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2013

Faculty of Economics and Business Administration

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## Research Memorandum 2013-36


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**Abstract**

In this paper we provide a smoothed perturbation analysis (SPA) of the sensitivity of a discrete time Parisian option with respect to the barrier level. The analysis put forward is of interest in a broader context than that of exotic options as we provide an SPA analysis for a problem where the critical event for the SPA estimator is based on an entire sample path, which is a novelty in the literature. Numerical examples illustrate the performance of the estimator.

**Keywords:** Parisian Option, Option pricing, Sensitivity analysis, Smoothed perturbation analysis

## 1 Introduction

A Parisian option on a stock with price process $\{S_t : t \geq 0\}$ and maturity $T$, $T > 0$, pays off some amount $\varphi(S_T)$ if the price process does not spend more than $\beta T$ consecutive time units above some specified threshold $\theta$, for $\beta \in (0,1)$. Computing the price of a Parisian option is a challenging task and only approximate solutions are known, see, for example, [1, 13]. In practice, trading is usually on a discrete time scale and the main reason that continuous time models are dominant in option theory is mathematical, as continuous time models can be modeled by stochastic differential equations and powerful tools for the analysis of stochastic differential equations exist. As pointed out in [3], the price of a discretely monitored option can be easily evaluated by means of Monte Carlo simulation. For risk-managers, evaluating the sensitivity of the option premium w.r.t. various parameters such as volatility, interest rate, maturity time or strike price is of importance. Sensitivities of option premiums are known in the literature as Greeks (they are denoted by Greek letters) and, due to their importance, they have received much attention in the mathematical finance literature in the last years. For more details we refer to [10].

In this paper we analyze the sensitivity of the value of a Parisian option with respect to $\theta$ for a discrete time model and we will adjust the above definition of the Parisian option accordingly More specifically, let $\{S_i : 0 \leq i \leq n\}$ be the price process monitored at discrete points in time. Specifically, beginning at $t_0 = 0$, we make $n$ observations, $t_1 = h, t_2, \ldots, t_n = nh$, equally spaced with interval $h$, that is, $t_i = ih$, $1 \leq i \leq n$ , with $t_n = nh = t$ , and $S_i := S_{t_i}$, for $i = 1, \ldots, n$. For the Parisian option with an up-and-out barrier, the option pays $\varphi(S_n)$ if the stock price does not stay more than $\alpha \in \{2, \ldots, n\}$ consecutive observations above $\theta$. We define $\tau_\theta(\alpha)$ as the first time instance of the event that $\alpha$ consecutive observations of the stock price have fallen into the payoff region:

$$\tau_\theta(\alpha) = \inf\{i \in \{\alpha - 1, \ldots, n\} | S_{i-k} > \theta \text{ for all } k = 0, \ldots, \alpha - 1\}, \tag{1}$$

setting $\tau_\theta(\alpha) = \infty$ if the set on the right hand side in (1) is empty. To simplify notation, we will suppress the depiction of $\alpha$. Let $s_0$ denote the initial value of the stock. Let $Z_i$, $1 \leq i \leq n$, be an i.i.d. collection of standard normal random variables. A realization of the price path at the monitoring times $t_i$, $i = 1, \dots, n$, is given by

$$S_{i+1} = S_i \exp\left(\mu h + \sigma\sqrt{h} Z_i\right), \tag{2}$$

where, following the Equivalent Martingale Measure construction, we let $\mu = r - \sigma^2/2$, with $r$ denoting the risk free interest rate and $\sigma$ being the volatility. Then the price of a discrete Parisian call option is given by

$$\mathbb{E}\big[e^{-rt}\varphi(S_n)\mathbf{1}\{\tau_\theta > n\}\big]. \tag{3}$$

In this paper we provide an unbiased estimator for the derivative of the value of the Parisian option with respect to the barrier level $\theta$. The analysis put forward is of interest in a broader context than that of exotic options as we provide an SPA analysis for a problem where the critical event for the SPA estimator is based on an entire sample path.

The paper us organized as follows. Section 2 provides a literature review. Section 3 is devoted to the smoothed perturbation analysis of the Parisian option. An importance sampling extension of the SPA estimator is discussed in Section 4 Numerical experiments are provided Section 5. We conclude with indications on further research.

## 2 Literature Review

A standard method for approximately computing a derivative is the finite-difference (FD) method, that is, one uses the following approximation (for $\Delta \to 0$):

$$\frac{dV(\theta)}{d\theta} \approx \frac{V(\theta+\Delta) - V(\theta-\Delta)}{2\Delta}; \tag{4}$$

see, e.g., [15]. Note that this method requires re-simulation since both $V(\theta+\Delta)$ and $V(\theta-\Delta)$ are obtained by simulation. Another drawback of the FD method is that, although the estimates converge, as $\Delta \to 0$, to the derivative $V'(\theta)$ when $V$ is differentiable in $\theta$, there is no clear indication on how small (close to 0) $\Delta$ should be, and this is directly affecting the unbiasedness of the estimate.

Given the shortcomings of the FD method, two so-called direct methods, i.e., no re-simulation is needed, known as *infinitesimal perturbation analysis* (IPA) and *score-function method* (SF), respectively, were proposed; see, e.g., [4]. IPA essentially requires path-wise differentiation while the SF method requires differentiation of the density. Both methods lead to unbiased gradient estimates. However, IPA is applicable only when the sample path function is Lipschitz continuous w.r.t. $\theta$, which is not the case in (3) since indicator functions induce discontinuities on the boundary of the corresponding sets. It is worth noting that under appropriate additional conditions IPA can be applied to this type of problems, see [12, 11, 9]. In general, however, to be able to deal with discontinuities of sample path functions, an extension of IPA, called *smoothed perturbation analysis* (SPA), has been applied that integrates out discontinuities. For details for this method we refer to [8], and for more details on the gradient estimation in general we refer to [7]. For a recent application of SPA to sensitivity analysis of option, we refer to [14], where sensitivities of step options are treated.

Estimation of Greeks can also be achieved by means of *Malliavin weighting function* (MF). This is a quite modern technique, based on Malliavin calculus, and determines a class of weighting functions which provide gradient estimates for the Greeks when multiplied with the payoff function. For some pioneering work on Malliavin Greek estimation we refer to [5, 6]. The MF approach is essentially an extension of the SF method. More specifically, it has been shown in [2] that the score function appears as the Malliavin weighting function which induces the smallest total variance. Computational issues related to such methods have been addressed in [2]. Nevertheless, like SF, the MF method cannot contain the case when the boundary of the feasibility set varies with respect to the parameter of interest, which leaves SPA as the method of choice.

# 3 Parisian Options

Section 3.1 is devoted to the derivation of the SPA estimator. An algorithm for the efficient identification of critical event is provided in Section 3.2.

