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# Serie research memoranda

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# Unemployment dynamics, and the propagation of aggregate and reallocation shocks using flow data for the Netherlands

Frank A.G. den Butter<sup>‡</sup>, Kees van Montfort <sup>◊</sup> and Gerben T.J. Weitenberg<sup>§</sup>

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## Abstract

This paper models the propagation at the macro level of four types of shocks using the SVAR approach. Time series data for the Netherlands on job creation, job destruction, the number of vacancies and labour supply are used to identify aggregate demand and supply shocks, and reallocation demand and supply shocks as different sources of unemployment dynamics. Each of these four types of shocks appears to have at least some influence on (changes) in unemployment both on the short and on the long run, although the long run influence of the aggregate labour supply shock is estimated to be very limited.

**Keywords:** SVAR model, labour market flows, aggregate and reallocation shocks

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## 1. Introduction

The traditional models of the real business cycle theory make a distinction between two types of shocks, namely aggregate demand and supply (or technology) shocks (Kydland and Prescott, 1982; Long and Plosser, 1993). The aggregate demand shocks are assumed to have a temporary influence on economic activity, whereas aggregate supply shocks are identified as permanent shocks. In this way both types of shocks represent the classical dichotomy between cyclical and structural developments in the economy. Following the seminal work of Blanchard and Quah (1989) a number of empirical studies have investigated the sources and propagation of these aggregate demand and supply shocks using the methodology of structural vector autoregressive models (SVAR models).

However, new developments in equilibrium search theory (Pissarides, 1988, 1990; Mortensen, 1986) and in the flow approach to the labour market (see e.g. Blanchard and Diamond, 1992) show that for the analysis of the sources and propagation of shocks, and of how these shocks affect the labour market, and more in particular unemployment, it does not suffice to consider aggregate shocks only. Reallocation shocks appear to play an important role as well. Against this background this paper identifies and estimates a SVAR model for unemployment dynamics, where four types of shocks are distinguished, namely aggregate demand and labour supply shocks, and reallocation shocks both at the demand and the supply side of the labour market (see Balakrishnan & Michelacci (1997) for an analysis in the same vein). Hereto we use a set of time series of data on worker and job flows for the Netherlands, which is newly constructed on the basis of administrative data sources (Broersma, Den Butter and Kock, 1998). The remainder of the paper is organized as follows. Section 2 discusses the background of the four types of shocks and their different influence on job creation and job destruction. Section 3 summarizes the construction method of the time series data. In section 4 we specify the four-variate SVAR model and discuss the identification procedure and the estimation results. Section 5 concludes.

## 2. Four types of shocks

In principle all shocks that hit a firm from the demand side can be regarded as idiosyncratic, i.e. firm specific. Yet usually idiosyncratic shocks are associated with shocks that are uncorrelated with respect to firms and, therefore, hit each firm in a different way. Due to changes in tastes and technology some firms are hit by a positive shock and are successful in expanding their production, whereas other firms are hit by an adverse shock and are bound to downsizing, or even go bankrupt. Hence, on the aggregate a compilation of such idiosyncratic shocks leads to a reallocation of production amongst the firms. Therefore at a macro level these shocks take the form as reallocation shocks. An example of such a shock is the introduction of an environmental tax, which gives fiscal advantages to firms with a clean production technology over firms with polluting production technology. A positive reallocation shock will increase the cross-sectional variance of the profitability of jobs. This leads to the creation of additional jobs in sectors and firms where the shock has a positive effect, while in firms which are negatively hit by the shock job destruction will occur. It implies that the reallocation shock will enhance both job destruction and job

creation. We will utilize this property of a reallocation shock as identifying restriction in our model.

On the other hand, the situation may occur that all idiosyncratic shocks to firms are completely correlated. In that case the shocks hit all firms in the same direction. At the macro level we then have an aggregate demand shock. A positive aggregate demand shock will result in higher profitability of jobs and therefore in an increase of job creation. Due to the enhanced economic activity jobs also remain productive over a longer time period so that job destruction decreases. Inversely, a negative aggregate demand shock will enhance job destruction and induce a decrease in job creation. So this opposite influences of an aggregate demand shock on job creation and destruction constitute our identifying restrictions of such shock in the model.

