A note on the paper: Solving the job-shop scheduling problem optimally by dynamic programming

van Hoorn, J.J.; Nogueira, A.; Ojea, I.; Gromicho, J.A.S.

2015

document version
Early version, also known as pre-print

Faculty of Economics and Business Administration

A note on the paper: Solving the job-shop scheduling problem optimally by dynamic programming

Research Memorandum 2015-9

Jelke J. van Hoorn
Agustín Nogueira
Ignacio Ojea
Joaquim A.S. Gromicho

vrije Universiteit amsterdam

A note on the paper: Solving the job-shop scheduling problem optimally by dynamic programming

Jelke J. van Hoorn[a,b,1], Agustín Nogueira[c,2], Ignacio Ojea[c,*,3], and Joaquim A. S. Gromicho[a,b,4]

[a]Department of Econometrics and OR, Vrije Universiteit Amsterdam, The Netherlands
[b]ORTEC, Zoetermeer, The Netherlands
[c]Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Buenos Aires, Argentina

[*]Fellow of CONICET, Argentina
[1]Jelke.vanHoorn@ortec.com
[2]aanogue87@gmail.com
[3]iojea@dm.uba.ar
[4]j.a.dossantos.gromicho@vu.nl

Abstract

We point out a flaw in the arguments given in [6] for proving the correctness of the algorithm there proposed, by showing an example that contradicts some claims of that article. More importantly, we amend the flaw, providing a new and simpler proof of the correctness of the algorithm.

1. Introduction

The job-shop scheduling problem is given by n jobs and m machines. Each job has to visit all the machines following a specific order. The order in which each job has to be processed along the machines, and the time it requires in each machine are known. Machines can process only one job at a time and the same job can not be processed simultaneously in two different machines. The goal is to minimize the *makespan*, i.e.: the completion time of the complete set of jobs.

The job-shop scheduling problem is one of the most studied combinatorial optimization problems, but it remains a very challenging problem. Even 'simplified versions' of the job-shop scheduling problem are NP-Hard (see, for example, [4]).

In [6] an algorithm is proposed for solving the job-shop scheduling problem optimally using a dynamic programming strategy. This is, according to our knowledge, the first exact algorithm for the Job Shop problem which is not based on integer linear programming and branch and bound. Despite the correctness of the dynamic programming algorithm presented in [6], the proof of correctness given there is unfortunately flawed. The contribution of the present paper is threefold: first we show by means of a simple example where the flaw lies. Secondly, we present a correct and to some extent more intuitive proof. Thirdly, we establish the contribution of the aforementioned paper as correct and worth merit enabling subsequent research to improve on it.

For a brief review on the bibliography on job shop scheduling problems, we refer the reader to [6]. However, since the appearance of [6], many articles have been published, dealing with the job shop scheduling problem. To mention a few: in [1] neighbourhood strategies are considered. In [2] an enumerative parallelized algorithm is developed. Both [3, 11] propose modified genetic algorithms. Constrained and mixed integer programming are studied in [8], whereas a differential evolution algorithm is proposed in [10]. Finally, an heuristic method is developed for a variant of the job shop scheduling problem with tree-structured precedence constrains in [5]. All of these articles take [6] into account, but fortunately it is used there in ways that are not affected by the flaw that we notice and repair.

In Section 2 we introduce the notation and the main definitions for the problem formulation. In Section 3 we state the dynamic programming formulation for the job-shop scheduling problem, and the algorithm therefore obtained, whereas in Section 4 we briefly review the arguments given in [6] for proving the correctness of the algorithm, and present a counterexample that show that some of the claims of [6] do not hold. Finally, in Section 5 a new proof is given for the correctness of the algorithm. Taking this into account, it is very important to remark that any work based exclusively on the correctness of the algorithm would not be affected by the present paper. Only some proofs, and not the results, given in [6] should be revised.

2. Notation and Preliminaries

We denote $\mathcal{J} = \{j_1, \dots, j_n\}$ the set of jobs and $\mathcal{M} = \{m_1, \dots, m_m\}$ the set of machines. Each job consists of m operations that should be processed in a given order. We denote $\mathcal{O} = \{o_1, o_2, \dots, o_n, \dots, o_{nm}\}$ the set of all the operations. The first n operations correspond to the first operation of each job, whereas operations $o_{n+1}, \dots, o_{2n}$ correspond to the second operations of each job, and so on. In this way, j_i is formed by operations: $\{o_{kn+i}\}_{k=0,\dots,m-1}$. For each operation o, we denote $m(o)$ the machine in which o should be processed and $j(o)$ the job where o belongs. Observe that $j(o_i) = i \mod n$. Finally, we denote $p(o)$ the processing time of o in $m(o)$.

Following this notation an instance of the job shop scheduling problem is given by the numbers n and m of jobs and machines, and two vectors of length $n \times m$, containing $m(o)$ and $p(o)$ for each operation o.

Definition 2.1. A schedule is a function $\psi : \mathcal{O} \to \mathbb{N} \cup \{0\}$, where $\psi(o)$ gives the starting point of operation o. A schedule ψ is feasible if:

1. For all $o_k, o_l \in \mathcal{O}$ such that $j(o_k) = j(o_l)$ and $k < l$, $\psi(o_k) + p(o_k) \leq \psi(o_l)$.
2. For all $o_k, o_l \in \mathcal{O}$ such that $m(o_k) = m(o_l)$ we have that $\psi(o_k) + p(o_k) \leq \psi(o_l)$ or $\psi(o_l) + p(o_l) \leq \psi(o_k)$.

The goal of the job shop scheduling problem is to find a feasible schedule that minimizes

$$C_{\max}(\psi) = \max_o \{\psi(o) + p(o)\}.$$

Schedules are represented by Gantt charts, like the ones showed in Figure 1. Each row represents a machine, while the x axis is time. The bars represent operations, and colors are used to identify jobs.

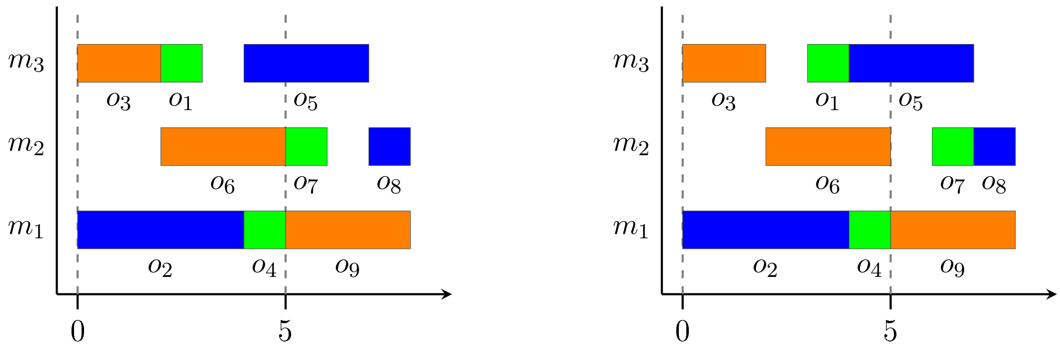


FIGURE 1. Active (left) and non-active (right) schedules.

Each schedule ψ can be associated to a sequence of operations by sorting the operations following some fixed criteria (for example: according to the starting time given by ψ). The following proposition is proved in [6] and establishes the criteria that we adopt for associating schedules to sequences of operations.

Proposition 2.1. *For every feasible solution for the job-shop scheduling problem there is one and only one sequence of operations defining the schedule such that the completion time of the operations along the sequence is non-decreasing and in which the order of the machines is increasing for two consecutive operations with equal completion time.*

Proposition 2.1 says that for each schedule we have one and only one sequence of operations. However, the converse is not true. Figure 1, for example, shows two possible schedules for the sequence: $o_3, o_1, o_2, o_4, o_6, o_7, o_4, o_9, o_8$. It is clear that any sequence admits an infinite number of schedules, since when all the operations in a tail of the sequence are moved to the right the same amount of time units, the relative order between them is not altered.

