# Measuring Dynamic Interaction in Movement Data

Jed A. Long<sup>1</sup>\*, Trisalyn A. Nelson<sup>1</sup>

<sup>1</sup> Spatial Pattern Analysis & Research (SPAR) Laboratory Department of Geography, University of Victoria

\*Corresponding author email & address: jlong@uvic.ca Department of Geography, University of Victoria PO Box 3060 STN CSC Victoria, BC V8W 3R4, Canada

# Pre-print of published version.

## **Reference:**

Long, JA and TA Nelson. 2013. Measuring dynamic interaction in movement data. Transactions in GIS. 17(1). 62-77.

DOI:

http://dx.doi.org/10.1111/j.1467-9671.2012.01353.x

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Running Head: Measuring Dynamic Interaction

Keywords: dynamic interaction, movement data, correlation, GPS, space-time, local

statistics, spatial analysis

#### 1 ABSTRACT:

2 The emergence of technologies capable of storing detailed records of object locations has 3 presented scientists and researchers with a wealth of data on object movement. Yet 4 analytical methods for investigating more advanced research questions from such detailed 5 movement datasets remain limited in scope and sophistication. Recent advances in the 6 study of movement data has focused on characterizing types of dynamic interactions, 7 such as single-file motion, while little progress has been made on quantifying the degree 8 of such interactions. In this article, we introduce a new method for measuring dynamic 9 interactions (termed **DI**) between pairs of moving objects. Simulated movement datasets 10 are used to compare **DI** with an existing correlation statistic. Two applied examples, team 11 sports and wildlife, are used to further demonstrate the value of the **DI** approach. The **DI** method is advantageous in that it measures interaction in both movement direction 12 13 (termed azimuth) and displacement. As well, the **DI** approach can be applied at local, 14 interval, episodal, and global levels of analysis. However the **DI** method is limited to 15 situations where movements of two objects are recorded at simultaneous points in time. 16 In conclusion, **DI** quantifies the level of dynamic interaction between two moving 17 objects, allowing for more thorough investigation of processes affecting interactive 18 moving objects.

19

20 1 Introduction

21 The study of individual movement has entered a new era whereby researchers 22 from various fields can benefit from fine resolution object movement data. Technical 23 developments associated with location aware technologies, such as GPS, are transforming 24 representations of movement. Despite improvements in spatially explicit movement 25 datasets, the scope and sophistication of research questions are limited by a lack of 26 methods and analysis (Wolfer et al. 2001). Laube et al. (2007) suggest that within 27 geography, reliance of geographic information systems (GIS) and spatial statistics on 2-28 dimensional representations may be limiting the development of more complex analyses 29 of movement, while disciplines outside of geography may be unaware of the power of 30 spatial (and space-time) analysis. To optimally utilize new movement datasets, analytical 31 techniques capable of addressing more advanced research questions are required. 32 Recently, the identification and measurement of dynamic interactions between 33 moving objects has become an active area of research, likely owing to readily available 34 fine granularity movement data. Dynamic interaction, a term from the wildlife ecology 35 literature, can be defined as the way the movements of two individuals are related 36 (Macdonald et al. 1980) or as inter-dependency in the movements of two individuals 37 (Doncaster 1990). Alternatively, the terms association (Stenhouse et al. 2005), relative 38 motion (Laube et al. 2005), and correlation (Shirabe 2006) have been used to refer to 39 dynamic interactions between moving objects in other examples. All of these terms refer 40 to the same general idea: identifying of how the movements of one individual are related 41 to another. Recent work on dynamic interactions has focused on methods for identifying 42 dynamic interaction patterns defined *a priori* (for example single file motion, Buchin et

al. 2010; or chasing behavior, de Lucca Siqueira and Bogorny 2011). However limited
work exists on quantifying the strength of dynamic interactions present in movement
data. With this in mind we are motivated to investigate methods for measuring the
strength of dynamic interactions when there is an expectation that such behavior occurs.
This approach differs from recent developments in movement analysis which focus on
identifying patterns, defined *a priori*, from large movement databases.

49 The objective of this work is to extend a previously developed statistic (Shirabe 50 2006) to a measure capable of quantifying the degree of dynamic interaction between 51 moving objects. The new method (termed **DI**) measures dynamic interaction in 52 coincidental movement segments, that is, it requires movement data of two individuals 53 recorded simultaneously. The **DI** method is separable into components measuring 54 dynamic interaction in movement direction (azimuth) and movement distance 55 (displacement), termed  $\mathbf{DI}_{\theta}$  and  $\mathbf{DI}_{d}$  respectively. Further,  $\mathbf{DI}$  is appropriate with the four 56 analysis levels (local, interval, episodal, and global – see Figure 1) identified by Laube et 57 al. (2007) with the beneficial property of local values (denoted here using lower-case – 58 **di**) that aggregate to the interval, episodal and global values. Lastly, **DI** is derived in a 59 way to allow for a time-lagged approach, but also extensions including time- and 60 distance-based weighting schemes.