These two types of demand shocks relate to economic activity and hence to labour demand. At the supply side of the labour market we also distinguish two types of shocks. Comparable to the aggregate demand shock is an aggregate supply shock. This shock relates to an autonomous increase or decrease in labour supply. Such shock can be caused by an increase in the working age population (potential labour supply) or/and by an autonomous or policy induced change in the labour participation rate. An identification criterion which is often used for this type of labour supply shock is the assumption that it has no influence on the unemployment rate in the long run (see Balakrishnan & Michelacci, 1997). In this paper we will be less restrictive in our identification of a labour supply shock. We note that a positive labour supply shock may enhance job creation as, due to the additional labour supply, opening of a vacancy for a new job becomes less costly. Inversely, a negative labour supply shock will induce a decrease in job creation. This short-run effect on job creation may, however, vanish in the long run because changes in wages will bring the economy back to its original equilibrium. Moreover, both the short-run and the long-run effects of an aggregate labour supply shock on job destruction are ambiguous.

The fourth type of shock we distinguish can be labelled a labour productivity shock. Here, in case of a positive productivity shock, the labour force becomes, given its size, more productive. In this case we can think of the implementation of labour saving technology. The cause of such positive shock may also be an increase in the level of education of the work force. When this labour productivity shock does not coincide with an aggregate demand shock it implies that the change in productivity is not matched by an equal change in labour demand. In that case an increase in productivity will lead to a decrease in employment and *vice versa*. Therefore, in our model we take as identifying restrictions for a labour productivity shock that a positive shock leads to a decrease in job creation and an increase in job destruction, whereas a negative productivity shock is associated with an increase in job creation and a decrease in job destruction.

Table 1 summarizes our discussion of the effects of the four types of shocks on job destruction (JD) and job creation (JC) by showing the long-run effects of positive shocks.

**Table 1** Long-run effects of a positive shock on job creation and destruction, *ceteris paribus* (E = employment)

Type of shock	$\Delta \frac{JC}{E}$	$\Delta \frac{JD}{E}$
Aggregate demand shock	↑	↓
Reallocation shock	↑	↑
Labour supply shock	?	?
Labour productivity shock	↓	↑

### 3. The data

Since we distinguish between four types of shocks we need four times series in order to enable identification of these shocks. The discussion of the previous section illustrates that data on job destruction (JD) and job creation (JC) are essential for identification in our model. The net result of job creation and job destruction gives the change in the total number of jobs ( $\Delta J = \Delta JC - \Delta JD$ ). In accordance with the definition of jobs in the data set the total number of jobs is equal to employment plus the total number of vacancies ( $J = E + V$ ). Now it is essential for the identification of our model to confront these flows and stocks of jobs with flows and stocks of workers. Data on stocks of workers are obtained in a straightforward way: **labour supply** is equal to employment plus unemployment ( $LS = E + U$ ). For the calculation of job creation and job destruction in connection with worker flows we have used a data set compiled by Broersma, Den Butter and Kock (1998) which is based on administrative sources. A major feature of the construction of the data set is that the links between worker flows and job flows are exploited as much as possible. The following two definition equations summarize how job creation and job destruction are compiled in this data set.

$$JC = VI_j + F_{eej} + F_{uej} + F_{nej} \quad (3.1)$$

$$JD = (F_{eu} - VI_{eu}) + (F_{en} - VI_{en}) + (F_{eej} + F_{eev} - VI_e) + VO_n, \quad (3.2)$$

where

JC = Job creation

JD = Job destruction

$VI_j$  = new vacancies

$F_{eej}$  = Employed that find a new job for which there was no vacancy

$F_{uej}$  = Unemployed who find a new job for which there was no vacancy

$F_{nej}$  = People not participating who find a new job for which there was no vacancy

- $F_{eu}$  = Employed who become unemployed (fires and quits)  
 $VI_{eu}$  = new vacancies as a result of fires and quits  
 $F_{en}$  = Employed becoming non-participants  
 $VI_{en}$  = new vacancies as a result of employed becoming non-participants  
 $F_{eev}$  = Employed who find a new job that had a vacancy  
 $VI_e$  = new vacancies as a result of employed that find a new job  
 $v_{on}$  = Autonomous scrapping of unfilled vacancies

It must be noted that in the construction of the data set some additional assumptions are needed on flows which are not directly available from primary data sources or which follow from definition equations. However, these assumptions do not interfere with the identifying restrictions of our model which we explain in the next section. Apart from the time series on job creation and job destruction we use total labour supply (LS) and the number of vacancies (V) as data for the identification of the four types of shocks. Moreover, employment (E) is taken as a scaling variable and unemployment as the dependent variable influenced by the shocks.