In order to identify sequences and schedules, we introduce, following for example [9], the notion of *active* schedules.

Definition 2.2. A feasible schedule ψ is *active* if the action of moving any operation one unit of time to the left makes it unfeasible.

In Figure 1, the left schedule is active, whereas the right one is not, since operations o_1 and o_7 have been moved unnecessarily to the right. Non-active schedules are also called *idle* schedules, since machines are idle even though jobs are available to process.

According to the notion of active schedule, given a sequence of operations, we will associate to it the schedule where the starting time for each operation is fixed as soon as possible, as long as it satisfies the restrictions with respect to the previous operations. Such a procedure guarantees that only active schedules are produced.

Remark 2.1. Given a sequence ς there is only one feasible active schedule ψ_ς associated to it. On the other hand, given the schedule ψ_ς, there is one and only one sequence ς' associated to it, according with Proposition 2.1. However, it is important to observe that ς' is not necessarily equal to ς. Consider, for the instance of Figure 1, the sequence $\varsigma = o_2, o_3, o_1, o_4, o_6, o_5, o_7, o_9, o_8$. ψ_ς is the schedule at the left of Figure 1, however ς is not ordered according to Proposition 2.1. The ordered sequence given by ψ_ς is $\varsigma' = \{o_3, o_1, o_2, o_6, o_4, o_7, o_5, o_8, o_9\}$.

We say that a sequence ς is *ordered*, if it is ordered according to the criteria established in Proposition 2.1. In other words: if $\varsigma' = \varsigma$. Otherwise, we say that ς is unordered. Furthermore, we denote $S(\varsigma)$ the subset of $\mathcal{O}$ containing all the operations that appear in the sequence ς. We say that ς is partial when $S(\varsigma) \neq \mathcal{O}$. On the other hand, ς is complete if $S(\varsigma) = \mathcal{O}$. In order to apply a dynamic programming strategy, we will build partial ordered sequences adding one operation at a time. The following definition states the basic notation for the dynamic programming formulation.

Definition 2.3. Given a sequence ς we define:

1. $\varepsilon(\varsigma) \subseteq \mathcal{O} \setminus S(\varsigma)$ is the set of operations o that can be added to ς such that all the operations in $j(o)$ that have to be scheduled before o belong to $S(\varsigma)$. Observe that $\varepsilon(\varsigma)$ depends only on $S(\varsigma)$, and not on the particular permutation ς.
2. We denote $\varsigma + o$ the sequence obtained by adding o at the end of ς. We say that $\varsigma + o$ is an expansion of ς. Observe that $\varsigma + o$ can be unordered, even if ς is ordered. We also denote $\psi(\varsigma, o)$ the starting time of o in $\varsigma + o$.
3. $\eta(\varsigma) \subseteq \varepsilon(\varsigma)$ is the set of operations such that $\varsigma + o$ is ordered. Observe that $\eta(\varsigma)$ depends on the sequence ς, and not only on the set of operations $S(\varsigma)$.
4. We say that $\varsigma_{\mathcal{O}}$ is a completion of ς if $S(\varsigma_{\mathcal{O}}) = \mathcal{O}$ and $\varsigma_{\mathcal{O}}$ is obtained from ς by sequentially adding one operation at a time.
5. For any (partial) sequence ς, $C_{\max}(\varsigma)$ stands for the completion time of ς.
6. Given $S \subseteq \mathcal{O}$, we denote $\Xi(S)$ the set of all ordered sequences ς such that $S(\varsigma) = S$.
7. For any sequence ς, we denote $\varsigma[i]$ the i-th operation of sequence ς.
8. For any sequence ς we denote $\Lambda(\varsigma)$ the last operation of ς.

3. The algorithm

In the seminal work of Held and Karp [7] several problems, including a simplified scheduling problem, are formulated as *sequencing* problems, and a dynamic programming approach is applied to them. Sadly, the strategy presented in [7] cannot be used for solving the job shop scheduling problem. The main difficulty for doing so is that the optimality principle does not hold for the job shop scheduling using the *natural* functional $C_{\max}$. Consequently, some technical work should be done in order to find a proper formulation for the application of dynamic programming. However, it is important to remark that we will not obtain a functional equation, as in classical dynamic programming, but a recursive strategy that will allow the progressive construction of the optimal solution.

Such a formulation for the job shop scheduling problem, and the exact algorithm that is derived from it are the main contributions of [6]. The complexity of the algorithm is exponential, but, more importantly, it is exponentially better than brute force.

Following a dynamic programming strategy, the algorithm proceeds in $n \times m$ stages. In stage i only ordered sequences of exactly i operations are considered. Some sequences are compared according to a criterion that is specified below, and that states a *domination* relationship between some sequences. When a sequence is *dominated* by another, it is discarded. For the sequences that are not discarded, all the possible ordered expansions are generated, obtaining sequences with $i + 1$ operations. At stage $n \times m$ an optimal solution is found.

In order to compare partial sequences, we define an *aptitude* value for a sequence ς and every operation $o \in \varepsilon(\varsigma)$:

$$\alpha(\varsigma, o) = \begin{cases} \psi(\varsigma, o) + p(o) & \text{if } o \in \eta(\varsigma), \\ C_{\max}(\varsigma) + p(o) & \text{otherwise.} \end{cases}$$

Observe that α is a lower bound for the completion time of o in any ordered completion of ς: if $o \in \eta(\varsigma)$, it can be added immediately, with completion time $\alpha(\varsigma, o)$, but it can also be added in a further step, with completion time greater that $\alpha(\varsigma, o)$; on the other hand, if $o \notin \eta(\varsigma)$ another operation o' has to be added before o in $m(o)$, with completion time at least $C_{\max}(\varsigma)$, and consequently when o is added its completion time is greater than $C_{\max}(\varsigma) + p(o)$.

We use α to compare partial solutions. It is noteworthy than only sequences involving the same operations can be compared. We denote $\vec{\alpha}(\varsigma)$ a vector containing the values of $\alpha(\varsigma, o)$, for every $o \in \varepsilon(\varsigma)$, ordered by job. Given two sequences ς^1 and ς^2 such that $S(\varsigma^1) = S(\varsigma^2)$, we say that $\vec{\alpha}(\varsigma^1) \lessdot \vec{\alpha}(\varsigma^2)$ if $\alpha(\varsigma^1, o) \leq \alpha(\alpha^2, o)$ for all o.

The following proposition is proved in [6], and it is the key of the proposed algorithm.

Proposition 3.1. *Let ς^1 and ς^2 be partial sequences in $\Xi(S)$, such that $\vec{\alpha}(\varsigma^2) \lessdot \vec{\alpha}(\varsigma^1)$ Then, every operation $o \in \mathcal{O} \setminus S$ of an* ordered *completion $\varsigma^1_{\mathcal{O}}$ of ς^1 can be scheduled at the same time in the schedule of ς^2. This leads to a feasible, though possibly non-active, complete schedule with makespan $C_{\max}(\varsigma^1_{\mathcal{O}})$.*

Remark 3.1. It is important to notice that the completion of ς^2 with the operations in $\mathcal{O} \setminus S$ in the order that they are scheduled in ς^1 can produce an *unordered* sequence.

Since the schedule obtained by completing ς^2 can be non-active, some operations can be moved to the left producing an active schedule ψ^2 with makespan lesser or equal than $C_{\max}(\varsigma^1_{\mathcal{O}})$. According to this result, we say that if $\vec{\alpha}(\varsigma^2) \lessdot \vec{\alpha}(\varsigma^1)$, ς^2 *dominates* ς^1.

It is possible to find sequences ς^1 and ς^2 such that $\alpha(\varsigma^1, o) = \alpha(\varsigma^2, o)$ for every o. In such cases some rule should be adopted in order to decide whether ς^1 dominates ς^2 or viceversa. It doesn't matter what rule is used, as long as the same criteria is applied to all the cases, for example take the lowest operation number of the first difference in the sequences.

