized. However, if the new equilibrium is derived by continuously deforming the old one and if this deformation process has a suitable economic interpretation there is reason to believe that this new equilibrium actually will be reached.

In this paper we will develop a technique which enables us to follow paths of approximate equilibria of a general equilibrium model under deformation. The technique we will use is a parametric version of the above mentioned simplicial algorithms and is based on the so-called continuous deformation algorithms as developed by Doup and Talman (1987). These continuous deformation algorithms are based on the homotopy principle, i.e. starting from a trivial system a path is followed terminating in a solution of the system of interest. The technique used in this paper is also based on this homotopy principle, however, instead of starting with a trivial system the algorithm starts



at a solution of the model at the initial situation and follows a path of approximate equilibria of the economy under deformation. In accordance with John (1981) we will call this parametric version the *homotopy technique* (see also Engles (1980)). Furthermore, we will apply this homotopy technique to an empirical general equilibrium model based on the Keller model for the Netherlands (Keller (1980)) and give some computational results.

The outline of the paper is as follows. Section 2 briefly discusses the concept of simplicial algorithms and describes the generalization to the homotopy technique. In section 3 we will elucidate the general equilibrium model used, while section 4 gives some computational results. Finally, section 5 will give some concluding remarks.

## 2. From variable dimension restart algorithms to the homotopy technique

### 2.1 Preliminaries

Before turning to a description of simplicial algorithms we will give some basic definitions and notation.

#### definition 2.1.1

Let  $w^1, \dots, w^{m+1}$  be  $m+1$  affinely independent points in  $R^n$ . The  $m$ -dimensional simplex or  $m$ -simplex  $\sigma(w^1, \dots, w^{m+1})$  is given by the convex hull of  $w^1, \dots, w^{m+1}$ , which are called the vertices of  $\sigma = \sigma(w^1, \dots, w^{m+1})$

#### definition 2.1.2

Let  $\sigma$  be an  $m$ -simplex in  $R^n$ . Then the  $k$ -simplex  $\tau$  with  $k \leq m$  is a face of  $\sigma$  if all vertices of  $\tau$  are also vertices of  $\sigma$ . We call an  $(m-1)$ -face of  $\sigma$  a facet of  $\sigma$ . A facet  $\tau$  of  $\sigma$  is said to be opposite to the vertex  $w^i$  if  $w^i$  is a vertex of  $\sigma$  but not a vertex of  $\tau$ ,  $i = 1, \dots, m+1$ .

#### definition 2.1.3

Two different simplices  $\sigma_1$  and  $\sigma_2$  are adjacent if either one is a facet of the other or if they share a common facet.

definition 2.1.4

Let  $C$  be an  $m$ -dimensional convex subset of  $\mathbb{R}^n$ . A collection  $G$  of  $m$ -simplices is a simplicial subdivision or triangulation of  $C$  if

- (i)  $C$  is the union of all simplices in  $G$
- (ii) the intersection of two simplices in  $G$  is either empty or a common face
- (iii) each facet of a simplex of  $G$  either lies in the boundary of  $C$  and is a facet of just one simplex in  $G$  or it does not lie in the boundary of  $C$  and is a facet of exactly two simplices in  $G$ .

definition 2.1.5

Let  $G$  be a triangulation of  $C$ . The diameter of a simplex  $\sigma$  of  $G$ , to be denoted by  $\text{diam } \sigma$ , is:

$$\text{diam } \sigma = \max (|x-y|, x, y \in \sigma)$$

and the mesh of a triangulation  $G$  is

$$\text{mesh } G = \sup_{\sigma \in G} (\text{diam } \sigma)$$

## 2.2 The variable dimension restart algorithm

In general equilibrium analysis the problem arises of finding an equilibrium of an economy. It is well known that such an equilibrium corresponds with a zero of an excess demand function. An excess demand function  $z(p)$  with  $p \in \mathbb{R}_+^{n+1} \setminus \{0\}$  and  $z(p) \in \mathbb{R}^{n+1}$  satisfies the following properties:

- (i) Walras' Law :  $p^T \cdot z(p) = 0$  for all  $p \in \mathbb{R}_+^{n+1} \setminus \{0\}$
- (ii) Weak Desirability Condition :  $z_j(p) \geq 0$  if  $p_j = 0$  for  $j=1, \dots, n+1$ .

