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Computing alternating offers and water prices in bilateral river basin management

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Abstract This contribution deals with the fundamental critique in Dinar et al. (1992, *Theory and Decision* **32**) on the use of Game theory in water management: People are reluctant to monetary transfers unrelated to water prices and game theoretic solutions impose a computational burden. For the bilateral alternating-offers model, a single optimization program significantly reduces the computational burden. Furthermore, water prices and *property rights* result from exploiting the Second Welfare Theorem. Both issues are discussed and applied to a bilateral version of the theoretical river basin model in Ambec and Sprumont (2002). Directions for future research are provided.

JEL (or AMS) references: C72, C78, D50, D58

Key Words: International River Management; Negotiation Theory; Game Theory; Computations; Non-transferable utility; Property rights; Walrasian equilibrium prices; Applied General Equilibrium model.

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1 Introduction

The international community has come to recognize that fresh water is scarce, witness the declarations at the Dublin Conference in 1992, the UN conference Johannesburg 2002 and the past tri-annual World Water Forums starting in 1997 and its fourth meeting March 2006. It is generally felt that the problem is not so much physical scarcity, as inefficient use and resource management and vested interests, in particular in case of the World's many international rivers. In some regions, flooding and pollution pose serious threats, whereas in water stressed regions, lack of agreement on how to share river waters and underground aquifers are a serious source of potentially violent conflict.[2]

International water law, i.e. the Helsinki Rules of 1966 and the UN Convention on the Law of the Non-Navigational Uses of International Watercourses of 1997, does not recognize claims by upstream countries of owing the water caught on its territory (absolute territorial sovereignty), confiscating headwaters by geopolitics or downstream nation's claims of 'historical rights' (unlimited territorial integrity), see e.g., Ambec and Sprumont (2002). Rather, international law states that the nations involved should mutually agree on sharing the river through negotiations, but it is left in the middle to what extend unilateral decisions can be made in the absence of agreement. Such negotiations are often deadlocked, because almost all governments in water stressed regions became aware of the water issues after having experienced serious shortages of water and a simple reshuffling of water is perceived as a 'zero sum' game where giving up water is regarded as unacceptable. Unless politics either deepen or broaden the water agenda, the situation is most likely to stay put or might even deteriorate ending in conflict.

Coalition formation, the division of gains within coalitions and unilateral decisions prior to the negotiations, threats traditionally belong to the realm of game theory, which is also recognized by global institutions involved in river management such as the World Bank, e.g.

[2] See e.g. UNESCO's initiative "$PC \rightarrow CP$ From Potential Conflict to Cooperative Potential", http://webworld.unesco.org/water/wwap/pccp/cd/pccp_publications.html

Carraro et al. (2005a,b). These references also contain an extensive overview of the many documented researches in economics and game theory addressing the water issue. However, these surveys also recognize that there are only three applications of formal negotiation theory in which negotiation procedures are explicitly modelled: Rausser and Simon (1992), Thoyer et al. (2001) and Simon et al. (2001). In these three references, a finite horizon is taken as a proxy for the fixed-point problem characterizing the unique subgame perfect equilibrium of their infinite-horizon bargaining game, because numerically solving fixed-point problems is computationally difficult.

Although it is eminent that game theory offers a methodology to address water issues, the game theoretic profession did not seem to respond to the critique in Dinar et al. (1992): stakeholders and policy makers are reluctant to game-theoretic transfers that are not related to water prices and, second, game theoretic solutions impose a huge computational burden upon the applied modeler. The computational burden in water issues arises because the physical economic problem has to be transformed into the so-called "utility-space", represented by the characteristic function form, before any of the game theoretic concepts can be applied and, then, requires a translation back into the original physical formulation. Also the computation of game theoretic concepts in utility space, especially fixed-point problems, adds to the complexity. This criticism still stands today.[3]

The first critique goes beyond the lack of water prices. Since most existing international treaties, such as the Jordan-Israeli Peace Accords of 1994, are formulated in terms of minimal transboundary flows, water quality and financial transfers, this hints at that the framing of negotiation theory should be preferably close to physical variables and notions understood by negotiation parties. Roemer (1988) was among the first to demonstrate how our understanding of two axiomatic bargaining solutions, including the one proposed in Nash (1950), benefits from taking physical reality as the primitive.

Recent theoretical work by Houba (2005a,b) for the alternating-offers model in Rubin-

[3] Personal communication with professor Dinar during Game Theory Practice 2006.

stein (1982) provides a promising way to effectively deal with the critique in Dinar et al. (1992). The bilateral case seems restrictive, but extends to multilateral negotiations requiring unanimity among all parties. The innovations are twofold.

First, the fixed-point problem characterizing the equilibrium proposals in the alternating-offers model is formulated directly in terms of physical variables and can be solved by computing the optimum of a single maximization problem for which excellent software is available.[4] Of course, every fixed point problem $f(x) = x$ can be reformulated as minimization of $(f(x) - x)^2$ and this square is minimized at every fixed point x of the function f. However, even for relatively small problems such procedure is known to be numerically cumbersome and might not produce any solution at all. The innovation in Houba (2005a,b) is a different reformulation that does allow for a robust numerical implementation. Furthermore, this method is computationally superior to methods relying on truncating the horizon. The objective of the single program is the same asymmetric Nash product as first reported in Binmore et al. (1986) for instantaneously fast negotiations and this insight therefore extends to time-consuming or sluggish negotiations. The bargaining weights provide a theoretical measure for bargaining power in sluggish negotiations.

Second, this single program generates the player-dependent Pareto-efficient proposals. Therefore, the Second Welfare Theorem applies: Every Pareto efficient allocation can be regarded as a Walrasian equilibrium with Walrasian market prices and suitable financial transfers. This provides a sound underpinning of the game theoretic solution in terms of water prices. Moreover, for every equilibrium proposals, these Walrasian prices coincide with the shadow prices in the optimal solution of the single program and these prices are automatically generated by the optimization software. The suitable financial transfers are equal to the difference in monetary value of the disagreement situation and the situation arising from agreement, both evaluated against the market prices. These transfers can be interpreted as transfers of *property rights*. This interpretation is well understood in

[4] For example, GAMS is popular in applied economics, see www.gams.com.

General Equilibrium models, but novel to game theory. Applied General Equilibrium (AGE) modelling is popular in applied economics and formulated in physical variables that are close to the policy makers' concerns and understanding. The AGE framework is flexible to accommodate sectors or regions in and across economies as well as extensions involving uncertainty and dynamics, see e.g., Ginsburgh and Keyzer (2002). Therefore, this framework is of relevance in modelling water related problems, as will be demonstrated for the bilateral version of the river basin management model proposed in Ambec and Sprumont (2002). Finally, this reinterpretation of Pareto efficiency assumes non-transferable utility in dealing with the negotiation problem instead of the more restrictive transferable utility

This paper discusses the relevance of the results obtained for exchange economies in Houba (2005a,b) in dealing with the fundamental critique in Dinar et al. (1992). First, the results for exchange economies are surveyed in Section 2. Then, production is added in the subsequent section. The Second Welfare Theorem, Walrasian equilibrium prices and transfers of property rights are discussed in Section 4. The bilateral version of the river basin model in Ambec and Sprumont (2002) in Section 5 serves as an illustration of the type of insights available for river basin management. Directions for future research are delegated to the final section.

