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Time-varying arbitrage and dynamic price discovery

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ABSTRACT

We introduce time-varying measures of price discovery based on underlying profit maximizing behavior by combining the heterogeneous agent modelling literature with the market microstructure literature. We set up a heterogeneous agent model with arbitrageurs and trend chasers (chartists), and allow agents to switch between the strategies conditional on recent forecasting performance. Estimation of the model on Canadian-US cross-listed stocks on high-frequency data shows that there is significant heterogeneity and switching, causing ample variation in the information processing capacity of markets.

1. Introduction

This paper develops an empirical heterogeneous agent model to explain time variation in price discovery for cross-listed assets. Building on a model that allows for both arbitrageurs and trend chasers, we demonstrate that price discovery becomes a function of which type of trader is active in the market. We empirically apply the model to a sample of Canadian firms that are cross-listed in the US. Using intraday data sampled at a one-second frequency, we document significant switching between arbitrage and trend chasing strategies. This induces time variation in price discovery and can explain e.g. an intraday pattern in the contribution to price discovery for each market.

Despite its mathematical elegance and convenience, evidence against the notion of a rational representative agent has been mounting; see e.g. Hong and Stein (2007). One response to this evidence against the representative agent model comes from the literature on heterogeneous agent models (HAMs). HAMs are endowment-based asset pricing models in which agents distribute wealth between a risky asset and a risk-free asset in order to maximize (mean-variance) utility. The innovation is that HAMs allow agents to have heterogeneous expectations about the future prospects of an asset; see Hommes (2006) for an overview. Typically, the models contain two types of traders: fundamentalists and chartists, who expect mean reversion and trend continuation, respectively. Furthermore, agents are allowed to switch between these two groups conditional on their relative performance. The literature on heterogenous agent models has demonstrated that allowing for agents to have heterogenous beliefs and allowing them to switch between these beliefs can generate several stylized facts of financial markets, such as excess volatility, volatility clustering, and heavy tails (see e.g. De Grauwe and Grimaldi, 2006).

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Empirically, HAMs have been applied to a wide range of assets, from explaining stock price behavior (Boswijk et al., 2007; Chiarella et al., 2014) to exchange rate dynamics (De Jong et al., 2010) to option pricing (Frijns et al., 2010); see Lux and Zwinkels (2018) and Chen et al. (2012) for an overview of the empirical literature. Although the models are non-linear in nature and therefore slightly more challenging to estimate, heterogeneity and time variation is introduced in a highly parsimonious way by using only one additional parameter. The studies consistently find that introducing heterogeneity and switching in otherwise standard asset pricing models increases the fit of the model significantly. In addition, the approach has been shown to outperform benchmark models in out-of-sample forecasting exercises (see e.g. Kouwenberg and Zwinkels, 2014).

In this paper, we apply a HAM in a novel setting, namely, to study the price discovery process for cross-listed assets. The novelty in this context is threefold. First, we develop a HAM that allows for agents to switch between arbitrage and trend chasing strategies in two (risky) assets markets,[1] and document that this model performs significantly better than a model that does not allow for heterogeneity. Second, to the best of our knowledge, this is the first paper that brings HAMs into the high frequency domain. Existing empirical studies using HAMs consistently use lower frequency data. However, to study price discovery, high-frequency data is crucial as informational advantages cannot be observed with low-frequency data. We document that high-frequency data in combination with a HAM can be used to explain time-variation in price discovery. Third, we link the empirical heterogeneous agent literature to the microstructure literature and demonstrate that price discovery measures are affected by the proportion of arbitrageurs/trend chasers present in the market. Hence, time variation in the proportion of arbitrageurs results in time variation in price discovery.[2]

We also contribute to the literature on price discovery (e.g. Garbade and Silber, 1979; Hasbrouck, 1995). The market microstructure literature has long been interested in the process of how information is impounded into prices. This question becomes particularly relevant when an asset is listed on multiple exchanges. In such a case, the question becomes which market is more efficient in incorporating information into prices and why. Thus far, the literature on price discovery has predominantly focused on the contribution of markets to price discovery and cross-sectional determinants of price discovery. We contribute to this line of literature by introducing an endogenous and time-varying measure of price discovery that is conditional on the type of trader active in the market at each point in time. Therefore, by combining a HAM with high-frequency data we not only introduce time variation in price discovery, but also provide an economic explanation for this time variation.[3]

We show that our empirical heterogeneous agent model reduces to a vector error correction model with time-varying coefficients conditional on the relative importance of arbitrage and trend chasing. Our empirical findings show that the error correction model that allows for switching between arbitrageurs and trend chasers significantly improves on a static error correction model. This demonstrates that there is indeed time variation in the weights on arbitrage and trend chasing strategies, and that the dynamics of these weights are driven by the past forecasting performance of the respective investment strategies. This time variation in weights leads to time variation in the speed of adjustment coefficients, which in turn leads to time variation in price discovery. Through simulation analysis, we demonstrate that this variation can be substantial. Empirically, we observe that the introduction of multiple strategies, and allowing for switching between these, results in considerable time variation in price discovery , and can explain e.g. an intraday pattern in price discovery.

The remainder of this paper is structured as follows. Section 2 provides an overview of the relevant literature on heterogeneous agent models and the literature on price discovery. In Section 3, we develop a model that allows us to explain time variation in price discovery. Section 4 discusses the data we employ in this study. In Section 5, we present the results of our analysis. Finally, Section 6 concludes.

2. Literature

Our paper builds on the heterogeneous agent modeling literature due to Brock and Hommes (1997, 1998). These models assume that agents have boundedly rational and heterogeneous expectations regarding the future value of an asset. Agents are typically assumed to be of the fundamentalist type, having expectations of mean reversion, or of the chartist type, relying on extrapolation of recently observed price patterns.[4] What makes these models unique is that investors can switch between the fundamentalist and chartists strategies and do so based on the relative past performance of these strategies. This switching between strategies is controlled by the so-called intensity of choice parameter, which is key in generating the nonlinear dynamics. Through numerical analysis, these studies not only demonstrate that these models can generate nonlinear dynamics and stylized facts, such as volatility clustering, excess volatility, and heavy tails in single markets (De Grauwe and Grimaldi, 2006), but also explain stylized facts observed between markets, such as comovements between stock prices, cross-correlation of volatility and other commonalities (Schmitt and Westerhoff, 2014).

[1] This implies several deviations from the benchmark setting: (1) multiple asset markets rather than one single risky asset; (2) alternative trader types to the archetypical fundamentalists and chartists; and (3) no need to define a fundamental value due to the arbitrage relation between markets.

[2] There are theoretical HAM studies that look into the effect of the market microstructure on price formation and stability; see e.g. Chiarella et al. (2009).

[3] This is in contrast to purely statistical approaches to introduce time variation based on, for example, state-space, generalized autoregressive score, or regime switching models.

[4] This choice is motivated by experimentally observed behavior; see e.g. Bloomfield and Hales (2002).

The models proposed by Brock and Hommes (1997, 1998) rely, however, on analytical results or simulations. Boswijk et al. (2007) introduce one of the first empirical specifications of a heterogeneous agent model that is closely related to the theoretical model of Brock and Hommes (1998). They apply this model to annual price ratios (price/dividend and price/earnings ratios) on the S&P 500 over the period 1871–2003. They document evidence of the existence of fundamentalists (those who follow strategies based on mean reversion in the price ratio) and chartists (those who follow strategies based prices deviating further from the fundamental price ratios). Over time, they document that there were indeed periods in time where the market became unstable with explosive price dynamics.

Following this work several studies have empirically applied heterogeneous agent models to explain dynamics of different asset classes. For instance, De Jong et al. (2009) use a heterogeneous agent model to capture shift-contagion during the Asian crisis and document that the switching between fundamentalism and chartism affected the shift-contagion that occurred during the crisis. Lof (2015) shows for the S&P500 that allowing for speculative agents next to standard fundamentalists dramatically improves the ability of the model to replicate market dynamics. Li et al. (2016) propose an asset pricing model with heterogeneous agents to explain stock-bond comovements and cross-market trading and show that heterogeneous agents indeed trade across markets and take the time-varying stock-bond comovements into consideration.

De Jong et al. (2010) use a HAM to model exchange rate dynamics during the EMS, and show that a model with agents that can switch between beliefs outperforms the random walk in terms of forecasting performance. Ter Ellen and Zwinkels (2010) apply a HAM to explain the price behavior of crude oil. Frijns et al. (2015a) focus on the pricing of options based on option traders that have heterogeneous expectations with regards to the volatility process. Allowing traders to have different expectations with regards to future volatility and switching between these expectations, they document a substantial improvement in terms of pricing error across a range of strike prices and maturities of the options. Lux and Zwinkels (2018) review the current empirical literature on heterogeneous agent models.

2.1. Price discovery

The second strand of literature that we relate to, is that of price discovery. Price discovery plays an important role in the market microstructure literature, especially when the same asset is listed in multiple markets. In this case an important question becomes which market is most informative regarding the true value of an asset or, stated differently, which market is most efficient in incorporating information into the market.