Furthermore, an excess demand function is homogeneous of degree zero in  $p$ . Consequently, we can normalize  $p$  and restrict  $z$  to  $S^n$ , the  $n$ -dimensional unit simplex which is defined as  $\{x \in \mathbb{R}^{n+1} \mid \sum_i x_i = 1, x_i \geq 0\}$ . Using Brouwer's fixed point theorem one can prove that an

excess demand function indeed has a zero point. The problem of finding this zero point is called the zero point problem (ZPP).

Whenever  $z(p)$  does not satisfy the weak desirability condition we will call  $z(p)$  a generalized excess demand function. Again using Brouwer's fixed point theorem it can be proven that for a generalized excess demand function there exists at least one  $p^*$  such that  $z(p^*) \leq 0$ . Note that such a point  $p^*$  satisfying  $z(p^*) \leq 0$  has the property that  $z_i(p^*) = 0$  if  $p_i^* > 0$  and  $z_i(p^*) \leq 0$  if  $p_i^* = 0$ , i.e.  $p^*$  and  $z(p^*)$  are complementary and  $p^*$  will be called a complementary point of  $z$ . The problem of finding this complementary point is called the non linear complementarity problem (NLCP).

Simplicial algorithms can be used to solve the zero point problem or the non linear complementarity problem. The basic idea of simplicial algorithms is to follow a piecewise linear path of price vectors  $p$  starting with an initial price vector  $p$  and ending within a finite number of steps with an approximate zero or complementary point of  $Z(p)$ .  $Z(p)$ , then, with  $p$  a point in the  $n$ -dimensional simplex  $\sigma(y^1, \dots, y^{n+1})$  of  $G$  with  $G$  some triangulation of  $S^n$ , is the piecewise linear approximation to  $z$  with respect to  $G$ , given by  $Z(p) = \sum \lambda_i z(y^i)$  with  $\lambda_1, \dots, \lambda_{n+1}$  the unique nonnegative numbers summing to one such that  $p = \sum \lambda_i y^i$ .

The variable dimension restart algorithm of van der Laan and Talman (1979) allows for an arbitrary starting point and generates a sequence of adjacent simplices of varying dimension until an approximating simplex is found. It is globally convergent and generally the approximate solution improves for finer subdivisions, since then the approximation  $Z(p)$  to  $z(p)$  improves.

Suppose now that our economy is subject to some kind of deformation. Furthermore, suppose that equilibrium of the economy under deformation is given by  $z(p,t) \leq 0$  and Walras' law,  $p^T \cdot z(p,t) = 0$  for all  $t$ , is still valid. The idea of the homotopy technique, now, is to embed the set  $S^n$  in  $S^n \times [0, \infty)$  and, starting with the simplex which yields an approximate solution at the zero level resulting from the variable dimension restart algorithm, generate a sequence of adjacent  $(n+1)$ -simplices in a triangulation of  $S^n \times [0, \infty)$ , such that each common facet of two adjacent simplices in the sequence yields an approximate

complementary point of  $Z(p,t)$ .  $Z(p,t)$ , then, is the piecewise linear approximation to  $z(p,t)$  with respect to the underlying triangulation of  $S^n \times [0,\infty)$ . The extra parameter  $t$  of  $z$  is for example some tax parameter or a time parameter. When  $H: S^n \times [0,\infty) \rightarrow R^{n+1}$  represents a one-parameter family of systems (i.e.  $H(\cdot,t)$ ,  $t \in [0,\infty)$ ), this sequence of approximate complementary points in fact corresponds with a homotopy path  $\gamma : [0,\infty) \rightarrow S^n \times [0,\infty)$  where  $\gamma(t)$  is a solution to  $H(\cdot,t)$  for  $t \in [0,\infty)$ .  $H$  is given by:

$$H_j(p,t) = Z_j(p,t) - p^T \cdot Z(p,t) \quad j=1, \dots, n+1.$$

### 2.3 The homotopy technique

In the previous section we have described how the variable dimension restart algorithm can be generalized to the homotopy technique in order to generate a path of approximate complementary points of  $z(p)$  parametrized on  $t$ . In this section we will take a closer look at the generated solution-path.

