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According to Pareto, the distribution of income depends on "the nature of the people comprising a society, on the organization of the latter, and, also, in part, on chance." An overlapping generations model of marriage, fertility and income distribution is developed here. The "nature of the people" is captured by attitudes toward marriage, divorce, fertility, and children. Singles search for mates in a marriage market. They are free to accept or reject marriage proposals. Married agents make their decisions through bargaining about work, and the quantity and quality of children. They can divorce. Social policies, such as child tax credits or child support requirements, reflect the "organization of the (society)." Finally, "chance" is modelled by randomness in income, opportunities for marriage, and marital bliss.

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Jeremy Greenwood, Nezih Guner and John Knowles*†

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Comments Welcome

Abstract

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Keywords: Fertility; Marriage and Divorce; Nash Bargaining; Income Distribution; Public Policy

Subject Area: Primary, macroeconomics; secondary, labor economics

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1. Introduction

Income Distribution: Just over one hundred years ago, Vilfredo Pareto (1896, p.305) plotted the number of people, N, earning more than a given income level, x, against this level of income. He found this relationship to be a straight line (when the variables were expressed in logarithms), and it has been immortalized as the Pareto distribution.[1] Pareto felt that the income distribution could be explained by "the nature of the people comprising a society, on the organization of the latter, and, also, in part, on chance (p. 304)." Economists have been fascinated with the distribution of income ever since. Champernowne (1953) derived the Pareto distribution as the limiting distribution of a Markov chain for individual income. The Champernowne (1953) model, while illustrative, is really an exercise in statistical mechanics.[2]

Economists know that an individual's position on the income scale is not just determined by his or her own dumb luck. Family background is also important. The correlation between a father's and a son's income is quite high. [This evidence is surveyed in Stokey (1998)]. Furthermore, family structure is important. Married men make more money than single men [Cornwell and Rupert (1997)]. A female-headed family with children has about one third the median income of a married family with children. Children from a single-parent household do much worse than children from a two-parent family on many dimensions: they are much

[1] Let $\ln N = \ln A - \alpha \ln x$. Then, $N = Ax^{-\alpha}$, which is the Pareto distribution (providing $\alpha > 0$).

[2] Pareto believed that a probability model (based on chance alone) couldn't generate enough skewness to provide the basis for society's income distribution. Champernowne's (1953) results show that this belief was misplaced.

more likely to be out of school, out of work, or experience an out-of-wedlock birth [McLanahan and Sandefur (1994)]. Other things equal, families with lower income also tend to have more children [Knowles (1998)]. Hence, resources per child are less in low income families, both because there is less income and because this income has to be spread over more family members. This point has been made forcefully by Kuznets (1989, p. 230-231):

> It makes little sense to talk about inequality in the distribution of income among families or households by income per family or household when the underlying units differ so much in size. A large income for a large family may turn out to be small on a per person or per consumer equivalent basis, and a small income for a small family may turn out to be large with the allowances for size of the family. Size distributions of income among families or households by income per family or household, reflecting as they do differences in size, are unrevealing ... It follows that, before any analysis of family can be undertaken, size distributions of families or households by income per family must be converted to distributions of *persons* (or consumer equivalents) by size of family or *household income per person* (or per *consumer*).

Therefore, to understand fully the determinants of the distribution of income in society it is important to understand the determinants of both marriage and fertility.

The Model: To do this, an overlapping generations model of the family is built here. The world is made up of males and females. The model has four key ingredients. First, marriage is modeled along the search-theoretic lines of

Mortenson (1988). Each period every adult must make a decision on whether or not to stay with his or her mate. If an adult rejects his or her mate, then he or she is free to look for another one in the future.

Second, in line with the work by Mansur and Brown (1980) and McElroy and Horney (1981), decisions within a marriage are arrived at via Nash Bargaining. There is evidence that allocations with the household are not decided in a manner consistent with a single decision maker who maximizes some common set of preferences for the family — the unitary preference model. For instance, when government child allowances where transferred from husbands to wives in Great Britain during the late 1970's resource allocations within the household became more tilted toward the wives — see Lundberg, Pollak and Wales (1997). The higher the ratio of eligible males to females in a population the more the resource allocations within a marriage favor the wife [Chiappori, Fortin and Lacroix (1998)]. These ...nding are consistent with Nash Bargaining where the allocation of resources within the family depends upon the relative bargaining power of husband and wife.

Third, as in Barro and Becker (1988) and Razin and Ben-Zion (1975), adults decide how many children to have. Fourth, following the work of Becker and Tomes (1993) and Loury (1981) parents must decide how much time and goods to invest in their children. In addition to Champernowne's (1953) luck, these parental investments determine the productivity of a child when he or she grows up.

In the equilibrium modelled, some adults are married while others are not, some women have large families and others small ones, some people are rich while others are poor.[3] The framework results in an equilibrium income distribution. An example is presented where a significant number of children live with a single mother. Some of these mothers are unwed, others are divorced.[4] These children grow up to earn much less than children raised in a two-parent family. The girls from a single-parent family are also more likely to experience an out-of- wedlock birth or a divorce than the girls from two-parent families. And, so the cycle perpetuates itself implying a low degree of intergenerational mobility. There is a

[3] Aiyagari, Greenwood and Guner (2000) have combined the Mortenson (1988) paradigm with the Becker and Tomes (1993) framework to model the plight of single-parent families. In their analysis the formation and dissolution of families and investment in children are endogenous. Family size is held fixed. Husband and wife play a noncooperative Nash game. The Barro and Becker (1988) model has been wedded with Becker and Tomes (1993) framework by Knowles (1999) to study the effects that redistribution policies may have on poverty when fertility and investment in children are endogenous. Here a single-sex model is employed. In a sense the current analysis goes one step further by combining all three things together. This task is not simple. Nash Bargaining plays a vital and natural role in such an extension. The choice to have children involves dynamic considerations. When deciding how many children to have, a married couple must take into account the possibility of a future breakup. A male's and female's attitude toward children, both within and out of marriage, may differ from one another. Nash bargaining allows for these differences to be reconciled. As will be seen, the form poised for household decision making has important implications for the study of marriage and fertility. Last, building a model of marriage and fertility is important for understanding the distribution of income. A large fraction of poor children live with a single parent and low-wage earners tend to have more kids. Modelling these facts is important for understanding the distribution of income.

[4] Regalia and Rios-Rull (1999) also develop a model of marriage and divorce. They use their model to analyze the rise in single motherhood since the 1970s. They attribute a significant fraction of this increase to the (relative) rise in female wages.

negative relationship between income and fertility, as in the data. That is, poor families tend to have more children. This exacerbates income inequality. Last, the equilibrium income distribution is approximately lognormal with a Pareto tail.

Computational General Equilibrium Analysis and Public Policy: The Cowles Commission was an advocate of the use of models in economics. It felt that economic models would be useful for dealing with the simultaneity problem in economics and predicting the consequences of out-of-sample variation in policies or other exogenous variables. Computational general equilibrium models can be used in this regard, in addition to conventional econometric analysis.[5] The natural policy experiments to consider in the current context are anti-poverty programs. To illustrate the potential uses of computational general equilibrium models of the family, the e¤ects of child tax credits and child support payments are investigated. With child tax credits, families will now have more income per child, other things equal. Thus, their children should be better o¤. But other things may not be

[5] The need for dynamic general equilibrium models of the family has been noted by labor economists. For instance, according to McElroy (1997, p. 53) while there has been much work on partial equilibrium models of the household "little analysis has been based upon the appropriate general equilibrium framework, the marriage market." Weiss (1997, p 120) in his survey on the literature on marriage and divorce states that when "examining the economic contributions, the main obstacles is the scarcity of equilibrium models which carefully tie the individual behavior with the market constraints and outcomes. Consequently, we do yet have a convincing model which explains aggregate family formation and dissolution." The study for such models for policy analysis has been noted. "A model of marital search would be a more accurate descriptor of AFDC entry and exits ... " than a model of job search, says Mo¢tt (1992, 26). Hoynes (1997, p. 95) echoes this sentiment stating that relative to the classic, but static, Beckerian model of marriage "a dynamic model of marital search is a natural extension, but has yet to be developed in the literature."

equal, if such a policy promotes a larger family size. In the calculations undertaken here, a child tax credit fails to elevate the level of well being in society due to an increase in family size. Child support payments insulate children against the drop in family income that occurs when their parents divorce. On the one hand, child support payments make divorce less attractive to married males and marriage less attractive to single ones, at least other things equal. On the other hand, child support payments will make divorce more attractive to married females and marriage more attractive to single ones. In the analysis undertaken here the e¤ects of child support on the equilibrium number is almost a wash. The number of marriages and the average level of income increase only slightly. The size of the e¤ect is found to depend crucially upon how decisions are undertaken within the marriage.

The model presented here is intended as a prototype. It is still too primitive to be used for public policy purposes. Before serious policy analysis can be undertaken, more needs to be known about the choice of various components into the model: the way decisions are undertaken within the family; the appropriate way to model the interface between parents and their children[6]; the choice of functional forms for tastes and technology, such as the human capital production function; the choice of parameter values. As research progresses, answers to these questions will begin to emerge. So, future generations of this type of model may yield reliable answers to public policy questions about the family.

[6] For instance, do parents care more about the investments they undertake in their children or their children's expected utility? Do parents choose their investments strategically to ensure certain outcomes in their o¤springs? Do children play strategically in order to get more transfers from their parents?

