Dynamics of generalised spatial interaction models

Nijkamp, P.; Poot, J.

1986

SERIE RESEARCH MEMORANDA

DYNAMICS OF GENERALISED SPATIAL

INTERACTION MODELS

Peter Nijkamp

Jacques Poot

Researchmemorandum 1986-6 februari 1986

VRIJE UNIVERSITEIT
FACULTEIT DER ECONOMISCHE WETENSCHAPPEN
AMSTERDAM

DYNAMICS OF GENERALISED SPATIAL INTERACTION MODELS

by

PETER NIJKAMP and JACQUES POOT

Paper to be presented at
the 9th Pacific Regional
Science Conference, Molokai,
Hawaii, August 12-15, 1985.

ABSTRACT

Dynamics of Generalised Spatial Interaction Models

This paper analyses dynamic properties of generalised spatial interaction models, with particular emphasis on Alonso's general theory of movements. Although the application of this theory addressed in the paper is a multiregional demographic stock-flow model, it can easily be shown that the approach can be generalised to all types of spatial interaction phenomena.

After an introduction to the Alonso model, it is demonstrated that various classes of spatial interaction models (e.g., gravity and entropy models; doubly constrained trip distribution models) are specific cases of the generalised Alonso model. Next, the equilibrium and stability conditions of the spatial distribution resulting from the stock-flow model are further analysed. Although an analytical expression for the so-called systemic variables of the model is only possible under very restrictive conditions, it is yet possible to study equilibrium and stability conditions more profoundly by rewriting the generalised spatial interaction model as a general non-linear dynamic Volterra-Lotka model, so that stable and unstable time trajectories can be examined. Furthermore, it appears to be possible to formulate more precise conditions that ensure local stability in particular cases.

In order to obtain more insight into the local and global stability of the generalised spatial interaction model, simulation experiments are carried out with a multiregional demographic stock-flow model for New Zealand. Various results are presented and discussed in the light of the above mentioned analysis.

I INTRODUCTION

The development of a system of regions (or cities) is characterised by a state of flux, both absolute and relative to each other. The time trajectory of a region is not only the result of its internal ecology and exogenous forces, but it is also affected by the interaction of the region with other regions. This interaction is multi-faceted and involves interregional flows of production factors and commodities, diffusion of technological advances and knowledge, external spillover effects and political conflict in supraregional decision making.

Such interaction may impede or promote regional development. Yet our understanding of the development process is unbalanced in that research in the past has emphasised the determinants of spatial interaction, whereas the consequences of interregional interdependencies have received far less attention. Unfortunately this meant that studies of spatial interaction generally adopted a static (or at best a comparative static) approach. This paper is a contribution to redressing the balance in favour of a systematic study of the consequences of spatial interaction for regional development. In the paper we investigate the regional dynamics implied by a general class of spatial interaction models, which follow from a theory of movements formulated by Alonso (1978).

The paper is structured in the following manner. The next section presents a dynamic formulation of the Alonso model. In section 3 the model is applied to migration in a multiregional demographic stock-flow system. Clearly, human migration is only one aspect of spatial interaction, but the methodology can be applied in principle to other types of spatial interaction, such as interregional commodity trade or commuting, provided appropriate functional forms for the consequences of the flows for the state of the system can be defined. Section 4 outlines the conditions under which an equilibrium population distribution is feasible in the presence of spatial interaction in a multiregional system. Although the identification of equilibrium conditions is interesting in itself, it is far more important to establish whether small perturbations generate equilibrating forces, since in a real-world spatial system a static equilibrium would be an exception rather than a rule. Thus a study of the stability of the system is warranted and this is also carried out in section 4. It should already be noted that because the most general formulation of Alonso's theory involves systemic variables which cannot be expressed in an analytically closed form, standard procedures to test local and global stability cannot be applied to the general case. It is nevertheless possible to formulate conditions which ensure local stability in specific cases, some of which are elaborated in the paper. An alternative approach is to test the model's dynamic properties by means of simulation. The results of some simulation experiments are reported in the penultimate section. The last section summarises the paper and suggests directions for further research.

II A DYNAMIC VERSION OF ALONSO'S GENERAL THEORY OF MOVEMENT

Alonso's (1978) paper proposes a theory of movement which is scientifically appealing and powerful due to its generality: it provides a systemic approach to movement of any type. The most common application is human migration, but other examples are international trade, shopping trips and sales of different brands of a product in different regions. Alonso's theory defines a class of spatial interaction models which can be shown to encompass many existing models of this type. The theory can be seen in particular as a generalisation of Wilson's family of spatial interaction models of the entropy type (Wilson, 1971; 1980). Theoretical reformulations and extensions can be found in Anselin and Isard (1979), Hua (1980) and Ledent (1981). Alonso's theory has been applied empirically to population migration in the United States (Porell and Hua, 1981; Porell, 1982), in Canada (Ledent, 1980; Fisch, 1981; Anselin, 1982) and in Japan (Tabuchi, 1984). An application of the theory to inter-urban labour migration in New Zealand is contained in Poot (1984b). A structurally identical model has been developed through a theory of supply and demand interaction in spatially separated markets (De Vos and Bikker, 1982; Bikker, 1982). The latter model, called the 3-Component model, has been applied to international trade flows and flows of patients to hospitals.

Alonso's theory, general as it may be, also contains some restrictive assumptions. First, the Alonso model pertains to a closed system, hence external forces (e.g., supraregional economic, political, social) are a datum. Secondly, there is no room in the model for a feedback loop in which the state of a region is itself affected by the interaction with other regions. Thirdly, the theory implies short-run equilibrium: when the model is seen as a supply and demand system, Alonso's definition of the systemic variables as balancing factors guarantees market clearance. In the model formulation that follows, the first and second assumption are relaxed, while the third remains to preserve internal consistency. This generalised Alonso model is stated in terms of migration, but an appropriate terminology may be substituted for other forms of interaction between regions.

It is assumed that migration takes place between a set of n regions over a certain predefined period. Although the theory does not require that each origin is also a destination, or even that origins and destinations are of the same type (compare with flows of patients to hospitals), this assumption is introduced for simplicity. Hence the flows of migrants can be represented by a square matrix $\mathbf{M}(t)$ in which $M_{ij}(t)$ is the flow from i to j $(i,j = 1,2,...,n)$ during period $(t, t+1)$. The main diagonal of $\mathbf{M}$ is ignored because of the difficulty in defining the spatial friction in intraregional flows relative to interregional flows.

The supply of migrants leaving region i during (t, t+1), or total out-migration $M_{i.}(t)$ is assumed to satisfy

$$M_{i.}(t) = \delta_i(t)\ O_i(t)\ R_i(t)^{\nu_i} \qquad i = 1,2,...,n \qquad (1)$$

with

$$O_i(t) = \prod_{k=1}^{K} X_{ik}(t)^{\alpha_i} \qquad i = 1,2,...,n \qquad (2)$$

This spatial interaction system can be interpreted as follows: out-migration is a result of internal unattractiveness, external pull forces and relative internal repulsiveness. Equation (2) shows that the push effect $O_i(t)$ has essentially a Cobb-Douglas specification of intrinsic unattractive characteristics of region i, $X_{ik}(t)$ $(k = 1,2,...,K)$. In the case of a heterogeneous population, $O_i(t)$ may also incorporate composition effects resulting from differences in migration propensities between socio-economic groups. Since $M_{i.}(t)$ is a flow and most variables $X_{ik}(t)$ would be stocks, we need to assume that the levels of these variables do not change significantly <u>within</u> the period (t, t+1). Next, variable $\delta_i(t)$ represents the influence of <u>external</u> effects; for example, $\delta_i(t)$ could reflect the propensity to emigrate resulting from the pull from outside the system of regions. Finally, $R_i(t)$ is the <u>internal</u> pull by the system as seen from origin i, i.e. the relative "repulsiveness" of i. Hence, $R_i(t)$ is the opportunity cost remaining in i, with ν_i being the elasticity of the supply of migrants from i with respect to the demand for migrants generated by the system.