As mentioned before, the homotopy technique generates a sequence of adjacent  $(n+1)$ -simplices which yields a piecewise linear path of approximate complementary points of  $Z(p,t)$ . Since solutions are possible with  $p_j = 0$  for some  $j$ , we need an algorithm that allows us to move on the boundary of  $S^n$  as well. So, we have to generalize to sequences of  $(n-u+1)$ -simplices where  $u$  is the number of prices equal to zero which can vary at each iteration. Now, starting with the  $(n-u+1)$ -simplex having as a facet the simplex yielding a complementary point of  $Z(p)$  at the zero level, a piecewise linear path of points  $x$  is followed where each linear piece corresponds to a line segment of solutions of a system of linear equations with respect to an  $(n-u+1)$ -simplex  $\psi$  in  $S^n \times [0,\infty)$ .

#### Definition 2.3.1

Let  $U$  be a subset of  $I_{n+1} = \{1, \dots, n+1\}$ .

We define  $S^n(U)$  as  $S^n(U) = \{ x \in S^n \mid x_i = 0 \text{ for } i \in U \}$ .

Definition 2.3.2

Let  $U$  be a subset of  $I_{n+1}$  and let  $\psi$  be an  $m$ -simplex,  $m = n-u, n-u+1$  in  $S^n(U) \times [0, \infty)$ . The simplex  $\psi(x^1, \dots, x^{m+1})$  with  $x^i = (p^i, t_i)$ ,  $p^i \in S^n(U)$ ,  $t_i \geq 0$ , is complete if the system of linear equations

$$\sum_{i=1}^{m+1} \lambda_i \begin{bmatrix} z(x^i) \\ 1 \end{bmatrix} + \sum_{i \in U} \mu_i \begin{bmatrix} e(i) \\ 0 \end{bmatrix} - \beta \begin{bmatrix} e \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (2.3.1)$$

(where  $e(i)$  is the  $i$ -th unit vector and  $e$  is a vector of ones), has a solution  $\lambda_i \geq 0$ ,  $i=1, \dots, m+1$ ,  $\mu_i \geq 0$ ,  $i \in U$  and  $\beta$ . Such a solution is called feasible and will be denoted by  $(\lambda, \mu, \beta)$ .

Assumption 2.3.3 (nondegeneracy)

If  $\psi$  is a complete  $m$ -simplex in  $S^n \times [0, \infty)$ , then for  $m=n-u$  the system (2.3.1) has a unique solution  $(\lambda, \mu, \beta)$  with  $\lambda_i > 0$ ,  $i=1, \dots, n+1$ ,  $\mu_i > 0$ ,  $i \in U$ , while for  $m=n-u+1$  at most one variable of  $(\lambda, \mu)$  is equal to zero at each feasible solution.

Now, by the nondegeneracy assumption, a complete  $(n-u+1)$ -simplex  $\psi(x^1, \dots, x^{n-u+2})$  contains a line segment of feasible solutions which can be followed by making a linear programming step in (2.3.1). This is exactly what the algorithm amounts to. Note that the system (2.3.1) is exactly the one-parameter family of systems,  $H$ , we referred to at the end of section 2.2. It can easily be seen that to each solution  $(\lambda, \mu, \beta)$  of (2.3.1) the point  $(p, t) = x = \sum \lambda_i x^i$  in  $\psi$  corresponds, which satisfies the following condition:

$$\begin{aligned} Z_j(p, t) &= \max_m Z_m(p, t) & \text{if } p_j > 0 & \text{ and} \\ Z_j(p, t) &\leq \max_m Z_m(p, t) & \text{if } p_j = 0 \end{aligned} \quad (2.3.2)$$