## 4. The model

### 4.1 Identification

First we give a technical introduction to the SVAR-model, which we use in order to disentangle and estimate the sources and propagation of the four types of shocks mentioned before. Therefore we begin with defining the following two vectors of length  $M$ :  $Y_t$  and  $\varepsilon_t$ . Vector  $Y_t$  consists of  $M$  input variables and vector  $\varepsilon_t$  represents the shocks to which the model is submitted. Assume  $Y_t$  is a vector of linearly regular covariant-stationary stochastic processes with mean zero. Then, with the use of Wold's decomposition theorem, each element of  $Y_t$  can be represented as a process driven only by a white noise process (see e.g. Whiteman (1983)). So, we have the innovative process of  $Y_t$ , which is denoted as follows:

$$Y_t = C(L)\eta_t \quad (4.1)$$

Where  $\eta_t$  is a vector of white noise processes with  $\text{Var}[\eta_t] = \Omega$ . The lag operator  $L$  is defined as follows  $Lx_t = x_{t-1}$ .  $L(Lx_t)$  is denoted as  $L^2x_t = x_{t-2}$ . Thus,  $L^px_t = x_{t-p}$ . By convention  $L^0x_t = x_t$ .  $C(L)$  is a polynomial of infinite order and is denoted as follows:

$$C(L) = \sum_{j=0}^{\infty} C_j L^j \quad (4.2)$$

where, the coefficients  $C_j$  are  $(M \times M)$  matrices and  $C_0$  is the identity matrix.

Assume further that  $Y_t$  and  $\varepsilon_t$ , can be represented by a Moving Average (MA) process:

$$\begin{aligned}
 Y_t &= B_0\varepsilon_t + B_1\varepsilon_{t-1} + \dots + B_j\varepsilon_{t-j} + \dots \Leftrightarrow \\
 Y_t &= (B_0 + B_1L + \dots + B_jL^j + \dots)\varepsilon_t \Leftrightarrow \\
 Y_t &= \sum_{j=0}^{\infty} B_jL^j\varepsilon_t = B(L)\varepsilon_t
 \end{aligned} \quad (4.3)$$

Thus,  $B(L)$  is a polynomial matrix of infinite order. The matrix coefficients  $B_j$  are the impulse response characteristics that determine the effect of the shocks on the system. The polynomial  $B(L)$  determines the effect of the disturbances on the  $Y_t$ . Furthermore it is assumed that the structural disturbances are uncorrelated white noise processes. This last assumption may not seem very realistic at first sight. It is clear to see that many shocks have both an aggregated as a reallocation component, like it was the case with the oil price shock in the seventies. However, this assumption of orthogonality is needed for the identification of the problem discussed in this paper. Assume that the reallocation component only depends on the size of the shock and not on the direction and assume further that fundamental shocks during a long period of time consist of a random mix of positive and negative aggregated shocks. Then there is almost no correlation between the aggregated and the reallocation shocks. After normalization it follows that  $\text{Var}[\varepsilon_t]$  is equal to the identity matrix  $I$ .

With equations (4.1) and (4.3) it follows that the vector of innovations and the vector of disturbances are related like  $\eta_t = B_0 \varepsilon_t$ . Furthermore  $B(L)$  can be computed with  $B_j = C_j B_0$ ,  $j > 0$ . Hence, to solve the system of equations it is sufficient to find  $B_0$ . This can be done in the following way:

$$\Omega = \text{var}(\eta_t) = \text{var}(B_0 \varepsilon_t) = B_0 \text{var}(\varepsilon_t) B_0' = B_0 B_0' \quad (4.4)$$

In short, the procedure is as follows.