2. Economic environment

Consider an economy populated by two groups of agents, females and males. Agents live for four periods: two periods as children, and two periods as adults. Let young and old refer to the …rst and second period of adulthood respectively. At any point in time, the female and male populations each consist of a continuum of children and a continuum of adults. Children become adults after they have been raised by their parents for two periods. Each adult is indexed by a productivity level. Let x denote the type (productivity) of an adult female, and z denote the type (productivity) of an adult male. Assume that x and z are contained in the sets $\mathcal{X} = \{x_1, x_2, \ldots, x_S\}$ and $\mathcal{Z} = \{z_1, z_2, \ldots, z_S\}$.

At the beginning of each period, there exists a marriage market for single agents. Any single agent can take a draw from this market. Agents are free to accept or reject a mate as they desire. If a single agent accepts a draw, s/he is married for the current period, provided of course, that the other person agrees too. Otherwise, the agent is single and can take a new draw at the beginning of the next period. Similarly, at the beginning of each period, married agents decide to remain married or get divorced. A divorced agent needs to remain single one period before having a new draw. Therefore, given the two-period overlapping generations structure remarriage is ruled out. Furthermore, assume that agents only match with people of the same generation.

Females are only fecund for the …rst period of their adult life. Therefore, each period, young married couples and young single adult females decide how many children to have. A child has equal chances of being a female or a male. Let k denote the number of children a female has. Assume that k is contained in the set $\mathcal{K} = \{0, 1, ..., K\}$. Children stay with their mothers, if their parents get divorced. A divorced male has to pay child support payments to his former wife after divorce.

Agents are endowed with one unit of (nonsleeping) time in each period. Females must split this time between work, child-care, and leisure. Males divide their time between work and leisure. A married male has to spend a fixed amount of time per child on homework.

Married agents derive utility from the consumption of a public household good, from human capital investment in their children, from leisure, and from marital bliss. Consumption of this household good depends upon the number of adults and children in the family. Parents must decide how much time and goods to invest in their children. This determines the level of human capital possessed by their children. Parents treat their children equally. Single males care only about their own consumption of goods and leisure and they do not worry about human capital investment in their children. When a male marries a female with children, however, he derives utility from the human capital investment in his stepchildren. A single mother must make the decision on her own about how much time and money to invest in her kids.

After two periods with their mother, children are endowed with productivity levels that depend on the human capital investment received throughout their childhood. Each period the oldest adult males and females die and are replaced by the oldest children who enter into the marriage market.

2.1. Preferences

Females: Let the momentary utility function for a woman be

$$
\begin{aligned}
F(c,e,k,1-l-t) &\equiv U(c)+V(e,k)+R(1-l-t-\iota_f k) \\
&\equiv \frac{c^{\nu_f}}{\nu_f}+\omega_f\frac{k^{\xi_f}}{\xi_f}\frac{e^{\vartheta_f}}{\vartheta_f}+\delta_f\frac{(1-l-t-\iota_f k)^{\varsigma_f}}{\varsigma_f}.
\end{aligned}
$$

Here c is the consumption of household production, which is a public good for the family, k is the number of children, and e is human capital investment per child. Females allocate l units of their time for work, and t units of it for child care or nurture. They also incur a …xed time cost of ι_f per child.

$Males$: A male's attitude toward children depends upon his marital status. Males spend n units of their time working. The utility function for a married male is described by

$$
\begin{aligned}
M(c,e,k,1-n;\chi) &\equiv U(c)+P(e,k;\chi)+S(1-n-\iota_m k) \\
&\equiv \frac{c^{\nu_m}}{\nu_m}+\chi\omega_m\frac{k^{\xi_m}}{\xi_m}\frac{e^{\vartheta_m}}{\vartheta_m}+\delta_m\frac{(1-n-\iota_m k)^{\varsigma_m}}{\varsigma_m},
\end{aligned}
$$

where

$$
\chi=\begin{cases} 1, & \text{if he is married, living with his own children,} \\ \lambda<1, & \text{if he is married, not living with his own children.} \end{cases}
$$

Married males incur a …xed time cost of ι_m per child. The functions V and P imply that the married male's attitudes toward the welfare of children is allowed to di¤er from the female's. The utility function for a single male can be expressed simply as $M(c,e,0,1-n;0)$; a single male does not realize any utility from the children borne through previous relationships.

2.2. Household consumption

Let p denote the number of adults in a household. Then, the consumption for a household with p adults and k children is given by

$$c = \Psi(p,k)[Y(l,n;x,z) - d] - \gamma I(q), \text{ for } q = m, s,$$

where

$$\Psi(p,k) = \left(\frac{1}{p+bk}\right)^{\eta}, \ 0 < \eta < 1, \ 0 < b < 1,$$

and

$$Y(l,n;x,z) = \begin{cases} (xl + zn), & \text{for a married couple,} \\ xl, & \text{for a single woman,} \\ zn, & \text{for a single man,} \end{cases}$$

and where the indicator function I returns a value of one for a married household and zero otherwise so that $I(m) = 1$ and $I(s) = 0$.

The function Y has a clear interpretation under the above parameterization. The variables x and z can be thought of as the market wages for type-x females and type-z males. The function Ψ translates household production into the consumption realized by adult family members. There are scale e¤ects in household consumption in the sense that each additional child costs less to feed and clothe than the one before. Still, it does cost more to maintain the extra child. Likewise, the second adult costs less than the …rst. The variable d represents the amount of household production that is used for investment in children. A single male will always set this to zero; because, either he has no children or he doesn't realize utility from them.

The parameter γ represents the quality of the match between a male and a female. Let $\gamma \in \mathcal{G} = \{\gamma_1, \gamma_2, ..., \gamma_m\}$ be a discrete random variable. For an unmarried couple this variable is drawn, after they are matched but before the marriage decision, according to distribution function $\Gamma(\gamma_h) = \Pr[\gamma = \gamma_h]$. For a married couple the variable γ then evolves over time according the process $\Delta(\gamma_n|\gamma_h) = \Pr[\gamma' = \gamma_n|\gamma = \gamma_h]$. Given the value drawn for γ', each party in a marriage decides whether to remain married.

2.3. Transmission of Human Capital

Human capital investment per child in a household with k children is given by

$$e = Q(t, d, k) \equiv \left(\frac{t}{k^{\square_1}}\right)^{\alpha} (\frac{d}{k^{\square_2}})^{1-\alpha}, \text{ for } 0 < \square_1, \square_2 < 1,$$

which transforms the child-care time of the mother, t, and the amount of the home produced good, d, into human capital investment, e. Recall that children are nurtured for two periods. At the end of every period the children of the oldest generation enter into the marriage market as single adults. The productivity levels for females are drawn from the distribution

$$\Xi(x_i|e_{-2} + e_{-1}) = \Pr[x = x_i|e_{-2} + \ e_{-1}],$$

and for males from

$$\Lambda(z_j|e_{-1} + e_{-1}) = \Pr[z = z_j|e_{-2} + \ e_{-1}],$$

where e_{-1} and e_{-2} indicate the human capital investment during the two periods of an agent's childhood. The distribution functions Ξ and Λ are stochastically increasing in $e_{-2}+ \ e_{-1}$ in the sense of …rst-order stochastic dominance. Thus, higher human capital investment in children by parents increases the likelihood that children will be successful in life.

The conditional distribution Ξ is represented by a discrete approximation to a lognormal distribution with mean, $\mu_{x|e}$, and standard deviation, $\sigma_{x|e}$. Similarly, suppose that Λ is also given by discrete approximation to a lognormal with mean, $\mu_{z|e}$, and standard deviation, $\sigma_{z|e}$. These conditional means are given by,

$$\mu_{x|e} = \mu_{z|e} = \varepsilon_1(e_{-2} + e_{-1})^{\varepsilon_2}, \text{ for } \varepsilon_2 \in (0,1),$$

where the ε's are the parameters governing the technology that maps human capital investment by parents into productivity levels.

After the first period of adulthood the productivity levels for females and males evolve according to the following transition functions:

$$X(x_j|x_i) = \Pr[x' = x_j|x = x_i],$$

and

$$Z(z_j|z_i) = \Pr[z' = z_j|z = z_i],$$

where x' and z' denote the next-period values. These Markov chains are constructed to approximate an AR(1) in logarithms.[7]

3. Decision Making

3.1. Household Activity — Single Old Adults

A single old female of type x with k children will solve the following problem:

$$G_2(x,k,z) = \max_{l,\ t,d} F\left(c,e,k,1-l-t\right) \qquad \text{P(1)}$$

[7] The discrete approximations for Ξ, Λ, X, and Z follow the procedure outlined in Tauchen (1986).

subject to

$$c = \Psi(1,k)[Y(l,0;x,0) + A(z,k) - d]$$

and

$$e = Q(t,d,k),$$

where

$$A(z,k) = azN^s(z,k)k.$$

Here z denotes her former husband's productivity and the function $N^s(z,k)$ denotes his labor supply. The function A determines how much child support a former husband has to pay, which is assumed to be a fraction, a, of his current income, $zN^s(z,k)$, per child. Obviously, for a single old female who was never married $z = 0$.

Denote a single mother's level of human capital investment in her children by

$$e = E_2^s(x,k,z).$$

This implies that $E_2^s(x,k,z)$=$Q(T_2^s(x,k,z), D_2^s(x,k,z), k)$, where $T_2^s(x,k,z)$ and $D_2^s(x,k,z)$ are the decision rules for t and d that arise from P(1).