$$\text{with } \max_m Z_m(p, t) = \beta$$

Theorem 2.3.4

Let  $z(p,t)$  be a continuous function on  $S^n \times [0,M]$ ,  $M \in \mathbb{R}_+$ . For all  $\epsilon > 0$  there is a mesh size  $\delta > 0$  such that for all points  $x=(p,t) \in S^n \times [0,M]$  satisfying (2.3.2) with respect to the triangulation of  $S^n \times [0,M]$  with mesh size  $\delta$  holds that

$$p z(p,t) - \epsilon < \beta < p z(p,t) + \epsilon$$

$$\beta - \epsilon < z_j(p,t) < \beta + \epsilon \quad \text{if } p_j > 0 \quad \text{and}$$

$$z_j(p,t) < \beta + \epsilon \quad \text{if } p_j = 0$$

with  $\beta = \max Z_j(p,t)$ .

Proof

Let  $\epsilon > 0$ , then there is a  $\delta > 0$  such that for all  $v=(p,t)$  and  $w=(q,s) \in S^n \times [0,M]$  holds that  $\max_i |v_i - w_i| < \delta$  implies  $\max_i |z_i(p,t) - z_i(q,s)| < \epsilon$  since  $z(p,t)$  is a continuous function on  $S^n \times [0,M]$ . Let  $x=(p,t)$  be a point in  $S^n \times [0,M]$  satisfying (2.3.2) and let  $\sigma(x^1, \dots, x^{n-u+2})$  be an  $(n-u+1)$ -simplex of the induced triangulation of  $S^n(U) \times [0,M]$  containing the point  $x$ . We have  $x^i = (p^i, t_i)$  for all  $i$ . There exist nonnegative numbers  $\lambda_i$   $i=1, \dots, n-u+2$  summing up to one such that  $x = \sum \lambda_i x^i$ , i.e.  $p = \sum \lambda_i p^i$  and  $t = \sum \lambda_i t^i$ . Since  $\beta = \max Z_j(p,t)$  we have that:

$$|\beta - p z(p,t)| = |p Z(p,t) - p z(p,t)| = |p (Z(p,t) - z(p,t))| =$$

$$|p (\sum \lambda_i z(p^i, t_i) - z(p,t))| < \epsilon$$

Furthermore, if  $p_j > 0$  we have

$$|z_j(p,t) - \beta| = |z_j(p,t) - Z_j(p,t)| = |\sum \lambda_i (z_j(p,t) - z_j(p^i, t_i))| < \epsilon$$

and if  $p_j = 0$  then

$$z_j(p,t) - \beta \leq z_j(p,t) - Z_j(p,t) = \sum \lambda_i (z_j(p,t) - z_j(p^i, t_i)) < \epsilon$$

□

Now, by theorem 2.3.4 we have, since  $z$  satisfies Walras' Law, that the algorithm indeed generates a path of approximate complementary points of  $z(p,t)$  with accuracy bounds on the levels given by  $2\epsilon$  whenever the mesh size adopted is  $\delta$ , since  $|\beta| < \epsilon$ . The smaller  $\epsilon$  is chosen the finer the subdivision we have to adopt. Note that if  $z(p,t)$  satisfies the weak desirability condition we even have a path of approximate zeros of  $z(p,t)$ .

At this stage it is necessary to consider the triangulation of  $S^n \times [0, \infty)$  in some more detail. The triangulation of  $S^n \times [0, \infty)$  we use is such that for every  $k$ ,  $k=0,1,2,\dots$ ,  $S^n \times \{k\}$  has a simplicial subdivision. Furthermore, there are only gridpoints on the subsequent levels  $S^n \times \{k\}$ ,  $S^n \times \{k+1\}, \dots$ . Since the distance between the levels is chosen in advance (e.g. in the case of a tax deformation, for instance a 1% increase of tax per level) and generally differs from the mesh  $\delta$  at the levels, the accuracy of  $2\epsilon$  only holds at the levels, while between the levels the accuracy differs according to the extent in which the distance between the level differs from the mesh size at the levels.