Corollary 3.1. *Let ς^1 be dominated by ς^2. Then, Proposition 3.1 implies that for any ordered completion $\varsigma^1_{\mathcal{O}}$ of ς^1, there is an ordered complete sequence $\varsigma^2_{\mathcal{O}}$ with equal or lower makespan. However Remark 3.1 indicates that such sequence is not necessarily obtained by iteratively expanding ς^2.*

Corollary 3.1 contains both the core of the algorithm, and its main subtleties. As we commented earlier, dominated sequences are dropped. This seems to be allowed by the corollary: we drop ς^1 because we know that for any completion of ς^1 another solution with equal or lower makespan can be produced. However, it is possible that such a better solution does not come directly from ς^2, but from another partial sequence ς^3 that is not comparable to ς^1 at stage $|S|$. Moreover, it is theoretically possible that the algorithm never generates ς^3, if some partial sequence of it is dropped at a previous stage. Taking this into account, it is not obvious that an optimal solution should be produced. It is clear that if a certain instance of the problem admits only one optimal solution, it will be never discarded. But if there are two or more optimal solutions, it would be possible that they dominate each other at different stages making the algorithm drop all of them. Fortunately, such a situation is not really possible, as we prove in Section 5.

Aside from the domination criterion, that leads us to the dynamic programming formulation, [6] states a *state reduction* procedure that allows to drop some additional partial sequences, reducing the number of sequences considered by the algorithm and, therefore, its practical performance. This procedure is not affected by the flaws of the proofs given in [6] and it is consequently omitted here.

We conclude this section stating a simplified scheme of the algorithm:

Require: An instance of the job shop scheduling problem
Ensure: An optimal solution for the instance.
  for all $o \in \varepsilon(\emptyset)$ **do**
    Define the set $\mathcal{F}(\{\varsigma\}) = \{\varsigma\}$ with $\varsigma = (o)$.
  for $i = 1$ **to** $n \times m$ **do**
    for all $S \subset \mathcal{O}: \ |S| = i$ **do**
      for all $\varsigma \in \mathcal{F}(S)$ **do**
        for all $o \in \eta(\varsigma)$ **do**
          $\varsigma' = \varsigma + o$.
          if ς' is not dominated by any sequence $\varsigma^2 \in \mathcal{F}(S \cup \{o\})$ **then**
            for all $\varsigma^2 \in \mathcal{F}(S \cup \{o\})$ **do**
              if ς' dominates ς^2 **then**
                remove ς^2 from $\mathcal{F}(S \cup \{o\})$
              add ς' to $\mathcal{F}(S \cup \{o\})$
  return the sequence $\varsigma \in \mathcal{F}(\mathcal{O})$ with minimum $C_{\max}$.

4. The original proof

The formulation of the job-shop scheduling problem as a sequencing problem is developed in [6] along with the dynamic programming algorithm. In order to prove the correctness of the algorithm, many new notions are introduced, and several preparatory results are proven. Unhappily, problems have been found on some of these preliminary steps. For the sake of brevity, we only comment here a key point that makes the main proof to fail, and present a counterexample to show this.

We have already defined the set $\Xi(S)$ containing all the ordered sequences using the operations in S. In [6] two subsets of $\Xi(S)$ are defined. $\widehat{\Xi}(S)$ is formed by all the sequences in $\Xi(S)$ that are not dominated by any other sequence in $\Xi(S)$, and $\overset{\triangle}{\Xi}(S)$ is the set of all the sequences ς in $\widehat{\Xi}(S)$ such that all the subsequences of ς are in $\widehat{\Xi}(S')$ for the corresponding set S'.

Proposition 3 in [6] states that the set $\overset{\triangle}{\Xi}(S)$ is never empty. Based on this result, Proposition 4 concludes that the sets $\mathcal{F}(S)$ generated by the algorithm are exactly $\overset{\triangle}{\Xi}(S)$. The following example shows both these assertions to be false, invalidating the line of argumentation of [6]. Fortunately, as it is shown in the next section, these problems are not essential, and can be avoided following a slightly different path.

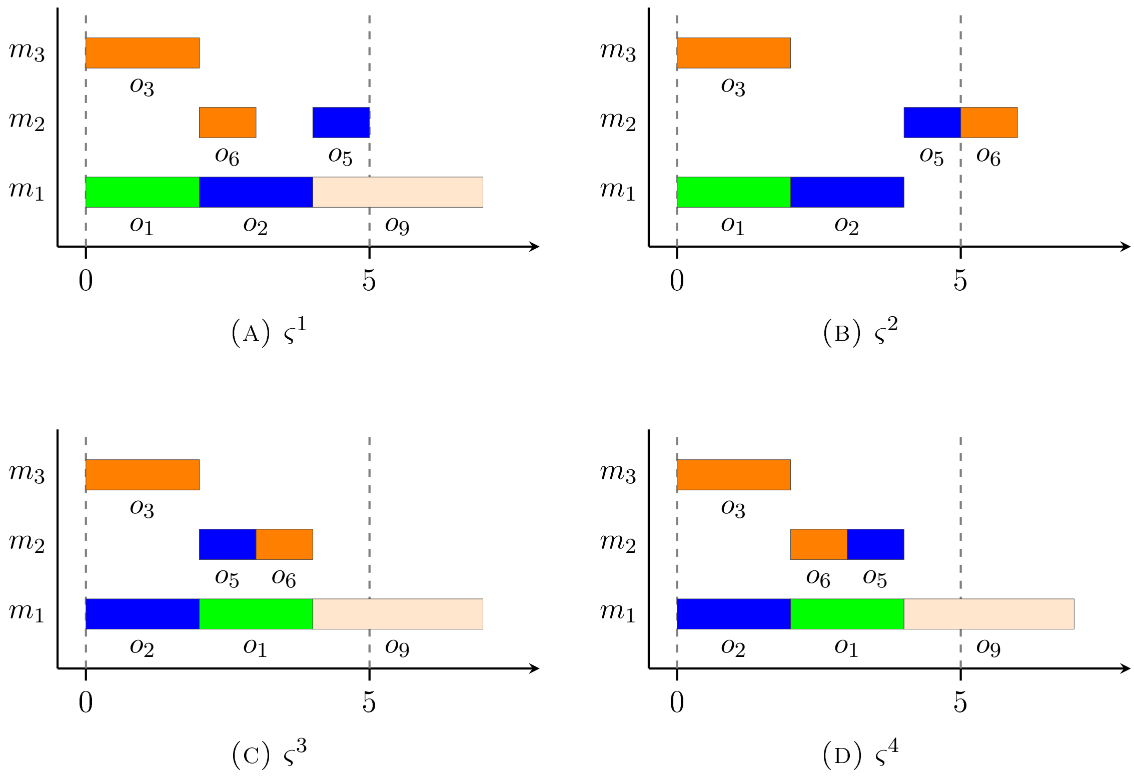
Consider the instance given by:

Operations	o_1	o_2	o_3	o_4	o_5	o_6	o_7	o_8	o_9
$p(o)$	2	2	2	4	1	1	1	3	3
$m(o)$	1	1	3	3	2	2	2	3	1

For this instance, consider the set $S = \{o_1, o_2, o_3, o_5, o_6\}$. It is easy to verify that $\Xi(S)$ is given by the four sequences:

$$\varsigma^1 = (o_1, o_3, o_6, o_2, o_5), \quad \varsigma^2 = (o_1, o_3, o_2, o_5, o_6),$$
$$\varsigma^3 = (o_2, o_3, o_5, o_1, o_6), \quad \varsigma^4 = (o_2, o_3, o_6, o_1, o_5),$$

represented by the following schedules:



(A) ς^1 (B) ς^2 (C) ς^3 (D) ς^4

FIGURE 2. Sequences in the set $\Xi(S)$

For these sequences we have:

$$\begin{array}{llll}
\alpha(\varsigma^1, o_4) = 6 & \alpha(\varsigma^2, o_4) = 6 & \alpha(\varsigma^3, o_4) = 8 & \alpha(\varsigma^4, o_4) = 8 \\
\alpha(\varsigma^1, o_8) = 8 & \alpha(\varsigma^2, o_8) = 8 & \alpha(\varsigma^3, o_8) = 6 & \alpha(\varsigma^4, o_8) = 7 \\
\alpha(\varsigma^1, o_9) = 7 & \alpha(\varsigma_2, o_9) = 9 & \alpha(\varsigma^3, o_9) = 7 & \alpha(\varsigma^4, o_9) = 7
\end{array}$$

So we conclude that $\varsigma^1 \lessdot \varsigma^2$ and $\varsigma^3 \lessdot \varsigma^4$. It is easy to check that the subsequences of ς^1 and ς^3 are in the corresponding $\widehat{\Xi}(S')$, so we conclude that:

$$\overset{\triangle}{\Xi}(S) = \{\varsigma^1, \varsigma^3\}.$$

Now, consider the set $S \cup \{o_9\}$. The algorithm could form sequences with set $S \cup \{o_9\}$ expanding, with the corresponding operation, not-dominated sequences with different sets of cardinal $|S|$. Particularly, with set: S, $(S \cup \{o_9\}) \setminus \{o_5\}$, $(S \cup \{o_9\}) \setminus \{o_6\}$ and $(S \cup \{o_9\}) \setminus \{o_1\}$. However, it is easy to see that the algorithm would only expand S, since the other alternatives are unfeasible or dominated at previous stages. Therefore, the algorithm would generate only the sequences $\varsigma^1 + o_9$ and $\varsigma^3 + o_9$. Observe that the expansions $\varsigma^1 + o_9$, $\varsigma^3 + o_9$ and $\varsigma^4 + o_9$ are shown in Figure 2.

For these sequences we have:

$$\begin{array}{cc} \alpha(\varsigma^1 + o_9, o_4) = 11 & \alpha(\varsigma^3 + o_9, o_4) = 8 \\ \alpha(\varsigma^1 + o_9, o_8) = 8 & \alpha(\varsigma^3 + o_9, o_8) = 10, \end{array}$$

which means that none of these sequences will be dropped. However, the sequence $\varsigma^4 + o_9$ is in $\Xi(S \cup \{o_9\})$, and we have:

$$\begin{array}{c} \alpha(\varsigma^4 + o_9, o_4) = 8 \\ \alpha(\varsigma_4 + o_9, o_8) = 7. \end{array}$$

This means that $\varsigma^4 + o_9$ dominates both $\varsigma^1 + o_9$ and $\varsigma^3 + 9$. Two main conclusions can be derived from this fact. The first one is that $\overset{\triangle}{\Xi}(S \cup \{o_9\})$ is empty, which contradicts Proposition 3 in [6]. The second is that the sets $\mathcal{F}(S)$ built by the algorithm are not the sets $\overset{\triangle}{\Xi}(S)$: since ς^4 would not be expanded to $\varsigma^4 + o_9$, this last sequence would not be available for comparison, and $\varsigma^1 + o_9$ and $\varsigma^3 + o_9$ would never be dropped. Therefore, we have that $\mathcal{F}(S \cup \{o_9\}) = \{\varsigma^1 + o_9, \varsigma^3 + o_9\}$, even when $\overset{\triangle}{\Xi}(S \cup \{o_9\}) = \emptyset$.

As we commented above, this example invalidates the course of action taken in [6]. However, the central ideas exposed there are still useful for proving the correctness of the algorithm, as proved in the next section.

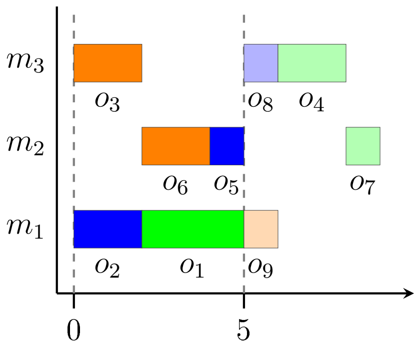
5. The new proof

Even though the proof provided in [6] is not correct, the algorithm does indeed provide an optimal solution. In this section we provide a new proof for the correctness of this algorithm.

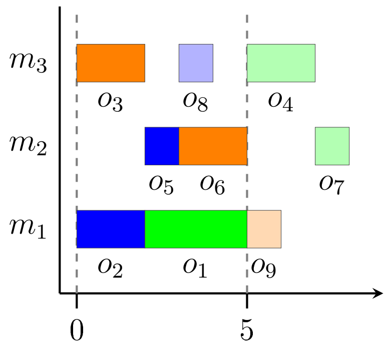
As we have seen above it is possible that a partial solution ς^1, in particular a partial solution of an optimal solution, can be dominated by another partial solution ς^2 which does not have an ordered completion which produces at least the same makespan as the best completion of the dominated solution. We can show such a solution exist by adding any completion of the dominated solution ς^1 to the dominating partial solution ς^2. When this schedule is converted to a non-idle schedule due to the domination criteria the makespan is at least the same or better. However, the ordered sequence of such a solution is possibly not a completion of the dominating solution ς^2.

To prove that an optimal solution is found we show that not all optimal solutions can be dominated this way and an optimal solution must be found by the Dynamic Programming algorithm. To show this we need to establish a few extra properties of the state space of the Dynamic Programming algorithm.

Let ς^1_O be an complete solution that is not found by the Dynamic Programming algorithm. Than there must be a partial solution ς^1_S of ς^1_O that is dominated by another partial solution ς^2_S with the same set of operations S. When ς^1_S is dominated by ς^2_S we can distinguish two cases for any completion ς^1_O of ς^1_S. Let ς^2_O be the ordered sequence of the schedule created by adding all operations of the completion ς^1_O of ς^1_S to the dominating solution ς^2_S. We call the solution ς^2_O *welded* from ς^2_S and the completion to ς^1_O of ς^1_S. Now we can distinguish two cases for ς^2_O



(A) Schedule of $o_2o_3o_6o_1o_5$ with completion $o_9o_8o_4o_7$



(B) Schedule of $o_2o_3o_5o_1o_6$ where the completion of fig. 3a is welded

FIGURE 3. Indirect domination where operation o_8 of a completion is scheduled before the last operation of the dominating sequence

I. The welded sequence $\varsigma^2_{\mathcal{O}}$ starts with the sequence ς^2_S. This implies that the operations of the completion from ς^1_S to $\varsigma^1_{\mathcal{O}}$ can be added after ς^2_S in an ordered way: otherwise, an operation o should be inserted *before* the last operation in ς^2_S, and the first $|S|$ operations of $\varsigma^2_{\mathcal{O}}$ would not be equal to ς^2_S. We call this *direct* domination. Note that the order of the operations in the completion may differ.

II. The welded sequence $\varsigma^2_{\mathcal{O}}$ does not start with the sequence represented by the partial solution ς^2_S. This implies that at least one operation $o \in \mathcal{O} \setminus S$ in schedule of $\varsigma^2_{\mathcal{O}}$ is advanced such that this operation occurs in the ordered sequence before the last operation $\Lambda(\varsigma^2_S)$ of the sequence represented by ς^2_S. This implies that $\alpha(\varsigma^2_S, o) = C_{\max}(\varsigma^2_S) + p(o)$ as otherwise the expansion of o could be done in an ordered way. In this case, solution $\varsigma^2_{\mathcal{O}}$ cannot be produced by successive expansions of ς^2_S, but by expanding another solution ς^3, that is not comparable to ς^1_S at stage $|S|$. We call this *indirect* domination.