2 Alternating Offers and the Single Program

The well-known alternating-offers model in Rubinstein (1982) is formulated in terms of the division of a single dollar or single issue. It is a standard result that, under certain assumptions, this model admits a unique SPE in stationary strategies in which the responding player is kept indifferent between accepting the equilibrium proposal and the equilibrium continuation after rejection. Furthermore, the assumption of a single issue can be easily replaced by simultaneous negotiations on multiple issues, for example consumption bundles in an exchange economy.

The alternating-offers model also allows for an interpretation of negotiations over an infinite stream of single dollars with discounting under stationary contracts, as pioneered by Fernandez and Glazer (1991) and Haller and Holden (1990) for wage bargaining. Recently, Houba and Wen (2006) point out that the Pareto frontier under an infinite stream of dollars and heterogeneous time preferences is supported by nonstationary contracts in which the impatient player obtains zero in the long run. Such contracts seem too unrealistic and the economic modeler has to impose restrictions upon the feasible divisions of such streams. Stationary contracts represent just one of many choices and such contracts impose a constant division over time that are by default Pareto inefficient.

For river basin management, the interpretation in terms of an everlasting stream of surpluses is appropriate, because rivers typically are renewable resources that are exploited by its users over time. Reservoirs such as lakes, dams, cisterns or aquifers are of relevance in river basin management and these require the introduction of stock variables that link subsequent economies. However, we postpone such variables until Remark 2 in Section 5. Even with stock variables, stationary contracts seem appropriate because many international agreements specify minimal annual river flows or cost sharing of annual operation and maintenance costs of operating installed infrastructure. Nonstationary contracts are dealt with in Remark 1 in Section 3.

To establish minimal notation, we consider an extension of the alternating-offers model in Rubinstein (1982) in which each of two agents discounts his per-period utility from a sequence of consumption bundles in an infinite stream of exchange economies (without stock variables). The exchange economy consists of two agents, called countries, are indexed $i = 1, 2$. The economy has $n \geq 2$ commodities, monotonic and concave utility functions $u_i : \mathbb{R}^n_+ \rightarrow \mathbb{R}$, $i = 1, 2$, a vector of initial endowments $\omega^i \in \mathbb{R}^n_+$ for country i and total endowments $\omega = \omega^1 + \omega^2 > 0$.[5] A feasible allocation is denoted as $z = (z^1, z^2)$, $z^1, z^2 \in \mathbb{R}^n_+$, such that $z^1 + z^2 \leq \omega$. We assume that $z = (\omega^1, \omega^2)$ is Pareto inefficient meaning that

[5] We could allow for $\omega^1 + \omega^2 \leq \omega$ that would describe cases where cumulative property rights over several underdeveloped resources are less than is physically feasible. For example, the Israeli-Jordan Peace Treaty of 1994 further develops the excess seasonal flows of the Yarmuck River.

the bargaining problem below is essential. Exchange economies form a special class of AGE models, see e.g., Ginsburgh and Keyzer (2002).

We regard exchange economies as multi-issue negotiation problems. Initial endowments or property rights are typically ill-defined in river basin management. In Section 5, we address the origin of these initial endowments in such situations and, until then, we assume these are given. The feasibility constraint is better known as the *aggregate commodity balance* and, whenever embedded in the single program, its shadow prices will play an important role in the application of the Second Welfare Theorem discussed in Section 4. In river basin management, it includes the so-called *water balances* that are determined by the hydrological experts, see e.g., Albersen et al. (2003).

Time is discrete and indexed by $t \in \mathbb{N}$. The feasible allocation in period t is denoted as $z^t = (z^{1,t}, z^{2,t})$. The subject of the negotiations is a feasible allocation $z = (z^1, z^2)$ that should be understood as an everlasting, binding and stationary contract, i.e., $\{(z^{1,\tau}, z^{2,t})\}_{\tau=0}^{\infty}$ with $z^{i,\tau} = z^i$ for both i and period $\tau = 0$ being the first period that the contract is implemented. In every period $t \in \mathbb{N}$ prior to agreement, each country consumes $z^{i,t} = \omega^i$. This means that country i's disagreement utility is given by $d_i = u_i(\omega^i)$, $i = 1, 2$. Country i's utility from $T \geq 0$ periods of disagreement followed by agreement on $z = (z^1, z^2)$ is given by

$$\left(1 - \delta_i^T\right) d_i + \delta_i^T u_i\left(z^i\right),$$

where $\delta_i \in (0, 1)$ is country i's discount factor. Furthermore, each constraint is binding.

At t odd, country 1 proposes the feasible allocation and, then, country 2 accepts or rejects. Accept ends the negotiations. If rejected, then each country i consumes ω^i before the negotiations move to the next (even) round. At t even, the countries' roles are reversed. The equilibrium concept is subgame perfectness (SPE).

It is a well-known result that the alternating offers model admits a unique SPE in stationary strategies (SSPE), see for a survey e.g., Muthoo (1999) and Houba and Bolt (2002). Stationary strategies prescribe country-dependent feasible allocations denoted as $x = (x^1, x^2)$, respectively, $y = (y^1, y^2)$ for country 1 and 2. In such SSPE, accept y is a best response for country 1 if and only if $u_1(y^1) \geq (1-\delta_1)\, d_1 + \delta_1 u_1(x^1)$. Similarly, accept x is a best response for country 2 if and only if $u_2(x^2) \geq (1-\delta_2)\, d_2 + \delta_2 u_2(y^2)$. Taking these equilibrium conditions and the feasibility constraints into account, we have that any pair of SSPE allocations (x, y) simultaneously solves the following pair of *convex* programs as a fixed point:

$$
\begin{aligned}
x &= \arg\max_z u_1\left(z^1\right), \qquad (1)\\
&\quad \text{s.t.} \quad z^1 + z^2 \leq \omega, \quad u_2\left(z^2\right) \geq (1-\delta_2)\, d_2 + \delta_2 u_2\left(y^2\right),\\
y &= \arg\max_z u_2\left(z^2\right), \qquad (2)\\
&\quad \text{s.t.} \quad z^1 + z^2 \leq \omega, \quad u_1\left(z^1\right) \geq (1-\delta_1)\, d_1 + \delta_1 u_1\left(x^1\right),
\end{aligned}
$$

where y^2, respectively, x^1 are exogenous in (1) and (2). Both x and y are Pareto efficient.