61

< Approximate location Figure 1 >

62 **2 Related Work** 

This research is motivated by an existing technique (Shirabe 2006) for measuring
the strength of dynamic interactions (termed correlations) present in movement data. The
use of the term correlation by Shirabe stems from the fact that the statistic takes the form

of a Pearson product-moment correlation coefficient. Consider two moving objects  $M^a$ and  $M^b$ , whose spatial coordinates (x, y) are recorded coincidentally at discrete times t = 1 $\dots$  *n*, termed *fixes*. Now consider for any *M* with  $t = 2 \dots n$ ,  $V = [M_t - M_{t-1}] = [v_t]$ , is a vector time series of *M* with *n*-1 vector segments. A correlation statistic for movement data defined this way takes the form (Shirabe 2006):

71 
$$r(\mathbf{V}^{a}, \mathbf{V}^{b}) = \frac{\sum_{t=1}^{n-1} (\mathbf{v}_{t}^{a} - \overline{\mathbf{v}}^{a}) \cdot (\mathbf{v}_{t}^{b} - \overline{\mathbf{v}}^{b})}{\sqrt{\sum_{t=1}^{n-1} |\mathbf{v}_{t}^{a} - \overline{\mathbf{v}}^{a}|^{2}} \sqrt{\sum_{t=1}^{n-1} |\mathbf{v}_{t}^{b} - \overline{\mathbf{v}}^{b}|^{2}}}$$
 (1)

72 Where  $\overline{\mathbf{v}} = \frac{1}{n-1} \sum_{t=1}^{n-1} \mathbf{v}_t$  are mean coordinate vectors of  $\mathbf{V}$ . The correlation statistic ( $\mathbf{r}$ )

is defined over the interval [-1, 1] with a score of 1 being perfect positive correlation and
a score of -1 perfect negative correlation, with 0 denoting no correlation.

75 The statistic  $-\mathbf{r}$ , could be advanced in three ways. First, it is dependent on the 76 mean vector of each path, and thus measures correlations in movement deviations from 77 their respective means. The statistic,  $\mathbf{r}$ , cannot be used for testing direct *interactions* 78 between two moving objects unless their corresponding mean vectors are identical or 79 near identical. An improved statistic would not rely on this overall mean value. Second, r 80 is unable to disentangle the effects of correlations in movement azimuth and distance, 81 while being sensitive to both. Decomposing such a statistic into components based on 82 movement direction (termed azimuth) and distance (displacement) would be beneficial, 83 as it would allow interactions in these two independent components of movement to be 84 analyzed separately. A third improvement would be a statistic that measures the 85 interaction of each individual movement segment (i.e., local level - Laube et al. 2007). By definition, **r** produces a single resulting value for the entire path (i.e., global level -86

5

87	Laube et al. 2007). When movement patterns are characterized by periods of interactive
88	and non-interactive behavior, or varying levels of interactive behavior, a local level
89	statistic will allow a finer treatment of dynamic interactions.

90 Measurements of dynamic interaction in movement data have also been 91 developed by wildlife researchers interested in a finer understanding of wildlife 92 movement processes. The types of interactions studied in wildlife are classified as either 93 static or dynamic interactions (Doncaster 1990; Macdonald et al. 1980). Static interaction 94 relates to how two individuals use space coincidentally, while dynamic interaction 95 reflects how the movements of two individuals are related, for example attraction 96 (Macdonald et al. 1980). Typically, measures of dynamic interaction summarize the 97 proximity of simultaneous movement points. Doncaster (1990) introduced one such 98 measure of dynamic interaction based on the variance/covariance matrix of the spatial 99 coordinates of simultaneous wildlife telemetry fixes; others have used Euclidean distance 100 as an indicator of interaction (Bandeira de Melo et al. 2007; Stenhouse et al. 2005). 101 Stenhouse et al. (2005) further investigated dynamic interaction in grizzly bears (termed 102 associations) by measuring dynamic interaction in movement direction (azimuth  $-\theta$ ) 103 defined as:

104 
$$f_t\left(\theta_t^a, \theta_t^b\right) = \frac{\left|-\left(\theta_t^a - \theta_t^b\right) - 180\right|}{180} \quad (2)$$

Equation (2) ranges from 0 - 1, with values of 1 when direction of movements is identical and zero when completely opposite (i.e., at  $180^{\circ}$ ).

107 Measuring dynamic interactions in moving object databases is also directly 108 related to a larger body of literature on identifying similar movement trajectories (Sinha 109 and Mark 2005; Vlachos et al. 2002; Yanagisawa et al. 2003). Similarity indices are 110 commonly employed as a first-step for identifying broader patterns or for detecting 111 clusters in larger movement databases (Benkert et al. 2008; Gao et al. 2010). Moving 112 object pairs that are highly interactive could also be said to follow a similar trajectory in 113 many of these applications, and the methods for detecting dynamic interactions in 114 movement data could be useful for detecting similar movement trajectories.

115 Recently, many new techniques have been developed for categorizing various 116 dynamic interaction patterns commonly found in movement data. Laube et al. (2005) 117 developed a method for detecting RElative MOtion (REMO) classes based upon 118 interpreting patterns of movement direction in groups of moving objects. For example, 119 trend-setting, when one object moves with anticipation of the movement of others, is 120 identifiable using the REMO approach. Noyon et al. (2007) use changes in inter-object 121 distance and velocity to identify relative behavior such as collision avoidance. Benkert et 122 al. (2008) present an algorithm for finding flock patterns in movement databases; which 123 tests whether a group of moving objects are contained in a circle radius r over a given 124 time interval. The study of flocking behavior is useful in the study of wildlife and crowd 125 dynamics (Batty et al. 2003). Buchin et al. (2010) have developed a method for 126 identifying single-file motion in groups of moving objects. Single-file motion is detected 127 using free-space diagrams, derived from the Fréchet distance metric for comparing 128 polygonal curves (Alt and Godau 1995). Related to single-file motion is the detection of 129 chasing behavior, identifiable using the algorithm proposed by de Lucca Siqueira and 130 Bogorny (2011). The methods mentioned above are capable of identifying specific types 131 of dynamic interactions in movement data as defined *a priori*. However, such methods

are unable to quantify the strength of dynamic interactions present, thus motivating thedevelopment of quantitative measures of dynamic interaction.