In-migration into region j is defined similarly:

$$M_{.j}(t) = \varepsilon_j(t)\ D_j(t)\ A_j(t)^{\mu_j} \qquad j=1,2,...,n \qquad (3)$$

with

$$D_j(t) = \prod_{l=1}^{L} Y_{jl}(t)^{\beta_l} \qquad j=1,2,...,n \qquad (4)$$

Here $D_j(t)$ represents the aggregate effect of intrinsic attractive characteristics of region j, $Y_{jl}(t)$ (j=1,2,...,L). External forces are affecting in-migration through $\varepsilon_j(t)$. $A_j(t)$ is the attractiveness of destinations j relative to all internal potential destinations, with μ_j being the elasticity of the demand for migrants in j with respect to the total supply of migrants generated by the system.

Following Anselin and Isard (1979), the total push out of region i is defined as

$$\overline{O}_i(t) = \frac{\delta_i(t)\, O_i(t)\, R_i(t)^{\gamma_i}}{R_i(t)} \qquad i=1,2,\dots,n \tag{5}$$

and, in the same way, the total pull into j is equal to

$$\overline{D}_j(t) = \frac{\varepsilon_j(t)\, D_j(t)\, A_j(t)^{\mu_j}}{A_j(t)} \qquad j=1,2,\dots,n \tag{6}$$

To measure the facility (or ease) of undertaking a migration between i and j, an index $F_{ij}(t)$ is used. This index is inversely related to the transportation cost, psychic cost and search cost involved in a migration process and thus reflects the distance between regions. However, note that the F matrix with entries $F_{ij}(t)$ is not necessarily symmetric.

Central to the theory of movement is that the flow of migrants between i and j is assumed to satisfy a <u>gravity</u> law: migration is proportional to the total pull, total push and the facility of moves. Hence

$$M_{ij}(t) = c(t)\ \overline{O}_i(t)\ \overline{D}_j(t)\ F_{ij}(t) \qquad i,j = 1,2,\dots,n \quad i \neq j \tag{7}$$

with c (t) being a cross-section proportionality constant. It is well known that the gravity formulation satisfies a number of optimality principles (Niedercorn and Bechdolt, 1969; Nijkamp, 1975; Colwell, 1982).

To close the model, the <u>systemic</u> variables $R_i(t)$ and $A_j(t)$ need to be defined in a way which guarantees internal consistency in that the adding-up conditions

$$\sum_{j=1,\, j\neq i}^{n} M_{ij}(t) = M_{i.}(t) \qquad i=1,2,\dots,n \tag{8}$$

and

$$\sum_{i=1,\, i\neq j}^{n} M_{ij}(t) = M_{.j}(t) \qquad j=1,2,\dots,n \tag{9}$$

are satisfied. Using (1), (3) and (5) - (9), it is straightforward to derive that

$$R_i(t) = c(t) \sum_{j=1}^{n} \overline{D}_j(t)\ F_{ij}(t) \qquad i=1,2,\ldots,n \quad j\neq i \tag{10}$$

and

$$A_j(t) = c(t) \sum_{i=1}^{n} \overline{O}_i(t)\ F_{ij}(t) \qquad j=1,2,\ldots,n \quad i\neq j \tag{11}$$

Hence $R_i(t)$ and $A_j(t)$ are weighted averages of total pull and push respectively. Since $\overline{D}_j(t)$ is a function of $A_j(t)$ and $\overline{O}_i(t)$ of $R_i(t)$, (10) and (11) need to be solved recursively. It can be easily shown that a unique solution exists when a scaling condition is introduced, that is $\prod R_i(t) = \prod A_j(t) = 1$, and that finding the systemic variables is equivalent to the biproportional adjustment problem of finding the matrix M with given marginal totals, which is biproportional to the matrix F. The systemic variables are therefore also called balancing factors.

A number of serious statistical complications are involved in estimating the parameters of the Alonso model. These will not be discussed here, but are elaborated in Porell and Hua (1981) and De Vos and Bikker (1982). A major condition, for instance, in order to estimate the pseudo-elasticity of out-migration and in-migration with respect to the system's pull and push respectively is that we would normally need to introduce the cross-section restriction that $v_i = v$ and $\mu_j = \mu$ for all i and j. This restriction is assumed to hold in the remainder of this paper.

Even when v and μ are assumed constant over regions, the Alonso model still encompasses a large class of spatial interaction models of which most existing models can be shown to be special cases. There are a number of methodological, theoretical, logical, and practical criteria which one may use to judge specific models within this general class (Van Lierop and Nijkamp, 1980) and these criteria may be helpful to choose the most suitable specific model for a certain application.

The dynamic properties of the general class of spatial interaction models are dependent on the actual specification. It is therefore useful to review some specifications which have been particularly popular in empirical applications. First, when $v=\mu=1$, the well known simple single-equation gravity model results.

Using (5), (6) and (7), the model is

$$M_{ij}(t) = c(t)\ \delta_i(t)\ \varepsilon_j(t)\ O_i(t)\ D_j(t)\ F_{ij}(t) \tag{12}$$

This migrant allocation model has been calibrated extensively with cross-section data in various countries, for example: United States, Greenwood (1969); Canada, Vanderkamp (1971); England and Wales, Langley (1974); Australia, Langley (1977); Japan, Inoki and Suruga (1981); New Zealand, Hampton and Giles (1976). The product $c(t)\ \delta_i(t)\ \varepsilon_j(t)$ is in cross-section studies the proportionality constant of the gravity model, which shows that in such studies the external effects are assumed identical across origins and destinations.

The least plausible consequence of the simple gravity model is that any specific (i,j) flow is independent of the characteristics of the n-2 other regions. When, for example, a large industrial project commences in a certain region, this will increase direct labour demand and may have a regional multiplier effect which induces further demand for labour. This would attract more workers from other regions but at the same time reduce some of the flows between these regions, which now have become relatively less attractive, *ceteris paribus*.

A specific model in which changes in the characteristics of a certain region induce a substitution effect, while leaving the total in-migration and out-migration of other regions unaffected, is the doubly constrained trip distribution model (Wilson, 1980). This model results when $\nu=\mu=0$. It has the unattractive property that the drop in migration to a region with declining opportunities is fully compensated by an increase in migration to alternative destinations, so that the propensity to migrate remains unchanged.

A model which we would expect to exhibit more realism is the production-constrained or supply-determined model in which $\mu=1$, but which has no restriction on ν. In this case

$$\frac{M_{ij}(t)}{M_{i.}(t)} = \frac{\varepsilon_j(t)\ D_j(t)\ F_{ij}(t)}{\sum_j \varepsilon_j(t)\ D_j(t)\ F_{ij}(t)}\ , \tag{13}$$

which follows from (5) - (7) and (10). The left-hand side of (13) may be redefined as $\pi_{ij}(t)$, the probability that a migrant from i chooses destination j, and (13) shows that this probability is a function of the <u>relative</u> attractiveness of j.

Although the Alonso model pertains to aggregate flows, equation (13) also follows from a behavioural theory of spatial choice in which the pull factors and the facility of migration are arguments of a stochastic disaggregate utility function such as the one in McFadden's (1974) conditional logit model. It can be demonstrated that there exists a close formal relationship between the multinomial logit model and the gravity-type model (see Van Lierop and Nijkamp, 1979). In addition, it has recently been shown by Heckman (1981) and Leonardi (1985) that dynamic disaggregate choice theory and dynamic spatial interaction models may emerge from the same class of utility theories. A treatment of stochastic spatial interaction models can also be found in Leonardi (1983).