Of greater significance is the equivalence between any fixed point (x, y) of (1)-(2) and the solution to a single convex program, as first established in Houba (2005b). The equivalence is based upon the observation that any pair of SSPE allocations, the proposed allocations x and y have the same asymmetric Nash product associated with the bargaining weight $\alpha = \ln \delta_2 / (\ln \delta_1 + \ln \delta_2)^{-1}$ for country 1. To see this, note that

$$\left(u_1\left(x^1\right) - d_1\right)^{\alpha} \left(u_2\left(x^2\right) - d_2\right)^{1-\alpha} = \delta_1^{\alpha} \left(u_1\left(x^1\right) - d_1\right)^{\alpha} \left(u_2\left(y^2\right) - d_2\right)^{1-\alpha},$$

and, because $\delta_1^{\ln \delta_2} = e^{(\ln \delta_1) \cdot \ln \delta_2} = \delta_2^{\ln \delta_1}$,

$$\left(u_1\left(y^1\right) - d_1\right)^{\alpha} \left(u_2\left(y^2\right) - d_2\right)^{1-\alpha} = \delta_1^{\alpha} \left(u_1\left(x^1\right) - d_1\right)^{\alpha} \left(u_2\left(y^2\right) - d_2\right)^{1-\alpha}.$$

This asymmetric Nash product is the objective function in the single program. The constraints in the single program are obtained by combining the constraints in (1) and (2). However, a minor modification is needed, because the endogeniety of both y^2 and x^1 in each second inequality constraint in (1) and (2) would violate the convexity of the program. The convexity can be restored by introducing the additional variables s_i, $i = 1, 2$, replacing the utility functions $u_1(x^1)$ and $u_2(y^2)$ in these constraints and the Nash product at the costs of adding the additional constraints $s_2 \leq u_2(y^2)$ and $s_1 \leq u_1(x^1)$. Then, the single convex program is given by

$$\max_{s \geq d; x,y} (s_1 - d_1)^{\alpha} (s_2 - d_2)^{1-\alpha}, \tag{3}$$

$$\text{s.t.} \quad \begin{array}{rll} s_1 & \leq u_1(x^1) & \\ s_2 & \leq u_2(y^2) & \\ (1-\delta_1) d_1 + \delta_1 s_1 & \leq u_1(y^1), & \\ (1-\delta_2) d_2 + \delta_2 s_2 & \leq u_2(x^2), & \\ x^1 + x^2 & \leq \omega, & (p^x) \\ y^1 + y^2 & \leq \omega, & (p^y) \end{array}$$

where p^x and p^y denote vectors of shadow prices. The following result states the equivalence of the fixed point (1) and (2) with program (3), which is the main result in Houba (2005b).

Proposition 1 *(s_1^*, s_2^*, x^*, y^*) is a solution of (3) if and only if $(x, y) = (x^*, y^*)$ is a pair of SSPE allocations to (1)-(2).*

Remark 1 *Reinterpretation of the n commodities in (3) can represent nonstationary contracts. Suppose the k-th component represents consumption of, say, water or money at period k. Then, the contract distinguishes consumption of a single commodity in n different periods. A normalized and constant stream of this single commodity would correspond to $\omega_k = 1$. Discounted utility over these n periods can be captured by introducing some per-period utility function $\hat{u}_i : \mathbb{R}_+ \to \mathbb{R}$ and redefining the utility function $u_i(z^i)$ as $\sum_{k=1}^{n} \delta_i^k \hat{u}_i(z_k^i)$. This would also allow for restrictions on the contract space through additional constraints. For example, stationarity imposes $z_k^i = z_1^i$ for all k, or limiting the growth of country i's consumption over time to a maximum of $\lambda * 100$ percent imposes $z_{k+1}^i \leq (1+\lambda) z_k^i$ for all k. Many of such restrictions preserve the convexity of the program.*

In the optimum, all constraints are binding. Therefore, $s_1^* = u_1(x^{*1})$ and $s_2^* = u_2(y^{*2})$ imply that the additional variables represent the SSPE utility levels for country i in the role of the proposer. Since the program is convex, the Maximum Theorem implies that the shadow prices p^x and p^y are nonnegative. Program (3) lends itself for implementation in many of the optimization packages available today, such as e.g., GAMS. Since this program is convex, these packages offer robust computational algorithms designed to efficiently compute an accurate numerical approximation of the unique optimum. This almost exact numerical solution is superior to approximation of the fixed point of (1)-(2) through a T-period finite horizon truncation that involves solving a sequence of T single programs of n variables and n linear constraints being either (1) or (2). Rausser and Simon (1992), Thoyer et al. (2001) and Simon et al. (2001) assume a random proposer at the final bargaining round that eliminates the deadline effect and speeds up the convergence.

The single program states the formula for the Nash bargaining solution for *all* parameter values δ_1 and δ_2 in a modified exchange economy of "double" size. It also generalizes the well-known result in Binmore et al. (1986) for instantaneous negotiations to time-consuming sluggish negotiations. Instantaneous negotiations correspond to taking the limit of vanishing time between bargaining rounds. Formally, let $\Delta > 0$ denote the time between any two subsequent bargaining rounds and consider discount factors equal to δ_1^{Δ} and δ_2^{Δ}. Vanishing time means taking the limit Δ goes to 0, i.e., $\lim_{\Delta \to 0} \delta_1^{\Delta} = \lim_{\Delta \to 0} \delta_2^{\Delta} = 1$. Then, instantaneous negotiations correspond to the asymmetric Nash bargaining solution and feature $x = y$. Therefore, these can be implemented by less variables and constraints and solved as

$$\max_{z} \left(u_1\left(z^1\right) - d_1\right)^{\alpha} \left(u_2\left(z^2\right) - d_2\right)^{1-\alpha}, \quad \text{s.t.} \quad z^1 + z^2 \leq \omega. \tag{4}$$

So, the additional computational costs, in terms of additional variables and constraints, of solving (3) for sluggish negotiations instead of (4) for instantaneous negotiations amounts to $n + 2$ variables and $n + 4$ constraints, of which n are linear.

The bargaining problem in utility representation

Houba (2005a) establishes similar results for convex bargaining problems in the utility representation. This has relevant theoretical value, because every bilateral negotiation problem that can be transformed into such convex bargaining problem can also be solved with a single convex program. Furthermore, it also provides valuable insights for non-convex bargaining problems.

Formally, a bargaining problem in utility representation is denoted as the pair (S, d) with $S \in \mathbb{R}^2$ the nonempty, compact set of feasible utility pairs and $d \in S$ the disagreement point. The curve $s_i = f_i(s_j)$, $i, j = 1, 2$, $i \neq j$, describes the Pareto frontier of S.