## 3.1 General Analysis

Starting point for the analysis is the representation for the pay off of the Parisian option put forward in (3). For $\Delta > 0$, note that

$$\begin{aligned}&\mathbb{E}\big[e^{-rt}\varphi(S_n)\mathbf{1}\{\tau_\theta > n\}\big] - \mathbb{E}\big[e^{-rt}\varphi(S_n)\mathbf{1}\{\tau_{\theta-\Delta} > n\}\big]\\ &\quad = \mathbb{E}\big[e^{-rt}\varphi(S_n)\big(\mathbf{1}\{\tau_\theta > n\} - \mathbf{1}\{\tau_{\theta-\Delta} > n\}\big)\big].\end{aligned}$$

Let $\mathbf{S} = (S_1, \ldots, S_n) \in \mathbb{R}^n$ denote a price path and write $\mathbf{S}_i \in \mathbb{R}^{n-1}$ for price path $\mathbf{S}$ with the $i$th value removed, that is, $\mathbf{S}_i = (S_1, \ldots, S_{i-1}, S_{i+1}, \ldots, S_n)$, for $1 \leq i \leq n$. In the following we search for time instances $\eta$, such that, given $\mathbf{S}_\eta$, the activation of the boundary depends on the outcome of $S_\eta$. If this is the case, then we call $S_\eta$ a *critical event*. More specifically, suppose that for a sample path $\mathbf{S}_\eta$ we do not observe a barrier activation for the payoff boundary at $\theta$ and a payoff will occur, then $S_\eta$ is a critical event if the following two conditions hold:

(a) inserting $S_\eta$ to $\mathbf{S}_\eta$ at position $\eta$, with $S_\eta > \theta$, a payoff will not occur; and

(b) inserting $S_\eta$ to $\mathbf{S}_\eta$ at position $\eta$, with $S_\eta \leq \theta$, a payoff will occur.

The term $\mathbf{1}\{\tau_\theta > n\} - \mathbf{1}\{\tau_{\theta-\Delta} > n\}$ only has $\{0, 1\}$ as possible values. More specifically, the difference of indicators is equal to 1 on the event that the option does pay off for $\theta$ but does not pay off for $\theta - \Delta$. Indeed, since a barrier activation at the payoff boundary at $\theta$ implies a barrier activation at $\theta - \Delta$, $\mathbf{1}\{\tau_\theta > n\} - \mathbf{1}\{\tau_{\theta-\Delta} > n\}$ cannot be equal to $-1$.

Define the event $\mathcal{B}_{\Delta,\eta} = \{S_\eta \in (\theta - \Delta, \theta]\}$, and

$$\mathcal{A}_{\theta,\eta} = \{\text{the first critical event occurs at time } \eta\}.$$

With these definitions, we have

$$\mathbf{1}\{\tau_\theta > n\} - \mathbf{1}\{\tau_{\theta-\Delta} > n\} = \mathbf{1}\left\{\bigcup_{i=1}^{n} \mathcal{A}_{\theta,i} \cap \mathcal{B}_{\Delta,i}\right\}.$$

The event on the above LHS can be phrased as follows: there is at least one critical event in $\mathbf{S}$ and corresponding price falls into $(\theta - \Delta, \theta]$. By construction, $\mathcal{A}_{\theta,i} \cap \mathcal{A}_{\theta,j} = \emptyset$ for $i \neq j$, we arrive at

$$\begin{aligned}&\mathbb{E}\big[e^{-rt}\varphi(S_n)\big(\mathbf{1}\{\tau_\theta > n\} - \mathbf{1}\{\tau_{\theta-\Delta} > n\}\big)\big]\\ &\quad = \sum_{i=1}^{n} \mathbb{E}\big[e^{-rt}\varphi(S_n)\mathbf{1}\{\mathcal{A}_{\theta,i} \cap \mathcal{B}_{\Delta,i}\}\big].\end{aligned} \tag{5}$$

Given an expectation indexed with respect to $\eta \in \{1, \ldots, n\}$, for the gradient estimator, we analyze the conditional expectation with respect to $\mathbf{S}_\eta$, specifically

$$\lim_{\Delta \downarrow 0} \frac{1}{\Delta} \mathbb{E}\big[e^{-rt}\varphi(S_n)\mathbf{1}\{\mathcal{A}_{\theta,\eta} \cap B_{\Delta,\eta}\}|\mathbf{S}_\eta\big].$$

By conditional expectation,

$$\begin{aligned}
&\lim_{\Delta\downarrow 0}\frac{1}{\Delta}\mathbb{E}\big[e^{-rt}\varphi(S_n)\mathbf{1}\{\mathcal{A}_{\theta,\eta}\cap B_{\Delta,\eta}\}|\mathbf{S}_\eta\big]\\
&=\lim_{\Delta\downarrow 0}\frac{1}{\Delta}\mathbb{E}\big[e^{-rt}\varphi(S_n)\mathbf{1}\{A_{\theta,\eta}\}\big|\mathcal{B}_{\Delta,\eta}\,,\,\mathbf{S}_\eta\big]\,\mathbb{P}(\mathcal{B}_{\Delta,\eta}|\mathbf{S}_\eta)\\
&=\ \lim_{\Delta\downarrow 0}\mathbb{E}\big[e^{-rt}\varphi(S_n)\mathbf{1}\{\mathcal{A}_{\theta,\eta}\}\big|\mathcal{B}_{\Delta,\eta}\,,\,\mathbf{S}_\eta\big]\cdot\lim_{\Delta\downarrow 0}\frac{1}{\Delta}\mathbb{P}(\mathcal{B}_{\Delta,\eta}|\mathbf{S}_\eta)\\
&=\ \mathbb{E}\big[e^{-rt}\varphi(S_n)\mathbf{1}\{\mathcal{A}_{\theta,\eta}\}\big|S_\eta=\theta\,,\,\mathbf{S}_\eta\big]\cdot\mathbb{P}(S_\eta\in d\theta|\mathbf{S}_\eta).
\end{aligned}$$

Repeating the above argument for $\Delta>0$ leads to the same expression, which yields

$$\begin{aligned}
&\partial_\theta\mathbb{E}\big[e^{-rt}\varphi(S_n)\,\mathbf{1}\{\mathcal{A}_{\theta,\eta}\cap\mathcal{B}_{\Delta,\eta}\}\big|\mathbf{S}_\eta\big]\\
&=\mathbb{E}\big[e^{-rt}\varphi(S_n)\mathbf{1}\{\mathcal{A}_{\theta,\eta}\}\big|S_\eta=\theta\,,\,\mathbf{S}_\eta\big]\cdot\mathbb{P}(S_\eta\in d\theta|\mathbf{S}_\eta).
\end{aligned}\tag{6}$$

The expression in (6) can be facilitated for simulation in two different ways.