Figure 3 shows an example of two partial solutions where the partial solution $o_2o_3o_6o_1o_5$ in fig. 3a is dominated by the partial solution $o_2o_3o_5o_1o_6$ in fig. 3b. Also a completion is shown where the domination is indirect as can be seen in fig. 3a where operation o_8 should be in the sequence before operations o_1 and o_6. The partial solution $o_2o_3o_5o_8o_1$ leading to the complete solution in fig. 3b would be the partial solution belonging to the same stage as the two partial solutions depicted.

When we have indirect domination (Case II) we can deduce some special properties.

Proposition 5.1. *If we have indirect domination between ς^1 and ς^2 as described in Case II, there is at least an operation $o \in \mathcal{O} \setminus S$ that is scheduled in the welded solution $\varsigma^2_{\mathcal{O}}$ such that o is finished in $\varsigma^2_{\mathcal{O}}$ before it is scheduled to start in $\varsigma^1_{\mathcal{O}}$.*

Proof. As we have indirect domination there is at least one operation o that is scheduled in $\varsigma^2_{\mathcal{O}}$ before $\Lambda(\varsigma^2)$. As operation o could not be scheduled as expansion of ς^2_S leading to an ordered schedule we have the following

$$\psi(\varsigma^1_{\mathcal{O}}, o) + p(o) \geq \alpha(\varsigma^1_S, o) \geq \alpha(\varsigma^2_S, o) = C_{\max}(\varsigma^2_S) + p(o).$$

From this we can conclude that

$$\psi(\varsigma^1_{\mathcal{O}}, o) \geq C_{\max}(\varsigma^2_S) = \mathcal{O}(\varsigma^2_{\mathcal{O}}, \Lambda(\varsigma^2)) + p(\Lambda(\varsigma^2)) \geq \psi(\varsigma^2_{\mathcal{O}}, o) + p(o).$$

$\square$

Corollary 5.1. *Operation o of Proposition 5.1 can be scheduled twice in $\varsigma_{\mathcal{O}}^2$ with a makespan equal or lower as that of $\varsigma_{\mathcal{O}}^1$.*

Proof. On one hand, operation o of Proposition 5.1 can be scheduled after $C_{\max}(\varsigma_S^2)$. On the other hand, it can be scheduled in the ordered sequence such that is finished before $C_{\max}(\varsigma_S^2)$. Therefore operation o can be scheduled twice consecutively in $\varsigma_{\mathcal{O}}^2$. $\square$

We have seen that all operations of a completion of a dominated solution can be scheduled at the same time or earlier in the schedule of a dominating solution. We can also deduce another important property of domination, which considers not the operations individually but the location within the sequence. For this we denote with $\varsigma[i]$ the i-th operation of the sequence ς and we denote with $C_o(\varsigma)$ the finish time of operation o in solution ς.

Proposition 5.2. *Let partial solution ς_S^1 of solution $\varsigma_{\mathcal{O}}^1$ be dominated in stage $i = |S|$ by solution ς_S^2. Let $\varsigma_{\mathcal{O}}^2$ be the solution welded from the completion of ς_S^1 to $\varsigma_{\mathcal{O}}^1$ and ς_S^2. Then we have that for any $j > i = |S|$ that $C_{\varsigma_{\mathcal{O}}^2[j]}(\varsigma_{\mathcal{O}}^2) \leq C_{\varsigma_{\mathcal{O}}^1[j]}(\varsigma_{\mathcal{O}}^1)$.*

Proof. When the completion from $\varsigma_{\mathcal{O}}^1$ to ς_S^1 is scheduled (possibly idle) at the times of $\varsigma_{\mathcal{O}}^1$ after ς_S^2 the proposition trivially holds. When this schedule is converted to a non-idle schedule operations are only moved backward in time. If this conversion is done in unit steps at the time it can be easily seen that the condition holds after each step. When an operation is moved backward by 1 without changing the order of operations the proposition naturally holds. When two operations must be switched to keep the ordering they have the same finish time just before the second operation is moved backward so the order of the operations can be changed without changing any finish time at any index. So at each index $j > i$ the finish time can only decrease. $\square$

When a partial solution ς_S^1 is dominated by ς_S^2 we have the guarantee that for each completion $\varsigma_{\mathcal{O}}^1$ another (welded) solution $\varsigma_{\mathcal{O}}^2$ with equal or lower makespan exists, however, we do not yet have the guarantee that such a solution is found. It is possible, with indirect domination, that a dominating solution ς_S^2 did not have an ordered completion with equal or lower makespan as $\varsigma_{\mathcal{O}}^1$. To show that we cannot dominate all optimal solutions we need the following proposition.

Proposition 5.3. *Let be $\varsigma_{\mathcal{O}}^1$ a solution and let its partial solution ς_S^1 be dominated indirectly by ς_S^2 in stage $i = |S|$. Let $\varsigma_{\mathcal{O}}^2$ be the solution welded from ς_S^2 and the expansion of ς_S^1 to $\varsigma_{\mathcal{O}}^1$. Now let for $k \geq 2$ $\varsigma_{S_k}^k$ be a partial solution of $\varsigma_{\mathcal{O}}^k$ that is directly or indirectly dominated by another partial solution $\varsigma_{S_k}^{k+1}$. Let $\varsigma_{\mathcal{O}}^{k+1}$ be the solution welded from $\varsigma_{S_k}^{k+1}$ and the expansion from $\varsigma_{S_k}^k$ to $\varsigma_{\mathcal{O}}^k$. When all dominations occur at or before stage i, thus $|S_k| \leq i$, we have $\varsigma_{\mathcal{O}}^k \neq \varsigma_{\mathcal{O}}^1$ for all welded solutions with $k \geq 2$.*

Proof. Since ς_S^2 dominates ς_S^1 indirectly there exist an operation $o \in \mathcal{O} \backslash S$ that is scheduled in $\varsigma_{\mathcal{O}}^2$ before the last operation $\Lambda(\varsigma_S^2)$. This operation o is scheduled in $\varsigma_{\mathcal{O}}^1$ such that $\psi(\varsigma_{\mathcal{O}}^1, o) \geq C_{\Lambda(\varsigma_S^2)}(\varsigma_S^2)$. First we conclude that the index of $\Lambda(\varsigma_S^2)$ is at least $i+1$ in $\varsigma_{\mathcal{O}}^2$. Using Proposition 5.2 and the fact that all dominations occur before stage $i+1$ we conclude that for all solutions $\varsigma_{\mathcal{O}}^k$ with $k \geq 2$ we have for operation $\varsigma_{\mathcal{O}}^k[i+1]$ that $C_{\varsigma_{\mathcal{O}}^k[i+1]}(\varsigma_{\mathcal{O}}^k) \leq C_{\Lambda(\varsigma_S^2)}(\varsigma_{\mathcal{O}}^2)$. When $C_o(\varsigma_{\mathcal{O}}^k) \leq C_{\Lambda(\varsigma_S^2)}(\varsigma_{\mathcal{O}}^2)$ we can conclude that $C_o(\varsigma_{\mathcal{O}}^{k+1}) \leq C_{\Lambda(\varsigma_S^2)}(\varsigma_{\mathcal{O}}^2)$. When $o \notin S_k$ this follows directly from the domination and when $o \in S_k$ this follows from the fact that we have an ordered sequence, $|S_k| \leq i$ and that $C_{\varsigma_{\mathcal{O}}^k[i+1]}(\varsigma_{\mathcal{O}}^k) \leq C_{\Lambda(\varsigma_S^2)}(\varsigma_{\mathcal{O}}^2)$. So in all solutions $\varsigma_{\mathcal{O}}^k$ with $k \geq 2$ we have that operation o finishes before it even starts in $\varsigma_{\mathcal{O}}^1$, and therefore $\varsigma_{\mathcal{O}}^k \neq \varsigma_{\mathcal{O}}^1$. □

Corollary 5.2. *When partial solution ς_S^1 of solution $\varsigma_{\mathcal{O}}^1$ is dominated in the DP algorithm before the last stage in stage $i = |S|$ ($i < |\mathcal{O}|$) there exists a partial solution in stage $i+1$ with a completion with a makespan not higher as $\varsigma_{\mathcal{O}}^1$.*

Proof. We have two cases:

(a) If ς_S^2 dominates $\varsigma_{\mathcal{O}}^1$ directly, the expansion of ς_S^2 with the first operation of the completion from ς_S^1 to $\varsigma_{\mathcal{O}}^1$ is ordered, and then the algorithm will perform the expansion, generating a partial sequence of $\varsigma_{\mathcal{O}}^2$ at stage $i+1$, which concludes the proof.