Of particular empirical interest is the time trajectory of the probabilities $\pi_{ij}(t)$. The following decomposition provides further insight:

$$\frac{M_{ij}(t)}{P_i(t)} = \pi_{ij}(t) \frac{M_{i.}(t)}{P_i(t)} , \qquad (14)$$

where $P_i(t)$ is the population of region i at the beginning of period (t,t+1). The left-hand side of (14) is the transition probability that a resident from i moves to j. The conditions that, (i), the Π matrix is constant over time, and (ii), the emission of migrants is proportional to the size of the population are sufficient, although not necessary, to describe the migration and population redistribution process by means of a Markov chain with stationary transition probabilities. When this transition matrix is called **R** and when natural increase and external migration rates are assumed not to vary over regions, the following equation describes the dynamics of population distribution

$$p(t+1) = \mathbf{R}\ p(t) \qquad (15)$$

with $p'(t) = (p_1(t), p_2(t), ..., p_n(t))$ and $p_i(t) = P_i(t) / \sum_k P_k(t)$. The dynamic properties of (15) are well known: when the R matrix is irreducible, the process converges to a steady state distribution. Computation of this steady state distribution has little practical value, because the speed of convergence implied by the R matrix is usually so slow that some of the strong assumptions underlying the model are likely to be violated (Poot, 1984a, chapter 4). Moreover, extrapolation with (15) exaggerates the motion in the system by ignoring duration of stay effects (Brown, 1970). However, these weaknesses do not render the Markov model useless. Sophisticated multigroup multistate matrix models, essentially based on generalisations of (15) have been successfully developed and applied (Rogers 1966, 1968, 1980). Nevertheless, recent research in the Netherlands has demonstrated that, while the Π matrix defined above is remarkably stable over time, the propensity to migrate from regions exhibits considerable variation (Van der Knaap and

Sleegers, 1982). This suggests that a simple Markov model such as (15) would yield inaccurated predictions even in the short-run.

Which type of the spatial interaction models reviewed in this section is appropriate in a particular situation, is naturally an empirical matter. The choice of a specific model out of Alonso's general class depends on the elasticities of the systemic variables. Since these elasticities depend on the level of aggregation, the estimation technique, the type of migration, the observation period and the structural determinants incorporated in the model, we would expect a range of values of elasticities in empirical applications. The recent study by Tabuchi (1984) confirms this. In this context, Anselin (1984) suggested specification tests to discriminate between spatial interaction models for a specific set of empirical observations.

After this brief discussion of Alonso's general theory of movement, its extensions, and a set of various specific attributes of this model, we shall now turn in section III to a dynamic multiregional model that includes the above mentioned migration model as a particular component. In this way, the dynamics of an interwoven spatial system may be studied in a more appropriate manner.

III REGIONAL DEVELOPMENT AND INTERACTION

The development of a region (or a set of regions) may be described by a set of state variables, each with their own time trajectory. The domain of the state variables could be either a discrete or a continuous state space. The former type defines qualitative episodes in the event-history of a region (e.g., a period of industrialisation), whereas the latter measures quantitative phenomena (e.g., population, regional product, pollution). Over a short period we may assume that regional dynamics is part of one single episode and that quantitative transitions are smooth, but in the long run certain events may induce large perturbations (Johansson and Nijkamp, 1984).

Spatial interaction is normally assumed to take place in an environment of smooth transitions. Interregional flows are in empirical studies a function of systemic effects and intrinsic characteristics of regions, but these conditions are assumed unaffected by the flows they generate. Even when feedback effects are taken into account, the quantitative impact of such effects is usually inferred from cross-section information so that true dynamics cannot be identified (examples in the migration literature can be found in Greenwood (1981) and Mead (1982)). The reason for this deficiency of empirical research on regional dynamics and interregional interaction is that such research requires a comprehensive dynamic input-output framework (or a comparable interrelated system), which is generally difficult to operationalise due to data limitations.

However, theoretical research on regional dynamics has progressed much further and a variety of models has been posited (Gordon and Ledent, (1980); Carlberg, (1981); Smith and Papageorgiou, (1982); and several papers in Griffith and MacKinnon (1981) and Griffith and Lea (1983)). The dynamics of regional product growth and production factor movements may be formalised by means of catastrophy theory and differential equation analysis (Casetti, 1981; Dendrinos, 1982). A differential equation model which has been particularly popular in ecology and demography is the Lotka-Volterra model, which was originally formulated to describe the biological association of species through food webs (see also Pimm, 1982).

The study of the dynamics of a system involves both the identification of equilibria and the formulation of conditions under which the system is locally, or globally, stable. However, unless a number of strong assumptions are introduced which simplify the structure of the model, the dynamic properties are often not analytically tractable. It is particularly common in studies of spatial interaction to limit the model to the competition between two regions (e.g., Sonis and Dendrinos, 1984) or between a metropolis and its hinterland (e.g., Hudson, 1970). When such assumptions are considered undesirable, simulation can be used to investigate a limited number of cases.

Working with a simple dynamic structure by no means implies that the model would not be capable of reproducing the turbulent behaviour we may observe in the real world. First, stochastic elements may be introduced. Secondly, even the simplest deterministic nonlinear difference equation can exhibit a remarkable spectrum of dynamic behaviour, from stable equilibrium , to stable oscillations through to a chaotic pattern (May, 1974; Li and Yorke, 1975). Although such difference equations require a discrete measurement of time which may not be appropriate for biological populations (unless generations are non-overlapping), they are very common in applications involving both observations on stocks and flows (e.g. Samuelson's multiplier- accelerator model).

The difference equation approach is adopted here also, because calibration of the Alonso class of spatial interaction models involves the choice of a certain period (usually one or five years) as a unit of measurement for migration. For the sake of simplicity we shall focus on population size as a single quantitative state variable, although migration affects of course the region in many ways, for example through labour supply, housing demand, local government revenue, congestion, cultural pluralism.

The time trajectory of population size is given by the following fundamental growth equation:

$$P_i(t+1) = [1+g_i(t)]\, P_i(t) + M_{.i}(t) - M_{i.}(t) \qquad (16)$$

in which $g_i(t)$ is the rate of natural increase over the period (t,t+1), although external migration may be incorporated in the growth rate g_i (t) in an open system. The substitution of total in-migration and out-migration as defined by (1) and (3) into (16) yields, assuming cross-section equality of the coefficients of the systemic variables:

$$P_i(t+1) = [1 + g_i(t)]\, P_i(t) + \varepsilon_i(t)\, D_i(t)\, A_i(t)^{\mu} - \delta_i(t)\, O_i(t)\, R_i(t)^{\nu} \qquad (17)$$

where A_i (t) and R_i (t) are endogenous and defined in (10) and (11). What remains is to specify the intrinsic push and pull characteristics of region i. The behavioural theory of migration suggests that migration is both an adjustment of location-specific amenities and an investment in human capital for labour force participants. Hence there are many economic and other factors that may have an impact on migration flows, but often population size itself is taken as a proxy for such factors and dominates the explanatory variables in empirical models of spatial interaction (Anselin, 1984). Moreover, when combined with systemic effects, the fit of the pure gravity model to observed migration matrices is often reasonable. Hence, we assume that

$$O_i(t) = P_i(t)^{\alpha} \qquad i = 1,2,\ldots,n \qquad (18)$$

and

$$D_i(t) = P_i(t)^{\beta} \qquad i = 1,2,\ldots,n \qquad (19)$$

Substitution of (18) and (19), and the expressions for the systemic variables, into (17) results in

$$\begin{aligned} P_i(t+1) = {} & [1+g_i(t)]\, P_i(t) \\ & + c(t)^{\mu}\, \varepsilon_i(t)\, P_i(t)^{\beta} \{\sum_k \delta_k(t)\, P_k(t)^{\alpha}\, R_k(t)^{\nu-1}\, F_{ki}(t)\}^{\mu} \\ & - c(t)^{\nu}\, \delta_i(t)\, P_i(t)^{\alpha} \{\sum_k \varepsilon_k(t)\, P_k(t)^{\beta}\, A_k(t)^{\mu-1}\, F_{ik}(t)\}^{\nu} \\ & \qquad i = 1,2,\ldots,n \quad k \neq i \end{aligned} \qquad (20)$$