One of the first studies on price discovery is that of Garbade and Silber (1979), who examine the price discovery occurring on regional markets in the US versus the NYSE. They document that the NYSE is the dominant market and that the regional markets act as satellite markets. Hasbrouck (1995) revisits the issue of price discovery of the NYSE versus regional exchanges, and introduces the concept of the *information share*, which is defined as the contribution of a market to the variance of the efficient price. The information share is computed based on the estimates of a vector error correction model and a key input in the calculation of the information share is the speed of adjustment coefficient of each market. In his analysis, Hasbrouck (1995) confirms that the NYSE dominates in terms of price discovery. An alternative measure of price discovery is the so-called Gonzalo and Granger (1995) *component share*. This metric is based on a permanent-transitory decomposition of the price process, and looks at the contribution of each market to the efficient price. This metric relies on the speed of adjustment coefficients of a vector error correction model. Finally, Putnins (2013) introduces the *information leadership share*, which uses the information share and the component share together to identify the price series that first impounds new information.

With these metrics, numerous studies have examined price discovery in a range of settings. Studies either apply these metrics to examine price discovery for cross-listed stocks. For instance, Eun and Sabherwal (2003) examine price discovery among Canadian stocks that are cross-listed in the US market. They estimate price discovery for a number of cross-listed stocks and examine the cross-sectional variation in price discovery. They document that measures of market quality, such as the bid-ask spread and market liquidity, can explain the cross-sectional variation in price discovery. Various other studies have used these metrics on other assets as well. For instance, to describe the price discovery occurring between spot and futures markets (Booth et al., 1999), bond and CDS markets (Blanco et al., 2005), or foreign exchange markets (Tse et al., 2006).

While many of the initial studies focused on addressing the question of where price discovery took place, more recent studies focus on determinants of and time variation in price discovery. Frijns et al. (2015a), for instance, estimate annual measures of price discovery for Canadian stocks cross-listed in the US, and use a dynamic panel approach to infer causal relations between measures of price discovery and market quality. In terms of time variation, Frijns et al. (2015c) examine the dynamics of price discovery of Canadian/US cross-listed stocks and relate changes in price discovery to changes in various measures of market quality and high-frequency trading activity. Taylor (2011) focuses on the price discovery in the S&P 500, considering the index and several of its derivatives (futures, E-mini futures, ETFs) and uses a cubic spline to capture intraday variation in price discovery. Frijns et al. (2015b) focus on the effect of macroeconomic announcements on price discovery, and document a shift in price discovery after announcements. Their result of an effect of macroeconomic announcements on price discovery suggests that market participants in one market have better information processing capacity than those in the other market.

The studies that have looked at time variation in price discovery so far either focus on explaining time variation in price discovery due to market characteristics, or just document patterns in price discovery. In this paper, we propose a model where variation in price discovery is actually a function of who is active in the market.

3. Model

3.1. A Heterogeneous agent model for cross-listed assets

HAMs have been developed based on the notion of bounded rationality, building on, among others, experimental studies illustrating the limited rationality and instability of human expectation formation (see e.g. Bloomfield and Hales, 2002). The question that arises, though, is whether heterogeneous agent models can be directly applied to the high-frequency domain because it is debatable whether humans are operating at such speed. One way to interpret this, is that high-frequency traders (HFTs)[5] show quite similar trading patterns as described above for human agents. At a general level, one can argue that HFTs are based on algorithms written by humans; consequently, HFT algorithms can be viewed as heuristics. More specifically, research shows that cross-market arbitrage is an important strategy for HFTs next to market making; see Menkveld (2016). In addition, possible momentum ignition by HFTs remains a concern of policy makers; see SEC (2010). These are exactly the two strategies that we incorporate in our heterogenous agent model.

Let us consider a single asset that is cross-listed and traded on two markets.[6] In these markets, we assume the presence of active profit maximizing traders. In trying to maximize profits, traders can follow certain simple trading rules and switch between these rules at their discretion. We consider two simple trading strategies. For our cross-listed shares, traders can either follow an arbitrage strategy, trading on the observed price differential, or they can be trend chasers, trading on patterns observed in historical prices.[7]

To formalize, let p_t be the (2×1) vector of (log) prices. Since prices in both markets are on the same underlying asset, arbitrageurs can profit when prices get too far out of sync, i.e. if the price of the asset in market 1, p_{1t} is substantially higher (lower) than the price in market 2, the arbitrageur can buy (sell) the share in market 2 and sell (buy) it in market 1 and make an arbitrage profit. This arbitrage opportunity implies that the prices in both markets must be cointegrated and the cointegrating vector is defined as $\beta' = (1\ -1)$. However, prices may temporarily deviate for various reasons; Shleifer and Vishny (1997) introduce the notion of limits to arbitrage, which can be based on implementation costs, fundamental risks, and noise trader risks. In our case, implementation costs consist of the bid-ask spread. There are no fundamental risks, as cross-listed assets have the same underlying fundamental value. Noise trader risk is captured by the trend chasers, as they can push prices further away from their equilibrium value and thereby moving against the position of the arbitrageur. The profitability of the arbitrageurs will therefore depend on the bid-ask spreads as well as the speed by which prices error correct, i.e. the speed by which the price differential will disappear. If the price differential is highly persistent, then an arbitrage strategy will not be very profitable. However, if prices error correct quickly, then profits can also be realized quickly.

When traders in either market follow an arbitrage strategy, they trade on the observed price differential. For example, a trader following an arbitrage strategy in market 1 will sell when $(p_{1t} - p_{2t}) > 0$ and vice versa. Hence a trader (in either market) following an arbitrage strategy (A) will have an expectation about the price at time $t+1$ that is conditional on the price difference observed at time t, i.e.,

$$E_t^{A_i}[p_{it+1}] = p_{it} + \alpha_i \beta' p_t, \tag{1}$$

where i is the market in which the trader is active and α_i is expected to be negative (positive) for $i = 1$ $(i = 2)$, and its absolute size indicates the degree of error correction the trader expects.[8]

Besides following an arbitrage strategy, traders could also follow a trend chasing strategy (C) based on price continuations or reversals. Traders following this strategy base their trades on observed patterns in either market.[9] Hence they predict price changes of the following form:

$$E_t^{C_i}[p_{it+1}] = p_{it} + \sum_j \Gamma_j \Delta p_{t-j}, \tag{2}$$

[5] 20% of trades came from HFT in the US in 2005, which peaked at 60% in 2009. With the outbreak of the financial crisis, the importance of HFT started to decrease. As of 2014, the share of HFT in equity markets has come down to 50% of the total market in the US, respectively.

[6] Our model can easily be generalized to the case of M markets.

[7] In addition to the switching between strategies within a single stock, we could also consider a model where agents switch between stocks. Although it is not the purpose of this paper to develop and estimate such a model (as we focus on the switching behavior within a particular stock), we can assess the potential value of such a model by looking at commonality in switching behavior.

[8] The stocks we consider in our sample are US-Canadian cross-listed stocks. For these stocks, the error-correction occurs relatively fast and price discrepancies are removed relatively quickly.

[9] This is a slight deviation from the typical assumption in HAMs, but can be motivated from the fact that we are dealing with cross-listed shares. We allow traders to observe prices in both markets for several reasons. First, as share prices are publicly available information through exchange websites and trading platforms, they are easily observed by traders. Second, it is a reasonable assumption within our model, where a trader can either trade on a price discrepancy observed between two markets (arbitrageur) - which assumes the trader can observe the price in the other market or a trend chasing strategy where it would make sense to allow the same trader to observe the price in the other market as well. If traders indeed do not consider the trend information in the other market, then our model would reveal this by showing insignificant coefficients on cross-market effects.

where Γ_j are (2×2) matrices of AR-coefficients, $j = 0, \dots, J$ is the number of lags trend chasers consider in their strategy and Δ is the first difference operator.

We assume that traders are familiar with both arbitrage and trend chasing strategies, and since these strategies are relatively simple and costless to apply, can switch between strategies without any (significant) costs. As is common in the HAM literature, we assume that switching occurs on the basis of the relative performance of a certain strategy. We measure performance by considering the forecast error a trader would make by following a certain strategy (see also Kouwenberg and Zwinkels, 2014; Ter Ellen and Zwinkels, 2010). The forecast error, $\pi_t^{k_i}$, for strategy k in market i, can have many functional forms, but we specify it in a way that is common in the HAM literature, i.e.,

$$\pi_t^{k_i} = \sum_{l=0}^{L} |E_{t-l-1}^{k}[p_{it-l}] - p_{it-l}|^n, \tag{3}$$

where a larger value for n will punish more severely for large forecast errors. Note that this is a negative performance measure in the sense that a smaller π implies a higher performance.

Traders can change strategies based on the relative performance of each trading rule and are expected to switch when one rule is more profitable than the other. However, switching may not occur instantaneously as there may be some stickiness to past trading rules. Hence, we define a rule for switching between possible strategies based on Brock and Hommes (1998), i.e.,

$$\begin{aligned} w_t^{A_i} &= (1 + \exp[\lambda_i(\pi_{t-1}^{A_i} - \pi_{t-1}^{C_i})])^{-1}, \\ w_t^{C_i} &= 1 - w_t^{A_i} \end{aligned} \tag{4}$$

where λ_i is the so-called intensity of choice parameter and controls the intensity by which market participants switch between the different strategies. For instance, if $\lambda_i = 0$ traders do not consider the profitability of any of the strategies and thus $w_t^{k_i} = \frac{1}{2}\,\forall k$. On the other hand, when $\lambda_i \to \infty$ traders become infinitely sensitive to the relative performance of each strategy and completely switch to the strategy with the highest performance measure, such that either all traders are arbitrageur or all traders are trend chaser.