Analogous to the variable dimension restart algorithm it can be proven that each level  $S^n \times \{k\}$ ,  $k=1,2,\dots$ , is crossed within a finite number of steps. Moreover, each time a new level  $S^n \times \{k\}$  is intersected an approximate complementary point of  $Z(p,t)$ , the piecewise linear approximation to  $z(p,t)$ , is determined with  $t=k$ . If, for example,  $t=k$  represents an income tax of  $k\%$  on top of some basic income tax, an equilibrium is determined for varying values of the income tax, starting with the basic income tax and gradually raising the income tax percent by percent.

### 3. An applied general equilibrium model for the Netherlands

#### 3.1 The structure of the model

The model we applied the continuous deformation algorithm to is an empirical general equilibrium model for the Netherlands based on the so-called Keller model (Keller (1980)). The Keller model is a multi-sector model which is mainly constructed to simulate the effects of tax policy. It describes the marginal effects of tax changes using linear approximations of the relations around the initial situation. Because of its local character the results are only valid for small tax changes. The linearity makes an easy assessment of the effects of tax changes possible. The model we will use is a global version of the above described model. The homotopy technique can now be used to tackle the difficulties in computing the effects of tax changes, raised by the non-linearity of the model.

The model consists of a consumption block, a production block and a fisc. The consumption block comprises the public household, two private households (a low income household and a high income household) and a foreign sector. The production block consists of four firm sectors: food, durables, services and capital goods. We distinguish eight goods: food (F), durables (D), services (S), capital goods (C), unskilled labour services ( $L_1$ ), skilled labour services ( $L_2$ ), capital services (K) and foreign goods (M). Each household (including the foreign sector) maximizes a utility function which is of the nested CES type (see appendix A, see also Keller (1976)), subject to a budget constraint. All firms, operating under a nested CES production function, maximize profits. The fisc collects transaction taxes and distributes the proceeds among the households in the form of transfers according to some a priori specified scheme. Transaction taxes are exogenous and levied on transactions between households and firms. The transaction tax is the wedge between the demander's and the supplier's net price. When the market price for good  $i$  is given by  $p_i$ , the tax rate on the demand for good  $i$  is given by  $t_i^d$  and the tax rate on the supply of good  $i$  is given by  $t_i^s$ , the demander's net price is given by  $p_i(1+t_i^d)$  while the supplier's net price is given by  $p_i(1-t_i^s)$ .

Equilibrium is defined as an allocation of goods, a set of net prices and a transfer distribution such that:

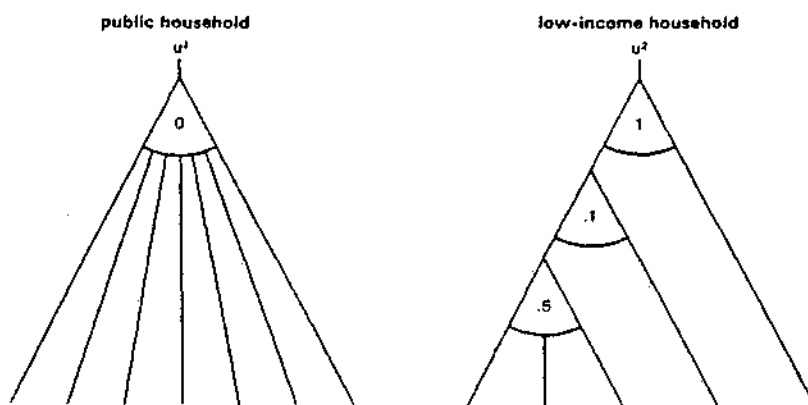


- (i) the utility of each household is maximized given their budgets
- (ii) profit of each firm is maximized given their net prices and technology
- (iii) total tax revenue equals total transfer payments
- (iv) aggregate demand equals aggregate supply

### 3.2 Further specification of the model

Before we can apply the continuous deformation algorithm to the model we have to specify the elasticities of substitution of the nested CES utility functions and the nested CES production functions. These elasticities are "guesstimated" (see Keller (1980)), i.e. they are based on values found in econometric studies of for example consumer demand systems. The utility and production structures and the values chosen for the elasticities of substitution are given in figures 3.2.1 and 3.2.2 below.