The maximized utility of a single old male is given by the following problem:

$$B_2(z,k) = \max_n M(c,0,0,1-n;0) \qquad \text{P(2)}$$

subject to

$$\begin{aligned} c &\equiv \Psi(1,0)[Y(0,n;0,z) - aznk] \\ &\equiv zn - aznk = zn(1-ak), \qquad 0 < a < 1, \end{aligned}$$

where k denotes the number of children for whom he has to pay child support. For a single old male who was never married $k = 0$.

3.2. Household Activity — Old Married Adults with k children

Nash Bargaining Problem: Consider a couple of type (x, z, γ, k, χ) that is married in the second period. Assume that they make their decisions by applying the Nash solution to a …xed-threat bargaining game. Their problem is to solve

$$\max_{l,t,n,d} \; [F(c, e, k, 1-l-t) - G_2(x, k, z)] \times [M(c, e, k, 1-n; \chi) - B_2(z, k)] \quad \text{P(3)}$$

subject to

$$c = \Psi(2, k)[Y(l, n; x, z) - d] - \gamma = \Psi(2, k)[xl + zn - d] - \gamma,$$

and

$$e = Q(t, d, k).$$

Here $B_2(z, k)$ and $G_2(x, k, z)$ and are the threat points for the husband and wife. They are the values of being single in the second period, and are given by the solutions for old single agent problems, P(1) and P(2).

Denote the level of human capital investment per child in a family with two old parents by

$$e = E_2^m(x, z, \gamma, k; \chi).$$

Let the resulting utility levels for an old husband and wife in a $(x, z, \gamma, k; \chi)$-marriage, or the values for M and F in P(3) evaluated at the optimal choices for l, t, n, d and the implied values for c and e, be represented by

$$H_2(x, z, \gamma, k; \chi),$$

and

$$W_2(x, z, \gamma, k; \chi). \quad \text{P'(3)}$$

3.3. Marriage — Old Adults

Consider an age-2 couple indexed by (x, z, γ, k, χ). Each party faces a decision: should s/he choose married or single life for the period. Clearly, a married female will *want* to remain married if and only if $W_2(x, z, \gamma, k; 1) \geq G_2(x, k, z)$; otherwise, it is in her best interest to get a divorce. Equally as clearly, a single female will *desire* to marry if and only if $W_2(x, z, \gamma, k; \lambda) \geq G_2(x, k, 0)$; otherwise, she'll go it alone. Similarly, a married male would wish to remain so if and only if $H_2(x, z, \gamma, k; 1) \geq B_2(z, k)$, while a single male will like to marry if and only if $H_2(x, z, \gamma, k; \lambda) \geq B_2(z, 0)$.

De…ne the indicator functions $I_2^q(x, z, \gamma, k)$ for $q = m, s$, which summarizes the matching decisions of married and single age-2 males, by

$$I_2^q(x, z, \gamma, k) = \begin{cases} 1, \text{ if } H_2(x, z, \gamma, k; \chi) \geq B_2(z, I(q)k), \text{ for } q = m, s, \\ 0, \text{ otherwise,} \end{cases} \quad \text{P(4)}$$

where $I(m) = 1$ and $I(s) = 0$. Note that χ is a function of the male's marital status *at the time of the decision*, since $\chi = 1$ if $q = m$ while $\chi = \lambda$ when $q = s$. Likewise, for $q = m, s$ let the indicator function $J_2^q(x, z, \gamma, k)$ de…ne the matching decisions for married and single age-2 females so that

$$J_2^q(x, z, \gamma, k) = \begin{cases} 1, \text{ if } W_2(x, z, \gamma, k, \chi) \geq G_2(x, k, I(q)z), \text{ for } q = m, s, \\ 0, \text{ otherwise.} \end{cases} \quad \text{P(5)}$$

3.4. Household Activity — Single Young Adults

Now, let the odds of drawing a single age-1 female of type x_i in the marriage market be represented by

$$\Phi_1(x_i)\text{, where } \Phi_1(x_i) \geq 0 \ \forall x_i \text{ and } \sum_{i=1}^{S} \Phi_1(x_i) = 1,$$

and the odds of meeting a single age-2 female of type x_i with k children in the marriage market be given by

$$\Phi_2(x_i, k)\text{, where } \Phi_2(x_i, k) \geq 0 \ \forall x_i \text{ and } \sum_{i=1}^{S} \sum_{k=0}^{K} \Phi_2(x_i, k) = 1.$$

Likewise, the odds of meeting a single age-i male of type z_i will be denoted by

$$\text{-}_j(z_i)\text{, where } \text{-}_j(z_i) \geq 0 \ \forall z_i \text{ and } \sum_{i=1}^{S} \text{-}_j(z_i) = 1.$$

A key step in the analysis will be to compute these matching probabilities.

The programming problem for an one-period-old single type-x_i female is

$$\begin{aligned} G_1(x_i) \quad = \quad & \max_{k} \{\max_{l,t,d}\{F(c, e, k, 1-l-t) + \beta \sum_{j=1}^{S} \sum_{l=1}^{S} \sum_{n=1}^{m} \max\{W_2(x_j, z_l, \gamma_n, k; \lambda) \\ & I_2^s(x_k, z_l, \gamma_n, k),\ G_2(x_j, k, 0)\} X(x_j|x_i)\text{-}_2(z_l)\,\Gamma(\gamma_n)\}. \end{aligned} \qquad \text{P(6)}$$

subject to

$$c = \Psi(1, k)[Y(l, 0; x, 0) - d] = \Psi(1, k)[x_i l - d],$$

and

$$e = Q(t, d, k).$$

In the above problem β is the discount factor. Here $\text{-}_2(z_l)\,\Gamma(\gamma_n)$ gives the probability that a single female of type x_i will meet a single male of type z_l and that their match will be of quality γ_n. Note that $W_2(x_k, z_l, \gamma_n, k; \lambda)$ is given by the solution to the Nash Bargaining problem P(3) for a type-$(x_k, z_l, \gamma_n, k; \lambda)$ marriage.

Marriage is an option only if her mate is willing; that is, when $I_2^s(x_k, z_l, \gamma_n, k) = 1$. The value $G_2(x_k, k, 0)$ of remaining single is given by the solution to the problem of an old single female, or by P(1).

Let the utility-maximizing decision rules for the quantity and quality of children that solve this problem be represented by

$$k = K^s(x_i),$$

and

$$e = E_1^s(x, k) = E_1^s(x, K^s(x_i)).$$

The analogous recursion for a single male is

$$\begin{aligned} B_1(z_j) \quad = \quad & \max_n \{M(c, 0, 0, 1-n; 0) + \beta \sum_{l=1}^{S} \sum_{i=1}^{S} \sum_{k=0}^{K} \sum_{n=1}^{m} \max\{H_2(x_i, z_l, \gamma_n, k; \lambda) \\ & J_2^s(x_k, z_l, \gamma_n, k),\ B_2(z_l, 0)\} \Phi_2(x_i, k) Z(z_l|z_j) \Gamma(\gamma_n)\}. \qquad \text{P(7)} \end{aligned}$$

subject to

$$c = \Psi(1, 0) Y(0, n; 0, z_j) = z_j n,$$

where $\Phi_2(x_i, k)\Gamma(\gamma_n)$ is the probability of meeting an old single female of type-x_i with k children and having a match quality of γ_n.

3.5. Household Activity — Young Married Adults

Nash Bargaining Problem: Consider now the problem of a young married couple. Applying the Nash Bargaining solution to the fixed-threat bargaining game facing a young couple in a type-(x_i, z_j, γ_h) marriage gives

$$\max_{l,n,t,d,k} \{\{F(c, e, k, 1-l-t) + \beta \sum_{k=1}^{S} \sum_{l=1}^{S} \sum_{n=1}^{m} \max[W_2(x_k, z_l, \gamma_n, k; 1)\ I_2^m(x_l, z_l, \gamma_n, k),$$

$$G_2(x_k,k,z_l)]\Delta(\gamma_n|\gamma_h)X\,(x_k|x_i)\,Z(z_l|z_j)-G_1(x_i)\}$$
$$\times\{M(c,e,k,1-n;1)+\beta\sum_{k=1}^{S}\sum_{l=1}^{S}\sum_{n=1}^{m}\max[H_2(x_k,z_l,\gamma_n,k;1)J_2^m(x_k,z_l,\gamma_n,k),$$
$$B_2(z_l,k)]\Delta(\gamma_n|\gamma_h)X\,(x_k|x_i)\,Z(z_l|z_j)-B_1(z_j)\}\} \qquad \text{P(8)}$$

subject to

$$c=\Psi(2,k)[Y(l,n;x_i,z_j)-d]-\gamma_h=\Psi(2,k)[x_il+z_jn-d]-\gamma_h, \qquad (3.1)$$

and

$$e=Q(t,d,k). \qquad (3.2)$$

The threat points $G_1(x_i)$ and $B_1(z_j)$ are given by the solutions to the problems for young single females and males. The female would like to remain married if $W_2(x_k,z_l,\gamma_n,k;1)\geq G_2(x_k,k,z_l)$ and get a divorce otherwise. Remaining married is only feasible, however, if it is mutually agreeable or $I_2^m(x_k,z_l,\gamma_n,k)=1$. Similarly, the male would like to remain married if $H_2(x_k,z_l,\gamma_n,k;1)\geq B_2(z_l,k)$, which is feasible if $J_2^m(x_k,z_l,\gamma_n,k)=1$.