Since the conditional expectation of the market price is the weighted average of the individual expectations, we obtain,

$$E_t[p_{it+1}] = \sum_k w_t^{k_i} E_t^k[p_{it+1}]. \tag{5}$$

Filling in the elements yields

$$\begin{aligned} E_t[p_{it+1}] &= w_t^{A_i}(p_{it} + \alpha_i \beta' p_t) + w_t^{C_i}(p_{it} + \sum_j \Gamma_j \Delta p_{t-j}) \\ E_t[p_{it+1}] &= p_{it} + w_t^{A_i}\alpha_i \beta' p_t + w_t^{C_i}(\sum_j \Gamma_j \Delta p_{t-j}). \end{aligned} \tag{6}$$

Expressing (6) in terms of returns we obtain

$$E_t[r_{it+1}] = w_t^{A_i}\alpha_i \beta' p_t + w_t^{C_i}\left(\sum_j \Gamma_j r_{t-j}\right), \tag{7}$$

where $r_t = \Delta p_t$. Expressing (7) in terms of realizations and in vector form we get[10]

$$r_t = c + \alpha_t \beta' p_t + \sum_j \Gamma_{jt} r_{t-j} + \varepsilon_t, \tag{8}$$

with $\alpha_t = w_t^{Ai}\alpha$ and $\Gamma_{jt} r_{t-j} = w_t^{C_i}\Gamma_j r_{t-j}$. We assume that the error term is multivariate normal with time-varying variance, i.e. $\varepsilon_t \sim N(0, \Omega_t)$.

3.2. Price discovery

The model developed in the previous section provides us with a basis to compute measures of time-varying price discovery. The market microstructure literature recognizes several measures of price discovery. The most widely used measures of price discovery are the Gonzalo and Granger, 1995 component share and the Hasbrouck (1995) information share. A more recent measure, the information leadership share, was introduced by Yan and Zivot (2010) and Putnins (2013).

[10] This step can be interpreted as agents having a one-to-one mapping from expectations to demand for the risky asset. The excess demand results in a price change, set by a market maker. This is consistent with the model of De Grauwe and Grimaldi (2005). The noise term can be seen as capturing the actions of agents not captured by the model, such as liquidity traders (i.e., investors that do not trade based on risk-return considerations) or traders that trade purely in their domestic market.

In this paper, we focus on the component share, which is mainly for computational reasons. The information share is computed based on the long-run impact matrix obtained from an Impulse-Response Function (IRF) on the Vector Moving Average (VMA) process derived from Equation (8). What that VMA process looks like is not trivial in the case of time-varying coefficients. Similarly, IRFs on this VMA with time-varying coefficients are not trivial either, due to the time variation in the coefficients, which needs to be modelled somehow in the IRF. As the information leadership share is a function of the information share it is also affected by this computational difficulty.[11]

The Gonzalo and Granger (1995) component share (GG) relies on a permanent-transitory decomposition of the price process in both markets. With the cointegrating vector defined as $\beta' = (1\ -1)$ the Granger–Gonzalo component share is defined as

$$GG_t^1 = \frac{\alpha_{2t}}{|\alpha_{1t}| + \alpha_{2t}}, \qquad GG_t^2 = 1 - GG_t^1. \tag{9}$$

These component shares are thus solely a function of the speed of adjustment coefficients $\alpha_{it} = w_t^{Ai}\alpha_i$, and will consequently be conditional on the weights of arbitrageurs that are present in the market. Specifically, if the proportion of arbitrageurs in market 1 increases relative to market 2, $|\alpha_t^1|$ will increase such that market 1 will respond more to price differences between the markets. As a result, the component share of market 1 decreases and of market 2 increases.

4. Data

We use data for a sample of Canadian stocks that are traded on the Toronto Stock Exchange (TSX) and are cross-listed on the New York Stock Exchange (NYSE). These markets provide an ideal setting to examine the topic of price discovery for several reasons. First, Canadian stocks that are cross-listed in the US are listed as ordinary shares and not as ADRs (American Depository Receipts). This implies that the assets in both markets are identical, which eliminates fundamental risk. Second, the two markets, Toronto Stock Exchange (TSX) and New York Stock Exchange (NYSE), have synchronized trading hours both trading from 9:30 am to 4:15 pm EST.

For our analysis, we focus on all Canadian stocks with cross-listings on the NYSE during the period October 2010 to January 2011 (for a list of the stocks included in the sample see the Appendix). This covers 65 stocks over 79 trading days. We obtain intraday data for these stocks from Thomson Reuters Tick History maintained by SIRCA.[12] We collect data sampled at a one-second frequency on trades and quotes for normal trading hours (9:30 am–4:00 pm). However, in our analysis we drop the first and last five minutes of the trading day to stay clear of the opening and closing algorithm. In addition, we collect data on the Canadian/US Dollar exchange rate, also sampled at a one-second frequency to make the prices comparable.

In our main analysis of price discovery, we focus on the midpoints of the bid and ask quotes as per Grammig et al. (2005). These midpoints have been shown to be less affected by microstructure noise, such as the bid-ask bounce, that is commonly observed in transaction prices. We obtain the midquotes for both prices of the asset traded on the TSX and the NYSE, and convert the Canadian price into US Dollars using the midpoint of the bid and ask quotes of the exchange rate.

In Table 1, we present descriptive statistics for the stocks in our sample. All statistics are computed on the data sampled at a one-second frequency. We report statistics for log price changes in both markets, as well as the log price difference between the Canadian and US Market. As we cover 65 stocks in the sample, we report a distribution of the summary statistics. As can be seen from Table 1, the mean and median returns for the different stocks are virtually zero. This is expected for data sampled at a frequency of one second. However, we do note some variation across the sample in the standard deviation of the returns, as well as the minimum and maximum values, with the standard deviation ranging from as low as 0.003% to 0.092% in Canada and 0.001% to 0.088% in the US (note that the numbers reported in Table 1 are multiplied by 100).

As for the price difference between the Canadian and US price, we see that across the majority of the distribution this is very close to zero as expected. We also note that for the majority of the distribution, the standard deviation and minimum and maximum values fall within relatively narrow defined ranges. Overall, the stocks in the sample seem to have statistical properties that are very much in line with what one would expect to observe in high-frequency data.

We estimate the multivariate model given by Eq. (8) using full-information maximum likelihood. Because there is no trading during closing hours, we need to account for the gap between the closing of day t and subsequent opening of day $t+1$. Therefore, we estimate the model separately for each of the 79 days in the sample based on approximately 21,000 daily seconds, and present the average coefficients and robust standard errors over the 79 days (this is inline with e.g. Hasbrouck (2003)). This is also helpful in terms of computation time because of the large number of observations combined with the multivariate model. We first estimate the static model, assuming $\lambda = 0$, and use the estimated coefficients as starting values for the switching model in which λ is estimated as a free parameter.

[11] An interesting extension of our paper would be to explore what the information share and the information leadership share look like for our heterogeneous agent model. This, however, is beyond the scope of the current paper as our aim to demonstrate that, first, a HAM can successfully be implemented with high-frequency data in the context of price discovery; and second, the implementation of a HAM using high-frequency data can generate time variation in price discovery using a commonly used measure of price discovery.

[12] Securities Industry Research Centre of Asia-Pacific.

Table 1
Descriptive statistics.

	Min	25%	50%	75%	Max
	$100^{*}\Delta P^{Ca}$				
Mean	−0.0001	0.0000	0.0000	0.0000	0.0000
Median	0.0000	0.0000	0.0000	0.0000	0.0000
stdev	0.0030	0.0039	0.0047	0.0062	0.0918
Min	−3.8341	−0.0882	−0.0610	−0.0411	−0.0290
Max	0.0239	0.0375	0.0571	0.0862	3.7948
	$100^{*}\Delta P^{US}$				
Mean	0.0000	0.0000	0.0000	0.0000	0.0000
Median	0.0000	0.0000	0.0000	0.0000	0.0000
stdev	0.0012	0.0023	0.0029	0.0046	0.0876
Min	−4.0906	−0.1372	−0.0643	−0.0415	−0.0165
Max	0.0167	0.0331	0.0580	0.1064	4.5383
	$100^{*}(P^{Ca}-P^{US})$				
Mean	−1.7759	−0.0005	0.0053	0.0122	1.2395
Median	−1.7743	0.0013	0.0073	0.0127	1.2399
stdev	0.0423	0.0561	0.0668	0.0850	1.4160
Min	−5.6603	−0.2000	−0.1521	−0.1194	1.0735
Max	−1.6613	0.1353	0.1621	0.2349	2.1537

Notes: This table presents the distribution of the descriptive statistics of log price changes in Canada and the Unites States, and log price differences between Canada and the United States, over the cross-section of stocks in the sample.

5. Results

In this section, we report the results for the model developed in Section 3. We start by showing results for a single stock to demonstrate the properties of the model and the dynamics it is able to generate. Subsequently, we document the results of all stocks in the sample by reporting summary statistics of the model estimates and price discovery measures.