- First we estimate a vector containing an autoregressive representation of  $Y_t$ . After inversion we get equation (4.1), with vector  $\eta_t$  and matrices  $C(L)$  and  $\Omega$ .
- Then  $B_0$  is computed with (4.4). This makes it possible to compute  $B_j = C_j B_0$  (for  $j > 0$ ). The shocks on the short run can be calculated by the relation  $\varepsilon_t = (B_0)^{-1} \eta_t$ , while the shocks on the long run can be obtained by choosing the lag operator equal to 1, and calculate  $\varepsilon_t = (B(1))^{-1} Y_t$ .

In the fourvariate SVAR-model the vector  $Y_t$  will be chosen as follows:

$$Y_t = \begin{pmatrix} A \frac{JC_t}{E_t} \\ A \frac{JD_t}{E_t} \\ \Delta \frac{\Delta LS_t}{LS_t} \\ A \frac{\Delta V_t}{LS_t} \end{pmatrix} \quad (4.5)$$

We have specified the components of  $Y_t$  in such a way that they are stationary time series, while  $JC_t/E_t$ ,  $JD_t/E_t$ ,  $\Delta LS_t/LS_t$  and  $\Delta V_t/LS_t$  are not stationary or cointegrated. However, it appeared that there was a major discontinuity in the data on job destruction and job creation in 1987, namely a permanent shift to a higher level which we accounted for by including a dummy variable.

Until now the system is not yet identified. Given  $\Omega$ , equation (4.4) imposes ten restrictions on sixteen elements of  $B_0$ :



$$B_0 = \begin{bmatrix} \sigma_1 & b_{12}\sigma_2 & b_{13}\sigma_3 & b_{14}\sigma_4 \\ b_{21}\sigma_1 & \sigma_2 & b_{23}\sigma_3 & b_{24}\sigma_4 \\ b_{31}\sigma_1 & b_{32}\sigma_2 & \sigma_3 & b_{34}\sigma_4 \\ b_{41}\sigma_1 & b_{42}\sigma_2 & b_{43}\sigma_3 & \sigma_4 \end{bmatrix} \quad (4.6)$$

To acquire a fully identified system six more restrictions on the elements of  $B_0$  are needed. We will choose different values for  $b_{21}$ . The remaining five-restrictions will be chosen as follows. Assume that changes in labour supply are independent of the short-term effects of the shocks that influence the demand side of the labour market. The result of this is that the short-term effects of the aggregated and reallocation shocks on the change in the labour supply is zero. This means  $b_{31}$  and  $b_{32}$  must be equal to zero. One could, of course, argue whether this is a realistic assumption. For example, increasing labour demand as a result of positive expectations on future demand may enhance labour participation of people who first were not looking for a job in the labour market. This encouragement effect can, on the other hand, be compensated by a decrease in participation because the higher wages, which are the result of the labour demand shock, lead to some substitution between work and leisure. Empirical research for the Netherlands does at least show that wage elasticities of (male) labour supply are rather low. However, we note that our assumption of independence is rather limited: the condition is that the two demand-shocks do not influence the labour supply *in the period that the shocks occur*. Hence, no restrictions are imposed to the effects these shocks might have in future periods (on the long-run). Thus,  $b_{31}^{(k)}$  and  $b_{32}^{(k)}$   $k > 0$ , are unrestricted.

Three more restrictions are needed. Identification of the labour productivity shock occurs by assuming that the relative influence of this shock on  $\Delta(JC/E)$  and  $\Delta(JD/E)$  is in line with the effect of the reallocation shock on these quantities. Hence,  $b_{12} = b_{14}/b_{24}$ . An argument in favour of this identification is that the reallocation shock and the labour productivity shock can both be seen as shocks that influence the structure of, respectively, the production process of firms and labour supply. Furthermore it is reasonable to assume they have identical effects on job creation and destruction. The final two restrictions make use of relations between vacancy flows, job creation and job destruction, which follow from the construction method of the data. Here it is assumed (based on empirical argumentation) that the short-run effect of the aggregated and reallocation shock on the number of vacancies (standardized with total labour supply) is related to job creation and destruction in the following way:  $b_{41} = (0,33 - 0,03b_{21})$  en  $b_{42} = (0,33b_{12} - 0,03)$ .