(b) If the domination is indirect, we consider now the sequence $\varsigma_{\mathcal{O}}^2$: if this sequence is generated, it is clear that, in particular, the subsequence of $\varsigma_{\mathcal{O}}^2$ containing the operations $\varsigma_{\mathcal{O}}^2[1], \ldots, \varsigma_{\mathcal{O}}^2[i+1]$ is built by the algorithm, and the result follows. On the other hand, if $\varsigma_{\mathcal{O}}^2$ is not generated, some partial sequence $\varsigma_{S_2}^2$ of $\varsigma_{\mathcal{O}}^2$ is dominated by some $\varsigma_{S_2}^3$. Iterating this process we find a chain of sequences $\varsigma_{\mathcal{O}}^k$ such that $\varsigma_{S_k}^{k+1}$ dominates $\varsigma_{S_k}^k$, as in Proposition 5.3. As all sequences $\varsigma_{\mathcal{O}}^k$ have a makespan not higher as $\varsigma_{\mathcal{O}}^1$ when any of the dominations in the chain occur in stage $i+1$ or higher the result follows. On the other hand when all dominations occur in stage i or lower there must exist a cycle in the chain of welded sequences $\varsigma_{\mathcal{O}}^k$ since there exists only a finite number of solutions. In order to prove that there is no cycle at all, we argue by contradiction: Let us assume a cycle in a chain of sequences as in Proposition 5.3 with all dominations in stage i or lower. Without loss of generality let $j \leq i$ be the largest stage where any domination occurs in this cycle. Any domination in this cycle at stage j is indirect as otherwise a partial solution of the dominating solution exists in stage $j+1$. Now observe such indirect domination, then Proposition 5.3 can be applied as all the dominations occur at stage j or lower. This directly leads to a contradiction with the existence of this cycle, and the results follows.

□

With these ingredients we can prove that the DP algorithm finds an optimal solution

Proposition 5.4. *The DP algorithm described in* [6] *finds an optimal solution for the job-shop scheduling problem.*

Proof. Suppose an optimal solution $\varsigma_{\mathcal{O}}^1$ is dominated, then there is a partial solution ς_S^1 of $\varsigma_{\mathcal{O}}^1$ that is dominated in stage $i = |S|$ by another partial solution ς_S^2. If $i < |\mathcal{O}|$ Corollary 5.2 provides a partial solution in stage $i+1$ with an optimal completion. Using this iteratively this provides an optimal solution in stage $|\mathcal{O}|$ where it can only be dominated directly by another optimal solution. So the DP algorithm described in [6] provides an optimal solution for the job-shop scheduling problem. □

References

[1] Amirghasemi, M., Zamani, R.; *A synergetic combination of small and large neighborhood schemes in developing an effective procedure for solving the job shop scheduling problem*; SpringerPlus (2014); **3**:193.

[2] Borissova, D., Mostakerov, I. ; *Open job shop scheduling via enumerative combinatorics*; International Journal of Mathematical Models and Methods in Applied Sciences (2015); 9:120-127.

[3] Brzeczek, T., Nowak, D.; *Genetic algorithm modification for production scheduling*; Foundations of Computing and Decision Sciences. **38**:4, (2013) pp. 299–309, doiI: 10.2478/fcds-2013-0015.

[4] Garey, M., Johnson, D.; *Computers and intractability: a guide to the theory of NP-Completeness*; W. H. Freeman and Company (1989).

[5] Ghiani, G., Grieco, A., Guerrieri, A., Manni, A., Manni, E.; *Large-scale assembly job shop scheduling problems with bill of materials: models and algorithms*; WSEAS Trans. on business and economics, Vol. 12 (2015); pp. 161-172.

[6] Gromicho, J., van Hoorn, J., Saldanha da Gama, F., Timmer, G.; *Solving the job-shop scheduling problem optimally by dynamic programming*; Computers and Operations Research (2012); doi: 10.1016/j.cor.2012.02.024.

[7] Held, M., Karp, R.; *A dynamic programming approach to sequencing problems*; J. of the S.I.A.M., **10**:1 (1962), 162-210.

[8] Sahraein, R., Namkshenas, M.; *On the optimal modeling and evaluation of job shops with a total weighted tardiness objective: Constraint programming vs. mixed integer programming*; Applied Mathematical Modeling, (2014); doi: 10.1016/j.apm.2014.07.032

[9] Sprecher, A., Kolisch, R., Drexl A.; *Semi-active, active and non-delay schedules for the resource-constrained proyect scheduling problem* (1995); Eur. J. of Op. Res., 80, p. 94-102.

[10] Tsai, J.T., Fang J.S., Chou J.H.; *Optimized Task Scheduling and Resource Allocation on Cloud Computing Environment Using Improved Differential Evolution Algorithm*; Computers and Operations Research (2013); doi:10.1016/j.cor.2013.06.012.

[11] Thammanoa, A., Teekeng, W.; *A modified genetic algorithm with fuzzy roulette wheel selection for job shop scheduling problems*; Int. Jour. of General Systems (2014); doi: 10.1080/03081079.2014.969252.

Research Memoranda of the Faculty of Economics and Business Administration

2011

2011-1	Yoshifumi Takahashi Peter Nijkamp	Multifunctional agricultural land use in sustainable world, 25 p.
2011-2	Paulo A.L.D. Nunes Peter Nijkamp	Biodiversity: Economic perspectives, 37 p.
2011-3	Eric de Noronha Vaz Doan Nainggolan Peter Nijkamp Marco Painho	A complex spatial systems analysis of tourism and urban sprawl in the Algarve, 23 p.
2011-4	Karima Kourtit Peter Nijkamp	Strangers on the move. Ethnic entrepreneurs as urban change actors, 34 p.
2011-5	Manie Geyer Helen C. Coetzee Danie Du Plessis Ronnie Donaldson Peter Nijkamp	Recent business transformation in intermediate-sized cities in South Africa, 30 p.
2011-6	Aki Kangasharju Christophe Tavéra Peter Nijkamp	Regional growth and unemployment. The validity of Okun's law for the Finnish regions, 17 p.
2011-7	Amitrajeet A. Batabyal Peter Nijkamp	A Schumpeterian model of entrepreneurship, innovation, and regional economic growth, 30 p.
2011-8	Aliye Ahu Akgün Tüzin Baycan Levent Peter Nijkamp	The engine of sustainable rural development: Embeddedness of entrepreneurs in rural Turkey, 17 p.
2011-9	Aliye Ahu Akgün Eveline van Leeuwen Peter Nijkamp	A systemic perspective on multi-stakeholder sustainable development strategies, 26 p.
2011-10	Tibert Verhagen Jaap van Nes Frans Feldberg Willemijn van Dolen	Virtual customer service agents: Using social presence and personalization to shape online service encounters, 48 p.
2011-11	Henk J. Scholten Maarten van der Vlist	De inrichting van crisisbeheersing, de relatie tussen besluitvorming en informatievoorziening. Casus: Warroom project Netcentrisch werken bij Rijkswaterstaat, 23 p.
2011-12	Tüzin Baycan Peter Nijkamp	A socio-economic impact analysis of cultural diversity, 22 p.
2011-13	Aliye Ahu Akgün Tüzin Baycan Peter Nijkamp	Repositioning rural areas as promising future hot spots, 22 p.
2011-14	Selmar Meents	How sellers can stimulate purchasing in electronic marketplaces: Using