### 134 **3 Derivation**

135	In developing a measure of dynamic interaction we consider the rather optimal	
136	data situation (as in Shirabe 2006) where two moving objects' ( $M^a$ and $M^b$ ) spatial	
137	coordinates (x, y) are recorded coincidentally at discrete times $t = 1 \dots n$ , termed <i>fixes</i> .	
138	For any <i>M</i> with $t = 2n$ , $V = [M_t - M_{t-1}] = [v_t]$ , is a vector time series of <i>M</i> with <i>n</i> -1	
139	vector segments. For each movement segment define two fundamental properties:	
140	direction ( $\theta$ ), termed azimuth, and length ( $d$ ), termed displacement. Azimuth ( $\theta$ ) is the	
141	angle between a movement segment and a constant axis, most commonly the horizontal	
142	axis (Figure 2a). Displacement $(d)$ is the Euclidean distance between two consecutive	
143	fixes in a movement segment (Figure 2a). We are interested in deriving a measure of	
144	dynamic interaction that separately quantifies interactions in azimuth and displacement	
145	(Figure 2b-e).	
146	< Approximate location of Figure 2 >	
147	3.1 Azimuth – $\theta$	

148 To investigate the interaction in movement azimuths we take the cosine of the149 angle between them. This is simply calculated as:

150 **di** 
$$_{\theta} = f_t(\theta_t^a, \theta_t^b) = \cos(\theta_t^a - \theta_t^b)$$
 (3)

151 where  $\theta_t$  is the angle of movement at time-step *t*. Here  $f_t$  has a range of [-1, 1] as desired.

- 152 The function  $\cos \left(\theta_t^a \theta_t^b\right)$  is 1 when movement segments have the same orientation, 0
- 153 when movement segments are perpendicular, and -1 when in complete opposing
- 154 directions. In practice if either object (or both) do not move (3) is undefined, because  $\theta_t$  is

undefined. Thus, we must consider two alternative scenarios; first if one object moves and one remains stationary, and second if both objects remain stationary. Here we make the assumption that if one moves and the other remains stationary the two objects exhibit no directional interaction, and if both are stationary they are positively interactive.

159 Considering these two alternative scenarios, a complete definition for (3) is:

160 
$$f_{t}\left(\theta_{t}^{a},\theta_{t}^{b}\right) = \begin{cases} 0 , \text{ one of } \theta_{t}^{a} \text{ or } \theta_{t}^{b} \text{ undefined} \\ 1 , \text{ both } \theta_{t}^{a} \text{ and } \theta_{t}^{b} \text{ undefined} \\ \cos\left(\theta_{t}^{a}-\theta_{t}^{b}\right) , \text{ otherwise} \end{cases}$$
(4)

161 3.2 Displacement - d

Interaction in movement displacement could be measured using a variety of functions. However, it is desirable to have the function  $(g_t)$  fall in the range of 0 - 1, where a value of 0 represents no interaction and 1 positive interaction. Note there is no consideration of negative interaction in displacement. Using this definition  $g_t$  can be thought of as a scaling function to  $f_t$ , and maintains the statistic on the range [-1, 1]. We propose the following function for  $g_t$ :

168 **di** 
$$_{d} = g_{t} \left( d_{t}^{a}, d_{t}^{b} \right) = 1 - \left( \frac{\left| d_{t}^{a} - d_{t}^{b} \right|}{d_{t}^{a} + d_{t}^{b}} \right)^{a}$$
 (5)

169 Where  $|\cdot|$  is the absolute value operator, and  $\alpha$  is a scaling parameter defaulting to 1. The

170 function  $g_t(d_t^a, d_t^b)$  approaches zero when  $d_t^a >>> d_t^b$  or vice-versa, and is 1 when  $d_t^a =$ 

171  $d_t^{b}$ . The effect of the scaling parameter ( $\alpha$ ) on the function  $g_t(d_t^{a}, d_t^{b})$  is demonstrated in

172 Figure 3. Parameter  $\alpha$  can be adjusted to place stricter or looser requirements on

173 similarity in displacement denoting interaction. As  $\alpha$  is increased larger differences in

174 displacement are still considered as positively interactive. A closer examination of (5)

175 reveals that it is undefined when  $d_{t}^{a} + d_{t}^{b} = 0$ , (i.e., both objects are stationary). If we 176 consider both objects remaining stationary as positive interaction, a more robust 177 definition of (5) is:

178 
$$g_{t}\left(d_{t}^{a}, d_{t}^{b}\right) = \begin{cases} 1 & , \quad d_{t}^{a} + d_{t}^{b} = 0\\ 1 - \left(\frac{\left|d_{t}^{a} - d_{t}^{b}\right|}{d_{t}^{a} + d_{t}^{b}}\right)^{a} & , \quad d_{t}^{a} + d_{t}^{b} > 0 \end{cases}$$
(6)

179 < Approximate location of Figure 3 >

180 Thus, for two corresponding movement segments, a measure of dynamic 181 interaction is the product between the azimuthal term  $(f_t)$  and displacement term  $(g_t)$ :

182 
$$\mathbf{di}_{t} \left( v_{t}^{a}, v_{t}^{b} \right) = \mathbf{di}_{\theta} \times \mathbf{di}_{d} = f_{t} \left( \theta_{t}^{a}, \theta_{t}^{b} \right) \times g_{t} \left( d_{t}^{a}, d_{t}^{b} \right) \quad (7)$$

We are motivated to use the functions  $f_t$  and  $g_t$  to provide a statistic that covers the range [-1, 1] as was done in Shirabe (2006). Positive values of  $\mathbf{di}_t$  correspond to cohesive or positively interactive movements, while negative values can be interpreted as repulsion or opposing movements. Values near zero should be interpreted as having no interaction. The **di** statistics measure dynamic interaction based on similarity in azimuth ( $\theta$ )

The **di** statistics measure dynamic interaction based on similarity in azimuth ( $\theta$ ) and displacement (d) of simultaneous movement segments but do not account for the proximity of moving objects. Thus, **di** represents a similarity index taken in a normalized plane (i.e., the distance between the two objects has no impact on the resulting value). We are motivated to use this type of formulation as the spatial proximity required for dynamic interaction to occur is application specific. It is up to the analyst to decide if two moving objects maintain a requisite proximity for dynamic interaction to occur, then such interaction can be measured using **di**. In cases where actual spatial contact is required, for 195 example when identifying points-of-interest in large movement databases (e.g., Benkert196 et al. 2007), the **di** method should not be employed.