The first interpretation is to simulate a version of the price process without sampling the price at $\eta$. With this model, the rate $\mathbb{P}(S_\tau\in d\theta|\mathbf{S}_\eta)$ in (6) is that of the Black-Scholes bridge process. More specifically, denote the density of $S_{i+1}$ given $S_i=s$ by $\phi(s,\cdot)$, then

$$\begin{aligned}
\mathbb{P}(S_\tau\in d\theta|\mathbf{S}_\eta)&=\mathbb{P}(S_\eta\in ds_\eta|S_{\eta-1}=s_{\eta-1},S_{\eta+1}=s_{\eta+1})\\
&=\frac{\phi(s_{\eta-1},s_\eta)\phi(s_\eta,s_{\eta+1})}{\phi(s_{\eta-1},s_{\eta+1})}\\
&=\frac{1}{\sqrt{\pi h}\sigma s_\eta}e^{-\frac{1}{\sigma^2h}\left(\ln s_\eta-\frac{1}{2}(\ln s_{\eta-1}+\ln s_{\eta+1})\right)^2}\\
&=:\phi(s_\eta|s_{\eta-1},s_{\eta+1}).
\end{aligned}\tag{7}$$

For $\eta=n$, the Brownian bridge reduces to the increment density $\phi(s_{n-1},s_n)$. The expected value on the LHS of (6) is then evaluated by adding $S_\eta=\theta$ to the price path. The resulting estimator becomes

$$\begin{aligned}
&\partial_\theta\mathbb{E}\big[e^{-rt}\varphi(S_n)\mathbf{1}\{\tau_\theta>n\}\big|\mathbf{S}_\eta\big]\\
&=\mathbb{E}\big[e^{-rt}\varphi(\tilde{S}_n)\mathbf{1}\{\mathcal{A}_{\theta,\eta}\}\phi(\theta|\tilde{S}_{\eta-1},\tilde{S}_{\eta+1})|\tilde{\mathbf{S}}_\eta\big].
\end{aligned}\tag{8}$$

**Remark 1** *Alternatively, we can interpret (6) as a concatenation of a Brownian Bridge with endpoints* $s_0$ *and* $S_\tau=\theta$*, and a price process beginning from* $S_\tau=\theta$*. Here, we simulate the stock price process until* $S_{\tau-1}$*, and simulating* $S_{\tau+k}$*, for* $k\geq 0$*, according to (2), with initial value* $S_\tau=\theta$*. With this underlying model, the rate* $\mathbb{P}(S_\tau\in d\theta|\mathbf{S}_\eta)$ *is the independent product of the rate of a jump from* $S_{\tau-1}$ *to* $\theta$ *with the rate of a jump from* $\theta$ *to* $S_{\tau+1}$*,* $\phi(S_{\eta-1},\theta)\phi(\theta,S_{\eta+1})$*. Both terms are transition densities for parts of price path: the first term belonging to the Brownian Bridge component whereas the second density to the restarted price paths. To exhibit the Brownian Bridge, we need to divide the product density function by* $\phi(s_0,\theta)$*, and thus we have this expression for the numerator. The combined estimator becomes*

$$\begin{aligned}
&\partial_\theta\mathbb{E}\big[e^{-rt}\varphi(S_n)\mathbf{1}\{\tau_\theta>n\}|\mathbf{S}_\eta\big]\\
&=\mathbb{E}\big[e^{-rt}\varphi(S_n)\mathbf{1}\{\mathcal{A}_{\theta,\eta}\}\phi(s_0,\theta)|\mathbf{S}_\eta\big].
\end{aligned}\tag{9}$$

*As the estimator in (9) is numerically more demanding than the estimator in (8), we will use the estimator in (8) in the following.*

The overall estimator, Equation (5), becomes

$$\begin{aligned}
&\partial_\theta V(\theta)\\
&=\sum_{i=1}^{n-1}\mathbb{E}\big[e^{-rt}\varphi(\tilde{S}_n)\mathbf{1}\{\mathcal{A}_{\theta,i}\}\phi(\theta|\tilde{S}_{i-1},\tilde{S}_{i+1})|\tilde{S}_i=\theta\big]\\
&\quad+\mathbb{E}\big[e^{-rt}\varphi(\theta)\mathbf{1}\{\mathcal{A}_{\theta,n}\}\phi(\tilde{S}_{n-1},\theta)\big|\tilde{S}_n=\theta\big],
\end{aligned}\tag{10}$$

which can be simplified if $\varphi(\theta) = 0$. Depending on the experimental setting, a critical event may only occur with small probability. This renders impractical the above estimator as most of sampled price paths provide a derivative contribution of zero.

## 3.2 Critical Events

The advantage of (10) is that the existence of a critical event can be checked on a per path basis. In the following we will provide the overall algorithm to search for critical events. To this end, we define a *run* as sequence of observations the price of which is above a payoff boundary. For example, $(j+1, \ldots, j+m)$ is a run if $S_{j+i} > \theta$ for $1 \leq i \leq m$, and $S_j \leq \theta$ and $S_{j+m+1} \leq \theta$, and we call $m$ the *length* of the run.

In the following we will illustrate the logic for determining critical events by means of an example, where we choose $\alpha = 4$, i.e., the option is activated if the stock price stays for at least four consecutive observations above the barrier level $\theta$.

Consider a price path, which contains a run of length eight $= 2\alpha$. Changing the value of the stock price at an arbitrary position may alter the present run but will not lead to a price path having no run of length of at least $\alpha$. In words, the path contains no critical event. Hence, we may disregard price paths with a run of length $2\alpha$ or higher.

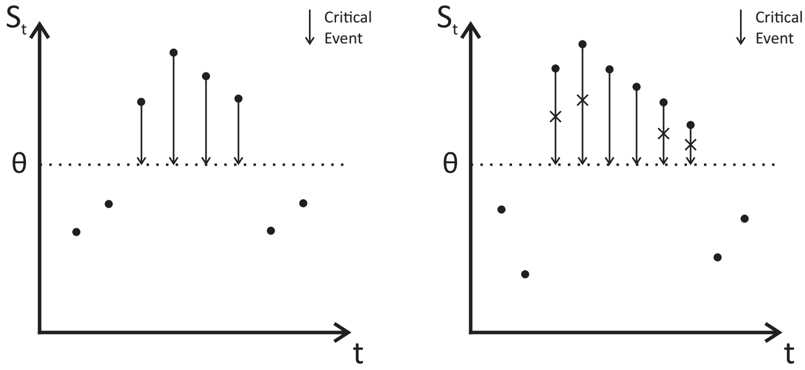
Now, consider a price path with two separate runs of length four. Again changing the value of the stock price at an arbitrary position will result in a price path that still contains a run of length four (if the stock price at the time instance between the two runs is changed, one can create a run of length nine). Hence, we may disregard price paths which contain more that one rune of $\alpha$ higher.

As next, we consider the situation that the price path contains a single run with length between $\alpha = 4$ and $2\alpha - 1 = 7$. Figures 1a and 1b provide two depictions of such a price path. The first figure depicts a run of four observations above the barrier level; the second, six observations. For sake of simplicity, we assume that these runs are the only runs in the respective price path. In both figures, the observed prices with uncrossed arrows pointing downwards constitute critical events, i.e., changing these stock prices will effect the activation of the option. For the path containing a run of length four, each observation contained in the run is critical. When there is a run of length six, decreasing the prices of the four outermost observations of the run will leave at least a run of length four and the barrier is still activated. The first critical event in the run in Figure 1b is at the fifth observation. The final critical event is at the sixth observation. The beginning and final critical events within a run is ascertained by successively incrementing the length of the run. Using our example of $\alpha = 4$, increasing the run length to five, six, and seven by adding future observations above $\theta$, does not alter the conclusion that we already have activated barrier. Hence, the $\alpha^{\text{th}}$ observation in the run is the final critical event. This can also be seen in reverse, by adding past observations with a price above the barrier level, with the first $l - \alpha$ observations not being critical events. Hence, if there exists one run of length $l$ such that $\alpha \leq l \leq 2\alpha - 1$, then the central $2\alpha - l$ observations are the sole critical events for the path.