Let the optimal decision rules for the quantity and quality of children in a type-(x_i,z_j,γ_h) young marriage be denoted by

$$k=K^m(x_i,z_j,\gamma_n),$$

and

$$e=E_1^m(x_i,z_j,\gamma_h,k)=E_1^m(x_i,z_j,\gamma_h,K^m(x_i,z_j,\gamma_h)).$$

Furthermore, let the expected lifetime utility for a young male and female arising out of a type-(x_i,z_j,γ_h)-marriage be represented by

$$H_1(x_i,z_j,\gamma_h), \qquad \text{P'(8)}$$

and

$$W_1(x_i, z_j, \gamma_h).$$

3.6. Marriage — Young Adults

Then the marriage decisions for a randomly matched young couple, (x, z, γ), are given by

$$I_1^s(x, z, \gamma) = \begin{cases} 1, & \text{if } H_1(x, z, \gamma) \geq B_1(z), \\ 0, & \text{otherwise.} \end{cases} \qquad \text{P(9)}$$

and

$$J_1^s(x, z, \gamma) = \begin{cases} 1, & \text{if } W_1(x, z, \gamma) \geq G_1(x), \\ 0, & \text{otherwise.} \end{cases} \qquad \text{P(10)}$$

For a marriage to occur it must be mutually agreeable, which requires that $I_1^s(x, z, \gamma) J_1^s(x, z, \gamma) = 1$.

4. Equilibrium

4.1. Population Growth

The average number of children per female, $\mathbf{k}$, is given by

$$\begin{aligned} \mathbf{k} &= \sum_{i=1}^{S} \sum_{j=1}^{S} \sum_{h=1}^{m} \Phi_1(x_i) \text{-}_1(z_j) \Gamma(\gamma_h) I_1^s(x_i, z_j, \gamma_h) J_1^s(x_i, z_j, \gamma_h) K^m(x_i, z_j, \gamma_h) \\ &+ \sum_{i=1}^{S} \Phi_1(x_i) [1 - \sum_{j=1}^{S} \sum_{h=1}^{m} \text{-}_1(z_j) \Gamma(\gamma_h) I_1^s(x_i, z_j, \gamma_h) J_1^s(x_i, z_j, \gamma_h)] K^s(x_i). \end{aligned}$$

To understand this formula, note that the probability of a type-(x_i, z_j, γ_h) marriage between young adults is $\Phi_1(x_i) \text{-}_1(z_j) \Gamma(\gamma_h) I_1^s(x_i, z_j, \gamma_h) J_1^s(x_i, z_j, \gamma_h)$. This match will generate $K^m(x_i, z_j, \gamma_h)$ kids. The odds that a woman will be type-x_i and remain single are $\Phi_1(x_i)\,[1-\sum_{j=1}^{S}\sum_{h=1}^{m} \text{-}_1(z_j)\Gamma(\gamma_h)I_1^s(x_i, z_j, \gamma_h)J_1^s(x_i, z_j, \gamma_h)]$. This woman will have $K^s(x_i)$ children. In a stationary equilibrium the growth rate of the population, g, will therefore be

$$g = \sqrt{\frac{\mathbf{k}}{2}}.$$

4.2. Matching Probabilities

Young Adults: The probabilities of meeting a young female and male of a given type in the marriage market are $\Phi_1(x)$ and $\text{-}_1(z)$. To determine these probabilities, let $\Upsilon^{mm}(x_i, z_j, \gamma_h, x_k, z_l, \gamma_n)$ represent the fraction of females who were married in both periods and transited from state (x_i, z_j, γ_h) to (x_k, z_l, γ_n). Likewise, let $\Upsilon^{ss}(x_i, x_k)$ denote the fraction of females who were single in both periods, and transited from x_i to x_k, and $\Upsilon^{ms}(x_i, z_j, \gamma_h, x_k, z_l)$ denote the fraction of females who su¤ered a marriage breakup, etc. Hence,

$$\begin{aligned}\Upsilon^{mm}(x_i, z_j, \gamma_h, x_k, z_l, \gamma_n) \equiv\ & \Phi_1(x_i)\text{-}_1(z_j)\Gamma(\gamma_h)I_1^s(x_i, z_j, \gamma_h)J_1^s(x_i, z_j, \gamma_h) \\ & \times I_2^m(x_k, z_l, \gamma_n, k^m)J_2^m(x_k, z_l, \gamma_n, k^m)\Delta(\gamma_n|\gamma_h)X(x_k|x_i)Z(z_l|z_j),\end{aligned}$$

$$\begin{aligned}\Upsilon^{ss}(x_i, x_k) \equiv\ & \Phi_1(x_i)\,[1-\sum_{j=1}^{S}\sum_{h=1}^{m}\Gamma(\gamma_h)\text{-}_1(z_j)I_1^s(x_i, z_j, \gamma_h)J_1^s(x_i, z_j, \gamma_h)] \\ & \times X(x_k|x_i)[1-\sum_{l=1}^{S}\sum_{n=1}^{m}\Gamma(\gamma_n)I_2^s(x_k, z_l, \gamma_n, k^s)J_2^s(x_k, z_l, \gamma_n, k^s)\text{-}_2(z_l)],\end{aligned}$$

$$\begin{aligned}\Upsilon^{ms}(x_i, z_j, \gamma_h, x_k, z_l) \equiv\ & \Phi_1(x_i)\text{-}_1(z_j)\Gamma(\gamma_h)I_1^s(x_i, z_j, \gamma_h)J_1^s(x_i, z_j, \gamma_h)X(x_k|x_i)Z(z_l|z_j) \\ & \times \{\sum_{n=1}^{m}\Delta(\gamma_n|\gamma_h)[1-I_2^m(x_k, z_l, \gamma_n, k^m)J_2^m(x_k, z_l, \gamma_n, k^m)]\},\end{aligned}$$

$$\begin{aligned}\Upsilon^{sm}(x_i, x_k, z_l, \gamma_n) &\equiv \Phi_1(x_i)\,[1 - \sum_{j=1}^{S}\sum_{h=1}^{m} \Gamma(\gamma_h) {-}_1(z_j) I_1^s(x_i, z_j, \gamma_h) J_1^s(x_i, z_j, \gamma_h)] \\ &\quad \times I_2^s(x_k, z_l, \gamma_n, k^s) J_2^s(x_k, z_l, \gamma_n, k^s) \Gamma(\gamma_n) X(x_k|x_i) {-}_2(z_l), \end{aligned} \tag{4.1}$$

where $k^m \equiv K^m(x_i, z_j, \gamma_h)$ and $k^s \equiv K^s(x_i)$.

Then, it is easy to see that the odds of meeting a young woman of type-x_r in the marriage market are given by

$$\begin{aligned}\Phi_1(x_r) &= \{ \sum_{i,j,l,h,n} \Xi(x_r|E_1^m(x_i, z_j, \gamma_h, K^m(x_i, z_j, \gamma_h)) + E_2^m(x_k, z_l, \gamma_n, K^m(x_i, z_j, \gamma_h); 1)) \\ &\quad \times \Upsilon^{mm}(x_i, z_j, \gamma_h, x_k, z_l, \gamma_n) K^m(x_i, z_j, \gamma_h) \\ &\quad + \sum_i \Xi(x_r|E_1^s(x_i, K^s(x_i)) + E_2^s(x_k, K^s(x_i), 0)) \Upsilon^{ss}(x_i, x_k) K^s(x_i) \\ &\quad + \sum_{i,j,l,h} \Xi(x_r|E_1^m(x_i, z_j, \gamma_h, K^m(x_i, z_j, \gamma_h)) + E_2^s(x_k, K^m(x_i, z_j, \gamma_h), z_l)) \\ &\quad \times \Upsilon^{ms}(x_i, z_j, \gamma_h, x_k, z_l) K^m(x_i, z_j, \gamma_h) \\ &\quad + \sum_{i,l,n} \Xi(x_r|E_1^s(x_i, K^s(x_i)) + E_2^m(x_k, z_l, \gamma_n, K^s(x_i); \lambda)) \\ &\quad \times \Upsilon^{sm}(x_i, x_k, z_l, \gamma_n) K^s(x_i) \} / \mathbf{k}. \end{aligned} \tag{4.2}$$

The probability of meeting a type-z_r young man is determined analogously:

$$\begin{aligned}{-}_1(z_r) &= \{ \sum_{i,j,l,h,n} \Lambda(z_r|E_1^m(x_i, z_j, \gamma_h, K^m(x_i, z_j, \gamma_h)) + E_2^m(x_k, z_l, \gamma_n, K^m(x_i, z_j, \gamma_h); 1)) \\ &\quad \times \Upsilon^{mm}(x_i, z_j, \gamma_h, x_k, z_l, \gamma_n) K^m(x_i, z_j, \gamma_h) \\ &\quad + \sum_i \Lambda(z_r|E_1^s(x_i, K^s(x_i)) + E_2^s(x_k, K^s(x_i), 0)) \Upsilon^{ss}(x_i, x_k) K^s(x_i) \\ &\quad + \sum_{i,j,l,h} \Lambda(z_r|E_1^m(x_i, z_j, \gamma_h, K^m(x_i, z_j, \gamma_h)) + E_2^s(x_k, K^m(x_i, z_j, \gamma_h), z_l)) \\ &\quad \times \Upsilon^{ms}(x_i, z_j, \gamma_h, x_k, z_l) K^m(x_i, z_j, \gamma_h) \\ &\quad + \sum_{i,l,n} \Lambda(z_r|E_1^s(x_i, K^s(x_i)) + E_2^m(x_k, z_l, \gamma_n, K^s(x_i); \lambda)) \end{aligned}$$

$\times \Upsilon^{sm}(x_i, x_k, z_l, \gamma_n) K^s(x_i)\}/\mathbf{k}.$

Old Adults: Next, how are the odds of meeting a single age-2 type-x female with k children, $\Phi_2(x, k)$, or of a single age-2 type-z male, $\text{-}_2(z)$ determined in stationary equilibrium? This depends upon the number of single agents who remain unmarried from the previous period. So, how many are there? Again, the number of married and single one-period-old type-x_i females are given by $\Phi_1(x_i) \sum_{j=1}^{S} \sum_{h=1}^{m} \text{-}_1(z_j) \Gamma(\gamma_h) I_1^s(x_i, z_j, \gamma_h) J_1^s(x_i, z_j, \gamma_h)$ and $\Phi_1(x_i) [1 - \sum_{j=1}^{S} \sum_{h=1}^{m} \text{-}_1(z_j) \Gamma(\gamma_h) I_1^s(x_i, z_j, \gamma_h) J_1^s(x_i, z_j, \gamma_h)]$. Given this supply of one-period-old single females, the quantity of two-period-old type-x_j single females will be $\sum_{i=1}^{S} X(x_j|x_i) \Phi_1(x_i) [1 - \sum_{j=1}^{S} \sum_{h=1}^{m} \text{-}_1(z_j) \Gamma(\gamma_h) I_1^s(x_i, z_j, \gamma_h) J_1^s(x_i, z_j, \gamma_h)]$.