5.1. Results for a single stock: Goldcorp Inc.

5.1.1. Estimation results

We start our analysis by showing the results for a single stock in our sample, Goldcorp, Inc. (GC) to illustrate the dynamics that can be generated and to show the time variation in price discovery that can be introduced by our model. To assess the contribution to price discovery of the prices of GC in the Canadian and US stock market, we first need to assess whether: 1. log prices contain unit roots; 2. log price changes are stationary; and 3. log prices in both markets are cointegrated. For GC, we find that prices indeed have a unit root, as we fail to reject the null hypothesis for the presence of a unit root in both markets (Augmented Dickey–Fuller test produces insignificant test statistics of −1.27 and −1.20 for the Canadian and US price, respectively). We also find that price changes are stationary as the Augmented Dickey–Fuller test produces significant (at the 1% level) test statistics of −359.40 and −116.20 for the price changes in the Canadian and US market, respectively. Finally, we find strong evidence for cointegration. The Johansen test for cointegration strongly rejects the null hypothesis of no cointegrating relation (Johansen Trace statistic of 45,576, significant at the 1% level), while we cannot reject the null hypothesis of at most one cointegrating relation (insignificant Trace statistic of 2.03).

Having met these necessary conditions, we proceed with the estimation of the vector-error correction model (VECM) for GC. We estimate the VECM for each day with a lag length of one,[13] and report the results for the average coefficients in Table 2. We also report Newey–West corrected standard errors based on the variation in daily coefficient estimates.

In the first column of Table 2, we report the results for the static model (i.e., taking $\lambda = 0$). The estimated cointegrating vector is close to $(1\ -1)'$ confirming the cointegration results. The upper part of the table shows the results for the price of GC in the Canadian market. We observe the expected positive coefficient on the error correction term, α of 0.588, which is highly significant, suggesting significant price corrections in the Canadian market following a price discrepancy between the Canadian and US price of the asset. In terms of the model, this result implies that arbitrageurs are indeed active in the Canadian market. We also observe that both trend chasing coefficients, γ_1^{Ca} and γ_2^{Ca}, are significant suggesting that agents with a trading strategy based on price extrapolation are indeed active in the Canadian market, and consider price trends in both markets. Specifically, chartists in Canada extrapolate the most recent change in the US price ($\gamma_2^{Ca} > 0$) whereas they have contrarian expectations with respect to the Canadian price ($\gamma_1^{Ca} < 0$).

The lower half of Table 2 reports the results for the price of GC in the US market. As expected, the speed of adjustment coefficient has a negative sign and is highly significant. This indicates that the price of GC in the US error corrects in response to a price discrepancy between the two markets. Hence, arbitrageurs are also active in the US market. However,

[13] We confirm that similar results are obtained for different lag lengths and show the robustness of our results to this choice later in the paper, when we assess all stocks in the sample.

Table 2
Estimation results for Goldcorp, Inc.

	Static	Dynamic
LLik	421,597	421,676
LLR		158.62***
		(17.62)
β	0.999***	0.999***
	(0.000)	(0.000)
	Canada	
α^{Ca}	0.588***	0.564***
	(0.027)	(0.028)
γ_1^{Ca}	−0.605***	−0.603***
	(0.015)	(0.016)
γ_2^{Ca}	0.532***	0.556***
	(0.015)	(0.016)
λ^{Ca}		24.32***
		(5.320)
c^{Ca}	−6.61E−5***	−6.00E−5***
	(0.000)	(0.000)
	United States	
α^{US}	−0.102***	−0.072***
	(0.006)	(0.005)
γ_1^{US}	−0.029*	−0.024
	(0.019)	(0.028)
γ_2^{US}	0.041***	0.049***
	(0.006)	(0.007)
λ^{US}		579.61***
		(202.52)
c^{US}	1.10E−5***	7.59e−6***
	(2.11E−6)	(2.77E−6)
	Component shares	
GG^{Ca}	0.1484	0.1125
GG^{US}	0.8516	0.8875

Notes: This table presents the estimation results of the static (the model with $\lambda^{US} = \lambda^{Ca} = 0$ such that $w_t^A = w^A = 0.5$) and dynamic (with λ as free parameters) model for the stock of Goldcorp, Inc. (GC). *LLik* denotes the Log-Likelihood of the model, *LLR* denotes the likelihood ratio test result, and *GG* denotes the (unconditional) Gonzalo-Granger component share. Numbers in parentheses are standard errors; *, **, *** denote significance at the 10, 5, and 1% level, respectively. Note that these are the time-series averages and Newey-West corrected standard errors over the 79 days in the sample.

the coefficient for the US market is smaller in magnitude than for the Canadian market. This observation will have important consequences for the contribution to price discovery of each market. When we consider the persistence in the returns, we note that there is evidence of trend chasing behavior in the US market with respect to both Canadian and US prices, where agents again tend to be price extrapolators with respect to US past prices ($\gamma_2^{US} > 0$), but contrarian with regard to Canadian past prices ($\gamma_1^{US} < 0$), although this evidence is weak. This again suggests that a strategy based on persistence in returns is being applied, but given that the persistence in the US is smaller in magnitude than the persistence in Canada, the profitability of a trend chasing strategy may be lower in the US.

The bottom rows of Table 2 show the unconditional Gonzalo–Granger component shares of GC, which measure the contribution to price discovery of each market. We observe that the component share is greater for the US (about 85% of price discovery takes place in the US, unconditionally) than it is for Canada. This is a consequence of the speed of adjustment coefficient in the US being smaller in magnitude than the speed of adjustment coefficient in Canada.

In the second column of Table 2, we report the results for the dynamic model (incorporating two switching parameters (λ) into the model). In this configuration, the agents look back one period to decide on their strategy, i.e., $L = 1$,[14] and focus on absolute forecast errors, i.e., $n = 1$.

The first thing we note is that this switching model significantly improves on the static model, producing a highly significant log-likelihood ratio (*LLR*) statistic of 158.62.[15] Hence, the switching model can better capture the behavior of agents, and thereby better capture the price dynamics of the cross-listed asset of GC.

We also report the coefficient values for the switching parameters, λ. These parameters turn out to be positive and highly significant in both the Canadian and US market, showing that investors in both markets follow a positive feedback strategy,

[14] We study the robustness of our results to this choice later in the paper when we consider all stocks in the sample.

[15] We compute the LLR as twice the difference between the log-likelihood of the unconstrained switching model and the constrained static model. This ratio has a χ^2-distribution with two degrees of freedom, and a critical value of 9.21 at the 1% level.

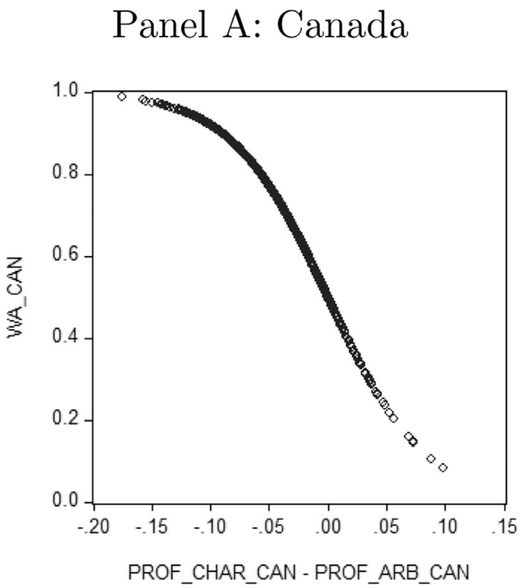

Fig. 1. Weights on Arbitrage as a function of the profit difference ($\pi^C - \pi^A$) for Goldcorp, Inc. *Note*: This figure presents the weights on the arbitrage strategy in Canada (Panel A) and the US (Panel B) as a function of the profit difference between the trend chasing and the arbitrage strategy ($\pi^C - \pi^A$) for the stock of Goldcorp, Inc. (GC).

chasing the strategy that has provided the highest past performance. The switching parameter, λ, is considerably larger in the US than in Canada. This implies that agents switch more aggressively between strategies in the US than in Canada.[16] This might be explained by the fact that the fundamentalist and chartist strategies are both better defined in Canada than in the US; the α and γ coefficients are both higher and more significant in Canada than in the US. In other words, the unconditional profitability of the two rules is higher in Canada than in the US, such that switching between strategies is less of a necessity in Canada than it is in the US market.

As for the other coefficients in the VECM, we note that the results are very similar compared with those of the static model. We only lose significance in γ_1^{US}, the persistence term on lagged Canadian price changes in the US market, suggesting that US investors do not extrapolate price patterns observed in the Canadian market.

[16] Typically, the intensity of choice parameter cannot be compared across time and space because its magnitude is conditional on the variation in the profit difference ($\pi^A - \pi^C$). In our case, however, the statistical characteristics of the two markets are very comparable, because they reflect the same underlying company. As such, we feel it is safe to make general statements about the order of magnitude of λ.