Equation (20) is a complex system of n nonlinear first order difference equations in the variables $P_1(t)$, P_2 (t), ..., $P_n(t)$. The first requirement to solve the system would be to eliminate the systemic variables R_k and A_k, but these variables cannot be analytically expressed in terms of population sizes and the factors F_{ij}. The only exception is the case that n=2, a "degenerate" model which is worked through in an appendix to this paper. This appendix shows that even in this simple situation the resulting system of difference equations is rather cumbersome. Thus, in the remainder of this paper we shall proceed by studying specific cases of spatial dynamics both analytically, where possible, and through simulation.

IV EQUILIBRIUM AND STABILITY ANALYSIS

Equilibrium is defined as the configuration in which the distribution of population over regions is stationary. Such a steady state distribution implies that all regions grow at the same rate. Naturally, zero population growth (ZPG) with $\Delta P_i = P_i(t+1) - P_i(t) = 0$ for all i, is a special case. The objective of this section is to identify the conditions under which the general spatial interaction model is compatible with a globally stable equilibrium.

It is obvious that the difference equation system (20) can display a wealth of dynamic behaviour, dependent on the natural growth rates, external influences and the values of the parameters. With respect to internal migration, population redistribution is a "zero-sum game", but the impact of net migration on a region's relative share of population may be reduced or amplified arbitrarily by natural increase. Hence it is only interesting to identify equilibrium under specific assumption about natural increase.

The simplest assumption possible with respect to natural increase (and external migration) is that the growth rates $g_i(t)$ are constant over time. However, if the growth rates of regions are not identical, it is always possible to choose the parameters of the spatial interaction component such that natural increase dominates migration and, consequently, population distribution would be unstable. The relative share of the region with the largest growth rate would be monotonically increasing. Hence it is more realistic to assume that there are limits to growth in a region, due to resource and technological constraints, externalities, etc. The most well known growth process with a ceiling (or a saturation level) is logistic growth, with the growth rate defined by

$$g_i(t) = r_i \; [\, 1 - P_i(t) \,/\, C_i \,] \tag{21}$$

In (21), C_i is the carrying capacity of region i and r_i is the tuning parameter which determines the speed of adjustment. In the absence of spatial interaction, (21) results in a stable population distribution vector χ with elements $\chi_i = C_i / \sum_i C_i$ (i.e. $0 < \chi_i < 1$, with $\sum_i \chi_i = 1$) when $0 < r_i < 2$ for all i, provided all initial populations are within the range $(0, C_i\,(1+1/r_i))$. When $r_i > 2$, the system may display cyclical or chaotic behaviour (May, 1974; 1976).

Even when natural growth is compatible with convergence to a stable equilibrium, a region's exposure to migration disturbs the growth path in general and hence instability may result. The easiest way to demonstrate this is through substitution of (21) into (20) and by assuming that the coefficients of the balancing factors, μ and v, are equal to zero, i.e. a doubly constrained spatial interaction model. This yields

$$P_i(t+1) = P_i(t) \ [1 + r_i (1 - P_i(t) / C_i)] + \varepsilon_i(t) \ P_i(t)^{\beta} - \delta_i(t) \ P_i(t)^{\alpha} \qquad i=1,2,...,n \qquad (22)$$

Note that in this case the growth paths of regions appear to be independent but in fact they are not: the external influences cannot be chosen arbitrarily but must satisfy the conservation condition that total in-migration in the system equals total out-migration. Under this condition, the doubly constrained spatial interaction model is characterised by absence of association in the sense of the Lotka-Volterra model. Equation (22) may or may not be compatible with equilibrium and when equilibrium exists this may or may not be at the level C_i. Assuming that equilibrium does exist with distribution vector χ, this distribution is only stable when <u>all</u> regions with a population greater than their equilibrium level loose through migration and <u>all</u> regions with a population less than their equilibrium level gain through migration. It is not difficult to show that (22) may generate both stable and unstable time trajectories for a specific region. For example, when for a given region k $\varepsilon_k(t) = \delta_k(t)$ for all t; $\beta = 2$ and $\alpha = 1$, it can easily be shown that $P_k = (r_k - \varepsilon_k) C_k / (r_k - \varepsilon_k C_k)$ is an, at least locally, stable equilibrium when $|1 - r_k + \varepsilon_k| < 1$. When $\alpha = 2$ and $\beta = 1$ the same result holds with ε_k replaced by $-\varepsilon_k$.

When the population distribution resulting from natural increase and migration is in a steady state, the equilibrium distribution vector χ is such that net migration is zero in <u>all</u> regions. Hence we need to search for conditions under which the migration matrix M as a function of P_i (i=1,2,...n) is symmetric. In (22) this is achieved by the appropriate choice of the external effects $\varepsilon_i(t)$ and $\delta_i(t)$.

More specific results about equilibrium can be derived for the simple unconstrained gravity model, i.e. the case that $v = \mu = 1$ (equation (12)). It is easy to verify that for this model the following conditions are sufficient for the existence of an equilibrium:

(i) External effects are identical for all regions

(ii) The facility of migration matrix, F, is symmetric

(iii) <u>Either</u> the distribution of populations over regions is homogenous, <u>or</u> the elasticities α and β are equal.

When the elasticities α and β are unequal, equilibrium can only be stable when the carrying capacity C_i of each region is the same, say C^*. In the remainder of this section we shall assume that the tuning parameter of the intrinsic natural growth rate, r_i ,is zero for regions and that the total population is given by $P < n\,C^*$.

In this situation, migration is an efficient growth regulator when conditions (i) and (ii) are satisfied and $\alpha > \beta$: an initial non-homogeneous distribution would, through migration, tend to a homogeneous distribution with $P_i = P / n$ for all i = 1,2,...,n. Under condition (i), the unconstrained gravity model is

$$M_{ij}(t) = c(t)\ \delta(t)\ \varepsilon(t)\ P_i(t)^{\alpha}\ P_j(t)^{\beta}\ F_{ij}(t) \tag{23}$$

For simplicity, this may be written as

$$M_{ij} = \phi\ P_i^{\alpha}\ P_j^{\beta}\ F_{ij} \tag{24}$$

with α, β and $\phi > 0$. Without loss of generality, let us assume an initial distribution with $P_1 < P / n$ as the smallest population and $P_n > P / n$ as the largest. Without natural increase,

$$\Delta P_i = \sum_{j=1, j\neq i}^{n} M_{ji} - \sum_{j=1, j\neq i}^{n} M_{ij} \tag{25}$$

Using (24) and assumption (ii),

$$\Delta P_i > (<)\ 0 \Leftrightarrow P_i^{\beta-\alpha} < (>)\ \frac{\Sigma\ P_j^{\beta} F_{ij}}{\Sigma\ P_j^{\alpha} F_{ij}} \quad (j\neq i) \tag{26}$$

It is not difficult to show that when $\beta > \alpha$, then $\Delta P_1 < 0$ and $\Delta P_n > 0$ and, conversely, that when $\beta < \alpha$, $\Delta P_1 > 0$ and $\Delta P_n < 0$. Whether the intermediate regions P_2 to P_{n-1} gain or loose population depends on their size relative to the interaction factors F_{ij}, but in general, when $\beta > \alpha$, large regions would gain population and small regions would loose; and vice versa when $\beta < \alpha$. The case $\alpha = \beta$ is trivial and implies global stability for any arbitrary population distribution.