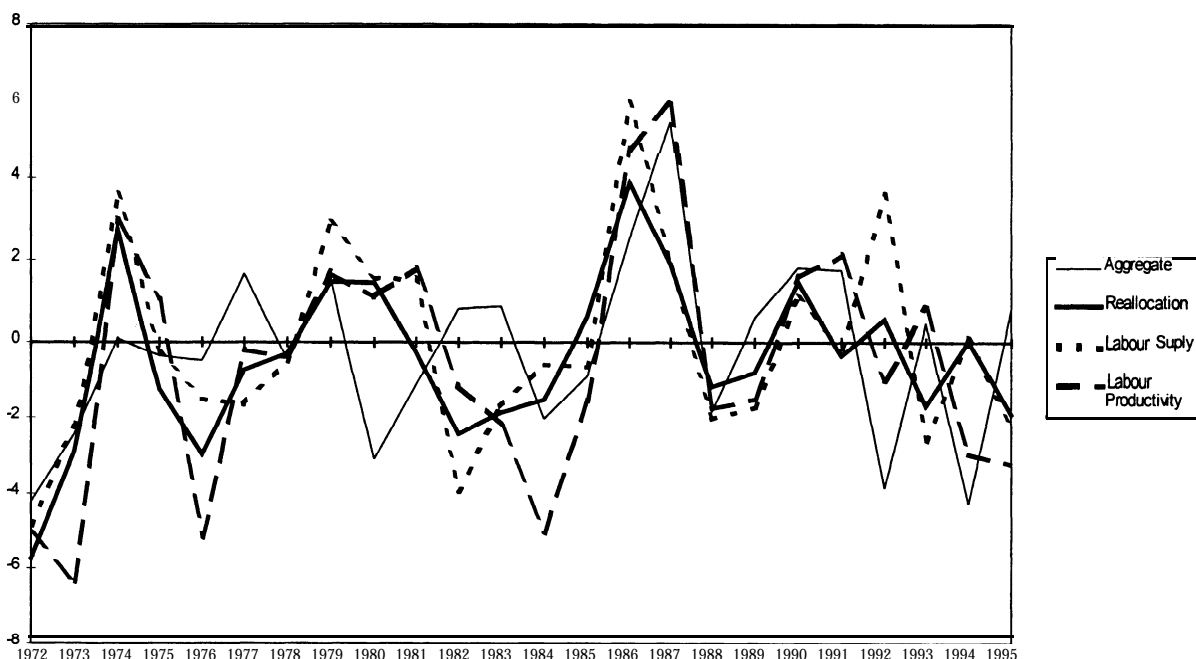
#### 4.2 Estimation

We have estimated the above specification of the SVAR with 1972 - 1995 as sample period. These estimation results give us the sizes of the four types of shocks and their propagation on the short run and on the long run to job creation, job destruction, the change in labour supply and the number of vacancies.

Now it is interesting to consider the actual sizes of the shocks. This is illustrated in figure 1. It appears that the shocks are about of equal size and that the largest negative shocks occur around 1973 and the largest positive shocks in 1987. In the latter year the data show a permanent upward shift in the pace of job destruction and job

creation, which, as mentioned before, we accounted for in the estimations by means of a dummy variable.

**Figure 1,** Time profiles of each of the four types of shocks in the reference period 1972-1995, where  $b_{21} = -30$  and where the matrix of long-run effects,  $B(l)$ , is used for the calculation of the shocks.



The next step is to determine the influence of each of the four disturbances on unemployment. As we need to difference all variables in our SVAR model in order to arrive at a vector of stationary series, we are only able to calculate the effects of the shocks on the second difference of the unemployment rate. Hereto we use the following relation between unemployment and our series from the SVAR model that holds approximately (see appendix):

$$\Delta \left( \frac{\Delta U_t}{LS_t} \right) = \left( \Delta \frac{JD_t}{E_t} - \Delta \frac{JC_t}{E_t} \right) \cdot \left( 1 - \frac{U_t}{LS_t} \right) + \Delta \frac{\Delta LS_t}{LS_t} + \Delta \frac{\Delta V_t}{LS_t} \quad (4.7)$$

Here  $\frac{U_t}{LS_t}$  is taken to be 0.06 because the unemployment rate was in the reference period on average about 6%.