Let

$$\aleph(x_i, k) = \begin{cases} 1, \text{ if } K^s(x_i) = k, \\ 0, \text{ otherwise,} \end{cases}$$

be an indicator function representing the number of children that a single one-year-old female of type-x_i has. Then, the odds of drawing a single two-period-old type-x_j female with k children in the marriage market will be

$$\begin{aligned} \Phi_2(x_j, k) &= \{\sum_{i=1}^{S} \aleph(x_i, k) X(x_j|x_i) \Phi_1(x_i) [1 - \sum_{j=1}^{S} \sum_{h=1}^{m} \Gamma(\gamma_h) \text{-}_1(z_j) I_1^s(x_i, z_j, \gamma_h) J_1^s(x_i, z_j, \gamma_h)]\} \\ &\div \{\sum_{j=1}^{S} \sum_{i=1}^{S} X(x_j|x_i) \Phi_1(x_i) [1 - \sum_{j=1}^{S} \sum_{h=1}^{m} \Gamma(\gamma_h) \text{-}_1(z_j) I_1^s(x_i, z_j, \gamma_h) J_1^s(x_i, z_j, \gamma_h)]\} \end{aligned} \tag{4.3}$$

The analogous formula for the odds of meeting a single two-period-old male of type-z_j, $\text{-}_2(z_i)$, reads

$$\text{-}_2(z_i) = \frac{\sum_{j=1}^{S} Z(z_i|z_j)\text{-}_1(z_j)[1-\sum_{i=1}^{S}\sum_{h=1}^{m}\Gamma(\gamma_h)\Phi_1(x_i)\, I_1^s(x_i,z_j,\gamma_h)J_1^s(x_i,z_j,\gamma_h)]}{\sum_{i=1}^{S}\sum_{j=1}^{S} Z(z_i|z_j)\text{-}_1(z_j)[1-\sum_{i=1}^{S}\sum_{h=1}^{m}\Gamma(\gamma_h)\Phi_1(x_i)\, I_1^s(x_i,z_j,\gamma_h)J_1^s(x_i,z_j,\gamma_h)]}. \tag{4.4}$$

It's now time to take stock of the situation so far.

Definition 4.1. *A stationary matching equilibrium can be represented by set of child quantity and quality allocation rules,* $K^m(x,z,\gamma)$, $K^s(x)$, $E_2^m(x,z,\gamma,k;\chi)$, $E_2^s(x,k,z)$, $E_1^m(x,z,\gamma,K^m(x,z,\gamma))$, and $E_1^s(x,K^s(x))$, *a set of accept/reject decision rules,* $I_2^m(x,z,\gamma,k)$, $I_2^s(x,z,\gamma,k)$, $J_2^m(x,z,\gamma,k)$, $J_2^s(x,z,\gamma,k)$, $I_1^s(x,z,\gamma)$, *and* $J_1^s(x,z,\gamma)$, *and a set of matching probabilities,* $\Phi_1(x)$, $\Phi_2(x,k)$, $\text{-}_1(z)$, *and* $\text{-}_2(z)$, *such that:*

1. *The child quality allocation rule* $E_2^s(x,k,z)$ *solves the old single female's household problem P(1).*

2. *The child quantity and quality allocation rules* $K^s(x)$ and $E_1^s(x,K^s(x))$ *solve the young single female's household problem P(6).*

3. *The child quality allocation rule* $E_2^m(x,z,\gamma,k;\chi)$ *solves the married old couple's Nash bargaining problem P(3).*

4. *The child quality and quantity allocation rules* $K^m(x,z,\gamma)$ and $E_1^m(x,z,\gamma,K^m(x,z,\gamma))$ *solve the young married couple's Nash bargaining problem P(8).*

5. *The old male's accept/reject choices,* $I_2^m(x,z,\gamma,k)$ *and* $I_2^s(x,z,\gamma,k)$, *are described by P(4), in conjunction with P(2) and P'(3).*

6. *The young male's accept/reject choice,* $I_2^s(x,z,\gamma)$, *is described by P(9), in conjunction with P(7) and P'(8).*

7. *The old female accept/reject choices,* $J_2^m(x, z, \gamma, k)$ *and* $J_2^s(x, z, \gamma, k)$*, are described by P(5), in conjunction with P(1) and P'(3).*

8. *The young female's accept/reject choice,* $J_1^s(x, z, \gamma)$*, is described by P(10), in conjunction with P(6) and P'(8).*

9. *The matching probabilities,* $\Phi_1(x)$, $\Phi_2(x, k)$, $-_1(z)$*, and* $-_2(z)$*, are governed by the stationary distributions described by (4.2) to (4.4).*

At a general level, not much can be said about the properties of the above model since the solution involves a complicated …xed-point problem. On the one hand, in order to compute the solution to a young single agent's choice problem one needs to know the equilibrium matching probabilities. On the other hand, calculating the equilibrium matching probabilities requires knowledge about the solutions to each of the decision problems.

5. Some Computational Analysis

5.1. Benchmark Equilibrium

To gain some insight into the model's mechanics, its solution will be computed numerically.[8] To do this, values must be assigned to the model's parameters. These are listed in Table 1. The parameter values are not chosen to tune the model to be in perfect harmony with any features of the real world. Instead, they are

[8] Part of the numerical procedure used to compute the model's solution is outlined in the Appendix. The algorithm for …nding the equilibrium type distributions, or the Φ's and - 's, is similar to that employed in Aiyagari, Greenwood, and Guner (2000). For more detail, see that source.

picked to generate an equilibrium that displays several interesting characteristics that will now be discussed.

TABLE 1: Parameter Values

Tastes	$\nu_f = 0.5,\ \omega_f = 1,\ \xi_f = 0.325,\ \vartheta_f = 0.2,$
	$\delta_f = 3, \iota_f = 0.05,\ \varsigma_f = 0.3,\ \beta = 0.67,$
	$\nu_m = 0.5, \omega_m = 1,\ \xi_m = 0.325,\ \vartheta_m = 0.35,$
	$\delta_m = 3,\ \iota_m = 0.0325,\ \varsigma_m = 0.3, \lambda = 1.$
Technology	$b = 0.30,\ \eta = 0.5,$
	$\alpha = 0.5,\ \square_1 = 0.4,\ \square_2 = 0.5,$
	$\varepsilon_1 = 15.15, \varepsilon_2 = 0.5,$
Stochastic Structure	$\mu_{x\|e} = \mu_{z\|e} = \varepsilon_1(e_{-2} + e_{-1})^{\varepsilon_2},\ \sigma_{x\|e} = \sigma_{x\|e} = 0.4,$
	$\rho_x = 0.7,\ \rho_z = 0.7,$
	$\Gamma(\gamma_1) = \Gamma(\gamma_2) = 0.5,\ \Delta(\gamma_1\|\gamma_1) = \Delta(\gamma_2\|\gamma_2) = 0.5,\ \gamma_1 = 2.5,\ \gamma_2 = 0,$
Simulation Control	$S = 15,\ K = 4,\ m = 2,$
Policy Variables	$a = 0.05.$

Properties of the Equilibrium: First, observe from Table 2 that at any point in time a significant proportion of the adult population is not married. In equilibrium some people are always single, others experience a divorce. At any time about 85% of the population is married.

Table 2: Marital Status

(Percentage Distribution)

	Young	Old
Married	86	85
Single	14	5
Divorced	–	10

Second, family income is related to marital status, as Table 3 illustrates. For example, family income for a household headed by a young single female is 17% of that for a married couple. This transpires for two reasons. To begin with, in a marriage there are two potential wage earners versus only one in a household with a single adult. Additionally, married males and females work more than unmarried ones — Table 4.

Table 3: Family Income

	Young	Old
Married	1.00	1.00
Single — female	0.17	0.14
Single — male	0.36	0.41
Divorced — female		0.24
Divorced — male		0.33

Table 4: Time Allocations

	Male			Female		
	Married	Single	Divorced	Married	Single	Divorced
Work	0.60	0.44	0.41	0.37	0.27	0.27
Nurture	0	0	0	0.21	0.10	0.10
Leisure	0.34	0.56	0.59	0.33	0.46	0.52
Fixed	0.06	0	0	0.09	0.17	0.12

Third, fertility is also related to marital status. Single women have a much higher fertility rate than married women do. A young married woman has 1.8 kids on average while a young single woman has 3.3. So, while 85% of the population is married, only 78.5% of children live in a household with two adults. On average a female has two children; therefore, the population is stationary.