Finally, we deal with paths that contain at most one or more runs of length $\alpha - 1$. Letting again $\alpha = 4$, Figure 2a illustrates a run of three prices above the barrier. For the observations immediately before and after the run, increasing these prices to $\theta$ will activate the barrier and these observations are thus border critical events. In Figure 2b, the fragment of this path contains a run of length two before four prices that alternate below and above the barrier. Changing the value of the fifth observation the barrier can be activated. For the seventh observation in this sequence, the final price below the barrier, increasing this price to the barrier level will only lead to a run of three prices in this same region. Hence, if the path contains runs of length $\alpha - 1$, the immediate observed prices before and after the run are critical events. In addition, if there are two runs each of length $\leq \alpha - 2$ with combined run length $\geq \alpha - 1$, such that there is one observation below the barrier between the runs, then this observation is also a critical event.

We summarize the above discussion in the following algorithm, where we assume $\alpha > 2$. The required adjustments in case $\alpha = 1, 2$ are discussed in the sequel.

**Algorithm for Determining Critical Events for the Discrete Parisian Barrier Sensitivity**
*Suppose we have a price path consisting of $n$ observations and $\alpha > 2$:*




(a) A single run of length four. (b) A single run of length six.

Figure 1: The critical events for paths that contain a single run of prices with length between $\alpha = 4$ and $2\alpha - 1 = 7$.

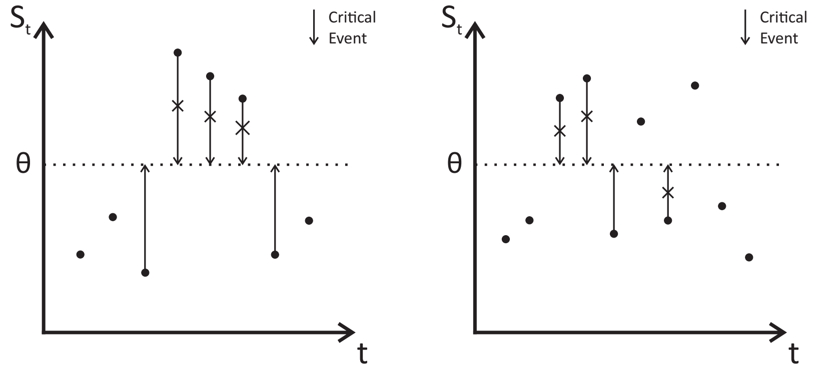
1. *Discard if the path contains at least one run of length* $2\alpha$*.*
2. *Discard if the path contains at least two or more runs of length* $\alpha$*.*
3. *There exists one run of length* $l$ *such that* $\alpha \le l \le 2\alpha - 1$*. The central* $2\alpha - l$ *observations are the sole critical events for the path. The first critical event occurs at observation* $l - (\alpha - 1)$ *within the run; the last occurs at the* $\alpha^{th}$ *observation.*
4. *Otherwise, if the path contains runs of length* $\alpha - 1$*, the immediate observed prices before and after the run are critical events. We call these critical events to be border critical events. In addition, if there are two runs each of length* $\le \alpha - 2$ *with combined run length* $\ge \alpha - 1$*, such that there is one observation below the barrier between the runs, then this observation is also a critical event.*

We conclude the discussion of the algorithm by commenting on the particular case $\alpha = 1, 2$. Step 1 to 3 are the same as above. However, Step 4 as to be adjusted in the following way. For $\alpha = 2$, in Step 4 of the algorithm, price paths with critical events have at most a single observation above the barrier, and critical events are border events, i.e., the prices on either side of these observation. For $\alpha = 1$, the paths has to stay below the payoff boundary in which all observations are critical events.

# 4 An Importance Sampling Approach

Depending on the experimental setting, a critical event may only occur with small probability, which renders the SPA estimator impractical as for most of the sampled price paths the derivative contribution is zero. To improve the performance of the estimator we will apply conditional sampling together with importance sampling in a fairly obvious way: first make the price process go above the barrier, then make it stay above the barrier for approximately $\alpha$ consecutive time periods, and then push the price process down but not below $K$. While technically the distribution of the price process can easily be modified in this way, the variance of such an estimator is likely to make it numerically infeasible as the density of the modified price path will be too for away from that of the original price path. To actually compute the optimal importance sampling estimator (i.e., with minimal variance) is a hard task and is a research topic on its own, which is far beyond the scope of this chapter. However, based on an intensive series of experiments we have constructed an importance sampling estimator with good performance. In the following we discuss the algorithm for the more challenging case $s_0 < \theta$.

In Phase I, we sample a price path conditioned on the event that the path is above the barrier $\theta$ with probability of approximately a half. We take into account that for $s_0$ and $\theta$, the event $S_1 > \theta$ will have




(a) Critical events may occur on either side of a run of length three.

(b) A critical event with three observations with a price above the barrier level.

Figure 2: The critical events for $\alpha = 4$, where the largest length for a run is $\alpha - 1 = 3$.

only a very small probability, which renders sampling $S_1$ conditioned on the event $S_1 > \theta$ numerically unstable. For our experiments, we use $10^{-8}$ as the threshold value. To circumvent this problem, we will set a lower limit for conditioned event. If $\mathbb{P}(S_1 > \theta) < 10^{-8}$, we search for the minimal value $\tau_1$ such that $\mathbb{P}(S_{\tau_1} > \theta) \geq 10^{-8}$. Once we have identified $\tau_1$, we simulate the price process until $\tau_1$ conditioned on $S_{\tau_1}$. Note that this phase can be skipped if $s_0 > \theta$. In Phase II, we change the drift of process to zero for $\alpha$ transitions. In Phase II we modify the drift of the process so as to be non-negative and continue generating the price-path until a run of $\alpha$ observations has been observed. Eventually, in Phase III, we set the drift such that the exercise prise equals in mean to the strike price. For the up-and-out put option, Figure 3 depicts a hypothetical path under the importance sample scheme, assuming $s_0 < K < \theta$.

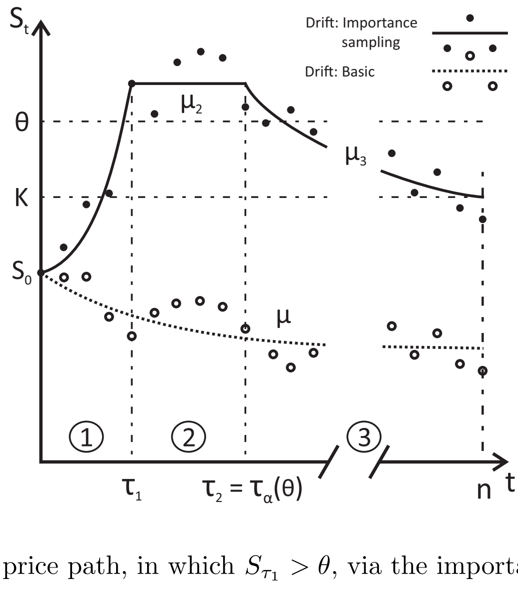



Figure 3: Hypothetical price path, in which $S_{\tau_1} > \theta$, via the importance sampling algorithm.