197 We have made assumptions in the equations for  $di_{\theta}$  and  $di_{d}$  regarding how to 198 analyze dynamic interactions when objects do not move (i.e.,  $\theta$  is undefined and d = 0). 199 In certain cases interpretation of these situations will be clear, for example, if one object 200 stops moving, does the other? However in practice, many applications may not facilitate 201 such straight-forward interpretation. For example, when studying urban travelers does 202 stopping at a red-light signify a change to dynamic interaction even if they will 203 eventually go straight? In light of these concerns, these assumptions can be modified to 204 accommodate different situations that may arise in various movement scenarios to fit a 205 given application.

206 *3.3 Global analysis* 

A global version of the **di** statistic can be used to measure the overall interaction in a set of movement segments. First, it is useful to recognize that we can identify global interaction in azimuth or displacement individually by summing the interaction values for each individual segment and dividing by the number of segments. This form of a global **DI** gives equal weight to each segment.

212 **DI** 
$$_{\theta} \left( V^{a}, V^{b} \right) = \frac{1}{n-1} \sum_{t=1}^{n-1} \mathbf{d}_{\theta}$$
 (8)

213 DI 
$$_{d}(V^{a}, V^{b}) = \frac{1}{n-1} \sum_{t=1}^{n-1} \mathbf{d} _{d}$$
 (9)

214 A global measure of overall dynamic interaction **DI** can also be derived.

215 **DI** 
$$(V^{a}, V^{b}) = \frac{1}{n-1} \sum_{t=1}^{n-1} (\mathbf{di}_{d} \times \mathbf{di}_{d}) = \frac{1}{n-1} \sum_{t=1}^{n-1} (\mathbf{di})$$
 (10)

It is important to note that in the local version  $d\mathbf{i} = d\mathbf{i}_{\theta} \times d\mathbf{i}_{d}$ , but with the global statistic, due to summation rules,  $\mathbf{DI} \neq \mathbf{DI}_{\theta} \times \mathbf{DI}_{d}$ . This can make interpretation of global values of **DI** less straightforward than with local values. However, if we were to alternatively define the global version as  $\mathbf{DI} = \mathbf{DI}_{\theta} \times \mathbf{DI}_{d}$ , then the equation defined by (10) would no longer hold. Thus, interpretation of **DI** values is best done separately for each component (i.e., **DI**, **DI**\_{\theta}, and **DI**\_d).

The global formulation is also appropriate for interval and episodal levels of analysis. Here we simply replace *n* with some interval or episode length *n*', where *n*' < *n*. This type of analysis can be illuminating when analyzing interactions in larger movement datasets, where varying levels of dynamic interaction may occur at different points in the movement paths.

#### 227 3.4 Time- and Distance-based Weighting

In instances where the sampling interval of the *n* fixes is unequal it is desirable to scale the statistic based on the temporal duration of each movement segment. In practice, this would give more weight to segments of longer duration and less weight to shorter segments. Temporal weighting may also be used to account for missing fixes, common to GPS-based tracking data. Let  $\Delta_t$  correspond to the temporal duration of segment *t*, where  $\sum_{t=1}^{n-1} \Delta_t = T$  is the total duration of the entire movement path. Then a time weighted

234 version of (10) is defined as:

235 **DI** 
$$(V^{a}, V^{b}) = \sum_{t=1}^{n-1} \frac{\Delta_{t}}{T} \operatorname{di}_{\theta} \times \operatorname{di}_{d}$$
 (11)

Viewed in light of the uncertainty associated with movement data, this form of temporalweighting may be counter-intuitive. That is, it may be logical to assign weights inversely

proportional to the duration between fixes; lower weights to segments with higher
uncertainty (i.e., more time between fixes) and higher weights to segments with higher
certainty or finer space-time resolution.

241 Similarly, we can define a distance-based weighting scheme for (10) where 242 movements with larger displacement have increased weight in calculation of the statistic. 243 Varying distance-based weights could be used when dynamic interactions of a specific 244 movement behavior are of interest. For example in the study of wildlife long directed 245 movements are often interspersed with shorter random movements distinguishing 246 migratory and foraging behavior (Turchin 1998). Distance weighting could be used to 247 tailor the measurement of dynamic interactions to either of migratory or foraging 248 behaviors in this case. A possible distance-based weighting scheme would be the average displacement of two segments:  $d_t^{avg} = (d_t^a + d_t^b)/2$ , and  $\sum_{t=1}^{n-1} d_t^{avg} = D$ . Based on the 249

average displacement a distance-weighted version of (10) is defined as:

251 **DI** 
$$(\mathbf{V}^{a}, \mathbf{V}^{b}) = \sum_{t=1}^{n-1} \frac{d^{avg}}{D} \mathbf{di}_{\theta} \times \mathbf{di}_{d}$$
 (12)

252 However, the average displacement of two objects movement segments is misleading 253 when one object has a large displacement and the other has a small displacement. Thus, 254 other distance measures are worth investigating for alternative distance-based weighting 255 schemes, keeping in mind that the sum of the weights should equal one. The equations 256 (11) and (12) can be combined to provide a time- and distance-based weighting scheme. 257 It is important to note that time- and distance-based weighting is really only useful when 258 interpreting global results when there is benefit to assigning segments weights based on 259 duration or distance.

Another interesting extension to studying correlations in movement paths is when movements interact with a temporal lag, for example when trend-setting occurs, as described by Laube et al. (2005). The **DI** statistic can be modified to evaluate dynamic interactions at a temporal lag. To measure dynamic interactions at a temporal lag, select a time lag – k, where k is generally taken to be a multiple of the fix interval (i.e., if fixes are taken at even intervals the time between consecutive fixes). Then we can, alternatively define  $di_{\theta}$  and  $di_{d}$  as:

267 **di**  $_{\theta} = f_{t}\left(\theta_{t}^{a}, \theta_{t+k}^{b}\right)$  (13)

268 **di**  $_{d} = g_{t} \left( d_{t}^{a}, d_{t+k}^{b} \right)$  (14)

269 The global statistics (**DI**, **DI**<sub> $\theta$ </sub>, **DI**<sub>d</sub>) can be computed as before, using the time lagged 270 versions of **di**<sub> $\theta$ </sub> (13) and **di**<sub>d</sub> (14).

271 **4 Data** 

#### 272 4.1 Simulated Data

273 Six simulated data sets are used to highlight the utility of the **DI** statistic and the 274 benefit of extensions it makes to **r** (Shirabe 2006). A single random walk (n = 10) is used 275 to generate a movement path that is the bases for the simulation examples. We used 276 manual permutations to the spatial coordinates of the original random walk to produce 5 277 new movement paths that represent 5 unique dynamic interaction scenarios (Table 1). 278 The first scenario simulates two objects moving with strong-positive dynamic interaction. 279 The second scenario uses the same two paths as the first scenario, but one is rotated at 280  $45^{\circ}$ , simulating strong interaction in displacement, and low interaction in azimuth. The 281 third scenario simulates positive interaction in azimuth and no interaction in 282 displacement. The fourth scenario simulates negative interaction in azimuth and no

283 interaction in displacement. The fifth scenario simulates no interaction in azimuth and 284 strong interaction in displacement. The sixth scenario uses a second independent random 285 walk to simulate random interactions between two moving objects. 286 < Approximate location of Table 1 > 287 4.2 Athletes – Ultimate Frisbee 288 In team sports players (objects) movements are expected to be highly interactive. 289 Often a defending player is tasked with "covering" an offensive player, and their 290 movements are in reaction to that offensive player. In the sport of ultimate frisbee, 291 offensive players move about the field in an attempt to get open for a pass from their 292 teammates. Defending players cover them, in an attempt to intercept or dissuade passes 293 from being completed. As such, in ultimate frisbee the movements of an offensive player 294 and their defender are highly interactive. We used 5 Hz sports-specific GPS devices 295 (GPSports, Fyshwick, Australia) to monitor the movements of two ultimate frisbee 296 players over a one minute segment during a training game. In this example, the two 297 players cover each other for the entirety of the one minute period. A total of n = 276 GPS 298 locations (out of a possible 300) were simultaneously recorded. Most of the missing 299 locations occur when the players are relatively stationary. At 5 GPS locations per second 300 this represents an extremely detailed movement dataset, appropriate for investigating the 301 intricate movements of athletes. 302 4.3 Grizzly Bears in Alberta, Canada 303 To further demonstrate **DI**, we investigate a previously published dataset

304 containing GPS telemetry locations of a number of grizzly bears in Alberta, Canada

305 (Stenhouse et al. 2005). Stenhouse et al. (2005) revealed that various bear combinations

306 showed evidence of dynamic interaction during different seasons, in particular male-307 female interactions were strongest during spring when mating activity occurs. To 308 demonstrate **DI**, we examine one specific male-female bear combination that exhibited a 309 relatively strong association during the mating season (male (G006) and female (G010) -310 see Fig. 4 in Stenhouse et al. 2005). Grizzly bear GPS collars were programmed to obtain 311 a location fix every four hours, however missing entries are frequent. As a result, only 312 112 simultaneous GPS fixes were obtained for the two bears during period from May 28, 313 2000 to July 08, 2000. In this example, we incorporate time-based weighting in order to 314 account for unevenness in fix intervals (ranging from 4 hours to over 6 days).

315 **5 Results** 

316 5.1 Simulated Data

317 Using the six simulated datasets we compared global values for **DI**, **DI**<sub> $\theta$ </sub>, and **DI**<sub>d</sub> 318 with Shirabe's (2006)  $\mathbf{r}$  statistic (Figure 4) to reveal both the similarities and differences 319 between these two methods. In scenario 1, where both movements are highly interactive 320 in both displacement and azimuth, **DI** and **r** are very similar. In scenario 2 **DI** and **r** are 321 similar, however using the DI method we can identify that interaction is higher in 322 displacement ( $\mathbf{DI}_d = 0.977$ ), and lower in azimuth ( $\mathbf{DI}_\theta = 0.664$ ). In contrast, scenario 3 323 reveals a situation where **DI** and **r** exhibit substantially different results. Using **DI**<sub> $\theta$ </sub> and 324  $\mathbf{DI}_d$  we can further examine the nature of the interaction in both azimuth and 325 displacement, in this case  $\mathbf{DI}_d = 0.287$  and  $\mathbf{DI}_{\theta} = 0.992$ . High  $\mathbf{DI}_{\theta}$  independent of  $\mathbf{DI}_d$ 326 could be useful in measuring interactive movement patterns via different modes of 327 transportation (e.g., walking vs. biking), or scale independent movement behavior in 328 wildlife. Scenario 4 demonstrates an example where negative dynamic interaction is

329	present (i.e., repulsion). In this case, <b>DI</b> is small and negative ( <b>DI</b> = $-0.278$ ) due to low			
330	interaction in displacement ( $\mathbf{DI}_d = 0.280$ ), while $\mathbf{r}_{xy}$ is large and negative ( $\mathbf{r} = -0.805$ ).			
331	Scenario 5, shows the case where low <b>DI</b> is a function of low interaction in azimuth ( <b>DI</b> <sub><math>\theta</math></sub> )			
332	= -0.095), despite having a strong level of interaction in movement displacement ( $\mathbf{DI}_d$ =			
333	0.979), while $\mathbf{r}_{xy} = -0.532$ . Measurement of high vs. low $\mathbf{DI}_d$ independent of $\mathbf{DI}_{\theta}$ could be			
334	used in behavior analysis to identify objects with similar diurnal activity patterns (i.e.,			
335	temporal patterns of long and short movements). In Scenario 6, both <b>DI</b> and $\mathbf{r}$ show			
336	values near 0, as would be expected from two independent random motions. It is			
337	interesting to note that $\mathbf{DI}_d = 0.649$ is relatively high in this example, as the random			
338	walks used identical parameters for their displacement distributions.			
339	< Approximate location of Figure 4 >			
340	) 5.2 Athletes – Ultimate Frisbee			
341	In the Ultimate Frisbee example, the two players positively interact in movement			
342	azimuth ( $\mathbf{DI}_{\theta} = 0.682$ ) and movement displacement ( $\mathbf{DI}_{d} = 0.730$ ). The global statistic			
343	shows that a substantial level of interaction exists between the two athletes ( $\mathbf{DI} = 0.572$ ).			
344	Local analysis enables the identification of times/locations where the athletes exhibit			
345	more or less interactive movements (Figure 5). In the ultimate frisbee example, local			
346	analysis is more informative than the global measure, as the movement path consists of			
347	many (shorter) movement segments. Maps of local <b>di</b> can be combined with a time-series			
348	graph of $\mathbf{di}$ , $\mathbf{di}_{\theta}$ , and $\mathbf{di}_{d}$ related to times/locations during the game where the defending			
349	player did a poor job covering the offensive player. We use episodal level analysis to			
350	segregate the movement paths into episodes of high vs. low interaction in order to further			
351	investigate the interactive behavior of these two athletes. For example, from 0 - 20 and 38			

- 40 seconds (highlighted in blue in Figure 5), high and positive di values suggest the
defending player is providing good defensive coverage (for these two episodes DI =
0.757). While from 20 - 38 seconds (highlighted in red in Figure 5) di values are much
lower, an indication of less interactive movement and poor defensive coverage (for this
episode DI = 0.122).

357

#### < Approximate location of Figure 5 >

358 5.3 Grizzly Bears in Alberta, Canada

In the grizzly bear example it was revealed that the male and female bears showed 359 360 substantial interaction (DI = 0.578) over the 42 day period from May 28, 2000 to July 8, 361 2000, using time-based weighting (see equation (11)) to account for missing fixes. 362 Similarly, time weighted results for azimuth ( $\mathbf{DI}_{\theta} = 0.663$ ) and displacement ( $\mathbf{DI}_{d} =$ 363 (0.731) reveal that both azimuth and displacement were strongly related during this 364 period. Local analysis revealed that the strong interaction seen with the global results was 365 a function of highly cohesive movements during the middle of June, while at the 366 beginning of June the two animals show little interaction (see Figure 6). Again we 367 perform analysis at the episodal level for separate periods identified visually from the 368 local analysis as having low and high dynamic interaction (low interaction: May 28 – 369 June 09; high interaction: June 09 - 29). The period of high interactions has a time-370 weighted  $\mathbf{DI} = 0.492$ , while the period of low interaction has a time-weighted  $\mathbf{DI} = 0.029$ . 371 Highly interactive behavior by mating grizzly bears is common in this region, as males 372 will attempt to confine female movements to a 'mating area' (Hamer and Herrero 1990). 373 Interpretation of maps and graphs of **di** facilitates the identification of where and when 374 such behavior occurs.

375

#### 376 6 Discussion

377 **DI** has three fundamental advantages over an existing method (Shirabe 2006) for 378 measuring interactions (termed correlations) in movement data. First, the existing method 379 follows a traditional correlation coefficient structure and is thus dependent on the mean 380 vector of a movement vector time series. In most cases, this mean movement vector will 381 have little relevance in the context of the analysis. However, in cases where interactions 382 are expected to occur relative to some mean movement trajectory, the method from 383 Shirabe (2006) is still advantageous. For instance, two objects moving radially from a 384 point (at some acute angle) may exhibit dynamic interaction (e.g., Fig. 4a in Shirabe 385 2006). Second, **DI** is explicitly decomposed into components measuring interaction in 386 movement azimuth and displacement. This property enables analysts to identify 387 situations where movements are related in one component but not the other. For example, 388 in scenario 3,  $\mathbf{DI}_d$  is low, however strong interaction is present in  $\mathbf{DI}_{\theta}$ , indicating that the 389 objects move with similar azimuths but not displacements, a conclusion not discernable 390 from the  $\mathbf{r}_{xy}$  statistic. Lastly, the **di** statistics we have developed are calculated 391 independently for each simultaneous movement segment. The **di** values can be mapped 392 and analyzed in a time-series fashion providing a local level analysis. Local analysis 393 reveals spatial-temporal information about locations of increased or decreased interaction 394 along the movement trajectory. Furthermore, the local level statistics (di,  $di_{\theta}$ , and  $di_{d}$ ) are 395 easily aggregated to coarser levels of analysis (interval, episodal, and global). 396 Other research areas where measuring movement interactions could provide new

397 and unique insight include transportation, human-activity, and other wildlife and sporting

398	examples. In transportation applications measuring interactions in large movement	
399	databases could be used for generating information on commuter behavior. Examples	
400	from human-activity research where interactions are important include tourist behavior	
401	(e.g., Shoval and Isaacson 2007) or crowd dynamics (Batty et al. 2003). With wildlife	
402	movement data, the detection of interactions is important in the study of resource	
403	selection (Millspaugh et al. 1998) and social behavior (Bandeira de Melo et al. 2007;	
404	Kenward et al. 1993), but also for examining offspring dependency, and inter-/intra-	
405	species behavior. Finally, a number of sporting examples exist where measuring	
406	6 movement interactions could provide new and unique insight including soccer, America	
407	football, and ice hockey.	
408	We use simulated movement data to highlight the advantages of <b>DI</b> over an	
409	existing method in a small set of specific scenarios designed to show the range of	
410	dynamic interactions present in movement data. When two movements are highly	
411	interactive (e.g., scenario 1) both methods successfully identify the high level of dynamic	
412	interaction. Also, when two movements show opposing or repulsive movements (e.g.,	
413	scenario 4) both methods are able to identify this behavior. The value of the <b>DI</b> method is	
414	demonstrated in scenarios 3, 4, and 5, where interactions in either azimuth or	
415	displacement are coupled with no interaction in the other component. This type of	
416	analysis may be useful, for example, when object movement is dependent on a temporal	
417	factor. For instance, many wildlife species are active only at specific times of the day and	
418	remain dormant during other periods. Measuring positive dynamic interactions in	
419	displacement, irrespective of azimuth, may be useful in identifying whether or not	

420 different species or individuals operate with similar circadian cycles (Merrill and Mech421 2003).

422 The example from athletes playing ultimate frisbee demonstrates the value of 423 measuring dynamic interactions at the local and episodal levels of analysis. Local and 424 episodal analysis revealed periods of varying degrees of dynamic interaction, which can 425 be related to player performance (i.e., how well the defensive player was able to cover the 426 offensive player). In many team sports, player evaluation has traditionally been 427 conducted by human observers. More recently, data driven analyses have become 428 common in the evaluation of players in team sports (e.g., Fearnhead and Taylor 2011). 429 When a player's movement can be directly related to specific abilities, for instance the 430 soccer example in Laube et al. (2005), the measurement of dynamic interactions, using 431 the **DI** method can enhance player evaluation using novel sport-specific movement 432 datasets.

433 The **DI** method we have developed requires that movement locations be recorded 434 simultaneously. Such a tidy form of movement data (i.e., where objects locations are 435 recorded simultaneously) may not always be available, limiting the ability to implement 436 this method. In such cases, path interpolation methods (e.g., Tremblay et al. 2006) could 437 be used to estimate the locations of one object at coinciding times. Similarly, in many 438 applications the assumption that movement data are collected at a regular interval is not 439 satisfied (e.g., with movement data collected using cell-phone records). This is also the 440 case in many wildlife telemetry studies where missing fixes are common. In the grizzly 441 bear example, we demonstrate the value of temporal weighting the **DI** statistic to account 442 for uneven sampling intervals. Further, we highlighted how local and episodal analyses

443 can provide unique and valuable insights into the nature of dynamic interactions present 444 in movement datasets. Local analysis reveals the times and locations of dynamic 445 interactions not discernable from global level statistics. When comparing male and 446 female grizzly bears, the dynamic interactions were likely due to mating behavior. This 447 example demonstrates the value of quantifying dynamic interactions in wildlife 448 movement datasets, as they can be related directly to specific social activities. 449 When movement data are collected at too fine a granularity, the movement 450 process (e.g., dynamic interaction) can be masked by data noise (termed over-sampling, 451 Turchin 1998). In these cases, down-sampling can be used to reduce data redundancy in 452 the movement path and improve the process signal to noise ratio. The **DI** statistics can 453 then be computed on the re-sampled movement dataset, as another form of interval and/or 454 episodal analysis (e.g., Laube et al. 2007). Variations of this procedure at different 455 interval and episodal scales can lead to increasingly complex and cross-scale 456 investigations of dynamic interactions in moving object datasets. Recently, Laube and 457 Purves (2011) have discussed the impact that movement data granularity (i.e., sampling 458 resolution) has on metrics used to quantify and describe movement trajectories (e.g., 459 mean speed). The **DI** method is similarly impacted by the granularity at which 460 movement data are represented. For example, at a coarse granularity objects may exhibit positive dynamic interactions, while at a fine granularity their movements may show 461 462 negative dynamic interaction (see Figure 7). Both the granularity at which the data are 463 represented and analysis level selected will impact the results and subsequent 464 interpretation of **DI**. One of Laube and Purves (2011) main recommendations is that

21

465 movement data analysis be conducted across a range of scales (granularities and analysis
466 levels) to correctly understand observed patterns.