Fourth, children from a single-female family tend to do much worse. This is because their mother doesn't have much time or money to invest in them. A single mother has less time for work, nurture, and leisure because she has more children on average; i.e., more of her time is absorbed on the …xed costs of child rearing. Since she earns less money than a married couple, she has less resources to invest in her o¤spring also. Additionally, single women tend to have more children than do married women. The result of these facts is a lower level of human capital investment per child in a single female family — Table 5.

Table 5: Investment in Human Capital

	Young	Old
Married	1.00	0.99
Single female	0.30	0.29
Divorced female		0.37

Table 6 shows the e¤ect of family background on a female's income. A girl growing up in a household a single mother can expect to enjoy only two-thirds of the family income of one growing up with both parents. She is much more likely (44% versus 20%) to experience an out of wedlock birth or a divorce than the girl from a two-parent home too — Table 7.

Table 6: Effects of Childhood History on Female Income

Childhood History	$m \to m$	$m \to s$	$s \to m$	$s \to s$
Expected Wage	1.00	0.79	0.75	0.54
Expected Family Income	1.00	0.87	0.85	0.68

Table 7: Effects of Childhood History on Female Marital Experience

Adult History	$m \to m$	$m \to s$	$s \to m$	$s \to s$
Childhood History				
$m \to m$	0.80	0.09	0.08	0.03
$m \to s$	0.73	0.11	0.11	0.06
$s \to m$	0.71	0.11	0.11	0.06
$s \to s$	0.56	0.17	0.16	0.12

Back to Pareto: Figure 1 plots the economy's income distribution, both in cumulative distribution function form and à la Pareto. As can be seen the tail of the income distribution is fairly well approximated by a straight line, or is Pareto. (The lower panel plots two reference Pareto distributions with $A = 3.3$ and $\alpha = 2.0$ and $A = 28.0$ and $\alpha = 10$.) The rest is lognormal. (The upper line plots a lognormal with $\mu = 2.1$ and $\sigma = 0.87$ for a comparison.) As Kuznets (1989) has noted, it makes a di¤erence whether family or per capita income is used. The distribution of income is more skewed when per capita income is used because low income families tend to have more children.

Figure 2 shows the relationship between income and family size for both the model and the US. The data for the US comes from the Panel Study on Income Dynamics (PSID). The earnings variable is the present value of future lifetime household labor income at age 30, as calculated in Knowles (1999). In the data, fertility declines with labor income. The fertility variable is total number of children ever born to a woman, who is either head or spouse of the household head. The model replicates this relationship quite well.

When family income is adjusted for size, the situation portrayed in Table 3 changes. Single males are now the best o¤, since they have no dependents. Perhaps, this is why they work the least. The situation for unmarried females is now even bleaker. Income per family member is only 16% of the level realized in a married household — Table 8.

Table 8: Family Income per Member

	Young	Old
Married	1.00	0.97
Single — female	0.16	0.12
Single — male	1.28	1.48
Divorced — female		0.26
Divorced — male		1.18

5.2. Some Comparative Statics Exercises

To gain some insight into the structure of the model, several comparative statics exercises will be undertaken now.

Elasticity on Quality, ϑ_f: Suppose that the elasticity on the quality of children in the female's utility function is lowered from 0.2 to 0.19. What happens? The return at the margin from investing time and resources in children declines more rapidly now. Hence, parents will tend to invest less in their o¤spring. Instead, they will choose to have more children. That is, they now prefer quantity relative to quality. Married females now have 2.0 children on average (versus 1.8 earlier) while single ones have 3.7 (as compared with 3.3). The population's annualized growth rate increases to 0.73% [$= (1.075^{1/10} - 1) \times 100\%$]. Since there is less investment per child, the average quality of the mating pool drops. The fraction of married agents falls by about 3 percentage points.

The Fixed Time Costs of Childrearing, ι_f and ι_m: Let the …xed time cost of raising a child for a female drop. Speci…cally, let ι_f fall from 0.05 to 0.04. Since the cost of raising a child has fallen, there are more children in equilibrium.

Married females now have 2.1 children on average while single ones have 3.9. Since single females have the most children, the attractiveness of being a single mother increases. This, too, raises the average number of children per female. These factors lead the population's annualized growth rate to increase to 0.98%. The long-run quality of the mating pool drops. The increase in the quantity of children comes at the expense of their quality. All parents invest less per child. There are also more single mothers and they invest less in their children than do married ones. These tendencies operate to lower the long-run quality of the matching pool. As a result of these factors, in the new equilibrium the number of marriages falls by about 4 percentage points.

Leisure Elasticity, ς_f: What will happen if the utility function for women is made more elastic with respect to leisure? In particular, let $\varsigma_f = 0.35$ as opposed to 0.30. Women are now willing to work more — both at home and in the market — since the disutility from working is not rising as fast in terms of e¤ort. There is now more investment of both goods and time in children. Since married women work the most this increases the bene…t of marriage. The quality of the matching pool also rises. The upshot of this is that the number of young single mothers falls by about 0.7 percentage points. Married women have more children, since at the margin the disutility from raising more of them has dropped. The population's growth rate decreases slightly (because the number of young single women drops).

Consumption Elasticities, ν_f and ν_m: Consider the impact of making the utility function more curved in consumption. Reset $\nu_f = \nu_m = 0.4$, as opposed to the value of 0.5 adopted earlier. The number of marriages now rises by 8.5 percentage points. The population's growth rate increases to 0.6% per period. The question is, why? When the marginal utility from consumption declines faster, parents divert more of their income into children. They choose to increase both the quantity and quality of their o¤spring. Additionally, the extra consumption that males realize from single life is valued less. There will be less children living with a single parent. These considerations increase the long-run quality of the mating pool. The number of marriages rises, therefore, on these accounts.

Shock Structure: How does the structure of the shocks a¤ect the equilibrium? To explore this, the degree of persistence in the matching shock is increased. Now, $\Delta(\gamma_1|\gamma_1) = \Delta(\gamma_2|\gamma_2) = 0.9$. This leads to drop in the rate of marriage among the young (from 86 to 74%). When there is a bad match quality shock it will now persist into the future making marriage less attractive. Since there are more single mothers, the population's growth rate increases to about 0.5% per period. Likewise, increasing persistence in either or both of the type shocks has a similar e¤ect.

5.3. Nash Bargaining

How does Nash Bargaining work in the model? The Nash Bargaining solution attains a Pareto-optimal allocation between husband and wife — the details are in the Appendix. Therefore, there exists some set of weights ρ and $(1-\rho)$ such that solving a type-(x_i, z_j, γ_h) young couple's Nash Bargaining problem, P(8), is equivalent to solving the Pareto problem

$$\max_{l,n,t,d,k} \{(1-\rho)\{F(c,e,k,1-l-t) + \beta \sum_{k=1}^{S}\sum_{l=1}^{S}\sum_{n=1}^{m} \max[W_2(x_k, z_l, \gamma_n, k; 1)\ I_2^m(x_l, z_l, \gamma_n, k),$$
$$G_2(x_k, k, z_l)]\Delta(\gamma_n|\gamma_h) X(x_k|x_i)\, Z(z_l|z_j)\}$$
$$+\rho\{M(c,e,k,1-n;1) + \beta \sum_{k=1}^{S}\sum_{l=1}^{S}\sum_{n=1}^{m} \max[H_2(x_k, z_l, \gamma_n, k; 1) J_2^m(x_k, z_l, \gamma_n, k),$$

$$B_2(z_l, k)]\Delta(\gamma_n|\gamma_h)X\,(x_k|x_i)\,Z(z_l|z_j)\}\}$$

subject to (3.1) and (3.2). The Pareto weight ρ re‡ects the husband's bargaining power and is endogenously determined as a function of the state (x_i, z_j, γ_h).

Figure 3 shows how this weight behaves as a function of the state (x, z, γ). Take the case where the match quality variable has the high value. Observe that the male's bargaining strength increases with the level of his productivity, z, and decreases with his wife's, x. The same is true when the match quality variable takes on the low value.

Now, suppose that the model is solved holding the weight ρ …xed across states. For example let $\rho = 0.5$, which gives husband and wife an equal say in family decision making, so to speak. The number of marriages plummets in equilibrium from about 85 to 49%. Why? When the weights are …xed, utility can't be transferred from one party to the other in order to prevent a breakup and therefore not nearly as many marriages are sustainable. The degree of positive assortative mating is much higher than under the Nash Bargaining solution. Figure 4 shows the set of sustainable marriages in the economy with Nash Bargaining — i.e., the set of (x, z, γ) for which $I_1^s(x, z, \gamma)J_1^s(x, z, \gamma) = 1$. With a good match quality shock virtually all matches are sustainable. Even when the quality of the match is low most matches are sustainable. No female, however, wants a male from the low end of the distribution. Males aren't quite as choosy. When each party's bargaining power is held …xed, there is a high degree of assortative mating as Figure 5 illustrates. Now, when the quality of match is poor most marriages aren't sustainable.

6. Two Public Policy Experiments

Child tax credits are designed to elevate the welfare of all children in the economy. They transfer income away from families without children to families with them. Child support payments are targeted at those children who experience a family breakup because their parents get divorced. Here, to ease the devastating impact that a divorce can have on family income, governments require fathers to pay child support to their former wives. To illustrate how a model such as this can be used, consider the effects of these two public policies.

6.1. Child Tax Credits

Suppose that all families with children, both single and two-parent families, are eligible to collect a child subsidy. This subsidy provides a tax credit per child equal to 0.5% of the average level of income in the benchmark economy. It is financed by a lump-sum tax equal to 1.0% of income in the benchmark economy. What are the effects of this policy?