Any pair (s_1^*, s_2^*) of SSPE utilities simultaneously solves the following pair of programs as a fixed point:

$$
\begin{aligned}
s_1^* &= \arg\max_{s \geq d} s_1, \quad \text{s.t.} \quad s_1 \leq f_1(s_2), \quad s_2 \geq (1 - \delta_2) d_2 + \delta_2 s_2^*, && (5) \\
s_2^* &= \arg\max_{s \geq d} s_2, \quad \text{s.t.} \quad s_2 \leq f_2(s_1), \quad s_1 \geq (1 - \delta_1) d_1 + \delta_1 s_1^*. && (6)
\end{aligned}
$$

Each program implies that the proposing country maximizes his own utility among the set of feasible and acceptable utility pairs. In each optimum, both constraints are binding. Houba (2005a) establishes the following result for convex bargaining problems (S, d):

Proposition 2 *Let S be a convex set. Then, (s_1^*, s_2^*) is the unique pair of SSPE utilities of (5-(6) if and only if*

$$
\begin{aligned}
(s_1^*, s_2^*) &= \arg\max_{s \geq d} (s_1 - d_1)^{\alpha} (s_2 - d_2)^{1-\alpha}, && (7) \\
s.t. \quad s_1 &\leq f_1((1 - \delta_2) d_2 + \delta_2 s_2), \\
s_2 &\leq f_2((1 - \delta_1) d_1 + \delta_1 s_1).
\end{aligned}
$$

This proposition partly extends to the class of bargaining problems in utility representation that are strongly comprehensive or 'non-convex' as in Herrero (1989). Then, Program (7) always yields a pair of SSPE allocations, because in the optimum both constraints are binding. However, the reverse may not hold as Herrero (1989) shows: Uniqueness of the pair of SSPE utilities (s_1^*, s_2^*) may break down and multiple non-stationary SPE strategies may exist as well. The Nash product associated to different pairs of SSPE utility pairs (s_1^*, s_2^*) are also different and may be less than the maximal attainable Nash product. For instantaneous negotiations, the (limit) pair of SSPE utilities (s_1^*, s_2^*) in program (7) coincides with the maximal Nash product as axiomatized in Kaneko (1980). The limit set of all SSPE utilities of (5)-(6) is axiomatized in Herrero (1989). What is needed for uniqueness in (non-stationary) SPE strategies is the stronger uniqueness in fixed points of (5)-(6).

3 Production

The results for exchange economies might seem of limited interest for applied economics. The aim of this and later sections is to show the merits in applications. In this section, we address how to incorporate production.

Extending the exchange economy to allow for production activities is conceptually straightforward, see e.g. Varian (1984). Production plans require inputs from the economy in order to produce outputs. These are represented in a single vector $q \in \mathbb{R}^n$ with positive and negative elements, where positive (negative) elements represent outputs (inputs). Production technologies are often represented by the so-called production set $Q \subset \mathbb{R}^n$ that represents all technologically feasible input-output combinations. Often, production sets are represented by transformation functions. In our case, transformation functions more naturally fit the optimization framework. The function $F : \mathbb{R}^n \to \mathbb{R}$ is a transformation function representing Q if $q \in Q$ if and only if $F(q) \leq 0$. Efficient production corresponds to the $=$-sign. The possibility of inaction and no free lunch translate into $F(0) = 0$. The technology is convex if the function F is quasi-convex. Otherwise, the technology is called nonconvex.

Since we later discuss bilateral river basin management in which the economy of each riparian country involves water related production, we assume that water related production is carried out by many small producers that are mainly active in one country. For explanatory reasons, we aggregate all producers in one country by assuming one production set for each country.[6] So, each country exclusively controls some production technology and country i's production plan is a vector $q^i \in Q^i$. The subject of the negotiations becomes a feasible allocation $z = (z^1, z^2, q^1, q^2)$ meaning that

$$z^1, z^2 \in \mathbb{R}^n_+, \quad q^1 \in Q^1,\ q^2 \in Q^2 \quad \text{and} \quad z^1 + z^2 \leq \omega + q^1 + q^2,$$

where negative components of either q^1 or q^2 lower the amount of that particular good available for consumption. The aggregate commodity balance implies that the demand for

[6]If not, we would have the index set J_i of producers in country i and $q^j \in Q^j$ for every $j \in J_i$. In the text, we assume $J_i = \{i\}$.

each good is at most equal to its supply. In that respect, it would be natural to also include the demand for inputs on the left-hand side, but it is standard to have one vector per producer representing the possibly negative net output on the right-hand side of this balance. As mentioned, we describe country i's production set Q^i by the transformation function F_i meaning $q^i \in Q^i$ if and only if $F_i\left(q^i\right) \leq 0$.

In case of convex production technologies, we immediately have that the bargaining problem in utility representation is also convex, see e.g., Roemer (1988), and, hence, the equivalence stated in Proposition 2 immediately applies. Also in terms of the economic environment, the equivalence between the fixed point problem and program (3) remains in tact, where the single program remains convex. Including (convex) production per country requires the following modifications to the alternating-offers model, where we add an additional superscript x and y to the production plans to distinguish between country 1's and 2's proposal. The modifications to program (1) imply rewriting the commodity balance and adding both transformation functions such that the modified program includes the following constraints:

$$x^1 + x^2 \leq \omega + q^{x,1} + q^{x,2}, \quad F_1\left(q^{x,1}\right) \leq 0 \quad \text{and} \quad F_2\left(q^{x,2}\right) \leq 0.$$

Similar, the modified program (2) includes the following constraints:

$$y^1 + y^2 \leq \omega + q^{y,1} + q^{y,2}, \quad F_1\left(q^{y,1}\right) \leq 0 \quad \text{and} \quad F_2\left(q^{y,2}\right) \leq 0.$$

Of course, these modifications must also be made to obtain the modified program (3), which we omit.

Nonconvex production technologies can be implemented in the same manner. However, such technologies cause a breakdown of the convexity of the modified program (3), because such technologies are known to give rise to nonconvex bargaining problem in utility representation. As argued for the utility representation in Section 2, the theoretical results only partly extend. Its counterpart for program (3) with nonconvex production reads: The maximum of the modified single program (3) corresponds to one of possibly multiple SSPE strategies. The SSPE specified by this program is special in that it has the largest Nash product.

The modified program (3) with production can also be implemented in optimization software by modifying all commodity balances and adding twice (i.e., once for each proposal) all variables and constraints concerning production. Under convex production technologies, the software returns the unique optimum. However, for nonconvex programs it is fundamentally unclear whether a local or global optimum is found, even though most packages offer robust algorithms. Nevertheless, although this is a fundamental problem of any numeric optimization, it will be clear that the numerical solution returned, whether it is the global or a local optimum, has properties that are consistent with SSPE behavior.

4 Market Prices and Property Rights

This paper is motivated by the fundamental critique in Dinar et al. (1992), who report on the difficulties arising from applying cooperative game theory to several small-scale water issues. They state: "Clearly, the potential for additional income due to cooperation is higher when side payments are possible. However, the soundness of such transfers with no a prior reference to the price per unit of water may be questioned, especially considering the general resentment of farmers to adopt side payments as a policy." Since side payments or transfers are advocated by (cooperative) Game Theory as the universal remedy towards cooperation, the game theoretic society should treat this critique very seriously.