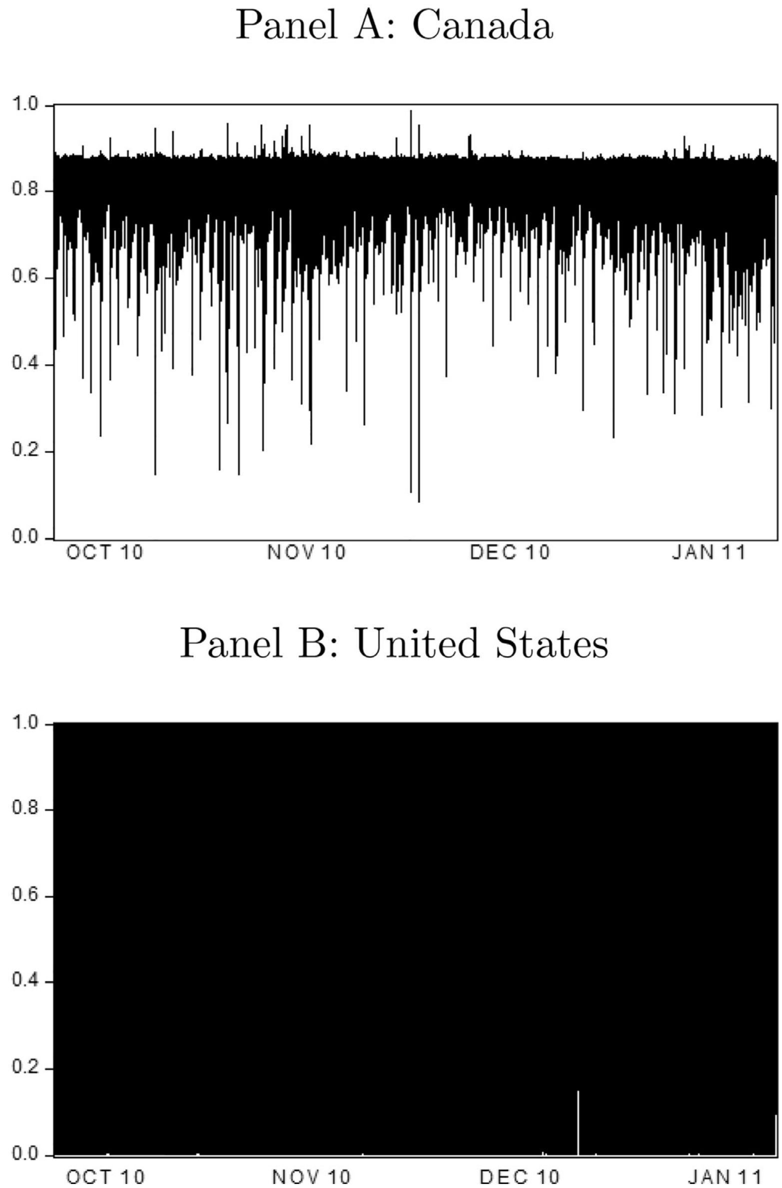

Fig. 2. Time Series plot of the Weights on Arbitrage for Goldcorp, Inc. *Note*: This figure presents a time series plot for the the weights on the arbitrage strategy in Canada (Panel A) and the US (Panel B) for the stock of Goldcorp, Inc. (GC).

As the switching model allows traders to shift between arbitrage and chartist strategies in both Canada and the US, we provide some properties of the weights allocated to the strategies in Figs. 1 and 2. Specifically, in Fig. 1 we report the weights on the arbitrage strategy in each market as a function of the profit difference between trend chasers and arbitrageurs in that market. The results presented in this figure confirm those presented in Table 2. First, switching is towards the strategy that has the higher profitability (lower forecast error) relative to the other strategy, i.e. agents act as positive feedback traders in both markets. Second, the switching is more aggressive in the US market than in the Canadian market, as highlighted by the steeper slope of the S-curve and the narrower range on the *x*-axis. Third, the time-variation in the weights is substantial as the weights range between zero and one for the US and between 0.1 and 0.95 for Canada.

To examine the dynamics in weights that this generates over time, we provide a time series plot for the weights on the arbitrage strategy in Fig. 2. We first observe that the aggressiveness of switching documented in the previous graphs is clearly observed in Fig. 2, where in the upper graph we can clearly see the time variation introduced by the switching in the Canadian market. However, for the US market switching turns out to be so aggressive that no clear pattern can be identified.

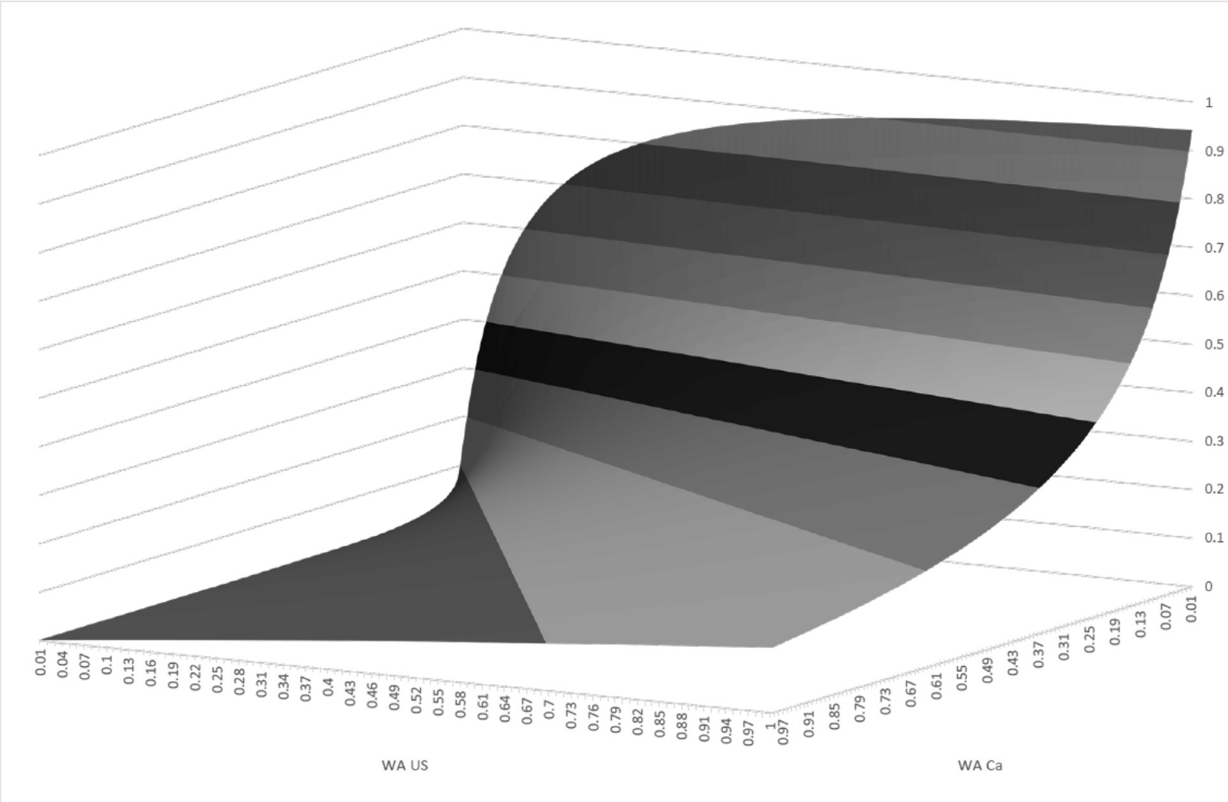

Fig. 3. Canadian Contribution to Price Discovery for Different Weight Combinations for Goldcorp, Inc. *Note*: This figure presents the Gonzalo–Granger component shares for the Canadian market for different levels of w^{Ca} and w^{US} for the stock of Goldcorp, Inc. (GC).

5.1.2. Time variation in price discovery

The results presented so far indicate that allowing for switching between arbitrage and chartist strategies in a model for the price dynamics of a cross-listed asset can improve the fit of the model significantly and also introduces considerable time variation in weights, while using a very parsimonious specification. We now explore the implications of this time variation for price discovery. Specifically, we focus on the Gonzalo–Granger component share (GG) as defined in Eq. (9).

As a demonstration of the degree of time variation in price discovery that can be introduced by our model, we run a simulation by varying the weights on the arbitrage strategy for Goldcorp, Inc. in both markets. To visualize the results of this analysis, we present a graph of the component shares for all possible weight configurations in Fig. 3. In this figure, the two horizontal axes represent the weights on the arbitrage strategy for the US and Canadian market, while the vertical axis represents the GG component share for the Canadian market. The plot shows the sensitivities of the GG measure to the weights on the arbitrage strategy in the two markets.

The graph shows that in the extremes, price discovery is completely dominated by one market. For instance, with no arbitrageurs in the US and only arbitrageurs in Canada, price discovery for Canada is close to zero. This is expected as the weight on the error correction term in Canada is equal to one, while the weight on the error correction term in the US is zero. Thus, all the error correction occurs in the Canadian market. When we move to the situation where the weights on the arbitrageurs is equal to one in both markets, we observe that the price discovery measure increases almost linearly. However, the GG measure remains well below 0.5, implying that the US remains the dominant market, explained by the fact that $\alpha^{Ca} > \alpha^{US}$. When the arbitrageurs in the Canadian market drop to zero the GG measure increases in a curvilinear fashion converging to one. Hence, the results for price discovery in the estimation of the model will be sensitive to the weight of arbitrageurs in the Canadian market. Finally, when we move to the point where there are no arbitrageurs in either market, we observe that the GG measure declines slowly at first but sharply as we get close to the case where there are no arbitrageurs in the US market. Overall, the graph shows that changes in price discovery will be most sensitive to changes in arbitrageurs in Canada when there are many arbitrageurs in the US, and to changes in arbitrageurs in the US, when the weight on arbitrageurs in Canada is relatively low.