The general conclusion is that under the conditions of the unconstrained gravity model with a symmetric matrix F, the homogeneous distribution vector χ, with $\chi_i = 1/n$ for all i, is locally stable when $\alpha > \beta$ but results in competitive exclusion (Hardin, 1960) when $\alpha < \beta$ with the largest region absorbing all population. However, note that α and β may not be chosen arbitrarily, for when $\alpha + \beta$ is very large, it is possible to generate an illogical situation in which population size becomes negative.

What can be said about equilibrium in the general Alonso model? Equilibrium may emerge when net migration is zero for all regions. Using (20) and assuming ZPG in the system, the equilibrium populations P_i would need to satisfy

$$P_i^{\beta-\alpha} = c^{\mu-\nu} \frac{\varepsilon_i \{\Sigma \delta_k P_k^{\alpha} R_k^{\nu-1} F_{ki}\}^{\mu}}{\delta_i \{\Sigma \varepsilon_k P_k^{\beta} A_k^{\mu-1} F_{ik}\}^{\nu}} \qquad i = 1,2,...,n \qquad (27)$$

for all i. However, in contrast with the unconstrained gravity model, there may be no solution to equation (27). Moreover, even when equilibrium exists it is unlikely to be characterised by a homogeneous population distribution. The only obvious steady state situation with $P_i = P / n$ for all i occurs when the F matrix is symmetric, $\alpha = \beta$, $\nu = \mu$ and the external effects (ε_i, δ_i) are pairwise identical, for in this case the systemic variables (R_i, A_i) are also pairwise identical.

Stability analysis is even more difficult than identifying an equilibrium. A simple result that $\alpha > \beta$ is sufficient for stability no longer holds. As the simulations of the next section show, $\alpha > \beta$ may still cause competitive exclusion when $\nu > \mu$.

For given parameters, we may proceed as follows. The non-linear system (27) may be solved numerically, for example by means of the Newton-Raphson method. This yields an equilibrium population distribution, say $\Omega = (\Omega_1, \Omega_2, ..., \Omega_n)$, which is conditional on $g_i = 0$ for all i. Next, the Jacobian of system (20) may be derived through differentiation and evaluated at Ω. Call this matrix Z, i.e.

$$Z = \begin{bmatrix} \frac{\partial P_1(t+1)}{\partial P_1(t)} & \cdots & \frac{\partial P_1(t+1)}{\partial P_n(t)} \\ \vdots & & \vdots \\ \frac{\partial P_n(t+1)}{\partial P_1(t)} & \cdots & \frac{\partial P_n(t+1)}{\partial P_n(t)} \end{bmatrix}_{P=\Omega} \qquad (28)$$

Denoting the equilibrium levels of in-migration and out-migration by $M_{i.}^*$ and $M_{.i}^*$ respectively, $M_{.i}^* = M_{i.}^*$ for all i. It is straightforward to show that the Jacobian satisfies

$$z_{ii} = 1 + (\beta - \alpha)\ M_{i.}^* / \Omega_i \qquad i = 1,2,...,n \tag{29}$$

and

$$z_{ij} = (\alpha\ \mu\ \delta_j\ \Omega_j^{\alpha-1} R_j^{\nu-1} A_i^{-1} F_{ji} - \beta\ \nu\ \varepsilon_j\ \Omega_j^{\beta-1} A_j^{\mu-1} R_i^{-1} F_{ij})\ M_{i.}^* \qquad i \neq j \tag{30}$$

The equilibrium values of the balancing factors R_i, A_j may be computed from (10) and (11) , and the Z matrix can be evaluated subsequently. The presence or absence of local stability is determined by the eigenvalues of Z: $\lambda_1, \lambda_2,..., \lambda_n$. Since Z is generally non-symmetric, the eigenvalues can have imaginary parts. The condition for stability is that

$$(\text{Re}\lambda_i)^2 + (\text{Im}\lambda_i)^2 < 1 \tag{31}$$

for all eigenvalues, although it is possible to have stability when one or more roots are equal to one. A simple example of this is the unconstrained gravity model of the interaction between two regions (n = 2). Using model (24) and equations (29) and (30), the Jacobian is

$$Z = \begin{bmatrix} 1 + (\beta - \alpha)\ M^* / \Omega & (\alpha - \beta)\ M^* / \Omega \\ (\alpha - \beta)\ M^* / \Omega & 1 + (\beta - \alpha)\ M^* / \Omega \end{bmatrix} \tag{32}$$

with $M_{1.}^* = M_{2.}^* = M^*$ and $\Omega_1 = \Omega_2 = ... = \Omega_n$. The matrix Z has two distinct eigenvalues when $\alpha \neq \beta$: $\lambda_1 = 1$ and $\lambda_2 = 1 + 2\ (\beta - \alpha)\ M^* / \Omega$. Hence, the unconstrained gravity model is certainly unstable when $\beta > \alpha$. Stability requires that $\alpha < \beta$ and that the difference between α and β is less than the reciprocal of twice the average propensity to migrate in the system. The latter condition is generally fulfilled when the system is not in an extreme state of flux with large proportions of the population migrating.

V SOME SIMULATION EXPERIMENTS

Since the Alonso model is analytically difficult to deal with, we tested the sensitivity of the outcomes of the Alonso model to the choice of parameters by means of a number of simulation experiments. The objective of this exercise is to demonstrate that parameter values which are obtained from a cross-section calibration of the model may or may not produce dynamic patterns which are plausible in the light of observed trends.

The parameters are chosen on theoretical grounds in all but one of the simulations, but we start with an empirical case study. Poot (1984b) estimated the parameters of the Alonso model with 1971-76 cross-section census data on the inter-urban migration of male workers in New Zealand. The model was statistically satisfactory and a number of economic and quality-of-life determinants of migration were identified. These results confirmed that the migration of workers can be seen as both an investment in human capital and an adjustment of location-specific amenities. A simple demographic specification of the Alonso model, in which the intrinsic push and pull factors were just population size, was statistically less adequate, but nevertheless allocated 79 percent of migrants correctly in the migration matrix. This specification is taken here as the starting point. The parameter values are: $\alpha = 0.7$, $\beta = 0.6$, $v = 0.6$, and $\mu = 0.9$. For comparison, Ledent (1980) found for a similar model of 1971-76 interprovincial migration in Canada: $\alpha = 0.9$, $\beta = 0.8$, $v = 0.5$ and $\mu = 0.2$. Hence the most significant difference between the two case studies is the coefficient of the systemic variable A_j. The New Zealand results are compatible with a production-constrained migration process, whereas the Canadian example suggests a demand-constrained process.

Before simulation, the interaction factors F_{ij} need to be computed. These factors represent, as discussed earlier, the ease with which migration between i and j can take place, and are, as such, inversely related to the generalised cost of migration. It is common to take some measure of distance as a proxy for these costs, although there may be other variables affecting migration costs. For example, the New Zealand case study showed that migration between the four main centres was less costly than inter-urban migration in general, *ceteris paribus*. For simplicity, we ignore such additional variables here and asume that $F_{ij} = D_{ij}^{\xi}$ where D_{ij} is the time it takes to travel comfortably by car between i and j. The New Zealand data suggested that $\xi = -0.7$ (Ledent found -0.9 for Canada).