In this section, we discuss the merits of the Second Welfare Theorem in General Equilibrium modelling in dealing with this fundamental issue. Since (cooperative) game theory developed autonomously from microeconomics, it does not refer to nor does it exploit the implications of the Second Welfare Theorem. For that reason, we discuss these implications in detail before turning our attention to bilateral river basin management in the next section.

The Second Welfare Theorem for economies with production states: Any Pareto efficient allocation is attainable as a price quasi-equilibrium or Walrasian equilibrium with transfers, see e.g., Varian (1984), Mas-Colell et al. (1995) and Ginsburgh and Keyzer (2002). Mas-Colell et al. (1995) show that this theorem holds under convex and locally nonsatiated preferences and convex production technologies. The transfers can be achieved through many appropriate physical reallocations of the initial endowments or through financial lump-sum transfers evaluated against the Walrasian equilibrium prices. In terms of a Walrasian economy, these transfers take place before price-taking behavior by all agents and such behavior ensures that the law of supply and demand will lead to the Walrasian equilibrium prices supporting the Pareto efficient allocation under consideration. The Walrasian prices can also be obtained as the shadow prices of a welfare program. The single program (3) can be seen as a welfare program associated with the Nash social welfare function as axiomatized in Kaneko (1982). The implication to water management is clear: Pareto efficient allocations can be reinterpreted in terms of lump-sum financial transfers and supporting water prices.

In further discussing these issues, we consider country 1's SSPE proposal $(x^{\star 1}, x^{\star 2}, q^{\star x,1}, q^{\star x,2})$ and assume that it is part of the optimum of the modified Program (3) with the vector of shadow prices $p^{\star x}$. The allocation $(x^{\star 1}, x^{\star 2}, q^{\star x,1}, q^{\star x,2})$ is feasible and, being a SSPE proposal, is Pareto efficient, see Houba (2005b). The shadow prices $p^{\star x}$ can be regarded as the Walrasian equilibrium prices and these prices clear all the markets: aggregate demand $x^{\star 1} + x^{\star 2}$ equals aggregate (net) supply $\omega + q^{\star x,1} + q^{\star x,2}$. Valued against $p^{\star x}$, country i's allocated consumption $x^{\star i}$ is worth $p^{\star x} \cdot x^{\star i}$ and can be regarded as country i's expenditure on all goods. This expenditure is financed from this country's market income obtained from selling against $p^{\star x}$ its endowments ω^i and producing $q^i \in Q^i$. This income is worth $p^{\star x} \cdot \omega^i + p^{\star x} \cdot q^{\star i}$.

In general, a country's allocated (or allowed) expenditure and its market income will not be equal and this means that either a country is allowed to expend more than it earns, or less. This difference can be interpreted as country i's implicitly received lump-sum subsidy, or tax levied on this country. Formally, in country 1's SSPE proposal, country i receives the net lump-sum transfer $T_i^* = p^{\star x} \cdot x^{\star i} - p^{\star x} \cdot \omega^i - p^{\star x} \cdot q^{\star i}$, which is a subsidy if positive and a tax if negative. Pre-tax market income is equal to $p^{\star x} \cdot \omega^i + p^{\star x} \cdot q^{\star i}$ and after-tax market income is $m^{\star i} = p^{\star x} \cdot \omega^i + p^{\star x} \cdot q^{\star i} - T_i = p^{\star x} \cdot x^{\star i}$. Note that in the optimum of the modified program (3) there is a balanced budget for the fictitious tax authority. This follows directly from the aggregate commodity balance that appears as $p^x \cdot (x^1 + x^2 - q^{x,1} - q^{x,2} - \omega)$ in the Lagrangian of the optimization problem and this term is equal to 0 in the optimum.

In the Walrasian equilibrium all trade is voluntary and the markets respect property rights in the sense that, valued against the Walrasian equilibrium prices, each consumer's expenditure is equal to his market income. Formally, $T_i^\star = 0$ in a Walrasian equilibrium. In any SSPE agreement, each country's expenditure and post-tax income satisfies the same property, but from moving from pre-tax to post-tax market income a change in property rights occurs valued $T_i^\star$ that is most likely different from zero. In the context of negotiations, the countries are rational and any agreement is reached on a voluntary basis. So, any such voluntary agreed upon contract implies agreement upon a redistribution of property rights.

As mentioned, the Pareto efficient allocation $(x^{\star 1}, x^{\star 2}, q^{\star x,1}, q^{\star x,2})$ can be thought of as arising from a Walrasian economy in which all parties act as price takers. Country i behaving as a price-taking consumer facing market prices $p^{\star x}$ and having after-tax income $m^{\star i} = p^{\star x} \cdot x^{\star i}$ solves

$$x^{\star i} = \arg\max_{x^i \geq 0} u_i\left(x^i\right), \quad \text{s.t.} \quad p^{\star x} \cdot x^i \leq m^{\star i},$$

where we take uniqueness of the maximizer for granted. So, country i acting as a price-taking consumer voluntarily purchases $x^{\star i}$ such that $\nabla u_i\left(x^{\star i}\right) = p^{\star x}$, where ∇u_i denotes the gradient of u_i in case of differentiability. Monotonicity of the utility function guarantees $p^{\star x} \cdot x^i = m^{\star i}$. This latter condition should also be fulfilled in the first-order conditions of the modified program (3). As mentioned, for convex programs the shadow prices p^x (and p^y) are nonnegative. This result generalizes to economies with non-convex production, because then the monotonicity of the utility functions guarantees the non-negativity of $p^{\star x}$ and $p^{\star y}$ through $p^{\star x} = \nabla u_i\left(x^{\star i}\right) \geq 0$ and $p^{\star y} = \nabla u_i\left(y^{\star i}\right) \geq 0$.

Similar to the Robinson Crusoe economy, country i is also producer i. Country i behaving as a price-taking producer facing market prices $p^{\star x}$ solves

$$q^{\star x,i} = \arg\max_{q^{x,i}} p^{\star x} \cdot q^{x,i}, \text{s.t.} \quad F_i\left(q^{x,i}\right) \leq 0,$$

where we once more take uniqueness for granted.[7] Similar as before, country i as a price-taking producer voluntarily chooses the production plan $q^{\star i}$ such that $\nabla F_i\left(q^{x,i}\right) = p^{\star x}$, which should also be fulfilled by the first-order conditions of the modified program (3). Under convex production, firm i always makes a nonnegative profit that accrues to consumer i's pre-tax market income. However, nonnegative profits are not automatically ensured under non-convexities. Then, we need to modify the profit maximization problem taking into account a lump-sum producer's subsidy $S^{\star i} = -p^{\star x} \cdot q^{\star x,i} \geq 0$ received by producer i to favour the producer's decision towards $q^{\star x,i}$ instead of inaction at $q^{x,i} = 0$. Of course, proper accounting requires that producers's subsidies and consumers' subsidies are counted just once. The producer subsidy $S_i^\star$ ensures that consumer i receives a net profit of 0 from operating the production plant, but this consumer pays for $S_i^\star$ through $T_i^\star$, which requires a minor adjustment of the national accounts.