figure 3.2.1 utility structures of households



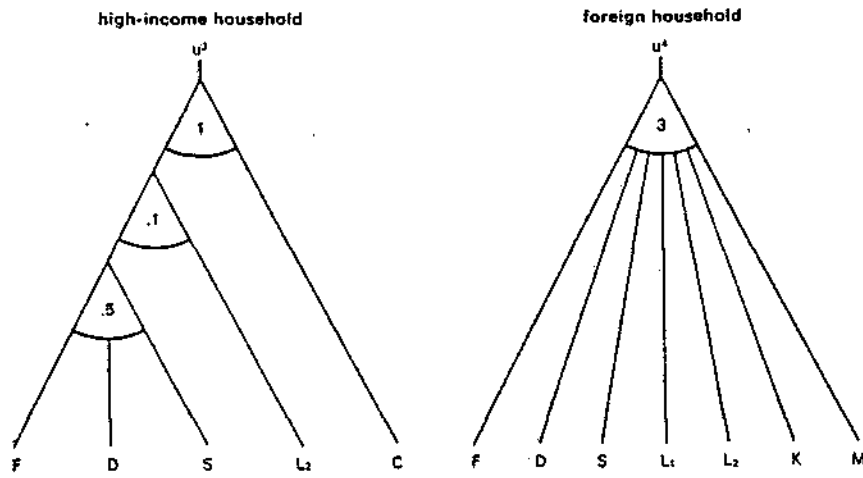
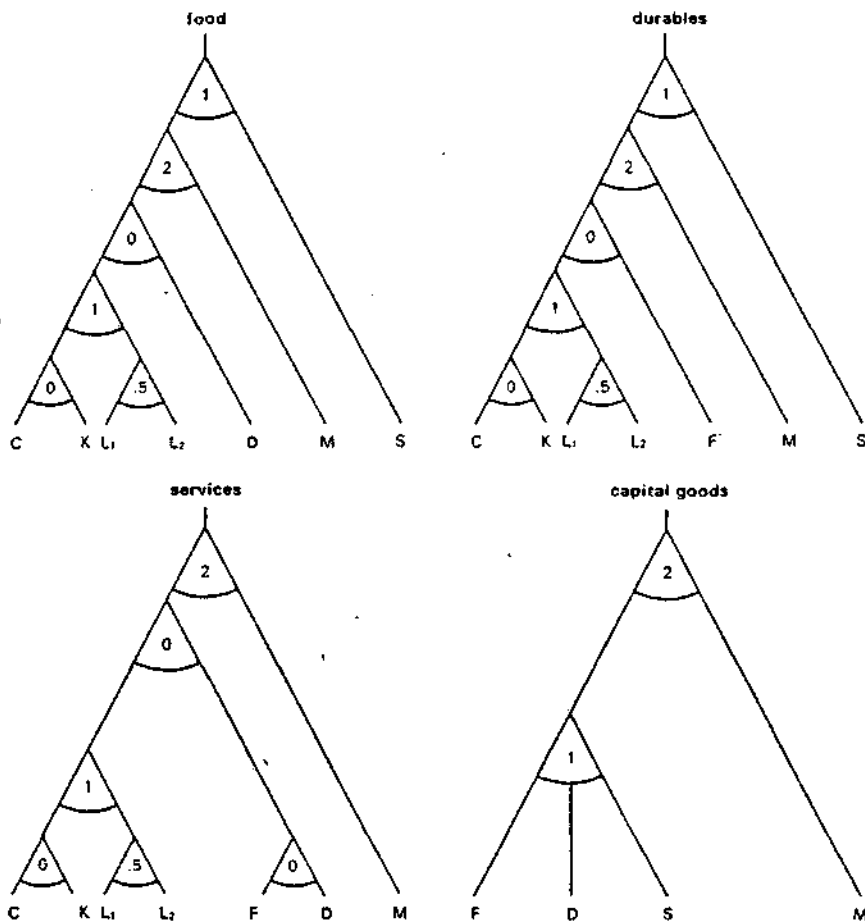


figure 3.2.2 production structures of firms



Furthermore, the nested CES functions are calibrated to a benchmark data set (see Cornielje (1984)). The benchmark data set we choose is a set of data describing the situation in the Netherlands in 1973 derived from the so-called Nationale Rekeningen (1976) and is given in table 3.2.3 below.

table 3.2.3 benchmark data set

Total accounts, the Netherlands, 1973 (positive entries refer to expenditures, negative entries to receipts).