The predictions of the spatial interaction model should be seen in the context of the long term process of population redistribution that takes place in New Zealand, a country consisting of two large and a few small islands. North-South flows dominate East-West flows due to the country's relatively linear shape. Since about the turn of the century there has been a significant population drift North, to the Auckland province with a large urban core and surrounding fertile regions in which primary production is the principal source of income. The Auckland metropolitan area may be considered a growth pole, with a population which recently exceeded the total population of the South Island. The southern half of the North Island hosts one large city, New Zealand's capital Wellington, a "transit" city which has had a net internal migration loss for some decades. Net migration to the South Island is also negative. When the country is considered as consisting of three regions, Auckland, the rest of the North Island and the South Island, recent population redistribution due to migration over a five year period may be typified by a Markov transition matrix **T** and a distance matrix **D**:

$$\mathbf{T} = \begin{bmatrix} 0.9512 & 0.0320 & 0.0168 \\ 0.0686 & 0.9045 & 0.0269 \\ 0.0285 & 0.0293 & 0.9422 \end{bmatrix}; \qquad \mathbf{D} = \begin{bmatrix} 0 & 675 & 1230 \\ 675 & 0 & 555 \\ 1230 & 555 & 0 \end{bmatrix}$$

For example, the probability to migrate from Auckland to the rest of the North Island (an average distance of 675 minutes) is 3.2 percent over a five year period (excluding return migration). The population with which the simulations started is 3.1 million with distribution vector $\chi' = (0.45 \;\; 0.27 \;\; 0.28)$. Simulation is based on the system described in equation (20). The parameters ε_i and δ_i ($i = 1,2,3$) were computed from levels of in-migration and out-migration resulting from the transition matrix **T** and by assuming that the balancing factors $R_i(t)$ and $A_j(t)$ are initially equal to one. After initialisation, the balancing factors and the proportionality factor c(t) are computed with the RAS method and new levels of in-migration and out-migration are computed. This yields a new population distribution and the process is repeated. The results are reported in Table 1.

Case 0 in Table 1 shows the population redistribution process resulting from the elasticities as estimated from the 1971-76 migration data. Natural increase was assumed zero in order to concentrate on the spatial interaction component of population growth. Of more interest than the reported percentages in the table are the trends that can be observed: Case 0 shows a gradual increase in the share of the largest region (Auckland) with both other regions declining. This is plausible in the light of observed historical trends and we may tentatively conclude that in the absence of negative externalities, affecting the carrying capacity of the region, the estimated cross-section elasticities produce likely time series results.

Cases I to XII report the predictions of theoretical situations with empirically possible but relatively more extreme parameter values than case 0. In case I, $\alpha > \beta$ and in the unconstrained gravity model this would result in convergence to a homogeneous distribution. Instead, case I is close to a production-constrained gravity model (v is near zero and μ is near one) and this results in almost immediate stability at the initial

CASE	α	β	γ	μ	g_i	T=0	1	2	3	4	5	10	20	40	60	80	100
						45	46	46	47	47	47	49	52	56	58	60	61
0	0.7	0.6	0.6	0.9	0	27	27	27	26	26	26	24	22	19	18	17	17
						28	27	27	27	27	27	27	26	25	24	23	22
						45	46	46	45	45	45	45	45	45	45	45	45
I	0.9	0.1	0.05	0.95	0	27	27	27	27	27	27	27	27	27	27	27	27
						28	27	27	28	28	28	28	28	28	28	28	28
						45	47	47	48	49	50	54	60	68	73	76	78
II	0.9	0.1	0.95	0.05	0	27	26	26	25	24	24	21	17	12	10	9	8
						28	27	27	27	27	26	25	23	20	17	15	14
						45	47	47	47	48	48	50	51	53	54	54	54
III	0.9	0.1	0.95	0.95	0	27	26	26	26	25	25	24	23	22	22	22	22
						28	27	27	27	27	27	26	26	25	24	24	24
						45	46	47	47	47	47	49	51	52	53	54	54
IV	0.9	0.1	0.05	0.05	0	27	27	26	26	26	26	25	23	23	22	22	22
						28	27	27	27	27	27	26	26	25	25	24	24
						45	46	46	45	45	45	45	45	43	40	36	26
V	0.1	0.9	0.05	0.95	0	27	27	27	27	27	27	27	26	23	17	0	0
						28	27	27	28	28	28	28	29	34	43	64	74
						45	47	47	48	50	51	59	85	100	100	100	100
VI	0.1	0.9	0.95	0.05	0	27	26	26	25	24	23	16	0	0	0	0	0
						28	27	27	27	26	26	25	15	0	0	0	0
						45	46	47	48	48	49	54	71	97	100	100	100
VII	0.1	0.9	0.95	0.95	0	27	26	26	25	25	24	19	0	0	0	0	0
						28	27	27	27	27	27	27	29	3	0	0	0
						45	46	47	47	47	48	51	60	89	100	100	100
VIII	0.1	0.9	0.05	0.05	0	27	27	26	26	26	25	23	15	0	0	0	0
						28	27	27	27	27	27	26	25	10	0	0	0
						45	46	47	47	47	48	49	51	52	52	52	52
IX	2	1	0.5	0.5	0	27	27	26	26	26	25	24	23	23	23	23	23
						28	27	27	27	27	27	27	26	25	25	25	25
						45	46	47	47	48	48	53	66	95	100	100	100
X	1	2	0.5	0.5	0	27	27	26	26	25	25	21	10	0	0	0	0
						28	27	27	27	27	27	26	24	5	0	0	0
						45	43	41	39	37	36	35	35	35	35	35	35
XI	0.9	0.1	0.95	0.95	logistic	27	28	29	30	31	31	32	32	32	32	32	32
						28	29	30	31	32	33	33	33	33	33	33	33
						45	43	41	38	36	35	33	33	33	33	33	33
XII	0.1	0.9	0.95	0.95	logistic	27	28	29	31	32	32	33	33	33	33	33	33
						28	29	30	31	32	33	34	34	34	34	34	34

Table 1 Simulation results for a three-region generalised spatial interaction model

distribution. The balancing factors are such that after three periods net migration is zero in all three regions. However, the demand constrained model with $\nu = 0.95$ and $\mu = 0.05$ (case II) produces a gradual dominance of the largest region, despite α being greater than β. It seems that such a trend "contrary to expectations" requires ν and μ to be unequal. When $\nu = \mu$, the relation between α and β determines the outcome: convergence to a stable distribution when $\alpha > \beta$ (cases III, IV and IX) and a trend towards competitive exclusion when $\alpha < \beta$ (cases VII, VIII and X). Case V shows that, when $\nu \neq \mu$ and $\alpha < \beta$ the initially largest region does not necessarily gradually absorb the others: in case V with $\nu < \mu$ the whole population is eventually concentrated in the third region (the South Island). When $\nu > \mu$ and $\alpha < \beta$, as in case VI, the systemic effect and the gravity effect reinforce each other and this results in a rapid tendency toward competitive exclusion by the initially largest region.

The last two cases, XI and XII, reflect a situation in which natural increase follows a logistic growth path. The parameter r_i is taken equal to 0.5 ($i = 1,2,3$) and the carrying capacity of each region is considered equal to a population of two million. Under these circumstances, logistic natural growth dominates spatial interaction even when the latter is by itself unstable ($\alpha < \beta$) and a stable population distribution results. This may be explained by the choice of the level of external effects δ_i and ε_i which were given realistic values and corresponded with an average propensity to migrate of no more than 10 percent. By an appropriate choice of parameters it is straightforward to simulate oscillating distributions which converge either to competitive exclusion or to a non-absorbing equilibrium. In this way, disequilibrium trajectories discussed in section 4 may be generated.