	Tibert Verhagen Paul Vlaar	information as a risk reduction signal, 29 p.
2011-15	Aliye Ahu Gülümser Tüzin Baycan-Levent Peter Nijkamp	Measuring regional creative capacity: A literature review for rural-specific approaches, 22 p.
2011-16	Frank Bruinsma Karima Kourtit Peter Nijkamp	Tourism, culture and e-services: Evaluation of e-services packages, 30 p.
2011-17	Peter Nijkamp Frank Bruinsma Karima Kourtit Eveline van Leeuwen	Supply of and demand for e-services in the cultural sector: Combining top-down and bottom-up perspectives, 16 p.
2011-18	Eveline van Leeuwen Peter Nijkamp Piet Rietveld	Climate change: From global concern to regional challenge, 17 p.
2011-19	Eveline van Leeuwen Peter Nijkamp	Operational advances in tourism research, 25 p.
2011-20	Aliye Ahu Akgün Tüzin Baycan Peter Nijkamp	Creative capacity for sustainable development: A comparative analysis of European and Turkish rural regions, 18 p.
2011-21	Aliye Ahu Gülümser Tüzin Baycan-Levent Peter Nijkamp	Business dynamics as the source of counterurbanisation: An empirical analysis of Turkey, 18 p.
2011-22	Jessie Bakens Peter Nijkamp	Lessons from migration impact analysis, 19 p.
2011-23	Peter Nijkamp Galit Cohen-blankshtain	Opportunities and pitfalls of local e-democracy, 17 p.
2011-24	Maura Soekijad Irene Skovgaard Smith	The ‘lean people’ in hospital change: Identity work as social differentiation, 30 p.
2011-25	Evgenia Motchenkova Olgerd Rus	Research joint ventures and price collusion: Joint analysis of the impact of R&D subsidies and antitrust fines, 30 p.
2011-26	Karima Kourtit Peter Nijkamp	Strategic choice analysis by expert panels for migration impact assessment, 41 p.
2011-27	Faroek Lazrak Peter Nijkamp Piet Rietveld Jan Rouwendal	The market value of listed heritage: An urban economic application of spatial hedonic pricing, 24 p.
2011-28	Peter Nijkamp	Socio-economic impacts of heterogeneity among foreign migrants: Research and policy challenges, 17 p.

2011-29	Masood Gheasi Peter Nijkamp	Migration, tourism and international trade: Evidence from the UK, 8 p.
2011-30	Karima Kourtit Peter Nijkamp Eveline van Leeuwen Frank Bruinsma	Evaluation of cyber-tools in cultural tourism, 24 p.
2011-31	Cathy Macharis Peter Nijkamp	Possible bias in multi-actor multi-criteria transportation evaluation: Issues and solutions, 16 p.
2011-32	John Steenbruggen Maria Teresa Borzacchiello Peter Nijkamp Henk Scholten	The use of GSM data for transport safety management: An exploratory review, 29 p.
2011-33	John Steenbruggen Peter Nijkamp Jan M. Smits Michel Grothe	Traffic incident management: A common operational picture to support situational awareness of sustainable mobility, 36 p.
2011-34	Tüzin Baycan Peter Nijkamp	Students' interest in an entrepreneurial career in a multicultural society, 25 p.
2011-35	Adele Finco Deborah Bentivoglio Peter Nijkamp	Integrated evaluation of biofuel production options in agriculture: An exploration of sustainable policy scenarios, 16 p.
2011-36	Eric de Noronha Vaz Pedro Cabral Mário Caetano Peter Nijkamp Marco Paínho	Urban heritage endangerment at the interface of future cities and past heritage: A spatial vulnerability assessment, 25 p.
2011-37	Maria Giaoutzi Anastasia Stratigea Eveline van Leeuwen Peter Nijkamp	Scenario analysis in foresight: AG2020, 23 p.
2011-38	Peter Nijkamp Patricia van Hemert	Knowledge infrastructure and regional growth, 12 p.
2011-39	Patricia van Hemert Enno Masurel Peter Nijkamp	The role of knowledge sources of SME's for innovation perception and regional innovation policy, 27 p.
2011-40	Eric de Noronha Vaz Marco Painho Peter Nijkamp	Impacts of environmental law and regulations on agricultural land-use change and urban pressure: The Algarve case, 18 p.
2011-41	Karima Kourtit Peter Nijkamp Steef Lowik Frans van Vught Paul Vulto	From islands of innovation to creative hotspots, 26 p.

2011-42	Alina Todiras Peter Nijkamp Saidas Rafijevas	Innovative marketing strategies for national industrial flagships: Brand repositioning for accessing upscale markets, 27 p.
2011-43	Eric de Noronha Vaz Mário Caetano Peter Nijkamp	A multi-level spatial urban pressure analysis of the Giza Pyramid Plateau in Egypt, 18 p.
2011-44	Andrea Caragliu Chiara Del Bo Peter Nijkamp	A map of human capital in European cities, 36 p.
2011-45	Patrizia Lombardi Silvia Giordano Andrea Caragliu Chiara Del Bo Mark Deakin Peter Nijkamp Karima Kourtit	An advanced triple-helix network model for smart cities performance, 22 p.
2011-46	Jessie Bakens Peter Nijkamp	Migrant heterogeneity and urban development: A conceptual analysis, 17 p.
2011-47	Irene Casas Maria Teresa Borzacchiello Biagio Ciuffo Peter Nijkamp	Short and long term effects of sustainable mobility policy: An exploratory case study, 20 p.
2011-48	Christian Bogmans	Can globalization outweigh free-riding? 27 p.
2011-49	Karim Abbas Bernd Heidergott Djamil Aïssani	A Taylor series expansion approach to the functional approximation of finite queues, 26 p.
2011-50	Eric Koomen	Indicators of rural vitality. A GIS-based analysis of socio-economic development of the rural Netherlands, 17 p.
2012-1	Aliye Ahu Gülümser Tüzin Baycan Levent Peter Nijkamp Jacques Poot	The role of local and newcomer entrepreneurs in rural development: A comparative meta-analytic study, 39 p.

2012

2012-2	Joao Romao Bart Neuts Peter Nijkamp Eveline van Leeuwen	Urban tourist complexes as Multi-product companies: Market segmentation and product differentiation in Amsterdam, 18 p.
2012-3	Vincent A.C. van den Berg	Step tolling with price sensitive demand: Why more steps in the toll makes the consumer better off, 20 p.
2012-4	Vasco Diogo Eric Koomen Floor van der Hilst	Second generation biofuel production in the Netherlands. A spatially-explicit exploration of the economic viability of a perennial biofuel crop, 12 p.

2012-5	Thijs Dekker Paul Koster Roy Brouwer	Changing with the tide: Semi-parametric estimation of preference dynamics, 50 p.
2012-6	Daniel Arribas Karima Kourtit Peter Nijkamp	Benchmarking of world cities through self-organizing maps, 22 p.
2012-7	Karima Kourtit Peter Nijkamp Frans van Vught Paul Vulto	Supernova stars in knowledge-based regions, 24 p.
2012-8	Mediha Sahin Tüzin Baycan Peter Nijkamp	The economic importance of migrant entrepreneurship: An application of data envelopment analysis in the Netherlands, 16 p.
2012-9	Peter Nijkamp Jacques Poot	Migration impact assessment: A state of the art, 48 p.
2012-10	Tibert Verhagen Anniek Nauta Frans Feldberg	Negative online word-of-mouth: Behavioral indicator or emotional release? 29 p.