	Firms				Households			
	Food	Durables	Services	Capital goods	Public	Low income	High income	Foreign
<i>Net expenditures on goods in net prices (10<sup>6</sup> Dfl.)</i>								
Food	-45456	834	1225	616	138	20266	4254	20003
Durables	3440	-118085	8305	36899	3202	22453	8465	43021
Services	12732	23814	-97299	4214	2102	26038	11899	20381
Capital goods	1520	3510	8410	-48134	8322	15136	11236	0
Unskilled labour	7224	23760	35387	0	9364	-39879	0	619
Skilled labour	3054	10329	10073	0	11531	0	-17822	291
Capital services	3611	6463	11388	0	1470	-6879	-11582	6340
Imports	13875	49375	22511	6405	0	0	0	-90315
<i>Net expenditures on taxes (10<sup>6</sup> Dfl.)</i>								
Food	-317	-14	208	4	19	2822	592	-1434
Durables	-5	-45	-6	2998	514	3229	1217	-202
Services	109	141	1051	225	195	1441	659	60
Capital goods	0	0	0	0	0	0	0	0
Unskilled labour	883	5464	5940	0	3014	21154	0	0
Skilled labour	241	1616	1116	0	2637	0	11846	0
Capital services	752	3436	2602	0	0	557	3464	0
Imports	620	1094	137	0	0	0	0	0

Now we are ready to apply the algorithm. Starting in the initial equilibrium we can follow the equilibrium continuously in the form of a set of market prices for increasing value of some tax parameter. Note that the tax proceeds are redistributed among the households. From the market prices we can derive the induced effects of tax changes on the net prices of both the households and the firms. Of course the changes in the net prices of the households and the firms as such are not very interesting, but the framework allows us to consider equity and efficiency effects using for example measures like the compensating variation (see also Keller (1980)).

#### 4. Computational results

We run the algorithm for changes in the following taxes:

- (i) tax on the supply of skilled labour
- (ii) tax on the supply of unskilled labour
- (iii) tax on the supply of foreign goods by the foreign household

The accuracy of the results is given by  $10^{-4}$ . Remind that the algorithm follows a path of equilibria of the economy under a tax-deformation, the accuracy between the levels being higher the closer the levels are to each other.

It appears that, under a rise in the tax on the supply of skilled labour services, the market prices are rather stable (see table 4.1.1). The stability of the market-prices of food, durables, services and capital services is due to the small open economy assumption combined with a rather price elastic demand of the foreign sector which has a stabilizing effect on the market-prices. When we raise the tax on the supply of unskilled labour (see table 4.1.2) we see that the market price of skilled labour rises. This is due to the fact that raising taxes induces a transfer of purchasing power from the private sector to the public sector. Since the public sector is relatively skilled labour intensive the aggregate demand for skilled labour increases. Note that a decrease in the import-price corresponds with an appreciation, since the world price of imports is fixed and we have floating exchange rates. The decrease in the import price is due to the fact that the public sector is relatively import un-intensive. So, a transfer of purchasing power to the public sector lowers the aggregate demand for imports.

table 4.1.1 tax on the supply of skilled labour services

marginal price-change in % after x percent tax change

x	food	durab	serv	capg	unskl	sklab	caps	imp	taxrev
1	-0.04	-0.05	-0.03	-0.05	0.06	-0.12	-0.05	-0.08	0.36
2	-0.04	-0.04	-0.02	-0.04	0.06	-0.19	-0.03	-0.07	0.36
3	-0.03	-0.04	-0.02	-0.04	0.05	-0.17	-0.04	-0.06	0.35
4	-0.03	-0.04	-0.02	-0.04	0.04	-0.17	-0.04	-0.06	0.35
5	-0.03	-0.04	-0.02	-0.04	0.04	-0.16	-0.04	-0.05	0.35