For both conditional expectations, the estimate is over $N$ paths. In the estimation, antithetic paths are generated. This means that for the standard normally distributed increments $Z = (Z_i)_{i=1}^n$ that for the random component of the price path, we can also use the increment sequence $-Z$ to form a second path. Each conditional expectation has its own set of i.i.d. price paths. For Phase II and Phase III, the change of measure sampling technique uses a BSM marginal distribution with the same implied volatility parameter.

In the following we provide a detailed description of the construction.

**Phase I**

From $\ln S_k \sim N(\ln s_0 + \mu kh, \sigma k)$, we find

$$\mathbb{P}\left(S_k > \theta\right) = 1 - \Phi\left(\frac{\ln(\theta/s_0) - \mu kh}{\sigma\sqrt{kh}}\right).$$

Denoting the right $\gamma$-tail of the standard normal distribution by $z_{1-\gamma}$, the smallest value of $k$ such that $\mathbb{P}\left(S_k > \theta\right) \geq \gamma$, which is denoted by $\tau_1$, is given by

$$\tau_1 = \left\lceil \frac{\left(\sigma z_{1-\gamma} - \sqrt{\sigma^2 z_{1-\gamma}^2 + 4\mu \ln(\theta/s_0)}\right)^2}{4h\mu^2} \right\rceil,$$

where $\lceil x \rceil$, $x \in \mathbb{R}$, denotes the smallest integer larger than of $x$. Note that in case $\mu = 0$, the expression for $\tau_1$ simplifies to

$$\tau_1 = \left\lceil \frac{\ln^2\left(\frac{\theta}{s_0}\right)}{\sigma^2 z_{1-\gamma}^2 h} \right\rceil.$$

As next we turn to the simulation of $S_{\tau_1}$. We generate a sample of $S_{\tau_1}$ as follows. We first sample a standard normal variate $\hat{Z}$ conditioned on the event that $\hat{Z} \geq z^{min}$, with $z^{min}$ given by

$$z^{min} = \frac{\ln(\theta/s_0) - \mu\tau_1 h}{\sigma\sqrt{\tau_1 h}}.$$

The actual realization of $S_{\tau_1}$ is then attained by (2) with $S_i = s_0$, $Z_i = \hat{Z}$, and $h$ replaced by $\tau_1 h$.

The path between $s_0$ and $S_{\tau_1}$, the Brownian Bridge distribution for $\ln S_i$ is determined by fixing the quantities $\ln S_{i-1}$ and $\ln S_{\tau_1}$. Following (7), the density function $\chi(s_i|s_{i-1}, s_{\tau_1})$ is given by

$$\chi(s_i|s_{i-1}, s_{\tau_1}) = \frac{1}{\sqrt{2\pi\frac{\tau_1 - i}{\tau_1-(i-1)}h}\sigma s_i} \cdot \exp\left(-\frac{1}{2\sigma^2\frac{\tau_1-i}{\tau_1-(i-1)}h}\left(\ln s_i - \left(\frac{\tau_1 - i}{\tau_1-(i-1)}\ln s_{i-1} + \frac{1}{\tau_1-(i-1)}\ln s_{\tau_1}\right)\right)^2\right),$$

for $1 \leq i \leq \tau_1 - 1$. Consequently, recursive generation of $(S_i, 1 \leq i \leq \tau_1 - 1)$, with $(Z_i, 1 \leq 1 \leq \tau_1 - 1)$ as a sequence of standard normal random variables, is given by the expression

$$\ln S_i = \frac{\tau_1 - i}{\tau_1 - (i-1)}\ln S_{i-1} + \frac{1}{\tau_1-(i-1)}\ln S_{\tau_1} + \sqrt{\frac{\tau_1 - i}{\tau_1-(i-1)}}\sigma Z_i.$$

**Phase II**

To aid in maintaining price path observations above the payoff boundary, we set the drift component in this phase, denoted by $\mu_2$, between successive observations to be non-negative. In particular, if $\mu \leq 0$, $\mu_2 = 0$, or otherwise the value of the instantaneous drift component is kept, i.e., $\mu_2 = \max\{\mu, 0\}$. Hence, the distribution of observations is given by $\ln S_{i+1} \sim N(\ln S_i + \mu_2 h, \sigma\sqrt{h})$.

The phase ends at the stopping time $\tau_2$, where $\tau_2$ is chosen to balance the aspect that a run of length $\alpha$ should be observed and that a long enough time sequence remains to move the security price below $\theta$.

For this reason, we let $\tau_2 = \min\{\tau_1 + n/2, n - \tau_1, \tau_\theta(\alpha)\}$. The term $\tau_\theta(\alpha)$ is the instance when a run of length $\alpha$ has been observed for the first time.

The Radon-Nikodym derivative for this phase is given by the product of the Radon-Nikodym derivatives of the increments that occur along this phase. Let $\chi^\mu(s_{i-1}, s_i)$ be the log-normal density function, with drift component $\mu$, then the increment derivative, $\Lambda^{\mu,\mu_2}(s_{i-1}, s_i)$, for changing the drift component from $\mu$ to $\mu_2$ is the ratio of the two densities

$$\Lambda^{\mu,\mu_2}(s_{i-1}, s_i) = \frac{\chi^\mu(s_{i-1}, s_i)}{\chi^{\mu_2}(s_{i-1}, s_i)} = \exp\left(-\frac{\mu_2 - \mu}{\sigma^2} \ln\left(\frac{s_i}{s_{i-1}}\right) - \frac{1}{2\sigma^2}(\mu^2 - \mu_2^2)h\right).$$

The Radon-Nikodym derivative for Phase II is then

$$\Lambda^{\mu,\mu_2}(s_{\tau_1}, s_{\tau_2}) = \exp\left(-\frac{\mu_2 - \mu}{\sigma^2} \ln\left(\frac{s_{\tau_2}}{s_{\tau_1}}\right) - \frac{1}{2\sigma^2}(\mu^2 - \mu_2^2)(\tau_2 - \tau_1)h\right).$$

**Phase III**

To ensure with probability 1/2 that a price path has a non-zero payoff function, we ascertain the drift coefficient $\mu_3$ from the mean stock price at observation $\tau_2$. At expiration, $\mathbb{E}[S_n|S_{\tau_2}] = S_{\tau_2}\exp(\mu_3(n-\tau_2)h)$. If we set the conditional expectation to equal the exercise price, then $\mu_3 = (\ln K - \ln S_{\tau_2})/((n - \tau_2)h)$. This drift coefficient is both state and time dependent.