To summarize, since each SSPE proposal is Pareto efficient it can be supported by Walrasian equilibrium prices as an immediate consequence of the Second Welfare Theorem. The associated Walrasian equilibrium prices are the shadow prices of the single program (3) and these resolve the lack of (water) prices. Although shadow prices are implicitly present in transforming the physical economy into the (often transferable) utility representation in game theoretic applications, their presence seems to be ignored. Also the richer interpretation of agreements in terms of reallocation of property rights remains behind a veil when taking the utility representation as the primitive of the analysis. In negotiations, parties benefit from voluntarily agreeing upon a redistribution of property rights, even in the absence of such rights as will be clear from the next section.

The Walrasian equilibrium prices suggest the possibility to decentralize all consumer

[7] Under constant returns to scale, the Walrasian equilibrium prices are such that the firms make zero profit and then a set of maximizers exists.

and producer decisions through markets and suitable taxation. In river basin management, introducing water markets is often advocated as a solution to inadequate water management. Of course, whether it is advisable to do so should depend upon whether or not these agents have market power to manipulate market prices, which is a separate matter and outside the realm of the Walrasian model.

5 Bilateral Joint River Basin Management

The previous sections established that the single program can be implemented in economies with production and that each SSPE proposal can be interpreted as a Walrasian equilibrium with equilibrium prices. In this section, we illustrate the potentials of this framework to bilateral river basin management in a two-country version of the model proposed in Ambec and Sprumont (2002).

Consider a river that runs through two countries, where country 1 lies upstream of country 2 and all water users within the same country are aggregated as a single consumer. A more detailed model would allow for explicit production and water users that differ in their spatial location, representing different regions or cities, and differ in their use, such as agriculture, industrial and domestic. The territory of country i, $i = 1, 2$, captures $e_i > 0$ of water that is available for use. The subject of the negotiations concerns the allocation of water and an explicit financial transfer. Therefore, each country derives utility from consuming water and from holding money. Country i's utility from consuming z_i of water and the possibly negative transfer t_i is given by $u_i(z_i, t_i) = b_i(z_i) + t_i$, where b_i is monotonically increasing, homogenous ($b_i(0) = 0$), differentiable and strictly concave. The function b_i can also be regarded as an implicitly described production technology that can be separated as described in Section 4 at the cost of an additional variable and constraint. Following Ambec and Sprumont (2002), money is transferred utility meaning that the two-country economy does not have initial holdings of money. Total endowments are $\omega = (e_1, e_2, 0)$.

In terms of economic goods, the model distinguishes between good 1 representing water that is physically located in country 1, good 2 representing water located in country 2 and good 3 representing money. In Walrasian economies, each consumer expresses a demand for each of the available goods, but in case of rivers country 1 cannot consume good 2 and country 2 cannot consume good 1. To minimize on subscripts and superscripts, we continue denoting country i's consumption of good i simply as z_i.

Since water disposed of by country 1 flows downhill and transforms good 1 into good 2 we should see the river as a giant production process governed by physical processes. In the model under consideration, the river accumulates e_2 on country 2's territory and, therefore, the river production of downhill water between "locations" 1 and 2 takes place on country 1's territory. So, it is country 1 that produces good 2 with good 1 as input. Although some countries spend a significant proportion of the gross national income on pumping water uphill, such as Israel and the Kingdom of Jordan, we refrain from pumping as in Ambec and Sprumont (2002). We only note that pumping should be treated as a production process. In the present setup, country 2 cannot produce good 1.

With respect to production of good 2, country 1 produces q_2 from input q_1 and, under costless transformation, we have $q_1 + q_2 \leq 0$ and $q_1 \leq 0$, which implies a convex production technology. The aggregate commodity balance is given by

$$\begin{array}{ll} z_1 \leq e_1 + q_1 & (p_1) \\ z_2 \leq e_2 + q_2 & (p_2) \\ t_1 + t_2 \leq 0, & (p_3) \end{array} \tag{8}$$

where p_1, p_2 and p_3 refer to the shadow prices for the three goods. Consumption of $z_1 \leq e_1$ by country 1 and efficiency in production implies $q_2 = -q_1 = e_1 - z_1 \geq 0$. Substitution yields the mathematically equivalent feasibility constraints in Ambec and Sprumont (2002):

$$\begin{array}{ll} z_1 \leq e_1, & (p_1) \\ z_2 \leq e_2 + e_1 - z_1, & (p_2) \\ t_1 + t_2 \leq 0, & (p_3) \end{array} \tag{9}$$

In terms of the latter balance, the subject of the negotiations is a feasible allocation (z_1, z_2, t_1, t_2). The allocation should be extended to also include q_1 and q_2 if river "production" is explicitly incorporated, which is would be the convenient approach in applications where the hydro-

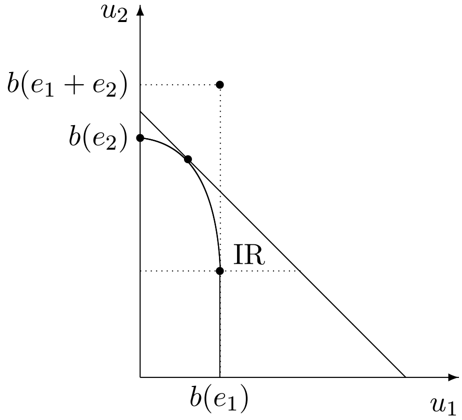

Figure 1: The bargaining frontier, the disagreement point $(b(e_1), b_2(e_2))$ and the set of individual rational payoffs (IR). The aspiration levels $(b(e_1), b_2(e_1+e_2))$ are infeasible.

logical part of the model is provided by hydrologists.[8]

The disagreement point

An essential ingredient of any bargaining problem is the disagreement point. For water issues, there is some modelling freedom in the choice of such point. An obvious choice is a disagreement point based upon property rights according to international law. An alternative choice for the disagreement point would be to assumes that this point is based upon the countries' unilateral decisions concerning water issues. We discuss both alternatives.