VI CONCLUSIONS

This study of the dynamic properties of the generalised spatial interaction model has led to various important results concerning the strength and the weakness of the Alonso model.

The strength of the Alonso model is that it provides a general analysis framework and classification scheme, which turns out to be extremely useful in identifying the properties of various specific families of existing spatial interaction models.

The weakness of the original Alonso model is that it is only a static allocation model which pays no attention to multiregional dynamic spillover effects, while it is also difficult to derive this model from an integrated equilibrating behavioural demand-supply theory (although probabilistic choice theory may provide at least some micro-behavioural choice foundation). In this context, we may assume that the systemic variables act as some sort of pseudo-shadow prices ensuring a certain market clearance.

Various empirical analyses based on the Alonso model have demonstrated its practical usefulness in a static context, but so far little attention has been paid to spatio-temporal feedback mechanisms and statistical-econometric problems emerging from spatio-temporal auto- and cross-correlation. In a dynamic framework, the use of LISREL-models on autoregressive and/or autocorrelation schemes may also provide new ways of treating the generalised spatial interaction model.

Unless a large number of, fairly restrictive, assumptions is introduced, it is in general impossible to derive analytical expressions for the parameter values to be estimated, so that it is hardly possible to study the time trajectory of the generalised spatial interaction model in an analytical sense. Consequently, simulation experiments are in general necessary.

The formal specification of the generalised spatial interaction model implies that it is not *a priori* evident that a spatial (economic or demographic) system is tending toward an equilibrium pattern. On the contrary, disequilibrating tendencies in these models are quite possible and also plausible from a real-world viewpoint. In this respect, the generalised spatial interaction model provides a new angle for studying structural changes in a complex dynamic spatial system.

APPENDIX

The reduced form of the two-region Alonso model

The general structural form of the Alonso model cannot be transformed into a reduced form because the equations for the systemic variables $R_i(t)$ and $A_j(t)$ cannot be solved analytically. However, when only two regions are considered, the reduced form can be derived. Although this simple solution is a degenerate case of the Alonso model, the resulting expressions are nevertheless complicated and demonstrate the intricate relationship between the characteristics of regions, the systemic variables and the interaction factors in the general model.

When the variable time is deleted, the two-region Alonso model is as follows:

$$M_{12} = \delta_1 P_1^{\alpha} R_1^{\nu} = \varepsilon_2 P_2^{\beta} A_2^{\mu} \tag{A.1}$$

$$M_{21} = \delta_2 P_2^{\alpha} R_2^{\nu} = \varepsilon_1 P_1^{\beta} A_1^{\mu} \tag{A.2}$$

$$R_1 = c\, M_{12} A_2^{-1} F_{12} \tag{A.3}$$

$$R_2 = c\, M_{21} A_1^{-1} F_{21} \tag{A.4}$$

$$A_1 = c\, M_{21} R_2^{-1} F_{21} \tag{A.5}$$

$$A_2 = c\, M_{12} R_1^{-1} F_{12} \tag{A.6}$$

$$R_1 = R_2^{-1} \tag{A.7}$$

$$A_1 = A_2^{-1} \tag{A.8}$$

It is easy to see from (A.3) and (A.4) or, alternatively, from (A.5) and (A.6) that c is the reciprocal of the product of the geometric average of M and F:

$$c = (M_{12} M_{21})^{-0.5} (F_{12} F_{21})^{-0.5} \tag{A.9}$$

Substituting this back into (A.3), and using (A.8), yields

$$R_1 = (M_{12} / M_{21})^{0.5} (F_{12} / F_{21})^{0.5} A_1 \tag{A.10}$$

and

$$A_1 = (M_{21} / M_{12})^{0.5} (F_{21} / F_{12})^{0.5} R_1 \tag{A.11}$$

Next, we may rewrite (A.1) and (A.2) as follows

$$R_1 = (\varepsilon_2 P_2^{\beta} / \delta_1 P_1^{\alpha})^{1/\nu} A_1^{-\mu/\nu} \tag{A.12}$$

and also

$$A_1 = (\delta_2 P_2^{\alpha} / \varepsilon_1 P_1^{\beta})^{1/\mu} R_1^{-\nu/\mu} \tag{A.13}$$

By equating (A.10) to (A.12) and (A.11) to (A.13) we solve for A_1 and R_1 respectively:

$$A_1^{1+\mu/\nu} = \varepsilon_1^{0.5} \varepsilon_2^{1/\nu-0.5} \delta_1^{-1/\nu} P_1^{-\alpha/\nu} P_2^{\beta/\mu} (F_{21}/F_{12})^{0.5} \tag{A.14}$$

and

$$R_1^{1+\nu/\mu} = \delta_1^{0.5} \delta_2^{1/\mu-0.5} \varepsilon_1^{-1/\mu} P_1^{-\beta/\mu} P_2^{\alpha/\nu} (F_{12}/F_{21})^{0.5} \tag{A.15}$$

Equations (A.14) and (A.15) may be simplified as

$$A_1 = k_1 (P_2^{\beta} / P_1^{\alpha})^{1/(\nu+\mu)} \tag{A.16}$$

and

$$R_1 = k_2 (P_2^{\alpha} / P_1^{\beta})^{1/(\nu+\mu)} \tag{A.17}$$

with k_1 and k_2 being constants determined by the parameters. Note that when a region grows, both systemic variables have declining values as expected. The systemic variables can be substituted in (A.1) and (A.2). This results in a rather cumbersome reduced form expression for M_{12} and M_{21} when k_1 and k_2 are expressed in terms of parameters. Hence, even in the simple two-region model the dynamics of population redistribution would need to be analysed, for general parameter values, by means of simulation experiments.

REFERENCES

ALONSO, W. (1978), "A Theory of Movements", in: *Human Settlement Systems*, ed. N.M. Hansen, Mass.: Ballinger, pp. 197-211.

ANSELIN, L. (1982), "Implicit Functional Relationships between Systemic Effects in a General Model of Movement", *Regional Science and Urban Economics*, Vol. 12, pp. 365-380.

ANSELIN, L. (1984), "Specification Tests and Model Selection for Aggregate Spatial Interaction: An Empirical Comparison", *Journal of Regional Science*, Vol. 24, pp. 1-15.

ANSELIN, L. and ISARD, W. (1979), "On Alonso's General Theory of Movement", *Man, Environment, Space and Time*, Vol. 1, pp. 52-63.

BIKKER, J.A. (1982), *Vraag-aanbodmodellen voor Stelsels van Geographisch Gespreide Markten*, Amsterdam: VU Boekhandel / Uitgeverij (with English summary).

BROWN, L.A. (1970), "On the Use of Markov Chains in Movement Research", *Economic Geography*, Vol. 46, pp. 393-403.

CARLBERG, M. (1981), "A Neoclassical Model of Interregional Economic Growth", *Regional Science and Urban Economics*, Vol. 11, pp. 191-203.

CASETTI, E. (1981), "A Catastrophe Model of Regional Dynamics", *Annals of the Association of American Geographers* , Vol. 71, pp. 572-579.

COLWELL, P.F. (1982), "Central Place Theory, and the Simple Economic Foundations of the Gravity Model", *Journal of Regional Science*, Vol. 22, pp. 541-546.

DENDRINOS, D.S. (1982), "On the Dynamic Stability of Interurban/Regional Labor and Capital Movements", *Journal of Regional Science*, Vol. 22, pp. 529-540.

DE VOS, A.F. and BIKKER, J.A. (1982), "Interdependent Multiplicative Models for Allocation and Aggregates; A Generalisation of Gravity Models", Research Report 80, Interfaculty of Actuarial Sciences and Econometrics, Free University of Amsterdam.

FISCH, O. (1981), "Contributions to the General Theory of Movement", *Regional Science and Urban Economics*, Vol. 11, pp. 157-173.