On the upside, the beneficial effects of the policy are twofold. First, poor families will get extra income that should allow them to invest more time and resources in their children. Second, it should make marriage a more attractive option for males, since single males are taxed without receiving any subsidy. On the downside, the attractiveness of marriage for females, however, might decline. Second, the beneficial aspects of this policy for children may be dissipated by larger family size.

The long-run health of the economy is not helped by this policy. First, the percentage of single mothers increases by about 4.5 percentage points. The percentage of children living with a young single mother rises by about 7 percentage points. This transpires because young single mothers tend to have more children than married ones, and because the policy promotes fertility. The (annualized) population growth rate rises from 0.13 to 1.07%. Single mothers now have 3.9 children as compared with 3.3 for the benchmark economy. Married women now average 2.1 children (versus 1.8 previously).

To understand the model's mechanics, it pays to artificially decompose the experiment into short- and long-run effects. For the short-run effects consider the impact of the child tax credit holding fixed the type distributions for young agents, or Φ_1 and $-_1$. This shuts down the effects on the economy from any induced changes in parental human capital investments. The percentage of single mothers rises by 2 percentage points. Both single and married women have more children (3.8 and 2.0). Married couples also substitute quality for quantity of children. The rise in female headship also reduces the average level of human capital investment in children. These effects operate to reduce the long-run quality of the mating pool, leading to a further 3 point rise in the percentage of single mothers.

Average income in the economy falls by about 11%. This occurs because there is now much less human capital investment in children. First, the increase in female headship is associated with a reduction in investment in children. Single mothers have less wherewithal — in terms of both time and goods — than married couples. Second, with an increase in the quantity of children there is a fall in their quality. As the price of having an extra child drops parents — married or otherwise — substitute quantity for quality. Figure 6 shows the impact of a child tax credit on the steady-state utility distributions for males and females. The policy makes males worse off in the sense that the utility distribution for the benchmark economy stochastically dominates the one for the economy with the child tax credit. This isn't the case for females. Women in the lower strata of the economy are better off with a child tax credit. The rest are slightly worse off. The poorest women have the largest number of children so a tax credit helps them the most. Since women value children more than men (single men don't value them at all), the overall effect of the tax credit on women's expected utility is less detrimental than it is for men.

6.2. Child-Support Payments

The per-child rate of support is set in the benchmark equilibrium at 5.0% of the male's income. What is the effect of this policy? The answer obtained by comparing the benchmark equilibrium to one without child support.

The removal of child support leads to a 0.65 point drop in the percentage of marriages. This is caused by both a rise in the number of young single females (0.8 percentage points) and an increase in divorces among the old (0.3 percentage points). Average income falls by about 1%. The rate of growth in the population rises ever so slightly from 0.13 to 0.19%. These effects seem moderate. The question is why.

One would expect that child support would make marriage and divorce less attractive for males and more attractive for females. The net impact will depend on which party is more likely to walk from a marriage. When child support is eliminated, marriages between high-type males and low-type females turn out to be more likely to break up. Without child support, a high-type male demands more than his low-type wife is willing to bear. Marriages between low-type males and high-type females, however, are less likely to dissolve. With child support in place, high-type females ask for more than a low-type male is willing to contribute to a marriage. The net e¤ect on the equilibrium number of divorces is very small. Some of the drop in the equilibrium number of marriages derives from the fact that divorced mothers now invest less in their children (about 7% drop in e) and this drives down the long-run quality of the mating pool. This can be seen by examining the impact of removing child support, which is done by holding the type distributions for young agents, or Φ_1 and - $_1$, …xed. Again, this turns o¤ the e¤ects on the economy from any induced changes in parental human capital investments. When this is done the number of marriages drops by 0.45 percentage points. Hence, about 0.20 percentage points of the fall in the number of marriages is due to the drop in the long-run quality of the mating pool.

Nash Bargaining, again: The elimination of child support leads to some interesting reallocations within the family. When child support is eliminated an older female has a lower threat point. So her husband has relatively more bargaining power. Let $\mathcal{B}_2$ and $\mathcal{C}_2$ denote the combinations of (x, z, γ, k) that generate viable marriages among the old in the benchmark and no-child support equilibriums. The old male's weight increases for each and every $(x, z, \gamma, k) \in \mathcal{B}_2 \cap \mathcal{C}_2$. The average weight for males rises from 0.57 to 0.60. Older females do indeed work more.[9] Their leisure falls by almost 4 percentage points. Almost all of this is due to increased work in the market. (These changes are also due in part to the fact that high-type women constitute a larger fraction of marriages now.) Now, con-

[9] To calculate the average one needs to know how many type-(x, z, γ, k) marriages there are. The distribution of marriages will be di¤erent for the benchmark and no-child support economies. The average was computed using the distribution from the benchmark economy — so as to not contaminate the changes in the male's weights with the shift in the distribution.

sider the impact on a young male's weight. Denote by $\mathcal{B}_1$ and $\mathcal{C}_1$ the combinations of (x, z, γ) that generate viable marriages among in the benchmark and no-child support steady states. Surprisingly, a young male's weight decreases for each and every $(x, z, \gamma) \in \mathcal{B}_1 \cap \mathcal{C}_1$! Why? A young female realizes that the gains from being married when she is old are lower when there is no child support in place. Hence, she will be more reluctant to marry when she is young. She demands more from her young suitor. Figure 7 shows the decline in the young male's weight, ρ, that occurs when child-support is withdrawn — the …gure shows the average weight for each type of married male. On average, the young male's weight falls from 0.61 to 0.60. Therefore, some of the gains that males realize when child support is removed are redistributed back to females. A young married female's leisure rises by 1.8 percentage points, on average.

Last, the manner in which households undertake their decision-making appears to be important for analyzing the consequences of economic policy. To see this, suppose that the Nash bargaining weights are held at their benchmark values when child support payments are eliminated. Now, the equilibrium number of marriages plummets by 10 percentage points. Average income drops by 18%. A marriage is no longer as ‡exible as before. One party is less able to transfer utility to the other in order to keep the marriage viable.

7. Conclusion

An overlapping generations model of marriage, divorce, and the quantity and quality of children is developed here to study the distribution of income. Singles meet in a marriage market and are free to accept or reject marriage proposals from the opposite sex. Likewise, married agents must decide whether or not to remain with their current spouses. Within a marriage, decisions about how much to work, the number of children, and the amount of time and money to invest per child are decided by Nash Bargaining. In the model's general equilibrium, some adults are married while others aren't. Some females have children in wedlock, others out of it. Marital status and income are related. Families headed by a single mother are the poorest. Likewise, fertility and income are also related. Fertility declines with income. Single mothers have the most children. Children raised by a single mother have a greater tendency (relative to other children) to grow up poor due to a lack of human capital investment. The distribution of income is more skewed when family size is taken into account.

Can social policies be designed to improve the society's welfare? Future generations of the prototype model may shed insight on such questions. To illustrate how the model could be used in such a context the impact of child tax credit and child support payments are considered. When the number of children is held ...xed, child tax credits increase the amount of income per child. But, the number of children cannot be held ...xed since the policy promotes an increase in family size. It also reduces the attractiveness of marriage for females. On net, child tax credits fail to elevate the well being of society.

Child support payments are aimed to insulate children from the drop in family income that occurs when their parents divorce. Child support payments should make divorce more attractive for females and less attractive for males. The e¤ect on the equilibrium number of marriages is small. This is because child support payments reduce marital breakups between high-type males and low-type females, but promote breakups between low-type males and high-type females. This experiment highlights the fact that the form of household decision making may be important for designing public policy. Child support payments transfer resources away from husbands toward wives, other things equal. This strengthens the hand of married women vis à vis their husbands. With Nash bargaining utility can be transferred away from a husband to a wife to keep a marriage sustainable, so long as it is in the husband's interest to do so. But, to the extent that single males have the option to remain unmarried, part of this transfer will be undone by renegotiating the terms of marriage. Last, the model is still too crude to place confidence in the results for these two policy experiments. Future generations of the model, however, may be able to enlist in public service.

8. Appendix A: Algorithm for Nash Bargaining

Representing the Nash Bargaining Problem as a Pareto Problem: Consider the Nash Bargaining problem when the number of children, k, is held fixed. The solution to this problem is Pareto optimal, a fact demonstrated later. Therefore, for some Pareto weight $\rho(k) \in (0,1)$ it solves

$$\max_{0 \leq n,l,t \leq 1,d} \{(1-\rho(k))[F(c,e,k,1-l-t) - G(x,k,z) + \rho(k)[M(c,e,k,1-n;\chi) - B(z,k)]\}, \tag{P(11)}$$

subject to the constraints for household production and human capital investment. Given the presence of the inequality constraints this is a nontrivial Kuhn-Tucker problem. For instance, in some marriages the woman will work in the market, while in others she won't.