table 4.1.2 tax on the supply of unskilled labour services

marginal price-change in % after x percent tax change

x	food	durab	serv	capg	unskl	sklab	caps	imp	taxrev
1	-0.14	-0.14	-0.14	-0.15	-0.24	0.46	-0.18	-0.18	0.70
2	-0.14	-0.13	-0.15	-0.14	-0.34	0.52	-0.16	-0.16	0.68
3	-0.13	-0.13	-0.14	-0.13	-0.32	0.49	-0.16	-0.16	0.68
4	-0.13	-0.12	-0.14	-0.13	-0.31	0.46	-0.15	-0.15	0.68
5	-0.13	-0.13	-0.14	-0.13	-0.31	0.48	-0.16	-0.15	0.67

table 4.1.3 tax on the supply of foreign goods

marginal price-change in % after x percent tax change

x	food	durab	serv	capg	unskl	sklab	caps	imp	taxrev
1	-0.21	-0.21	-0.16	-0.24	-0.05	0.53	-0.42	-0.40	1.17
2	-0.26	-0.28	-0.16	-0.33	0.07	0.67	-0.32	-0.69	1.29
3	-0.31	-0.36	-0.18	-0.41	0.05	0.73	0.06	-0.91	1.33
4	-0.33	-0.39	-0.19	-0.45	0.05	0.78	0.16	-0.98	1.36

## 5. Conclusions

In this paper we have developed a technique to follow paths of equilibria of an economy under (tax-)deformation. The justification for the method is that it is efficient and that it gives a clue to which equilibrium is to be realized in the case of multiple equilibria. Whenever the path curves back we are indeed in a situation of multiple equilibria, which also points at discontinuity of the path as a function of the tax parameter under consideration. Implementation of the technique for a general equilibrium model gives us the results as shown in tables 4.1.1 to 4.1.3. It appears that the results are not always linear -in table 4.1.3 we see that the direction of the effect can even turn around- which gives a justification for the rather complex non-linear model used. Finally, we remark that it is in our framework also possible to change several taxes simultaneously. However, this can only be done according to some a priori specified rule. The derivation of multi-dimensional paths is the next issue to be considered in this area of research.

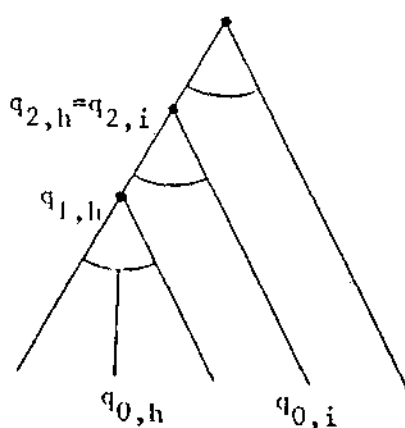
### Appendix A Nested CES functions

The concept of nested CES utility functions rests upon the assumption that utility is built up as a utility tree which consists of  $N+1$  levels (see figure A.1 below). At each level we distinguish several utility components ( $q_{k,i}$ ). On the highest level there is only one component which corresponds with overall utility. On the zero level the components correspond with the commodities of the economy. The utility components on level  $k$  are functions of the utility components on level  $k-1$ . These functions are of the CES-type. So we have:

$$q_{k,i} = \left[ \sum_{j \in i} \alpha_{k-1,j}^{1-\rho_{k,i}} q_{k-1,j}^{\rho_{k,i}} \right]^{1/\rho_{k,i}}$$

The first index refers to the level of aggregation, while the second index refers to the basic good/commodity the aggregate is related to. Furthermore,  $\rho_{k,i} = (\sigma_{k,i} - 1)/\sigma_{k,i}$  where  $\sigma_{k,i}$  is the elasticity of substitution associated with the matching utility component. For  $\sigma_{k,i} = 0$  the utility component at level  $k$  associated with commodity  $i$  reduces to a Leontief form, while for  $\sigma_{k,i} = 1$  we have the well known Cobb-Douglas form.

figure A.1 structure of a nested CES function



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