2013

2013-1	Tüzin Baycan Peter Nijkamp	The migration development nexus: New perspectives and challenges, 22 p.
2013-2	Haralambie Leahu	European Options Sensitivities via Monte Carlo Techniques, 28 p.
2013-3	Tibert Verhagen Charlotte Vonkeman Frans Feldberg Plon Verhagen	Making online products more tangible and likeable: The role of local presence as product presentation mechanism, 44 p.
2013-4	Aliye Ahu Akgün Eveline van Leeuwen Peter Nijkamp	A Multi-actor multi-criteria scenario analysis of regional sustainable resource policy, 24 p.
2013-5	John Steenbruggen Peter Nijkamp Maarten van der Vlist	Urban traffic incident management in a digital society. An actor-network approach in information technology use in urban Europe, 25 p.
2013-6	Jorge Ridderstaat Robertico Croes Peter Nijkamp	The force field of tourism, 19 p.
2013-7	Masood Gheasi Peter Nijkamp Piet Rietveld	Unknown diversity: A study on undocumented migrant workers in the Dutch household sector, 17 p.
2013-8	Mediha Sahin Peter Nijkamp Soushi Suzuki	Survival of the fittest among migrant entrepreneurs. A study on differences in the efficiency performance of migrant entrepreneurs in Amsterdam by means of data envelopment analysis, 25 p.

2013-9	Kostas Bithas Peter Nijkamp	Biological integrity as a prerequisite for sustainable development: A bioeconomic perspective, 24 p.
2013-10	Madalina-Stefania Dirzu Peter Nijkamp	The dynamics of agglomeration processes and their contribution to regional development across the EU, 19 p.
2013-11	Eric de Noronha Vaz Agnieszka Walczynska Peter Nijkamp	Regional challenges in tourist wetland systems: An integrated approach to the Ria Formosa area, 17 p.
2013-12	João Romão Eveline van Leeuwen Bart Neuts Peter Nijkamp	Tourist loyalty and urban e-services: A comparison of behavioural impacts in Leipzig and Amsterdam, 19 p.
2013-13	Jorge Ridderstaat Marck Oduber Robertico Croes Peter Nijkamp Pim Martens	Impacts of seasonal patterns of climate on recurrent fluctuations in tourism demand. Evidence from Aruba, 34 p.
2013-14	Emmanouil Tranos Peter Nijkamp	Urban and regional analysis and the digital revolution: Challenges and opportunities, 16 p.
2013-15	Masood Gheasi Peter Nijkamp Piet Rietveld	International financial transfer by foreign labour: An analysis of remittances from informal migrants, 11 p.
2013-16	Serenella Sala Biagio Ciuffo Peter Nijkamp	A meta-framework for sustainability assessment, 24 p.
2013-17	Eveline van Leeuwen Peter Nijkamp Aliye Ahu Akgün Masood Gheasi	Foresights, scenarios and sustainable development – a pluriformity perspective, 19 p.
2013-18	Aliye Ahu Akgün Eveline van Leeuwen Peter Nijkamp	Analytical support tools for sustainable futures, 19 p.
2013-19	Peter Nijkamp	Migration impact assessment: A review of evidence-based findings, 29 p.
2013-20	Aliye Ahu Akgün Eveline van Leeuwen Peter Nijkamp	Sustainability science as a basis for policy evaluation, 16 p.
2013-21	Vicky Katsoni Maria Giaoutzi Peter Nijkamp	Market segmentation in tourism – An operational assessment framework, 28 p.
2013-22	Jorge Ridderstaat Robertico Croes Peter Nijkamp	Tourism development, quality of life and exogenous shocks. A systemic analysis framework, 26 p.

2013-23	Feng Xu Nan Xiang Shanshan Wang Peter Nijkamp Yoshiro Higano	Dynamic simulation of China's carbon emission reduction potential by 2020, 12 p.
2013-24	John Steenbruggen Peter Nijkamp Jan M. Smits Ghaitrie Mohabir	Traffic incident and disaster management in the Netherlands: Challenges and obstacles in information sharing, 30 p.
2013-25	Patricia van Hemert Peter Nijkamp Enno Masurel	From innovation to commercialization through networks and agglomerations: Analysis of sources of innovation, innovation capabilities and performance of Dutch SMEs, 24 p.
2013-26	Patricia van Hemert Peter Nijkamp Enno Masurel	How do SMEs learn in a systems-of-innovation context? The role of sources of innovation and absorptive capacity on the innovation performance of Dutch SMEs, 27 p.
2013-27	Mediha Sahin Alina Todiras Peter Nijkamp	Colourful entrepreneurship in Dutch cities: A review and analysis of business performance, 25 p.
2013-28	Tüzin Baycan Mediha Sahin Peter Nijkamp	The urban growth potential of second-generation migrant entrepreneurs. A sectoral study on Amsterdam, 31 p.
2013-29	Eric Vaz Teresa de Noronha Vaz Peter Nijkamp	The architecture of firms' innovative behaviors, 23 p.
2013-30	Eric Vaz Marco Painho Peter Nijkamp	Linking agricultural policies with decision making: A spatial approach, 21 p.
2013-31	Yueting Guo Hengwei Wang Peter Nijkamp Jiangang XU	Space-time changes in interdependent urban-environmental systems: A policy study on the Huai River Basin in China, 20 p.
2013-32	Maurice de Kleijn Niels van Manen Jan Kolen Henk Scholten	User-centric SDI framework applied to historical and heritage European landscape research, 31 p.
2013-33	Erik van der Zee Henk Scholten	Application of geographical concepts and spatial technology to the Internet of Things, 35 p.
2013-34	Mehmet Güney Celbiş Peter Nijkamp Jacques Poot	The lucrative impact of trade-related infrastructure: Meta-Analytic Evidence, 45 p.
2013-35	Marco Modica Aura Reggiani Peter Nijkamp	Are Gibrat and Zipf Monozygotic or Heterozygotic Twins? A Comparative Analysis of Means and Variances in Complex Urban Systems, 34 p.

2013-36	Bernd Heidergott Haralambie Leahu Warren Volk-Makarewicz	A Smoothed Perturbation Analysis Approach to Parisian Options, 14 p.
2013-37	Peter Nijkamp Waldemar Ratajczak	The Spatial Economy – A Holistic Perspective, 14 p.
2013-38	Karima Kourtit Peter Nijkamp Eveline van Leeuwen	New Entrepreneurship in Urban Diasporas in our Modern World, 22 p.

2014

2014-1	John Steenbruggen Emmanouil Tranos Peter Nijkamp	Data from mobile phone operators: A tool for smarter cities? 22 p.
2014-2	John Steenbruggen	Tourism geography: Emerging trends and initiatives to support tourism in Morocco, 29 p.

2015

2015-1	Maurice de Kleijn Rens de Hond Oscar Martinez-Rubi Pjotr Svetachov	A 3D Geographic Information System for 'Mapping the Via Appia', 11 p.
2015-2	Gilberto Mahumane Peter Mulder	Introducing MOZLEAP: an integrated long-run scenario model of the emerging energy sector of Mozambique, 35 p.
2015-3	Karim Abbas Joost Berkhout Bernd Heidergott	A Critical Account of Perturbation Analysis of Markovian Systems, 28 p.
2015-4	Nahom Ghebrihiwet Evgenia Motchenkova	Technology Transfer by Foreign Multinationals, Local Investment, and FDI Policy, 31 p.
2015-5	Yannis Katsoulacos Evgenia Motchenkova David Ulph	Penalizing Cartels: The Case for Basing Penalties on Price Overcharge, 43 p.
2015-6	John Steenbruggen Emmanouil Tranos Piet Rietveld †	Can Motorway Traffic Incidents be detected by Mobile Phone Usage Data? 21 p.
2015-7	Gilberto Mahumane Peter Mulder	Mozambique Energy Outlook, 2015-2030. Data, Scenarios and Policy Implications. 47 p.
2015-8	John Steenbruggen Maarten Krieckaert Piet Rietveld † Henk Scholten Maarten van der Vlist	The Usefulness of Net-Centric Support Tools for Traffic Incident Management, 32 p.

2015-9	Jelke J. van Hoorn Agustín Nogueira Ignacio Ojea Joaquim A.S. Gromicho	A note on the paper: Solving the job-shop scheduling problem optimally by dynamic programming, 12 p.