Suppose in modeling water issues we opt for a disagreement point based upon international law. As Ambec and Sprumont (2002) argue, international law is ambiguous and they discuss two conflicting doctrines. One of these doctrines is *absolute territorial sovereignty* and it assigns e_i as the property rights for country i and the river's production technology to country 1. This would imply $\omega^1 = (e_1, 0, 0)$, $\omega^2 = (0, e_2, 0)$ and the disagreement point $d = (b_1(e_1), b_2(e_2))$. This point is always feasible, because

$$b_1(e_1) + b_2(e_2) \leq \max_{z_1 \in [0, e_1]} b_1(z_1) + b_2(e_1 + e_2 - z_1). \tag{10}$$

In order to have an essential bargaining problem, the inequality has to be strict. Another

[8]In terms of Section 4, we have $z^1 = (z_1, 0, t_1)$, $q^1 = (q_1, q_2, 0) \in Q^1 = \{q^1 \,|q_1 \leq 0, q_2 \leq -q_1|\}$, $z^2 = (0, z_2, t_2)$ and $q^2 \in Q^2 = \{0\}$.

doctrine is *unlimited territorial integrity* that assigns incompatible property rights to the countries, namely e_1 to country 1 and $e_1 + e_2$ to country 2. Also, the river's production technology accrues to country 2. This would translate into $\omega^1 = (e_1, 0, 0)$, $\omega^2 = (0, e_1 + e_2, 0)$ and the disagreement point $d = (b_1(e_1), b_2(e_1 + e_2))$. This point is not feasible, because

$$b_1(e_1) + b_2(e_1 + e_2) > \max_{z_1 \in [0, e_1]} b_1(z_1) + b_2(e_1 + e_2 - z_1). \tag{11}$$

This disagreement point is physically unattainable and, as suggested by Ambec and Sprumont (2002), can only be interpreted as the countries' aspiration levels. In case both countries are committed to these "virtual" aspiration levels, then the negotiations remain deadlocked and disagreement is the only outcome, see e.g. Crawford (1982) and Muthoo (1992). Figure 1 illustrates both doctrines.

As an alternative modelling approach that seems closer to reality, we may assume that each country takes unilateral decisions concerning water issues in the absence of bilateral river basin management. In terms of noncooperative bargaining theory, the disagreement point is endogenous. Several theoretical models are available, see e.g. Bolt and Houba (1998), Busch and Wen (1995) and Houba (1997).[9] These bargaining models also assume the model under consideration as the disagreement game in which both countries take unilateral decisions. Under SSPE behavior and the impossibility of commitment to disagreement actions prior to the negotiations, the disagreement point coincides with a Nash equilibrium, which is $(z_i, t_i) = (e_i, 0)$, $i = 1, 2$, in our case. This Nash equilibrium coincides with absolute territorial sovereignty. Although international law suggests that countries should mutually agree on Pareto improvements through negotiations, this law seems to lack a doctrine how to treat unilateral decisions in absence of agreement.

The Second Welfare Theorem and water pricing

The purpose of discussing this particular model is to arrive at water prices, money trans-

[9] Busch and Wen (1995) and Houba (1997) assume $\delta_1 = \delta_2$. As pointed out in Houba and Wen (2006a,c), the current bargaining literature on endogenous disagreement points under $\delta_1 \neq \delta_2$ contains serious technical difficulties.

fers and transfers of property rights through taxation. The interesting case assumes the non-trivial case in which the disagreement point corresponds to absolute territorial sovereignty: $d = (b_1(e_1), b_2(e_2))$. Application of the modified program (3) yields the single convex program

$$\max_{s \geq d; \left(x_1, x_2, q_1^x, q_2^x, t_1^x, t_2^x\right), \left(y_1, y_2, q_1^y, q_2^y, t_1^y, t_2^y\right)} (s_1 - d_1)^{\alpha} (s_2 - d_2)^{1-\alpha},$$

$$\begin{array}{rlr}
\text{s.t.} & & \\
s_1 & \leq b_1(x_1) + t_1^x & (\mu_1) \\
s_2 & \leq b_2(y_2) + t_2^y & (\mu_2) \\
(1-\delta_1) d_1 + \delta_1 s_1 & \leq b_1(y_1) + t_1^y, & (\lambda_1) \\
(1-\delta_2) d_2 + \delta_2 s_2 & \leq b_2(x_2) + t_2^x, & (\lambda_2) \\
x_1 & \leq e_1 + q_1^x, & (p_1^x) \\
x_2 & \leq e_2 + q_2^x, & (p_2^x) \\
t_1^x + t_2^x & \leq 0, & (p_3^x) \\
y_1 & \leq e_1 + q_1^y, & (p_1^y) \\
y_2 & \leq e_2 + q_2^y, & (p_2^y) \\
t_1^y + t_2^y & \leq 0, & (p_3^y) \\
q_1^x + q_2^x & \leq 0, & (\gamma^x) \\
q_1^y + q_2^y & \leq 0, & (\gamma^y)
\end{array}$$

where all Greek symbols between brackets denote shadow prices. As in Section 4, we only discuss country 1's SSPE proposal $(x_1, x_2, q_1^x, q_2^x, t_1^x, t_2^x)$. The part of the first-order conditions involving the partial derivatives of these six variables (maintaining the stated order) are given by

$$\begin{aligned}
\mu_1 b_1'(x_1) - p_1^x &= 0, \\
\lambda_2 b_2'(x_2) - p_2^x &= 0, \\
p_1^x - \gamma^x &= 0, \\
p_2^x - \gamma^x &= 0 \\
\mu_1 - p_3^x &= 0, \\
\lambda_2 - p_3^x &= 0.
\end{aligned}$$

Solving these equations yields

$$p_1^x = p_2^x = p_3^x b_1'(x_1) = p_3^x b_2'(x_2) > 0, \tag{12}$$

because utility is strictly increasing in money and, therefore, $p_3^x > 0$. Due to specificities of the model, we obtain the special case of a uniform water price for each location. From (12) we observe that only relative prices matter and we may take money as the numeraire by dividing all shadow prices by p_3^x, where we denote the normalized (uniform) water price as $p_w^x = p_1^x / p_3^x$. Assuming that the bargaining problem is essential, i.e., Pareto improvements exist as depicted in Figure 1, the total gains from bilateral river basin management are maximized if the upstream country is willing to trade some of its water for money. Since also all shadow prices are positive, we must have that all constraints are binding, including efficient river production. So, $q_1^x = x_1 - e_1 < 0$ and $q_2^x = -q_1^x > 0$ implies $x_2 = e_1 + e_2 - x_1 > e_2$. Furthermore, $b_1' (x_1) = b_2' (x_2)$ in (12) implies that the joint surplus $b_1 (x_1) + b_2 (x_2)$ is maximized in this SSPE proposal and, hence, x_1 coincides with the unique maximizer of the right-hand side of (11), as could be expected. Finally, $t_1^x + t_2^x = 0$ and $q_1^x + q_2^x = 0$ imply that aggregate spending equals aggregate income $p_w^x (x_1 + x_2) = p_w^x (e_1 + e_2)$. Similar properties hold for country 2's SSPE proposal and, in particular, $y_i = x_i$ for both $i = 1, 2$.