The upper bound in Phase II is chosen so that the denominator is not too small, which would otherwise increase the variance of the derivative estimator. If the drift $\mu < \mu_3$, we use the value of the drift coefficient, likelier for the put option, $K > S_n$. Therefore, for Phase III, $\mu = \min\{\mu_3, \mu\}$ and the distribution between subsequent increments is given by $\ln S_i \sim N(\ln S_{i-1} + \mu_3 h, \sigma\sqrt{h})$, $i = \tau_2 + 1, \ldots, n$. Analogous to Phase II, the Radon-Nikodym derivative for this phase is given by the expression

$$\Lambda^{\mu,\mu_3}(s_{\tau_2}, s_n) = \exp\left(-\frac{\mu_3 - \mu}{\sigma^2} \ln\left(\frac{s_n}{s_{\tau_2}}\right) - \frac{1}{2\sigma^2}(\mu^2 - \mu_3^2)(n - \tau_3)h\right).$$

If $\mu_3 = \mu$, the payoff function has a non-zero value with probability half is readily seen from

$$\mathbb{P}(S_n < K|S_{\tau_2}) = \mathbb{P}\left(\ln S_{\tau_2} + \mu_3(n - \tau_2)h + \sigma\sqrt{(n - \tau_2)h}Z < K\right) = \Phi(0),$$

after cancellations, where $Z$ is a standard normal random variable.

The above construction results in a path $(s_i : 0 \le i \le n)$, with $s_0$ fixed, the density of which is given as follows. For $\tau_1 > 1$, the density of the part of the price process belonging to Phase I, is attained from

$$\begin{aligned}
&\chi((s_1, \ldots, s_{\tau_1})|s_0, s_n) \\
&= \frac{\chi^\mu(s_0, s_1)\chi^\mu(s_1, s_n)}{\chi^\mu(s_0, s_n)} \cdot \frac{\chi^\mu(s_1, s_2)\chi^\mu(s_2, s_n)}{\chi^\mu(s_1, s_n)} \cdot \ldots \cdot \frac{\chi^\mu(s_{n-2}, s_{n-1})\chi^\mu(s_{n-1}, s_n)}{\chi^\mu(s_{n-2}, s_n)} \\
&= \prod_{i=1}^{\tau_1} \chi(s_i|s_{i-1}, s_n).
\end{aligned}$$

Combining this with the condition transition densities for Phase II and Phase III, we arrive the following expression for the overall density of a price path:

$$\begin{aligned}
&= \left(\frac{1\{s_{\tau_1} \le \theta\}}{\mathbb{P}(S_{\tau_1} \le \theta)} + \frac{1\{s_{\tau_1} > \theta\}}{\mathbb{P}(S_{\tau_1} > \theta)}\right) \chi((s_1, \ldots, s_{\tau_1})|\, s_0, s_n)\chi^\mu(s_0, s_{\tau_1}) \\
&\quad \cdot \prod_{i=\tau_1+1}^{\tau_2} \chi^{\mu_2}(s_{i-1}, s_i) \prod_{i=\tau_2+1}^{n} \chi^{\mu_3}(s_{i-1}, s_i).
\end{aligned}$$

**Sensitivity Estimator**

The importance sampling sensitivity for $\partial_\theta V(\theta)$ is attained from sampling a price path under the above change of measure and rescaling the outcome by the Radon-Nikodym derivatives:

$$
\begin{aligned}
&\frac{\partial}{\partial\theta} V(\theta)\\
&= \mathbb{P}(S_{\tau_1} > \theta)\\
&\quad \cdot \left\{ \sum_{i=1}^{n-1} \mathbb{E}\left[ e^{-rt} \upsilon(S_n) 1\{A_{\theta,i}\} \chi(\theta | S_{i-1}, S_{i+1}) \Lambda^{\mu,\mu_2}(S_{\tau_1}, S_{\tau_2}) \Lambda^{\mu,\mu_3}(S_{\tau_2}, S_n) \right| S_i = \theta, S_{\tau_1} > \theta \right]\\
&\qquad + \mathbb{E}\left[ e^{-rt} \upsilon(S_n) 1\{A_{\theta,n}\} \chi(S_{n-1}, \theta) \Lambda^{\mu,\mu_2}(S_{\tau_1}, S_{\tau_2}) \Lambda^{\mu,\mu_3}(S_{\tau_2}, S_n) \right| S_n = \theta, S_{\tau_1} > \theta \left.\right] \Big\}\\
&\quad + \mathbb{P}(S_{\tau_1} \leq \theta)\\
&\quad \cdot \left\{ \sum_{i=i}^{n-1} \mathbb{E}\left[ e^{-rt} \upsilon(S_n) 1\{A_{\theta,i}\} \chi(\theta | S_{i-1}, S_{i+1}) \Lambda^{\mu,\mu_2}(S_{\tau_1}, S_{\tau_2}) \Lambda^{\mu,\mu_3}(S_{\tau_2}, S_n) \right| S_i = \theta, S_{\tau_1} \leq \theta \right]\\
&\qquad + \mathbb{E}\left[ e^{-rt} \upsilon(S_n) 1\{A_{\theta,n}\} \chi(S_{n-1}, \theta) \Lambda^{\mu,\mu_2}(S_{\tau_1}, S_{\tau_2}) \Lambda^{\mu,\mu_3}(S_{\tau_2}, S_n) \right| S_n = \theta, S_{\tau_1} \leq \theta \left.\right] \Big\}.
\end{aligned}
$$

For the evaluation of the estimator, we take the sample average over independent replications.

# 5 Numerical Experiments

For the numerical experiments we choose an up-and-out put option begins at-the-money with $s_0 = K = 100$. The number of observations before expiration of the contract is $n = 252$, equally spaced over $t = 1$ year of trading days, i.e., $h = 1/252$.

As the procedure to simulate the barrier-level sensitivity for a Parisian option is demanding, we first compare our results with the FD estimator. For both SPA estimators, with and without importance sampling, we generate $2^{12} = 4096$ paths, and we increase the number of generations to $2^{16} = 65536$ for the FD estimator. The model parameters are set to the cases $(r, \sigma) = (0.03, 0.40)$, in which $\mu = -0.05$, i.e., a volatile market, and $(r, \sigma) = (0.07, 0.20)$ where $\mu = 0.05$. In this later scenario, the short rate has increased and the implied volatility decreased, resulting in a bull market. For each model setting, we have chosen two pairs of contract parameters, $(\alpha, \theta) = \{(110, 5), (105, 21)\}$. For the FD method, we set the step-size to equal $\Delta = 0.10$; see (4). The results are over 500 estimates. Table 1 presents the comparison for the parameter pair $(r, \sigma) = (0.03, 0.40)$, and Table 2 displays the comparison $(r, \sigma) = (0.07, 0.20)$.

Contract parameters: $s_0 = K = 100$; $n = 252$ observations over 1 yr.
Model parameters: $r = 0.03, \sigma = 0.40$.

| $(\theta, \alpha) = (110, 5)$ | | | $(\theta, \alpha) = (105, 21)$ | | |
|---|---|---|---|---|---|
| Basic | IS | FD | Basic | IS | FD |
| 0.3690 | 0.3654 | 0.3669 | 0.3164 | 0.3183 | 0.3200 |
| (0.0361) | (0.0297) | (0.0418) | (0.0358) | (0.0301) | (0.0395) |

Table 1: Mean and, in parentheses, standard deviation of the barrier sensitivity of the up-and-out Parisian put option via SPA estimated from $2^{12}$ paths compared to the FD method estimated from $2^{16}$ paths.