Replacing in equation (4.7) the terms of the right hand side by corresponding cells of  $B_0$ , we get the short run effects of the separate shocks on changes in unemployment growth. For instance, for the short run effect of the first shock (i.e. the aggregate demand shock) we substitute  $B_0(1,1)$ ,  $B_0(2,1)$ ,  $B_0(3,1)$  and  $B_0(4,1)$  in  $JC_t/E_t$ ,  $JD_t/E_t$ ,  $\Delta LS_t/LS_t$  and  $\Delta V_t/LS_t$  respectively. Using cells of the matrix  $B(l)$ , instead of cells of the matrix  $B_0$ , we get the long run effects on changes in unemployment growth.

The short run and long run effects of the shocks on changes in unemployment growth are given in table 2. It is noticeable that in all cases the short run effects are, in absolute value, larger than the long run effects, which is no surprise as we expect

changes in unemployment growth to return to zero after some time in an equilibrium framework. So it may even come as a surprise that, apart from the labour supply shock, all other types of shocks considered by us seem to have a rather substantial effect on unemployment dynamics even in the long run.

**Table 2** Short and long run effects of the four types of shocks on changes in unemployment growth, where the size of the impulses is unity and  $b_{21}=-30$

	Short run	Long run
Aggregate demand shock	-0.003 10	-0.00167
Reallocation shock	-0. 00328	-0. 00133
Labour supply shock	0. 000464	I-O.00010
Labour productivity shock	0.00532	0.00464

## 5. Conclusions

Combined worker and job flow data allow us to consider the sizes and propagation of reallocation shocks at the labour market in addition to the usual aggregate demand and supply shocks of real business cycle models. This paper uses flow data for the Netherlands based on administrative sources for specification and estimation of a four-variate structural vector autoregressive (SVAR) model which distinguishes four types of shocks, namely aggregate supply and demand shocks, a reallocation shock at the demand side of the labour market and a productivity shock at the supply side of the market. The specification of the model needs a number of identifying restrictions which we have derived as much as possible from economic argumentation. It appears that each of these four types of shocks has at least some influence both on the short and on the long run, although the long run influence of the aggregate labour supply shock is estimated to be very limited. The fact that in three out of four cases the long run effect is not close to zero is somewhat surprising as for technical reasons we have to calculate the impulse response effects on changes in unemployment growth. One expects that after a shock the economy would move to a new equilibrium with no the acceleration (or deceleration) in the unemployment rate. Apparently in reality the various types of shocks distinguished by us give rise to very complicated unemployment dynamics. Yet, from the point of view of policy analysis it is very essential to disentangle these various shocks. That is because these shocks constitute different sources of labour market dynamics where each type of shock may lead to a specific policy reaction. A scope for future research is even to consider other types of shocks as well. By way of example we mention “skill-biased” demand shocks which are, according to Mortensen and Pissarides (1999), and Marimon and Zilibotti (1999), behind differences in unemployment dynamics between the United States and Europe, given differences in the social security system.

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**Appendix**, derivation of expression 4.7

We mentioned already the relations:

$$J_t = E_t + V_t \text{ and } LS_t = E_t + U_t.$$

Using these relations it follows:

$$U_t = LS_t - E_t = LS_t - J_t + V_t.$$

We now write up the first order difference of  $U_t$ :

$$U_t - U_{t-1} = LS_t - LS_{t-1} - J_t + J_{t-1} + V_t - V_{t-1}.$$

Now we normalise this expression with  $LS_t$ :

$$\frac{U_t - U_{t-1}}{LS_t} = \frac{LS_t - LS_{t-1}}{LS_t} - \frac{J_t - J_{t-1}}{LS_t} + \frac{V_t - V_{t-1}}{LS_t}.$$

We then substitute  $J_t - J_{t-1}$  by  $JC_t - JD_t$  and we substitute  $\frac{1}{LS_t}$  by  $\frac{1}{E_t} \cdot \left(1 - \frac{U_t}{LS_t}\right)$ .

This leads us to the expression:

$$\frac{U_t - U_{t-1}}{LS_t} = \left(\frac{JD_t}{E_t} - \frac{JC_t}{E_t}\right) \cdot \left(1 - \frac{U_t}{LS_t}\right) + \frac{LS_t - LS_{t-1}}{LS_t} + \frac{V_t - V_{t-1}}{LS_t}.$$

By using first differences we get the final expression (4.7).