The analysis presented above was based on simulations. In that analysis, we considered weights over the range of zero to one. However, as we observed in Fig. 2, the actual weights on the arbitrage strategies do not cover this full range. In Fig. 4, we provide a time series plot of the Canadian contribution to price discovery (GG for Canada) for the stock of Goldcorp,

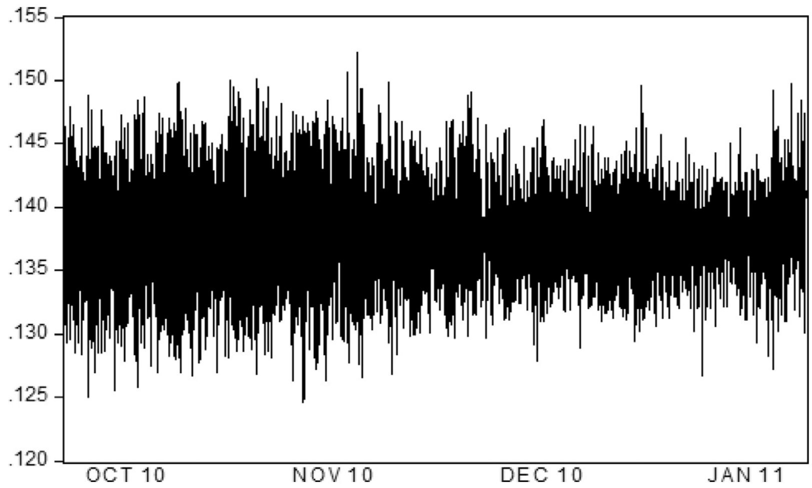

Fig. 4. Time Series Plot of Canada's Contribution to Price Discovery for Goldcorp, Inc. *Note*: This figure presents a time series plot of Canada's contribution to price discovery (the conditional Canadian Gonzalo–Granger component share) for Goldcorp, Inc.

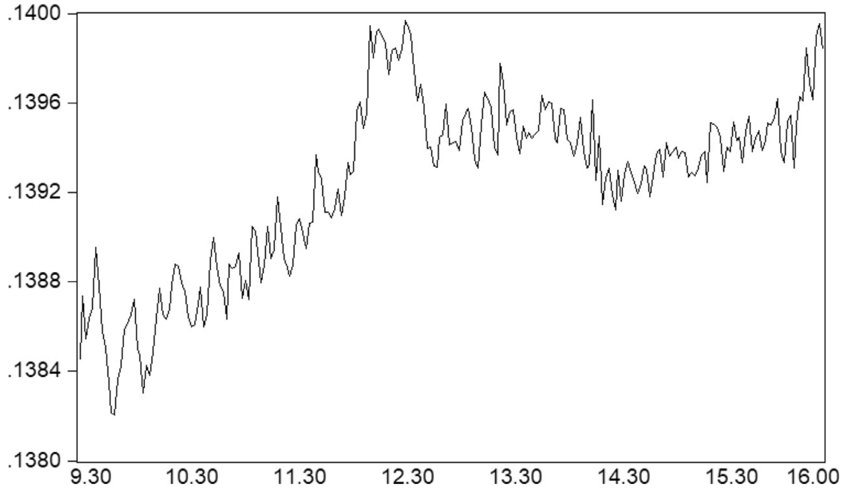

Fig. 5. Intraday Component Share Pattern for Goldcorp, Inc. *Note*: This figure presents the intraday pattern of the conditional Canadian Gonzalo–Granger component share for Goldcorp, Inc. The component share is calculated based on 100 non-overlapping seconds, averaged over the 79 days in the sample.

Inc. As we can see from this plot, there is considerable time variation in price discovery that is introduced by our switching model.[17]

Given that Fig. 4 provides a rather noisy picture of the time variation in price discovery that is being generated, it is hard to identify any trends or patterns. However, we can consider the intraday pattern in price discovery that is generated by the model. In Fig. 5, we plot the GG measure during the day. When considering this plot, we observe that there is a distinct intraday pattern in the contribution to price discovery for the stock of Goldcorp, Inc. We observe that the Canadian price discovery measure generally increases over the trading day, most visibly during the first couple of hours of the trading day. This finding is in line with Taylor (2011), who documents that changes in price discovery occur mostly during the start and end of the trading day. This suggests that the information processing capacity of the US market seems to be stronger near the open of the market, whereas the Canadian market gains a bit during the later hours of the trading day.

5.2. Full sample results

In the previous section, we presented the results for one stock, Goldcorp Inc., to demonstrate the dynamics that can be generated by our switching model. In this section, we present the full results for all 65 stocks in our sample. We do this by showing summary statistics for the various model estimates and measures we derive. We provide additional robustness

[17] Note that the mean of 13.9% is somewhat higher than the unconditional component share of 11.25% we documented in Table 2. This is due to the fact that the average arbitrageur weight is higher than 1/2, which was not taken into account in Table 2.

Table 3
Estimation results static model.

	Significance						Distribution				
	10% pos	10% neg	5% pos	5% neg	1% pos	1% neg	min	25%	50%	75%	max
	Cointegrating equation										
β	65	0	65	0	65	0	0.984	1.000	1.000	1.000	1.052
	Canada										
α^{Ca}	50	0	35	0	17	0	0.065	0.160	0.227	0.400	0.808
γ_1^{Ca}	0	65	0	65	0	64	−0.673	−0.585	−0.517	−0.472	−0.308
γ_2^{Ca}	65	0	65	0	65	0	0.150	0.311	0.384	0.460	0.673
c^{Ca}	45	1	36	0	16	0	0.000	0.000	0.000	0.000	0.000
	United States										
α^{US}	0	46	0	40	0	38	−0.032	−0.013	−0.006	−0.002	0.275
γ_1^{US}	17	11	13	11	11	10	−0.272	−0.019	0.017	0.082	0.202
γ_2^{US}	64	0	64	0	60	0	−0.003	0.012	0.023	0.030	0.063
c^{US}	13	5	7	2	2	1	0.000	0.000	0.000	0.000	0.000
	Component Shares										
GG^{Ca}							0.000	0.011	0.037	0.084	0.405
GG^{US}							0.595	0.916	0.963	0.989	1.000

Notes: This table presents the estimation results of the static model, i.e., the model with $\gamma = 0$ such that $w_t^A = w^A = 0.5$. *GG* denotes the (unconditional) Gonzalo–Granger component share. The six columns on the left show the number of stocks (out of 65) for which the estimated coefficient is significant at the 10, 5, and 1% level, respectively, based on Newey–West corrected standard errors. The five columns on the right present the distribution of estimated coefficients over the 65 stocks.

tests for our model by showing model results for different lag lengths in the VECM and lookback periods in the switching rule.

5.2.1. Model estimates

Before we can estimate the VECM for the various stocks in the sample, we again need to assess whether log prices contain unit roots in both markets, and whether they are cointegrated. As we estimate the model per stock, per day, we assess whether these conditions are met for each stock-day pair. We find that unit roots (according to an Augmented Dickey–Fuller test) are present in the Canadian and US market in 89.4% and 88.9% of the cases and that prices are cointegrated (as per Johansen's Trace statistic) in 95.6% of the cases. The days in which prices do not contain a unit root or are not cointegrated, are excluded from the estimation.[18,19]

For the stock-days that meet the unit root and cointegration conditions, we estimate the static VECM using a lag length of one, and report the results for this model in Table 3. In the first few columns of this table, we report the number of stocks for which the average coefficient over all days is significant at a given significance level (based on Newey–West corrected standard errors), while in the last few columns of the table, we provide an overview of the coefficient estimates, by showing their values at various percentiles as well as the minimum and maximum. Overall, we observe that the estimated cointegrating vector is close to $(1\ -1)'$ for all firms across most of the distribution, confirming the cointegration results. When we consider the error correction term for the Canadian market (upper part of the table), we observe the expected positive coefficient on the error correction term, α, across the distribution of parameter estimates. For 50 out of the 65 stocks, this coefficient is significant at the 10%, while for 17 stocks it is significant at the 1% level. This suggests that while price corrections occur in the Canadian market following a price discrepancy between the Canadian and US price of the asset, these price corrections are not all significant. Overall, this result implies that arbitrageurs are indeed active in the Canadian market.

We find very strong evidence of trend chasing behavior in the Canadian market, with evidence on price extrapolation based on US prices ($\gamma_2^{Ca} > 0$) for all stocks at the 1% level, and evidence on contrarian expectations with respect to the Canadian price ($\gamma_1^{Ca} < 0$) for virtually all stocks (64 stocks have negative coefficients significant at the 1% level). In terms of the distribution of the coefficients, we observe a range between minimum and maximum of the trend chasing coefficients of −0.673 to −0.308 for Canadian past stock price movements, and 0.150 to 0.673 for US past stock price movements.