From these results, the SPA methods are at least $2^4 = 16$ times as precise as the FD estimator, and the implementation of importance sampling provides a further reduction in standard deviation. Other choices of the step-size did not provide any significant improvement for the FD estimator. In particular, smaller values of the step-size deteriorate performance. The advantage of the SPA methods is that the critical event algorithm hypothesizes when an observed price is a critical event as opposed to needing prices to be within $(\theta - \Delta, \theta]$ at a specific observation.

| Contract parameters: $s_0 = K = 100$; $n = 252$ observations over 1 yr. | | | | | |
|---|---|---|---|---|---|
| Model parameters: $r = 0.07, \sigma = 0.20$. | | | | | |
| Basic | IS | FD | Basic | IS | FD |
| 0.1327 | 0.1307 | 0.1308 | 0.1572 | 0.1579 | 0.1597 |
| (0.0199) | (0.0127) | (0.0173) | (0.0258) | (0.0170) | (0.0183) |

Table 2: Mean and, in parentheses, standard deviation of the barrier sensitivity of the up-and-out Parisian put option via SPA estimated from $2^{12}$ paths compared to the FD method estimated from $2^{16}$ paths.

The coordinate choices for our experiments (i.e., the run length of observations in the payoff region before barrier activation $\alpha$ and barrier price $\theta$) are provided in Table 3. The barrier prices are the same as in the previous experiment, except that the value $\theta = 100$ is omitted (as we have assumed $s_0 \neq \theta$). The choices for $\alpha$ consider cases where (i) a small number of consecutive prices is needed before barrier activation, including the discrete barrier option $\alpha = 1$, as well as (ii) cases with lengthy runs up to $\alpha = 63$. This last case is an idealization of three months of trading days where the price must remain above the barrier level.

| | Contract parameters: $(\alpha, \theta)$ |
|---|---|
| $\alpha$ | $1, 2, \ldots, 10, 12, \ldots, 18, 21, \ldots, 33, 39, \ldots, 63.$ |
| $\theta$ | $101, 102, \ldots, 110, 112, \ldots, 130, 135, \ldots, 150.$ |

Table 3: The coordinate choices for $(\alpha, \theta)$, with $\alpha$ the run length before the barrier activation, and $\theta$ the barrier level.

We again generate $2^{12} = 4096$ price paths for each SPA estimate. Table 4 and 5 compare the means and standard deviations of the basic and importance sampling derivative estimators over 500 estimates for certain choice of $(\alpha, \theta)$. The presented choices of model parameter pairs are again $(r, \sigma) = (0.03, 0.40)$ and $(r, \sigma) = (0.07, 0.20)$. Specifically, Table 4 presents the results for $(r, \sigma) = (0.03, 0.40)$ . Table 5 presents the results for $(r, \sigma) = (0.07, 0.20)$ where $\mu = 0.05$.

Figure 4 plots the Parisian barrier sensitivity for $(r, \sigma) = (0.03, 0.40)$ and Figure 5 plots the sensitivity results for $(r, \sigma) = (0.07, 0.20)$ over the entire contract parameter grid. We plot the results attained by the importance sampling estimator as they are numerically more reliable.

As with the continuous step option, the discrete Parisian barrier derivative estimator has the same functional nature. For fixed payoff boundary $\theta$, an increase in the value of $\alpha$ shows a gradual diminishing of the sensitivity value. For fixed boundary $\alpha$, an increase in the pay off boundary $\theta$ leads to a remarkable reduction on the sensitivity value.

We conclude this section with a discussion on the relation between the basic and the importance sampling estimator. While both estimators are unbiased, the importance sampling estimator has generally smaller variance. The advantage of the importance sampling estimator over the standard estimator becomes significant for extreme choices of $\alpha, \theta$, and further pronounced for an increasing value of $\mu$. Compare, for example, the standard deviation of the standard estimator and the importance sampling estimator for $\theta = 130$ and $\alpha \leq 10$. For these parameter settings the probability of observing a critical event is rather low which leads to a rather high variance of the standard estimator. Due to the sample path modifications for the importance sampling estimator, the probability of observing a critical event increases, which results in a smaller variance.

# 6 Conclusions and Future Research

In this paper we established an SPA estimator for sensitivities of financial options for which the exercise rule depends upon the whole path of a stock price up to maturity and not only on the final value.

| | | | | | | |
|---|---|---|---|---|---|---|
| Contract parameters: $s_0 = K = 100$; $n = 252$ observations over 1 yr. | | | | | | |
| Model parameters: $r = 0.03, \sigma = 0.40$. | | | | | | |
| $\alpha$ | 1 | | 2 | | 5 | |
| $\theta$ | Basic | IS | Basic | IS | Basic | IS |
| 101 | 0.6710 | 0.6721 | 0.6838 | 0.6797 | 0.6279 | 0.6293 |
| | (0.0594) | (0.0487) | (0.0521) | (0.0406) | (0.0559) | (0.0371) |
| 105 | 0.6325 | 0.6334 | 0.5881 | 0.5927 | 0.5080 | 0.5094 |
| | (0.0527) | (0.0551) | (0.0487) | (0.0473) | (0.0470) | (0.0409) |
| 110 | 0.4692 | 0.4675 | 0.4337 | 0.4332 | 0.3690 | 0.3654 |
| | (0.0444) | (0.0447) | (0.0381) | (0.0376) | (0.0361) | (0.0297) |
| 120 | 0.2449 | 0.2445 | 0.2175 | 0.2178 | 0.1770 | 0.1760 |
| | (0.0273) | (0.0263) | (0.0235) | (0.0219) | (0.0226) | (0.0165) |
| 130 | 0.1147 | 0.1148 | 0.1018 | 0.1022 | 0.0783 | 0.7846 |
| | (0.0177) | (0.0153) | (0.0156) | (0.0126) | (0.0142) | (0.0094) |
| $\alpha$ | 10 | | 21 | | 63 | |
| $\theta$ | Basic | IS | Basic | IS | Basic | IS |
| 101 | 0.5480 | 0.5464 | 0.4280 | 0.4240 | 0.1931 | 0.1939 |
| | (0.0506) | (0.0355) | (0.0477) | (0.0289) | (0.0306) | (0.0180) |
| 105 | 0.4275 | 0.4287 | 0.3163 | 0.3183 | 0.1311 | 0.1330 |
| | (0.0462) | (0.0379) | (0.0358) | (0.0301) | (0.0251) | (0.0189) |
| 110 | 0.2992 | 0.2993 | 0.2143 | 0.2133 | 0.0796 | 0.0782 |
| | (0.0356) | (0.0270) | (0.0291) | (0.0233) | (0.0179) | (0.0114) |
| 120 | 0.1369 | 0.1355 | 0.0903 | 0.0891 | 0.0251 | 0.0265 |
| | (0.0219) | (0.0143) | (0.0180) | (0.0111) | (0.0084) | (0.0047) |
| 130 | 0.0580 | 0.0581 | 0.0349 | 0.0344 | 0.0081 | 0.0081 |
| | (0.0123) | (0.0075) | (0.0094) | (0.0052) | (0.0045) | (0.0020) |

Table 4: Mean and, in parentheses, standard deviation of the barrier sensitivity of the up-and-out Parisian put option with model parameters $r = 0.03$, and $\sigma = 0.40$, ($\mu = -0.05$).