GORDON, P. and LEDENT, J. (1980), "Modelling the Dynamics of a System of Metropolitan Areas: A Demoeconomic Approach", *Environment and Planning*, A, Vol. 12, pp. 125-133.

GREENWOOD, M.J. (1969), "An Analysis of the Determinants of Geographic Labor Mobility in the United States", *Review of Economics and Statistics*, Vol. 51, pp. 189-194.

GREENWOOD, M.J. (1981), *Migration and Economic Growth in the United States*, New York: Academic Press.

GRIFFITH, D.A. and LEA, A.C. (eds.) (1983), *Evolving Geographical Structures*, The Hague: Martinus Nijhoff.

GRIFFITH, D.A. and MACKINNON, R.D. (eds.) (1981), *Dynamic Spatial Models*, Alphen aan den Rijn, The Netherlands: Sijthoff & Noordhoff.

HAMPTON, P. and GILES, D.E.A. (1976), "Growth Centres, City Size and Urban Migration in New Zealand", *Annals of Regional Science*, Vol. 10, pp. 41-44.

HARDIN, G. (1960), "The Competitive Exclusion Principle", *Science*, Vol. 131, pp. 1292-1298.

HECKMANN, J.J. (1981), "Statistical Models for Discrete Panel Data", in: C.F. Manski and D. McFadden (eds.), *Structural Analysis of Discrete Data with Econometric Applications*, Cambridge, Mass.: MIT Press, pp. 114-178.

HUA, C. (1980), "An Exploration of the Nature and Rationale of a Systemic Model", *Environment and Planning*, A, Vol. 12, pp. 713-726.

HUDSON, J.C. (1970), "Elementary Models for Population Growth and Distribution Analysis", *Demography*, Vol. 7, pp. 361-368.

INOKI, T. and SURUGA, T. (1981), "Migration, Age, and Education: A Cross-Sectional Analysis of Geographic Labor Mobility in Japan", *Journal of Regional Science*, Vol. 21, pp. 507-517.

JOHANSSON, B. and NIJKAMP, P. (1984), "An Analysis of Urban Episodes by Means of Event History Analysis, Working Paper, IIASA, Laxenburg.

LANGLEY, P.C. (1974), "The Spatial Allocation of Migrants in England and Wales: 1961-1966", *Scottish Journal of Political Economy*, Vol. 21, pp. 259-277.

LANGLEY, P.C. (1977), "Inter-Regional Migration and Economic Opportunity, Australia, 1966-1971", *Economic Record*, Vol. 53, pp. 51-69.

LEDENT, J. (1980), "Calibrating Alonso's General Theory of Movement: The Case of Interprovincial Migration Flows in Canada", *Sistemi Urbani*, Vol. 2, pp. 327-358.

LEDENT, J. (1981), "On the Relationship between Alonso's Theory of Movement and Wilson's Family of Spatial-Interaction Models", *Environment and Planning*, A, Vol. 13, pp. 217-224.

LEONARDI, G. (1983), "An Optimal Control Representation of a Stochastic Multistage-Multiactor Choice Process", in: D.A. Griffith and A.C. Lea (eds.), *Evolving Geographical Structures*, The Hague: Martinus Nijhoff, pp.62-72.

LEONARDI, G. (1985), "Equivalenza Asintotica tra la Teoria della Utilita Casuali e la Massimizzazione dell' Entropia", in: A. Reggiani (ed.), *Territorio e Trasporti*, Milano: Franco Angeli, pp.29-66.

LI, T. and YORKE, J.A. (1975), "Period Three Implies Chaos", *American Mathematical Monthly*, Vol. 82, pp. 985-992.

MAY, R.M. (1974), "Biological Populations with Nonoverlapping Generations: Stable Points, Stable Cycles, and Chaos", *Science*, Vol. 186, pp. 645-647.

MAY, R.M. (1976), "Simple Mathematical Models with Very Complicated Dynamics", *Nature*, Vol. 261, pp. 459-467.

MCFADDEN, D. (1974), "Analysis of Qualitative Choice Behaviour", in: *Frontiers of Econometrics*, ed. P. Zarembka, New York: Academic Press, pp. 105-142.

MEAD, A.C. (1982), "A Simultaneous Equations Model of Migration and Economic Change in Nonmetropolitan Areas", *Journal of Regional Science*, Vol. 22, pp. 513-527.

NIEDERCORN, J.H. and BECHDOLT, B.V. Jr. (1969), "An Economic Derivation of the 'Gravity Law' of Spatial Interaction", *Journal of Regional Science,* Vol. 9, pp. 273-281.

NIJKAMP, P. (1975), "Reflections on Gravity and Entropy Models", *Regional Science and Urban Economics,* Vol. 5, pp. 203-225.

PIMM, S.L. (1982), *Food Webs,* London: Chapman & Hall.

POOT, H.J. (1984a), *Models of New Zealand Internal Migration and Residential Mobility,* Ph.D. Dissertation, Victoria University of Wellington, New Zealand.

POOT, H.J. (1984b), "Modelling the Inter-Urban Exchange of Workers in New Zealand", Discussion Paper No. 113, Australian National University, Centre for Economic Policy Research.

PORELL, F.W. (1982), "Intermetropolitan Migration and Quality of Life", *Journal of Regional Science,* Vol. 22, pp. 137-158.

PORELL, F.W. and HUA, C.M. (1981), "An Econometric Procedure for Estimation of a Generalised Systemic Gravity Model under Incomplete Information about the System", *Regional Science and Urban Economics,* Vol. 11, pp. 585-606.

ROGERS, A. (1966), "A Markovian Policy of Interregional Migration", *Papers of the Regional Science Association,* Vol. 17, pp. 205-224.

ROGERS, A. (1968), *Matrix Analysis of Interregional Population Growth and Distribution,* Berkeley: University of California Press.

ROGERS, A. (1980), "Introduction to Multistate Mathematical Demography", *Environment and Planning,* A, Vol. 12, pp. 489-498.

SMITH, T.R. and PAPAGEORGIOU, G.J. (1982), "Spatial Externalities and the Stability of Interacting Populations near the Center of a Large Area", *Journal of Regional Science,* Vol. 22, pp. 1-18.

SONIS, M. and DENDRINOS, D.S. (1984), "A Discrete Relative Growth Model: Switching, Role Reversal and Turbulence", Paper presented at the Sixth Advanced Studies Institute in Regional Science, Bamberg, Germany.

TABUCHI, T. (1984), "The Systemic Variables and Elasticities in Alonso's General Theory of Movement", *Regional Science and Urban Economics,* Vol. 14, pp. 249-264.

VANDERKAMP, J. (1971), "Migration Flows, Their Determinants and the Effects of Return Migration", *Journal of Political Economy,* Vol. 79, pp. 1012-1032.

VAN DER KNAAP, G.A. and SLEEGERS, W.F. (1982), *ORSMIN: de Structuur van de Migratiestromen in Nederland,* Deelrapport nr.2, Economic Geography Institute, Erasmus University, Rotterdam.

VAN LIEROP, W. and NIJKAMP, P. (1979), "A Utility Framework for Interaction Models", *Sistemi Urbani,* Vol. 1, no.1, pp. 41-64.

VAN LIEROP, W. and NIJKAMP, P. (1980), "Spatial Choice and Interaction Models: Criteria and Aggregation", *Urban Studies,* Vol. 17, pp. 299-311.

WILSON, A.G. (1971), "A Family of Spatial Interaction Models and Associated Developments", *Environment and Planning,* Vol. 3, pp. 1-32.

WILSON, A.G. (1980), "Comment on Alonso's Theory of Movement", *Environment and Planning,* A, Vol. 12, pp. 727-732.