In the lower half of Table 3, we report the results for the prices in the US market. In this market, speed of adjustment coefficients are expected to be negative, and indeed in the majority of the cases, we observe that it is. In some cases, we observe negative coefficients, but very close to zero, and in a few cases we observe positive coefficients. For 38 of the 65 stocks, we observe that the coefficient is significant and negative at the 1% level. The positive coefficients are, however, never significantly different from zero. Similar to what we observed for the stock used in our example, the coefficients on the error correction term in the US market are smaller in magnitude than in the Canadian market, and this again will have

[18] Unit roots and lack of cointegration may be due to low liquidity for a stock in a particular day. We confirm that unit roots are presented in all prices and cointegration is observed for all stock when this is assessed per stock over all days in the sample.

[19] We also note that the correlations between the price discrepancies of the various stocks are relatively high, around 90% on average. This lends support for a potential multivariate model that could consider these cross-stock interactions.

Table 4
Switching results.

	Significance						Distribution				
	10% pos	10% neg	5% pos	5% neg	1% pos	1% neg	min	25%	50%	75%	max
λ^{Ca}	3	3	2	0	0	0	−393.291	48.386	59.406	74.794	606.186
λ^{US}	3	3	1	1	0	0	−10885	−2207	−581	1183	16410
LLR	65		65		65		190.1	326.2	385.3	493.1	1018.4

Notes: This table presents the likelihood ratio test results (LLR) as well as the estimated intensity of choice parameters (λ) of the switching model. The six columns on the left show the number of stocks (out of 65) for which the estimated coefficient is significant at the 10, 5, and 1% level, respectively, based on Newey–West corrected standard errors. The five columns on the right present the distribution of estimated coefficients over the 65 stocks. The remaining parameter estimates are suppressed for reasons of brevity; they are qualitatively similar to the static results, though.

consequences for the GG component shares of the various stocks. When we consider the coefficients on the trend chasing parameters of the model, we observe slightly different results compared with what we observed for the price dynamics in the Canadian market. For the trend chasing coefficient of US prices relative to past US prices (γ_2^{US}), we observe that virtually all are positive and significant, suggesting that trend chasers in the US market use past US prices in an extrapolative way. However, when we consider the evidence for the trend chasing behavior with regards to Canadian past prices, we observe mixed results. For 11 stocks there is evidence at the 1% level of price extrapolation, while for 10 stocks there is evidence at the 1% level of contrarian behavior. For many other stocks, we find no significant evidence of trend chasing behavior with regards to Canadian past prices. Overall, this suggests that US trend chasers in many cases pay little attention to past prices in Canadian market (potentially because those prices are not very informative). We also note the huge difference in the order of magnitude between the trend chasing coefficients in Canada and the US, which is much smaller in the US, as observed for our example stock of Goldcorp, Inc.

The last rows of Table 3 show the unconditional Gonzalo–Granger component shares.[20] In all cases we observe that the *GG* measure is greater for the US than for Canada. This is a consequence of the speed of adjustment coefficient in the US being smaller in magnitude than the speed of adjustment coefficient in Canada. The *GG* measures for Canada range from a low of 0 to a high of 0.405. The dominance of the US market in terms of price discovery in recent times has also been documented by Frijns et al. (2015c), while cross-sectional variation in price discovery measures has been documented by Eun and Sabherwal (2003).

Table 3 reported the results for a static VECM, and documented that there is indeed evidence of both arbitrageurs and trend chasers being active in both markets, but that there are considerable differences between the price dynamics in Canada and the US. Furthermore, we documented that the US dominates in terms of price discovery. We now extend this static model by allowing for switching between arbitrage and chartist strategies, and report results for this switching model in Table 4. Since the results for the static VECM parameters are comparable to those reported in Table 3, we only report the results for the switching parameters and the Log-Likelihood Ratio (*LLR*). The LLR statistic again has a χ^2-distribution with two degrees of freedom as we introduce two switching parameters (λ) to the model and a critical value of 9.21 at the 1% level. The results are again reported for a lag length of one, a lookback period on the strategies of one period, and focus on absolute forecast errors, i.e., $n = 1$.[21]

As can be seen from Table 4, for all 65 stocks we observe a LLR statistic that exceeds the 1%-level critical value, ranging from 190.1 to 1,018.4, showing that for all stocks the model with switching between different investment strategies is a significant improvement on the static model. Hence, the model with switching can better capture the behavior of agents, and thereby better capture the price dynamics of the cross-listed assets in our sample.

The results for the signs of the switching parameters, λ, are mixed, with only 6 switching parameters significant (3 positive and 3 negative) at the 10% level in both Canada and the US. This suggests that while the switching parameters are significant for each of the daily estimates (as documented by the significant LLR statistics), there is relatively little consistency over time in the direction of switching. In other words, we also find dynamics in the dynamic expectation formation of agents.

As the switching model allows traders to shift between arbitrage and chartist strategies in both Canada and the US, we report some properties of the weights allocated to each strategy in Table 5.

As can be seen from Table 5, the average weight on the arbitrage strategy in the Canadian market is centered around 0.85, suggesting that, on average the arbitrage strategy is more profitable in the Canadian market. In the US market, the average weight on the arbitrage strategy is centred around 0.4, suggesting that the trend chasing strategy in the US is slightly more profitable than the arbitrage strategy, on average. This can be explained by the relatively low component share of the Canadian market, making the arbitrage strategy more viable in Canada. More interesting than the averages, however, is the variation in the weights on the arbitrage strategy both across stocks and over time. Across stocks, as seen by the values across the different percentiles, we observe that the weights can cover nearly the full spectrum of $< 0, 1 >$,

[20] Note that these are unconditional component shares based on the estimated α's, without taking the arbitrageur weights into account.

[21] We assess robustness of our results subject to different settings below.

Table 5
Descriptives Arbitrageur weights.

	Min	25%	50%	75%	Max
			Canada		
Mean	0.000	0.693	0.846	0.963	0.965
Median	0.000	0.724	0.864	0.964	0.965
Max	0.152	0.903	0.948	0.965	0.965
Min	0.000	0.074	0.127	0.419	0.965
stdev	0.000	0.004	0.021	0.064	0.106
ac	−0.188	−0.070	0.002	0.071	0.542
	United States				
Mean	0.005	0.100	0.403	0.761	0.965
Median	0.000	0.093	0.393	0.793	0.965
Max	0.144	0.895	0.965	0.965	0.965
Min	0.000	0.000	0.000	0.027	0.607
stdev	0.004	0.063	0.101	0.144	0.288
ac	0.002	0.072	0.118	0.227	0.673
	Correlation				
cc	−0.395	−0.020	0.001	0.028	0.240

Notes: This table presents the descriptive statistics of the estimated arbitrage weights w_t^A for both the Canadian and US market. AC denotes auto-correlation; CC cross-correlation. The five columns on the right present the distribution of estimated coefficients over the 65 stocks.

Table 6
Varying lookback periods, L, and trend chasing Lags, J.

	Panel A: Varying lookback periods, L							
	Significance			Distribution				
	10%	5%	1%	Min	25%	50%	75%	max
$LLR(L=5)$	65	65	65	98.91	207.45	257.37	295.77	445.13
$LLR(L=10)$	65	65	65	33.32	173.16	236.56	273.97	409.26
$LLR(L=30)$	65	65	65	11.60	132.33	198.80	238.12	391.62
	Panel B: Varying trend chasing lags, J							
	Significance			Distribution				
	10%	5%	1%	Min	25%	50%	75%	Max
$LLR(J=5)$	65	65	65	630.4	3309.4	4788.0	5608.9	7097.7
$LLR(J=10)$	65	65	65	678.7	3314.0	5185.4	5964.2	7512.7
$LLR(J=30)$	65	65	65	874.2	3556.8	5223.8	6023.1	7839.5

Notes: This table presents the likelihood ratio statistics LLR for different values of the lookback period, L (Panel A), and different values of the lags incorporated by the trend chasers, J (Panel B). The 3 columns on the left show the number of stocks (out of 65) for which the LLR statistic is significant at the 10%, 5%, and 1% level, respectively. The five columns on the right present the distribution of LLRs over the 65 stocks.

but also within a percentile, we observe considerable variation. These variations show that the introduction of switching in the model can induce considerable time variation in the weights on the arbitrage and chartist strategies. In Canada, we observe little persistence in the weight on the arbitrage strategy, however in the US the autocorrelations in the weights on the arbitrage strategy is positive across the distribution of stocks, showing some persistence, most likely due to persistence in the profitability of the arbitrage strategy. When we consider the cross-correlation in the switching parameter, we note that, on average, these are zero, suggesting that there is virtually no contemporaneous relation between the weight on arbitrageurs in Canada and the US.

5.2.2. Robustness tests

To illustrate the robustness of our findings, we estimate the model using several alternative specifications. Specifically, in the benchmark model presented above the agents had a lookback period of 1 second in the switching function, $L=1$. Furthermore, the chartists only looked at the price change in the most recent second to determine their expectations, $J=1$. This is the simplest configuration of the model, and thereby also the most conservative one, which already turned out to yield significant results. In this robustness check, we vary both L and J and study the effect on the switching parameter and model fit. For both dimensions we look at lags of 5, 10, and 30 s.