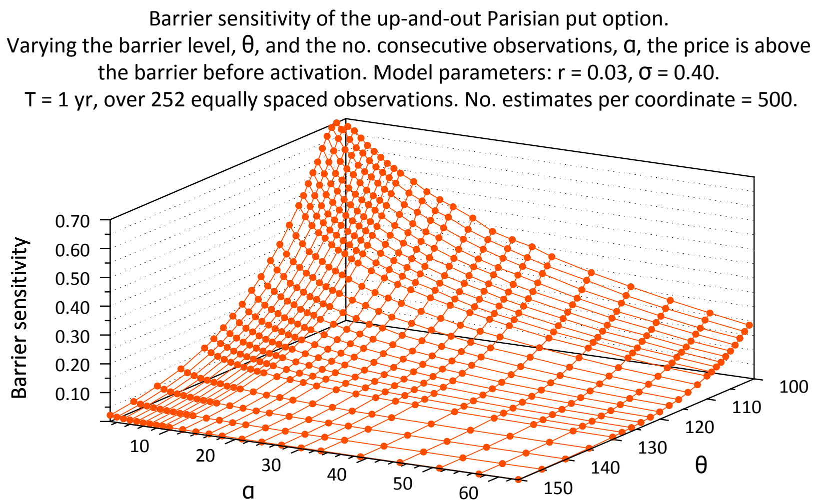



Figure 4: Mean over 500 estimates of the barrier sensitivity of the up-and-out Parisian put option for all terms of the variable pair $(\alpha, \theta)$. Model parameters: $(r, \sigma) = (0.03, 0.40)$.

Contract parameters: $s_0 = K = 100$; $n = 252$ observations over 1 yr.
Model parameters: $r = 0.07, \sigma = 0.20$.

| $\alpha$ | 1 | | 2 | | 5 | |
|---|---|---|---|---|---|---|
| $\theta$ | Basic | IS | Basic | IS | Basic | IS |
| 101 | 0.4454 | 0.4451 | 0.4380 | 0.4382 | 0.3983 | 0.4010 |
| | (0.0494) | (0.0416) | (0.0453) | (0.0339) | (0.0432) | (0.0278) |
| 105 | 0.3281 | 0.3257 | 0.3049 | 0.3047 | 0.2602 | 0.2618 |
| | (0.0396) | (0.0378) | (0.0334) | (0.0311) | (0.0329) | (0.0228) |
| 110 | 0.1774 | 0.1776 | 0.1616 | 0.1593 | 0.1327 | 0.1306 |
| | (0.0248) | (0.0217) | (0.0218) | (0.0163) | (0.0199) | (0.0127) |
| 120 | 0.0340 | 0.0337 | 0.0289 | 0.0287 | 0.0210 | 0.0212 |
| | (0.0084) | (0.0064) | (0.0075) | (0.0046) | (0.0070) | (0.0031) |
| 130 | 0.0040 | 0.0040 | 0.0030 | 0.0032 | 0.0023 | 0.0021 |
| | (0.0025) | (0.0014) | (0.0021) | (0.0010) | (0.0022) | (0.0006) |

| $\alpha$ | 10 | | 21 | | 63 | |
|---|---|---|---|---|---|---|
| $\theta$ | Basic | IS | Basic | IS | Basic | IS |
| 101 | 0.3490 | 0.3526 | 0.2802 | 0.2772 | 0.1303 | 0.1321 |
| | (0.0419) | (0.0264) | (0.0372) | (0.0220) | (0.0250) | (0.0141) |
| 105 | 0.2183 | 0.2197 | 0.1572 | 0.1579 | 0.0608 | 0.0609 |
| | (0.0317) | (0.0210) | (0.0258) | (0.0170) | (0.0155) | (0.0087) |
| 110 | 0.1030 | 0.1026 | 0.0672 | 0.0681 | 0.0200 | 0.0202 |
| | (0.0194) | (0.0106) | (0.0158) | (0.0088) | (0.0077) | (0.0035) |
| 120 | 0.0146 | 0.0148 | 0.0078 | 0.0080 | 0.0013 | 0.0014 |
| | (0.0067) | (0.0024) | (0.0045) | (0.0016) | (0.0016) | (0.0005) |
| 130 | 0.0014 | 0.0013 | 0.0006 | 0.0006 | $< 0.0001$ | $< 0.0001$ |
| | (0.0018) | (0.0004) | (0.0011) | (0.0002) | (0.0002) | ($< 0.0001$) |

Table 5: Mean and, in parentheses, standard deviation of the barrier sensitivity of the up-and-out Parisian put option with model parameters $r = 0.07$, and $\sigma = 0.20$, ($\mu = 0.05$).

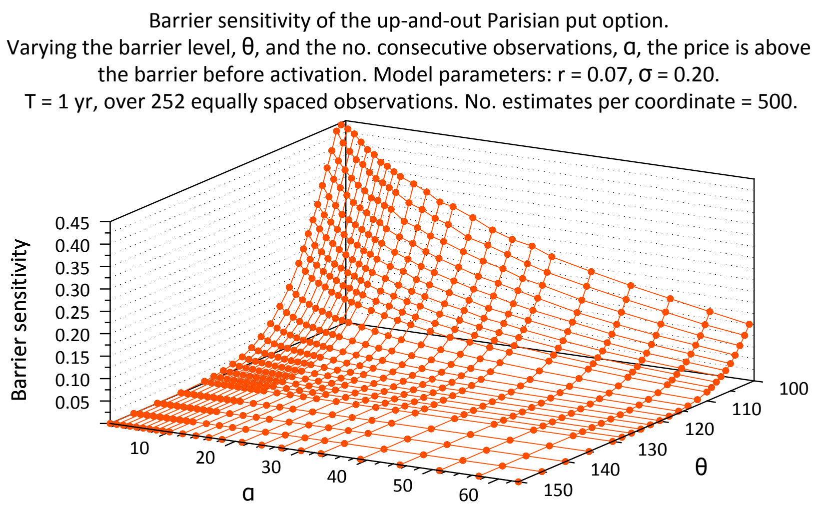



Figure 5: Mean over 500 estimates of the barrier sensitivity of the up-and-out Parisian put option for all terms of the variable pair $(\alpha, \theta)$. Model parameters: $(r, \sigma) = (0.07, 0.20)$.

Numerical experiments illustrated the properties of this sensitivity estimator. We also presented a version of our SPA estimator incorporating importance sampling. Extending the estimator to sensitivity analysis of the continuous time version of the Parisian option is topic of further research.

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