467

< Approximate location of Figure 7 >

#### 468 6 Conclusions

469 Movement data are being collected for a variety of research agendas involving the 470 study of humans, their vehicles, and wildlife. Central to analyzing movement data is the 471 measurement of dynamic interactions between pairs of moving objects. We have 472 developed a new statistic (**DI**) for measuring dynamic interactions in discrete movement 473 data (e.g., with a GPS). The basic properties of movement segments – azimuth and 474 displacement, are used to detect dynamic interactions in azimuth, displacement, and 475 overall movement. The **DI** method can be applied at four analysis levels (local, interval, 476 episodal, and global - Laube et al. 2007) associated with movement data, and results can 477 be aggregated across analysis levels. We introduce both time- and distance-based 478 weighting schemes that can be useful in specific situations. The measurement of dynamic 479 interactions at a temporal-lag, an example of trend-setting (Laube et al. 2005), can be 480 easily incorporated. Like many spatial analysis techniques the **DI** method is impacted by 481 the granularity at which movement data is represented. A detailed investigation of cross-482 scale effects is warranted to provide a better understanding of how the measurement of 483 dynamic interaction is impacted by changing data granularities.

In some situations the nature of movement interactions will not simply involve two moving objects, but rather involve two moving objects impacted by a third. Consider the grizzly bear example; the bears exhibit varying levels of dynamic interaction over the course of the time period. The level of interaction is likely affected by their position

488	relative to the location of other objects, including other bears, roads, or sources of		
489	attraction or repulsion (i.e., food or danger). Future research will develop approaches for		
490	measuring third-party interactions, whereby pairs of moving objects interact with respec		
491	to a third stationary or moving object.		
492	To those wishing to measure dynamic interactions with their own applications w		
493	have developed code for implementing <b>DI</b> in the statistical software package R (R		
494	Development Core Team 2011), for more information please visit:		
495	<insert here="" link="" to="" website=""></insert>		
496	Acknowledgements		
497	Funding for this work was provided by Canada's Natural Science and		
497 498	Funding for this work was provided by Canada's Natural Science and Engineering Research Council (NSERC) and GEOIDE through the Government of		
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Scenario	Azimuth $(\theta)$	Displacement (d)
1	Positive interaction	Interaction
2	Positive interaction	Interaction
	(rotated by 45°)	
3	Positive interaction	No interaction
4	Negative interaction	No interaction
5	No interaction	Interaction
6	Random	Random

Table 1: Simulated movement scenarios, depicting different types of dynamic interactions, used to examine the differences between the new interaction statistic (DI) and an existing method ( $\mathbf{r}$ ).



Figure 1: Diagram of four analysis levels used in movement data analysis (after Figure 2 in Laube et al. 2007). Local level statistics are calculated for each individual movement segment. Interval level analysis computes a running average statistic using a moving window. Episodal level analysis computes the statistic over a selected 'episode' or period of the dataset. Global level analysis computes the statistic over the entire movement path.



Figure 2: a) Diagram of movement properties azimuth ( $\theta$ ) and displacement (d). Examples of movement segments that exhibit: b) positive interaction in  $\theta$  and low interaction in d; c) negative interaction in  $\theta$  and high interaction in d; d) no interaction in  $\theta$  and high interaction in d; d) no interaction in  $\theta$  and high interaction in d; d) no interaction in  $\theta$  and high interaction in d; d) no interaction in  $\theta$  and high interaction in d; d) no interaction in  $\theta$  and high interaction in d; d) no interaction in  $\theta$  and high interaction in d; d) no interaction in  $\theta$  and high interaction in d; d) no interaction in  $\theta$  and high interaction in d; d) no interaction in  $\theta$  and high interaction in d; d) no interaction in  $\theta$  and high interaction in d; d) no interaction in  $\theta$  and high interaction in d; d) no interaction in  $\theta$  and high interaction in d; d) no interaction in  $\theta$  and high interaction in d; d) no interaction in  $\theta$  and high interaction in d; d) no interaction in  $\theta$  and high interaction in d; d) no interaction in  $\theta$  and high interaction in d; d) no interaction in  $\theta$  and high interaction in



Figure 3: Relationship between  $log(d_a/d_b)$  and **di**<sub>*d*</sub>, for values of  $\alpha = 1, 2, 3$ .



Figure 4: Results from global analysis of 6 simulated example scenarios, comparing the new **DI** method with the Shirabe (2006) correlation statistic –  $\mathbf{r}$ . Original path is solid and black, while the path in dashed grey portrays variations based on six simulated scenarios (see Table 1).



Figure 5: Local analysis showing maps of **di** values for a) player 1, and b) player 2, from the ultimate frisbee example. c) time series graphs of **di**,  $di_{\theta}$ , and  $di_{d}$  can be used to identify periods of high and low dynamic interaction. Highlighted in blue in the time series graphs (c) are periods where player 1 does a good job covering player 2 (**DI** = 0.757). Highlighted in red is a period where the player 1 does a poorer job covering player 2 (**DI** = 0.122).



b) di map of female grizzly bear (male bear in gray)

c) time series graphs of di,  $di_{\theta}$ , and  $di_d$ 

Figure 6: Local analysis showing maps of **di** values for a) the male grizzly bear (G006), and b) the female grizzly bear (G010), from the grizzly bear example. c) time series graphs of **di**,  $di_{\theta}$ , and  $di_{d}$  can be used to identify periods of high and low dynamic interaction. Highlighted in red in the time series graphs (c) is a period where the bears exhibit low dynamic interaction (**DI** = 0.029). Highlighted in blue is period where the bears exhibit strong dynamic interaction (**DI** = 0.492), in this example indicative of mating behavior.



Figure 7: A pair of moving objects that exhibit negative dynamic interaction when analyzed at a fine granularity (dashed line,  $\mathbf{DI} = -0.47$ ) but positive dynamic interaction when analyzed at a coarser granularity (solid line,  $\mathbf{DI} = 0.49$ ). This example illustrates how changes in data granularity can impact results and interpretation of **DI**.