Panel A of Table 6 presents the likelihood ratios, LLR, for different values of the lookback period, L. Overall, the likelihood ratio values are significant for all lags considered, but tend to decrease slightly as the lookback period increases, but remain highly significant in all cases. This suggests that agents overall use a relative short lookback period. Panel B of Table 6 presents the likelihood ratios, LLR, for different values of the number of lags in the trend chasing rule, J. Again, for all lags we observe that the likelihood ratio values are significant. In this case, the likelihood ratio values tend to increase

Table 7
Impulse response results.

From	To	Positive	Neutral	Negative
$\|CE\|$	$\|CE\|$	63	2	0
$\|CE\|$	GG^{Ca}	11	23	31
$\|CE\|$	$w^{A_{Ca}}$	51	13	1
$\|CE\|$	$w^{A_{US}}$	17	29	19
GG^{Ca}	$\|CE\|$	43	12	10
GG^{Ca}	GG^{Ca}	23	18	24
GG^{Ca}	$w^{A_{Ca}}$	4	18	43
GG^{Ca}	$w^{A_{US}}$	11	20	34
$w^{A_{Ca}}$	$\|CE\|$	54	7	4
$w^{A_{Ca}}$	GG^{Ca}	27	9	29
$w^{A_{Ca}}$	$w^{A_{Ca}}$	15	16	34
$w^{A_{Ca}}$	$w^{A_{US}}$	13	9	43
$w^{A_{US}}$	$\|CE\|$	10	14	41
$w^{A_{US}}$	GG^{Ca}	30	21	14
$w^{A_{US}}$	$w^{A_{Ca}}$	40	20	5
$w^{A_{US}}$	$w^{A_{US}}$	45	17	3

Notes: This table presents the relations obtained from the impulse-response analyses based on a VAR estimation of the variables. We consider shocks to the absolute price discrepancy between the Canadian and US market ($|CE|$); the component share for the Canadian market (GG^{Ca}); the weight on the arbitrage strategy in the Canadian market ($w^{A_{Ca}}$); and the weight on the arbitrage strategy in the US market ($w^{A_{US}}$). The columns give the number of stocks (out of 65) for which the permanent effect is positive and significant, neutral, or negative and significant.

as the number of lags increases. This implies that the model fit tends to increase with higher values of J and therefore that trend chasers take a several lags into account when forming expectations about future returns. Overall, the results show that in all cases the switching model significantly improves on the static model, lending strong support for the model we developed in Section 3.

5.2.3. Time-varying price discovery

So far we have shown that allowing for switching between arbitrage and chartist strategies in a model for the price dynamics of cross-listed assets can improve the fit of the model significantly and also introduces considerable time variation in weights, while using a very parsimonious specification. In this section, we explore the implications of this time variation for price discovery of the 65 cross-listed stocks. Specifically, we focus on the Gonzalo–Granger component share as defined in Eq. (9).

To demonstrate the time variation that can be generated by our model, we conduct an impulse response analysis. In Table 7, we report the results for the impulse response functions. Specifically, we report the results of shocks to the error correction term ($|CE|$), the Gonzalo–Granger component share for the Canadian market (GG^{Ca}), the weight on the arbitrageurs in Canada ($w^{A_{Ca}}$) and the US ($w^{A_{US}}$). In the table, the shocks are applied to the series in the first column and the responses are tracked on the variable in the second column. We report the number of stocks for which we observe positive, neutral or negative effects, based on whether the long-run cumulative impact is significantly positive, insignificant or significantly negative, respectively.

When we examine the impact of a shock in the error correction term, we observe that the responses in terms of the component shares are mostly negative (31 out of 65 stocks). Likewise, we observe that a shock in the error correction term results in an increase in the weight of arbitrageurs in the Canadian market for most stocks (51 out of 65), while the results for the weight of arbitrageurs in the US is mixed (for 17 stocks we observe a positive response, while for 19 stocks we observe a negative response). This suggests that if the gap between the prices in Canada and the US widens, it is more exploited by traders in Canada who switch towards an arbitrage strategy aimed at closing this gap. However, since the arbitrage is a reactive strategy to the prevailing price gap, the degree of price discovery in Canada decreases.

Shocks to the GG component share for the Canadian market generally lead to an increase in the error correction term (43 out of 65 stocks). We also observe that an increase in the Canadian component share leads to a decrease to both the weight on the arbitrage strategy in the Canadian and US market. This suggests that as the Canadian market becomes more informative, price discrepancies tend to be larger as the Canadian prices tend to move more independently of the US price. This likely causes the arbitrage strategy to be less profitable and causes traders in both markets to switch away from this strategy.

Shocks to arbitrage weights lead to an increase in the error correction term, while at the same time scare away arbitrageurs in the US market. The results on GG^{Ca} tend to be mixed. This seems to suggest that the decrease in the weight on the arbitrage strategy in the US, due to the increase in the weight on the arbitrage strategy in Canada has a positive effect on the price discrepancy between the two markets.

Table 8
Conditional component shares.

	Min	25%	50%	75%	max
Mean	0.035	0.037	0.051	0.163	1.000
Median	0.035	0.036	0.048	0.161	1.000
Max	0.038	0.077	0.278	0.527	1.000
Min	0.035	0.035	0.035	0.040	0.826
stdev	0.000	0.001	0.005	0.016	0.080
ac	−0.086	0.013	0.048	0.087	0.361

Notes: This table presents the descriptive statistics of the Canadian conditional component shares calculated over the full sample period using the estimation results of the switching model. AC denotes auto-correlation. The five columns on the right present the distribution of estimated coefficients over the 65 stocks.

For shocks in the weights of arbitrageurs in the US, we observe the reverse, i.e. a decrease in the price discrepancy between Canada and the US, and an increase in the weight of arbitrageurs in the Canadian market. Hence, an increase in arbitrageurs in the US seems to trigger more arbitrageurs in Canada and this causes the price discrepancy to decrease. Overall, the results of shocks to weights in arbitrageurs in Canada and the US suggest that the arbitrageurs in the US are more informative as they are followed by arbitrageurs in Canada, while arbitrageurs in the US do not follow the arbitrageurs in Canada.

The impulse-response analysis reported above was based on a numerical exercise. To provide a more realistic picture of the degree of variation that is actually introduced by our model, we report summary statistics for the conditional GG measures in Table 8. As before, we show several descriptive statistics for various percentile values to show variation across stocks and over time. As can be seen, the average GG measures are around 5%, with some variation across stocks. However, the informational dominance of the US is also very clear in these conditional measures. When we consider the minimum and maximum values within a specific percentile, we observe a considerable range, highlighting that considerable time variation can be generated by our model. We also find some evidence of persistence in price discovery, as autocorrelations are mostly positive.

6. Conclusions

This paper is, to our best knowledge, the first to bring an empirical heterogeneous agent model into the market microstructure domain. The objective is to use the heterogeneity approach to capture time-varying price discovery. This is of importance as it illustrates that heterogeneous agent models not only generate the stylized facts of financial markets, but also impact the information processing ability of markets.

We build an empirical heterogeneous agent model with two groups of traders, arbitrageurs and chartists, who are allowed to switch between groups. In an application to Canadian-US cross listed stocks, we find strong evidence of both heterogeneity and switching. The model generates ample variation in price discovery, and can generate interesting intraday patterns.

Appendix

List of companies	
	Kingsway Financial Services Inc.
Advantage Oil & Gas Ltd.	Kinross Gold Corporation
Barrick Gold Corporation	Manulife Financial Corporation
Agnico-Eagle Mines Limited	Magna International Inc.
Agrium Inc.	Nordion Inc.
Atlantic Power Corp.	North American Energy Partners Inc.
Yamana Gold Inc.	Nexen Inc.
BCE Inc.	Precision Drilling Corporation
IESI Ltd.	Pengrowth Energy Corp
Bank of Montreal	Potash Corporation of Saskatchewan Inc.
Bank of Nova Scotia	Provident Energy Ltd
Brookfield Properties Corporation	Penn West Petroleum Ltd.
Baytex Energy Corp	Ritchie Bros. Auctioneers Incorporated
CAE Inc.	Rogers Communications Inc.
Cameco Corporation	Royal Bank of Canada

(*continued on next page*)

List of companies	
Celestica Inc.	Shaw Communications Inc.
Canadian Imperial Bank of Commerce	Sun Life Financial Inc.
Canadian Natural Resources Limited	Silver Wheaton Corp.
Cott Corporation	Seaspan Corporation
Canadian Pacific Railway Limited	Stantec Inc.
Cenovus Energy	Suncor Energy Inc.
EnCana Corporation	TransAlta
Eldorado Gold Corp.	Thompson Creek Metals Company Inc.
Enbridge Inc.	Teck Resources Limited
Equal Energy Ltd.	Toronto-Dominion Bank
Enerplu Corp	Tim Hortons Inc.
Goldcorp Inc.	Talisman Energy Inc.
CGI Group Inc.	Thomson Reuters Corporation
Gildan Activewear Inc.	TransCanada Corporation
Gammon Gold, Inc.	TELUS Corporation
Harry Winston Diamond Corporation	Domtar Corporation H
IAMGOLD Corporation	US Gold Corp.
Ivanhoe Mines Ltd.	Valeants Pharmaceuticals Inc

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