According to the Second Welfare Theorem, country i's pre-tax income $p_w^x e_i$ and its expenditure or after-tax income is equal to $m_i^x \equiv p_w^x x_i + t_i^x$. Acting as a price-taking consumer, country i spends its income on water consumption and monetary liquidity by solving:

$$(x_i, t_i^x) \in \arg\max_{z_i, t_i} u_i (z_i, t_i), \quad \text{s.t.} \quad p_w^x z_i + t_i \leq m_i^x, \tag{13}$$

which yields $\frac{\partial u_i(x_i,t_i)}{\partial x_i} = b_i' (x_i) = p_w^x$ and $\frac{\partial u_i(x_i,t_i)}{\partial t_i} = 1$. Note that $b_i' (x_i)$ is also country i's marginal rate of substitution between water and money and it is equal to the relative price $p_w^x / 1$. The monotonic preferences imply the budget constraint is binding. Country 1 receives the amount of money $t_1^x = -t_2^x > 0$ for its delivery of $e_1 - x_1$ to country 2. This implies a unit price of water of $(e_1 - x_1) / t_1$ that is unrelated to do prices related to marginal costs and benefits. Note that it does not matter whether t_i represents money or some consumption good from which the countries obtain utility. Since the SSPE proposal is individually rational, we obtain that $t_i^x \geq b_i (e_i) - b_i (x_i) > 0$ for both $i = 1, 2$. Summation of these inequalities shows a nonempty range of t_1 that are feasible, because $t_1^x + t_2^x = 0$ and

$b_1(e_1) + b_2(e_2) - b_1(x_1) - b_2(x_2)$ is negative.

Next, consider producer 1 with its constant-returns-of-scale river production technology. Producer 1's profit under price-taking is equal to 0, because he buys input $q_1 = x_1 - e_1 < 0$ against price p_w^x and sells exactly this amount at exactly the same price to country 2. We refer to Albersen et al. (2003) for an example of non-trivial financial accounts based upon shadow pricing associated with non-convex physical processes.

Although money is usually regarded as a special economic good, it is just one of the goods in the economy, also according to the Second Welfare Theorem. This theorem provides an interpretation of the allocation $(x_1, x_2, q_1^x, q_2^x, t_1^x, t_2^x)$ in terms of market trade (or marginal cost/benefit pricing) against the price vector $(p_w^x, p_w^x, 1)$ and a redistribution of property rights equal to $T_i^\star = p_w^x x_i + 1 \cdot t_i^x - p_w^x e_i$, being the difference between expenditure and pre-tax income. Whether $T_i^\star$ is positive or negative is an empirical matter. This redistributive effect consists of the combined value of net trade in water $p_w^x(x_i - e_i)$ and the net trade in money $1 \cdot (t_i^x - 0)$ that is opposite in sign. The negotiation outcome has two effects: a redistributive element and Pareto improving trade that is incorporated in the pre-tax market income. Even though the initial rights in river basin management might be ill-defined, both countries agree on a redistribution of wealth representing implicitly defined property rights by establishing "new" property rights associated with $(x_1, x_2, q_1^x, q_2^x, t_1^x, t_2^x)$. Although the shadow prices can be thought of to represent Walrasian equilibrium prices as if established water markets are governed by the law of supply and demand, this interpretation assumes the countries refrain from exercising market power.

In general, the Second Welfare Theorem deals with non-transferable utility instead of transferable utility or money, as e.g., in Ambec and Sprumont (2002). For the bilateral case, the transferable utility value of cooperation, denoted as $v(1,2)$, is equal to the right-hand side of (10). The Pareto frontier is described by $f_i((1-\delta_j)d_j + \delta_j s_j) = v(1,2) - (1-\delta_j)d_j - \delta_j s_j$.

Then, direct application of program (2) yields

$$\max_{s_1,s_2} (s_1 - d_1)^{\alpha} (s_2 - d_2)^{1-\alpha},$$
$$\text{s.t.}$$
$$s_1 \leq v(1,2) - (1-\delta_2) d_2 - \delta_2 s_2,$$
$$s_2 \leq v(1,2) - (1-\delta_1) d_1 - \delta_1 s_1.$$

In this program, any reference to prices and marginal benefits has vanished from the model description. In this simple case, this crucial information can be retrieved from (10), but for less transparent applications a holistic approach in physical variables as in e.g. Roemer (1988) yields more information to policy makers.

Finally, Ambec and Sprumont (2002) suggest that rational countries should realize that their joint maximal aspiration level is bounded by the maximum on the right-hand side of (10) or (11) and propose the downstream incremental distribution that assigns the following utility levels to the two countries $s_1 = d_1$ and $s_2 = v(1,2) - d_1 > d_2$. This solution coincides with the SSPE outcome (in utilities) in the alternating offer model with disagreement point associated to absolute territorial sovereignty and a bargaining weight $\alpha = 0$ to country 1, or in terms of the primitives either $\delta_1 = 0$ or $\delta_2 = 1$. The alternating-offers perspective indicates that it is very unlikely that this axiomatic solution will prevail. Furthermore, even at $\alpha = 0$ the Second Welfare Theorem applies.

We conclude this section with a remark on stock variables.

Remark 2 *Optimal river basin management includes the optimal release and recharge of lakes and dams as reservoirs of water. Reservoirs would introduce stock variables to the model. A reservoir can also be seen as a production process that produces "future" water from "current" water and it can be represented as before by some production set Q. As an illustration, reinterpret z_1, respectively, z_2 as water at present and in the future, say the wet and dry season. Then, the reservoir produces future water q_2 from present water q_1 as input and, under absence of evaporation, we have $q_1 + q_2 \leq 0$ and $q_1 \leq 0$ as before. Then q_1 is the end stock of period* 1 *and q_2 the initial stock at period* 2.

6 Concluding Remark

This contribution deals with the two points of fundamental critique in Dinar et al. (1992) that alienate game theory from the language and concerns of policy makers: Easy to implement solutions that are based upon common notions of water pricing; and insight in the gains and losses for every stakeholder from policy reforms toward efficient river basin management. For bilateral negotiations modeled as alternating-offers, as pioneered in Rubinstein (1982), a powerful computational innovation in a physical representation of real-world issues is available that simultaneously allows for an interpretation of water prices as Walrasian equilibrium prices, which is a direct consequence of the Second Welfare Theorem. Unfortunately, the utility representation in Game Theory washes away any notion of water prices from the physical model, as illustrated in Section 5. The physical model can be regarded as the popular AGE framework that allows for further differentiation of water users, water related production and consumption goods by time (within the hydrological cycle), by space and uncertainty about extreme whether conditions (droughts and floodings), although the latter would assume the existence of contingent contracts and a discrete number of events.

Identifying water prices as Walrasian equilibrium prices should not be mistaken as naively suggesting to decentralize decisions through water markets. For that to be the best policy recommendation, it should be made clear first that all participants on these markets do not posses significant market power. For river basin management, also the role of governments is crucial even in case these do not trade themselves on the water market, because upstream countries might initiate development of plans for, say, expanding the area under irrigation affecting future downstream flows.

Although this paper identifies a promising route for further developing tools for water policy research, the bilateral case is just a first step. Future research should first of all be directed to deal with coalition formation among countries or among different stakeholder within and across countries. Also the issue of regulating water markets when some the parties have market power is a relevant issue. As in all areas of economic policy, lobbying is a matter of related interest. The bilateral model in Houba (2005a) captures such negotiations in which the (exogenously given) probability of success in lobbying means a higher probability to propose during the negotiations. This model reduces to the standard alternating-offers model after a transformation of the probabilities of becoming the proposer into modified discount factors and, therefore, the approach advocated in this contribution also applies.

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