Consider the case where an interior solution obtains. The …rst-order conditions for an interior solution are:

$$(1-\rho(k))F_c + \rho(k)M_c = -\rho(k)\frac{M_n}{\Psi z}, \tag{8.1}$$

$$(1-\rho(k))F_c + \rho(k)M_c = -(1-\rho(k))\frac{F_l}{\Psi x}, \tag{8.2}$$

$$[(1-\rho(k))F_e + \rho(k)M_e]Q_t = -(1-\rho(k))F_t, \tag{8.3}$$

and

$$\Psi[(1-\rho(k))F_c + \rho(k)M_c] = [(1-\rho(k))F_e + \rho(k)M_e]Q_d. \tag{8.4}$$

Observe that when $\rho(k) = [F + \mathfrak{W} - G]/\{[M + \mathfrak{H} - B] + [F + \mathfrak{W} - G]\}$ the solution to the Pareto problem P(11) will correspond with the solution to

$$\begin{aligned} &\max_{0 \leq n,l,t \leq 1,d}\{\ [F\left(c,e,k,1-l-t\right) + \mathfrak{W} - G\left(x,k,z\right)] \\ &\times[M(c,e,k,1-n;\chi) + \mathfrak{H} - B\left(z,k\right)]\}, \end{aligned} \qquad \text{P(12)}$$

subject to the constraints for household production and human capital investment, and where $\mathfrak{W}$ and $\mathfrak{H}$ are the continuation values associated with the married state. This fact is readily veri…able by comparing the …rst-order conditions associated with the two problems while imposing the condition $\rho(k) = [F + \mathfrak{W} - G]/\{[M + \mathfrak{H} - B] + [F + \mathfrak{W} - G]\}$. This shows that the solution to the Nash Bargaining problem is Pareto optimal.

Solving the Nash Bargaining problem: It is easier to solve numerically the Pareto problem P(11) than the Nash Bargaining problem P(12). The Nash bargaining problem can only be easily solved on the set of viable marriages. In advance it is hard to know what this set is. To compute the solution to the Pareto problem requires …nding the weight $\rho(k)$ that maximizes the product of the net gains from marriage, again holding …xed the number of children, k. So, the algorithm proceeds by making a guess for $\rho(k)$. The problem P(11) is then solved using this guess. This involves numerically solving the set of equations (8.1) to (8.4), or their analogues that incorporate the appropriate Kuhn-Tucker conditions — a married woman may not work in the market, for instance. This gives values for F and M. The weight is then updated using the formula

$$\rho(k) = \min\{\max\{\frac{[F + \mathfrak{W} - G]}{[M + \mathfrak{H} - B] + [F + \mathfrak{W} - G]}, \delta\}, 1 - \delta\},$$

for some small $\delta > 0$. Therefore, $0 < \rho(k) < 1$. The Pareto problem is then recomputed using the new weight. The algorithm proceeds until a …xed point is found. This gives the values of $M + \mathfrak{H} - B$ and $F + \mathfrak{W} - G$ for a …xed number of kids, k. Sometimes a …xed point cannot be found, because the marriage is not viable. For a marriage to be viable, $M + \mathfrak{H} - B$ and $F + \mathfrak{W} - G$ must both exceed zero. Observe that if $M + \mathfrak{H} - B < 0$ then $\rho(k) > 1$, while if $F + \mathfrak{W} - G < 0$ then $\rho(k) < 0$. Therefore, it is easy deduce which marriages are viable or not. For example, set $\rho(k) = 1 - \delta$ and solve the Pareto problem P(11). If $M + \mathfrak{H} - B < 0$ then there is no viable marriage from the male's perspective.

Last, when the number of kids is also a choice variable the algorithm then picks $k \in \mathcal{K}$ over the set of viable marriages to maximize the Nash Product:

$$\max_{k \in \mathcal{K}} [F + \mathfrak{W} - G][M + \mathfrak{H} - B].$$

Now, let k^* denote the solution to the above problem and de…ne ρ by $\rho = \rho(k^*)$. The is the weight used in the couple's Pareto problem outlined in Section 5.3.[10]

[10] The number of kids is discrete. It is still true, however, that the Nash Bargaining problem solves a Pareto problem. Suppose that $k^* = \arg\max_k [X(k)Y(k)]$, for some functions X and Y.

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Then k^* must also solve $\max_k [\rho X(k) + (1-\rho)Y(k)]$, when $\rho = Y(k^*)/[X(k^*)+Y(k^*)]$. Suppose not. Then the optimal value of the objective function would be proportional to $Y(k^*)X(k) + X(k^*)Y(k)$, a fact that is readily seen by substituting in for ρ. But, this can't be an optimum. By choosing $k = k^*$ the maximand could be increased since the Nash product maximizes $X(k)Y(k)$.

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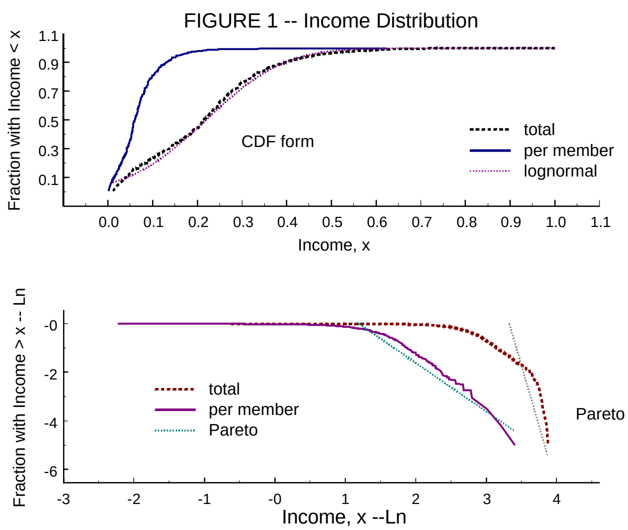


Figure 8.1:

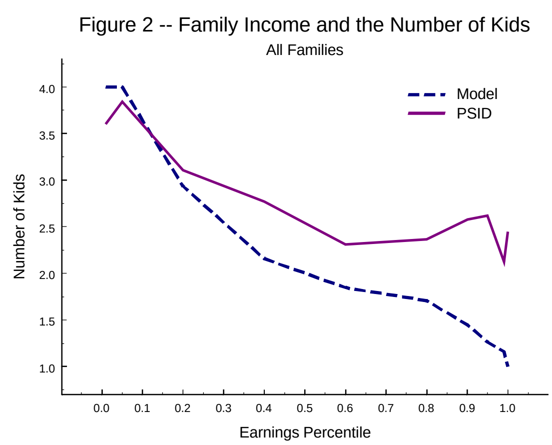


Figure 8.2:

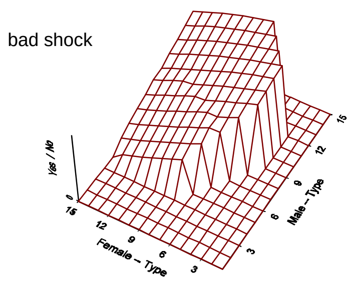
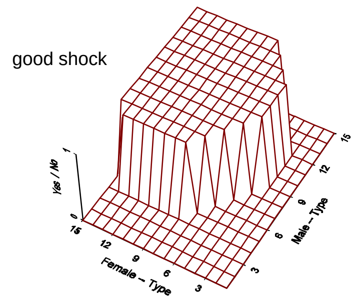


Figure 8.3:

FIGURE 4 -- Viable Marriages, Nash Bargaining



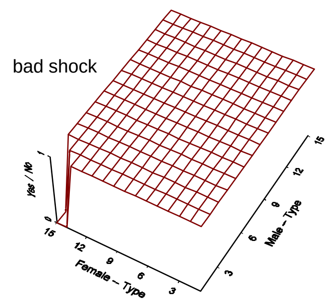


Figure 8.4:

FIGURE 5 -- Viable Marriages, Fixed Weight

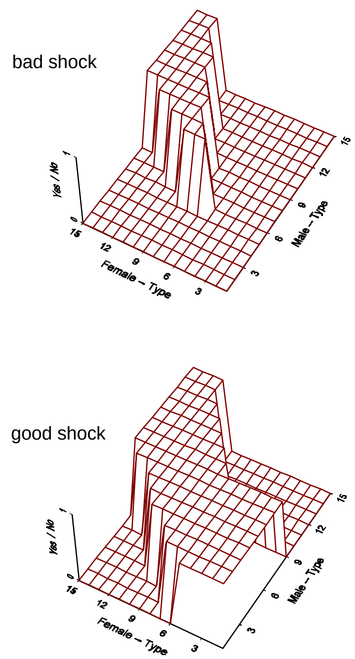


Figure 8.5:

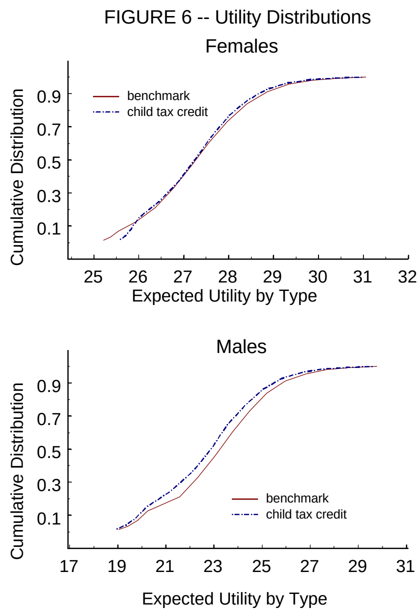


Figure 8.6:

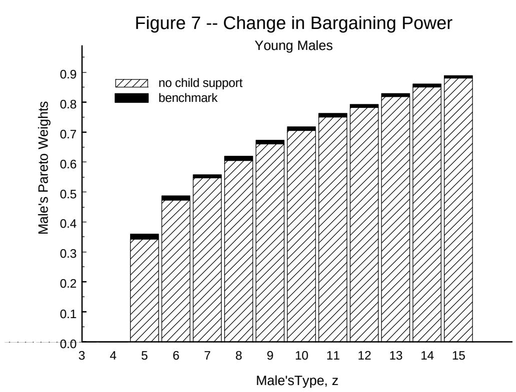


Figure 8.7: