Iterative Signal Detection for Large Scale GSM-MIMO Systems

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***Abstract*—Generalized spatial modulations (GSM) represent a novel multiple input multiple output (MIMO) scheme which can be regarded as a compromise between spatial multiplexing MIMO and conventional spatial modulations (SM), achieving both spectral efficiency (SE) and energy efficiency (EE). Due to the high computational complexity of the maximum likelihood detector (MLD) in large antenna settings and symbol constellations, in this paper we propose a lower complexity iterative suboptimal detector. The derived algorithm comprises a sequence of simple processing steps, namely an unconstrained Euclidean distance minimization problem, an element wise projection over the signal constellation and a projection over the set of valid active antenna combinations. To deal with scenarios where the number of possible active antenna combinations is large, an alternative version of the algorithm which adopts a simpler cardinality projection is also presented. Simulation results show that, compared with other existing approaches, both versions of the proposed algorithm are effective in challenging underdetermined scenarios where the number of receiver antennas is lower than the number of transmitter antennas.**

***Index Terms*—Generalized Spatial Modulations (GSM), Large Scale MIMO (LS-MIMO), compressed sensing (CS).**

I. Introduction

Large-scale multiple input multiple output (LS-MIMO) schemes, where a large number of antenna elements (AEs) are employed at the base station (BS), are considered one of strongest candidates for enabling the intended capacity and reliability improvements in future 5G systems [1]. However, one of the main drawbacks of conventional LS-MIMO is the need for an individual radio frequency (RF) chain per each AE which can result in a strong overhead in terms of energy consumption and hardware cost [2].

Multiple input multiple output antenna schemes relying on generalized spatial modulations (GSM) [3]-[5] have recently emerged as an attractive technique to achieve both spectral efficiency (SE) and energy efficiency (EE). While spatial multiplexing MIMO is aimed at SE and spatial modulations (SM) are targeted at EE [6][7], GSM can be regarded as a compromise between both, as only a subset of the available transmitting antennas is active at any given moment, thus reducing the number of RF chains required. As the information is encoded on the active antenna combination and also on the modulated symbols transmitted on the AEs, GSM can achieve higher SE than SM. There are two main types of GSM. In the first one [3][4], all active AEs transmit the same symbols while in the second one, which is the one we are concerned with in this paper, each AE can transmit a different modulated symbol, thus achieving higher SE [5].

The optimal maximum likelihood decoder (MLD) for GSM-MIMO transmissions requires an exhaustive search over all active antenna combinations and modulated symbols which makes it unviable for most applications. Although sphere decoding (SD) detectors achieving optimal MLD performance have been proposed for GSM-MIMO [8], they exhibit a computational complexity which still grows exponentially with the problem size. Therefore, many low-complexity suboptimal approaches for GSM-MIMO have been proposed recently. In [5], a GSM-MIMO detector is described, which applies linear decorrelation techniques for first detecting the active antennas and then demodulating the constellation symbols. The resulting approach has a lower complexity than the MLD but its bit error rate (BER) performance is far from optimal. A promising detector named ordered block minimum mean-squared error (OB-MMSE) was presented in [9]. Although it is capable of near-MLD performance, its complexity is affordable only when the number of possible active antenna combinations is small. One of the main challenges in deriving low complexity detectors is the common GSM-MIMO scenario where the number of transmitting antennas is larger than the receiving antennas. This results in an underdetermined detection problem which makes linear detectors like zero forcing (ZF) and MMSE unsuitable. In order to deal with the reduced receiver antennas set and exploit the sparsity property of GSM signals, several authors have resorted to efficient reconstruction tools available within the compressed sensing (CS) framework [10][11]. For example, in [12] the authors applied and evaluated the basis pursuit denoising (BPDN) formulation from [13] to the detection of GSM-MIMO signals. However, directly applying conventional CS algorithm to problems defined over discrete sets, such as in GSM-MIMO, will have a performance which is far from optimal unless knowledge of the discrete nature of the signal is directly exploited inside the reconstruction method [14][15]. Therefore in [16], a greedy algorithm named multipath matching pursuit with slicing (sMMP) was presented which combines the use of an inner integer slicing

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This work was partially supported by the FCT - Fundação para a Ciência e Tecnologia and Instituto de Telecomunicações under project UID/EEA/50008/2013.

step with the adoption of multiple promising candidates for minimizing the residual. This approach was adapted and evaluated for GSM-MIMO transmissions in [17], with good performance-complexity tradeoffs.

In this paper we propose a new iterative GSM-MIMO detector which relies on the application of the alternating direction method of the multipliers (ADMM) as a heuristic for solving the ML detection problem. ADMM is a well-known approach for convex optimization problems [18], and even though the GSM-MIMO detection problem is a nonconvex one, there are several successful examples published in the literature regarding nonconvex problems where ADMM was found to work with excellent performance [20]-[22]. What makes ADMM appealing for the GSM-MIMO detection context is its ability to split the MLD problem into a sequence of simpler steps comprising an unconstrained Euclidean distance minimization problem, an element wise projection over the signal constellation and a projection over the set of valid active antenna combinations. As this last subproblem can incur an excessive complexity cost when the valid antenna combination set is large we also propose a simpler algorithm version where this projection is relaxed into a simpler cardinality one. In either case, the resulting algorithm also includes a refinement step based on a projected MMSE estimate defined over a fixed support set whose goal is to find a solution closer to the optimal one. Simulation results show that both versions of the proposed GSM-MIMO detector can attain very competitive complexity-performance tradeoffs against other suboptimal approaches.

The remainder of the paper is organized as follows. Section II describes the GSM-MIMO system model and formulates the MLD problem. Section III derives the iterative detection algorithm and discusses several important aspects about its operation. Simulation results are presented in Section IV followed by the conclusions in Section V.

Notation: Matrices and vectors are denoted by uppercase and lowercase boldface letters, respectively. The superscripts $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and conjugate transpose of a matrix/vector, $\|\cdot\|_2$ is the 2-norm of a vector, $\|\cdot\|_0$ is its cardinality, $\text{supp}(\mathbf{x})$ returns the set of indices of nonzero elements in $\mathbf{x}$ (i.e., the support of $\mathbf{x}$), $\text{diag}(\cdot)$ represents the vector containing the diagonal elements of a matrix, $\lfloor\cdot\rfloor$ is the floor function, $\binom{N}{k}$ denotes the number of combinations of N symbols taken k at a time and $\mathbf{I}_n$ is the $n\times n$ identity matrix.

II. System Model and Problem Statement

Let us consider a GSM MIMO transmission in a flat fading channel employing N_t transmitter antennas and N_r receiver antennas. At any given time, only N_a AEs are active allowing a total of $N_{comb} = 2^{\left\lfloor \log_2 \binom{N_t}{N_a} \right\rfloor}$ antenna combinations available for mapping $\left\lfloor \log_2 \binom{N_t}{N_a} \right\rfloor$ information bits. If each active AE transmits a different M-QAM modulated symbol then a total of $\left\lfloor \log_2 \binom{N_t}{N_a} \right\rfloor + N_a \log_2 M$ bits are sent on each GSM symbol. The baseband received signal can be represented using

$$\mathbf{y} = \mathbf{Hs} + \mathbf{n} \tag{1}$$

where $\mathbf{y} \in \mathbb{C}^{N_r\times 1}$ is the received vector, $\mathbf{H} \in \mathbb{C}^{N_r\times N_t}$ is the channel matrix and $\mathbf{n} \in \mathbb{C}^{N_r\times 1}$ is the vector containing independent zero-mean circularly symmetric Gaussian noise samples with covariance $2\sigma^2\mathbf{I}_{N_r}$. The $N_t\times 1$ GSM signal vector $\mathbf{s}$ contains only N_a nonzero elements drawn from an M-sized complex valued constellation set $\mathcal{A}$.

The maximum likelihood detector (MLD) for model (1) can be formulated as

$$\min_{\mathbf{s}} \ f(\mathbf{s}) \triangleq \|\mathbf{y} - \mathbf{Hs}\|_2^2 \tag{2}$$

$$\text{subject to } \mathbf{s} \in \mathcal{A}_0^{N_t} \tag{3}$$

$$\text{supp}(\mathbf{s}) \in \mathbb{S}. \tag{4}$$

where we employ an extended constellation set defined as $\mathcal{A}_0 \overset{def}{=} \mathcal{A} \cup \{0\}$ and $\mathbb{S}$ denotes the set of possible supports of $\mathbf{s}$ (i.e., valid active antenna combinations). Although there are many possible suboptimal approaches for solving the detection problem, what complicates the adoption of most approaches is when the system has to operate in an underdetermined scenario, which is common in the downlink where the number of receiver antennas can be substantial smaller than the number of transmitter antennas, i.e., $N_r < N_t$.

III. Iterative GSM Detector Algorithm

A. Algorithm Description

In the MLD formulation, constraints (3) and (4) make the optimization problem nonconvex. To avoid excessive computational complexity, in this section we apply ADMM to (2)-(4). However, due to the nonconvex nature of the problem, ADMM is used as a heuristic in the sense that it will provide a solution faster than an optimal method but it is not guaranteed to be the optimal one. Since ADMM solves linearly constrained problems with a separable objective function, we rewrite problem (2)-(4) as

$$\min_{\mathbf{s},\mathbf{x},\mathbf{z}} \ \|\mathbf{y} - \mathbf{Hs}\|_2^2 + I_{\mathbb{S}}(\mathbf{x}) + I_{\mathcal{A}_0^{N_t}}(\mathbf{z}) \tag{5}$$

$$\text{subject to } \mathbf{s} = \mathbf{x} \tag{6}$$

$$\mathbf{s} = \mathbf{z} \tag{7}$$

where $I_{\mathcal{G}}(\mathbf{v})$ is the indicator function for a generic set $\mathcal{G}$ which is 0 if $\mathbf{v} \in \mathcal{G}$ and $+\infty$ otherwise. Note that the use of the indicator function allows us to integrate constraints (3) and (4) directly into the objective function (5), whereas the introduction of (6) and (7) make this function separable over three different variables ($\mathbf{s}$, $\mathbf{x}$ and $\mathbf{z}$). This enables a convenient splitting of the problem, as will be shown. We can write the augmented Lagrangian [18] for this problem as

$$L_{\rho_x,\rho_z}(\mathbf{s},\mathbf{x},\mathbf{z},\mathbf{u},\mathbf{w}) = \|\mathbf{y} - \mathbf{Hs}\|_2^2 + I_{\mathbb{S}}(\mathbf{x}) + I_{\mathcal{A}_0^{N_t}}(\mathbf{z}) + \\ + \rho_x\left(\|\mathbf{s}+\mathbf{u}-\mathbf{x}\|_2^2 - \|\mathbf{u}\|_2^2\right) + \rho_z\left(\|\mathbf{s}+\mathbf{w}-\mathbf{z}\|_2^2 - \|\mathbf{w}\|_2^2\right). \tag{8}$$

where $\mathbf{u},\mathbf{w} \in \mathbb{C}^{N_t \times 1}$ are scaled dual variables and ρ_x, ρ_z are the penalty parameters for constraints (6) and (7). The detection algorithm is then obtained using gradient ascent to iteratively solve the dual problem, with the minimization of the augmented Lagrangian accomplished independently for each of the primal variables **s**, **x** and **z**. This allows the original problem to be decomposed into the following simpler steps:

$$\mathbf{s}^{t+1} = \min_{\mathbf{s}} \left\{ \|\mathbf{y}-\mathbf{H}\mathbf{s}\|_2^2 + \rho_x \|\mathbf{s}+\mathbf{u}^t-\mathbf{x}^t\|_2^2 + \rho_z \|\mathbf{s}+\mathbf{w}^t-\mathbf{z}^t\|_2^2 \right\} \quad (9)$$

$$\mathbf{x}^{t+1} = \min_{\mathbf{x}} \left\{ I_{\mathbb{S}}(\mathbf{x}) + \rho_x \|\mathbf{s}^{t+1}+\mathbf{u}^t-\mathbf{x}\|_2^2 \right\} \quad (10)$$

$$\mathbf{z}^{t+1} = \min_{\mathbf{z}} \left\{ I_{\mathcal{A}_0^{N_t}}(\mathbf{z}) + \rho_z \|\mathbf{s}^{t+1}+\mathbf{w}^t-\mathbf{z}\|_2^2 \right\} \quad (11)$$

$$\mathbf{u}^{t+1} = \mathbf{u}^t + \mathbf{s}^{t+1} - \mathbf{x}^{t+1}. \quad (12)$$

$$\mathbf{w}^{t+1} = \mathbf{w}^t + \mathbf{s}^{t+1} - \mathbf{z}^{t+1}. \quad (13)$$

A closed form solution for (9) can be derived from $\nabla_{\mathbf{s}^H} L_{\rho_x,\rho_z}(\mathbf{s},\mathbf{x}^t,\mathbf{z}^t,\mathbf{u}^t,\mathbf{w}^t) = 0$ leading to

$$\mathbf{s}^{t+1} = \left(\mathbf{H}^H\mathbf{H} + (\rho_x+\rho_z)\mathbf{I}_{N_t}\right)^{-1} \left(\mathbf{H}^H\mathbf{y} + \rho_x\left(\mathbf{x}^t-\mathbf{u}^t\right) + \rho_z\left(\mathbf{z}^t-\mathbf{w}^t\right)\right). \quad (14)$$

The **x** and **z** update steps, can be expressed as

$$\mathbf{x}^{t+1} = \prod\nolimits_{\mathcal{D}}\left(\mathbf{s}^{t+1}+\mathbf{u}^t\right) \quad (15)$$

$$\mathbf{z}^{t+1} = \prod\nolimits_{\mathcal{A}_0^{N_t}}\left(\mathbf{s}^{t+1}+\mathbf{w}^t\right) \quad (16)$$

where $\prod_{\mathcal{D}}(\cdot)$ denotes the projection onto set $\mathcal{D}=\{\mathbf{s}:\mathrm{supp}(\mathbf{s})\in\mathbb{S}\}$ which is accomplished by keeping the N_a largest magnitude elements whose indices also match a valid antenna combination. Regarding the projection $\prod_{\mathcal{A}_0^N}(\cdot)$ in (16), it can be implemented as a simple rounding of each component to the closest element in $\mathcal{A}_0$. For a large set $\mathbb{S}$, i.e., a large number of valid active antenna combinations, the complexity associated to the projection (15) can become excessively high due to the exhaustive search required. As an alternative, we can relax constraint (4) into

$$\|\mathbf{s}\|_0 \le N_a \quad (17)$$

which turns the MLD problem into a cardinality one. In this case (15) is replaced by a projection over set $\mathcal{C}=\{\mathbf{s}:\|\mathbf{s}\|_0 \le N_a\}$ which can be easily implemented by zeroing the $N_t - N_a$ smallest magnitude elements. Note that, even though the number of active antennas N_a is fixed, we do not adopt an equality constraint in (17) as it would be problematic to formally define a corresponding projection capable of dealing with input vectors with a number of nonzero elements lower than N_a.

Algorithm 1 summarizes the sequence of steps required to obtain the estimate of the symbols, $\hat{\mathbf{s}}$, for the two versions of the proposed detector: the one based on search over valid antenna combinations (VAC-ADMM) and the one based on the simpler cardinality constraint (17) (C-ADMM). In the algorithm, I is the support of $\mathbf{x}^{t+1}$, $\bar{I}$ is the complement of set I (i.e., $\bar{I}=\{1,...,N_t\}\setminus I$), $\mathbf{H}_I$ is an $N_r \times N_a$ reduced matrix containing the columns of **H** selected according to the indices in I, Q is the maximum number of iterations, P is the number of iterations where a polishing technique is applied and $\hat{\mathbf{s}}_I$ ($\hat{\mathbf{s}}_I^{candidate}$) is the reduced $N_a \times 1$ vector containing the nonzero elements of $\hat{\mathbf{s}}$ ($\hat{\mathbf{s}}^{candidate}$) given by the support I.

Algorithm 1: Proposed GSM-MIMO Detectors

Input: $\mathbf{u}^0$, $\mathbf{w}^0$, $\mathbf{x}^0$, $\mathbf{z}^0$, $\mathbf{H}$, $\mathbf{y}$, ρ_x, ρ_z, Q, P
$f_{best} = \infty$.
$\mathbf{\Phi} \leftarrow \left(\mathbf{H}^H\mathbf{H} + (\rho_x+\rho_z)\mathbf{I}_{N_t}\right)^{-1}$.
for t=0,1,…Q-1 **do**
  $\mathbf{s}^{t+1} \leftarrow \mathbf{\Phi}\left(\mathbf{H}^H\mathbf{y} + \rho_x\left(\mathbf{x}^t-\mathbf{u}^t\right) + \rho_z\left(\mathbf{z}^t-\mathbf{w}^t\right)\right)$.
  $\mathbf{x}^{t+1} \leftarrow \prod_{\mathcal{D}}\left(\mathbf{s}^{t+1}+\mathbf{u}^t\right)$. (VAC-ADMM)
  $\mathbf{x}^{t+1} \leftarrow \left(\mathbf{s}^{t+1}+\mathbf{u}^t\right)$ with $N_t - N_a$ smallest magnitude elements set to 0. (C-ADMM)
  $I \leftarrow \mathrm{supp}\left(\mathbf{x}^{t+1}\right)$.
  $\mathbf{z}^{t+1} \leftarrow \prod_{\mathcal{A}_0^{N_t}}\left(\mathbf{s}^{t+1}+\mathbf{w}^t\right)$.
  If $t \ge Q$-P **then**
  $\hat{\mathbf{s}}_{\bar{I}}^{candidate} \leftarrow 0, \hat{\mathbf{s}}_I^{candidate} \leftarrow \prod_{\mathcal{A}^{N_a}}\left(\left(\mathbf{H}_I{}^H\mathbf{H}_I + 2\sigma^2\mathbf{I}_{N_a}\right)^{-1}\mathbf{H}_I{}^H\mathbf{y}\right)$. (polishing)
  else
  $\hat{\mathbf{s}}_{\bar{I}}^{candidate} \leftarrow 0$, $\hat{\mathbf{s}}_I^{candidate} \leftarrow \prod_{\mathcal{A}^{N_a}}\left(\mathbf{s}_I{}^{t+1}\right)$.
  end if
  If $f\left(\hat{\mathbf{s}}^{candidate}\right) < f_{best}$ **then**
    $\hat{\mathbf{s}}_{\bar{I}} \leftarrow 0$, $\hat{\mathbf{s}}_I \leftarrow \hat{\mathbf{s}}_I^{candidate}$.
    $f_{best} = f\left(\hat{\mathbf{s}}^{candidate}\right)$.
  end if
  $\mathbf{u}^{t+1} \leftarrow \mathbf{u}^t + \mathbf{s}^{t+1} - \mathbf{x}^{t+1}$.
  $\mathbf{w}^{t+1} \leftarrow \mathbf{w}^t + \mathbf{s}^{t+1} - \mathbf{u}^{t+1}$.
end for
Output: $\hat{\mathbf{s}}$.

B. Polishing

Due to the nonconvex nature of the MLD formulation that we are addressing, ADMM is applied as a heuristic in the sense that the derived algorithm is not guaranteed to find the exact solution. Therefore, we also propose one additional solution refinement step consisting in the minimization of the expected value of the Euclidean distance (2) (a reduced MMSE estimate) followed by the projection over the constellation symbols

$$\hat{\mathbf{s}}_I^{candidate} = \prod\nolimits_{\mathcal{A}^{N_a}}\left(\left(\mathbf{H}_I{}^H\mathbf{H}_I + 2\sigma^2\mathbf{I}_{N_a}\right)^{-1}\mathbf{H}_I{}^H\mathbf{y}\right). \quad (18)$$

This refinement step can be applied at the P last iterations, with $0 \le P \le Q$. Please note other alternative detectors could be adopted for this step, such as an SD applied to the reduced problem resulting from the estimated antenna combination.

Although, for simplicity, in this work we assume perfect knowledge of matrix **H,** in practice a channel estimation algorithm must be implemented. While the resulting estimation error always has an adverse effect on the performance of MIMO detectors, the proposed algorithm can be made more robust if the channel estimation error model is incorporated into the design of the **s**-update and polishing steps, following an approach similar to [23].

C. Extension for Soft Decoding

While not the main purpose of this paper, it is important to highlight that some form of forward error correction coding is often adopted in wireless communication systems as it can substantially improve the performance of the overall system, especially when it is based on OFDM. The proposed algorithm can be modified in order to yield soft decisions which are required to achieve high coding gains. Since running several iterations of the algorithm produces a list of candidate vectors, Λ, approximate log-likelihood ratios (LLR) can be computed for the k^{th} bit using [19]

$$L_k = \log \frac{\sum_{\hat{\mathbf{s}} \in \Lambda : b_k(\hat{\mathbf{s}})=1} \exp\left(-\|\mathbf{y}-\mathbf{H}\hat{\mathbf{s}}\|_2^2 / 2\sigma^2\right)}{\sum_{\hat{\mathbf{s}} \in \Lambda : b_k(\hat{\mathbf{s}})=0} \exp\left(-\|\mathbf{y}-\mathbf{H}\hat{\mathbf{s}}\|_2^2 / 2\sigma^2\right)} \tag{19}$$

where $b_k(\hat{\mathbf{s}})$ denotes the k^{th} bit of vector $\hat{\mathbf{s}}$.

D. Initialization and Penalty Parameters

Due to the nonconvex nature of the feasible set (defined by constraints (3) and (4)) in the MLD problem, ADMM is only applied as a heuristic and, thus, there are no theoretical guarantees that it converges to the optimal points. It has been observed that in nonconvex problems, both the initialization and penalties have an important impact on the quality of the solution found [18], being still active areas of research [22], [24]. Therefore, in the proposed detector the penalty coefficients, ρ_x and ρ_z, act as tuning parameters which can be chosen through numerical evaluation in order to achieve the best performance for a specific problem setting.

Regarding the initialization we propose the use of either a warm or a random start. In the case of warm initialization, we correlate the received vector with the columns of the channel matrix and normalize with the respective squared norm

$$\boldsymbol{\psi} = \left(\operatorname{diag}\left(\mathbf{H}^H\mathbf{H}\right)\right)^{-1} \mathbf{H}^H \mathbf{y} \,. \tag{20}$$

An initial support set is then estimated as

$$I = \operatorname{supp}\left(\boldsymbol{\psi}^H \boldsymbol{\psi}\right) \,, \tag{21}$$

followed by

$$\mathbf{s}_I^{warm} = \left(\mathbf{H}_I{}^H \mathbf{H}_I + 2\sigma^2 \mathbf{I}_{N_a}\right)^{-1} \mathbf{H}_I{}^H \mathbf{y} \,, \tag{22}$$

$\mathbf{s}_{\bar{I}}^{warm} = 0$, $\mathbf{x}^0 = \mathbf{s}^{warm}$, $\mathbf{z}^0 = \prod_{\mathcal{A}_0^{N_t}}\left(\mathbf{s}^{warm}\right)$, $\mathbf{u}^0 = 0$ and $\mathbf{w}^0 = \mathbf{s}^{warm} - \mathbf{z}^0$. In the case of random start, we simply replace $\mathbf{s}^{warm}$ in the previous procedure with a vector whose elements are randomly selected inside the constellation limits.

As previously stated regarding the application of ADMM as a heuristic to the GSM-MIMO detection problem, there are no guarantees that it will converge to the exact solution. Therefore, the adoption of a stopping criterion based on the primal and dual residuals (as explained in [18]) or on the detection of a stall condition (variables with negligible change after several iterations), must be complemented with a maximum number of iterations Q to guarantee the termination of the algorithm. Furthermore, the algorithm can be restarted multiple times with different initializations (warm and/or random) as this can increase the chance of finding the optimal solution [21].

TABLE I

NUMBER OF REAL FLOPS FOR DIFFERENT MIMO-GSM DETECTORS

Detector	Complexity
MLD	$(8N_rN_a + 4N_r - 1)N_{comb}M^{N_a}$
OB-MMSE	$12N_rN_a + 2N_t$ $+N_{comb}\left(N_a - 1 + 4N_a^3 + 12N_a^2N_r + 7N_a^2 + 14N_aN_r + 4N_r - 1\right)$
BPDN	$4N_t^3 + 12N_t^2N_r + 7N_t^2 + 6N_tN_r$
sMMP	$(8N_tN_r + N_t)(1-T^{N_a})/(1-T) + (5N_r - 2)T^{N_a}$ $+\sum_{k=1}^{N_a}\left(4k^3 + k^2(4N_r + 15) + k(20N_r - 5)\right)T^k$
VAC-ADMM	$4N_t^3 + N_t^2(4N_r + 15) + N_t(12N_r - 3) + Q\left(2N_{comb}(N_a - 1) + 35N_t\right)$ $+P\left(4N_a^3 + N_a^2(4N_r + 15) + N_a(12N_r - 3)\right)$
C-ADMM	$4N_t^3 + N_t^2(4N_r + 15) + N_t(12N_r - 3) + 32N_tQ$ $+P\left(4N_a^3 + N_a^2(4N_r + 15) + N_a(12N_r - 3)\right)$

E. Complexity

The steps with highest complexity in the proposed algorithm are the **s**-update step (14), which involves an $N_t \times N_t$ matrix inversion (although it is only computed in the beginning of the algorithm), and the **x**-update step (15) (for the VAC-ADMM algorithm version). In Table I we present the worst-case complexities (i.e., no earlier termination of the algorithm occurs) in terms of real-valued floating point operations (flops) of the proposed algorithms, as well as of MLD, OB-MMSE [9], BPDN [13] and sMMP [16] (T is the number of child candidates expanded at each iteration of sMMP). The expressions were obtained assuming that the sum, product and absolute value of complex numbers require 2, 6 and 3 flops, respectively. The complexities of OB-MMSE and BPDN were taken from [25] although in the case of BPDN the expression is optimistic as it does not consider the iterative nature of the inner algorithm (exact computation is dependent on the chosen optimization method). We also note that in the case of using

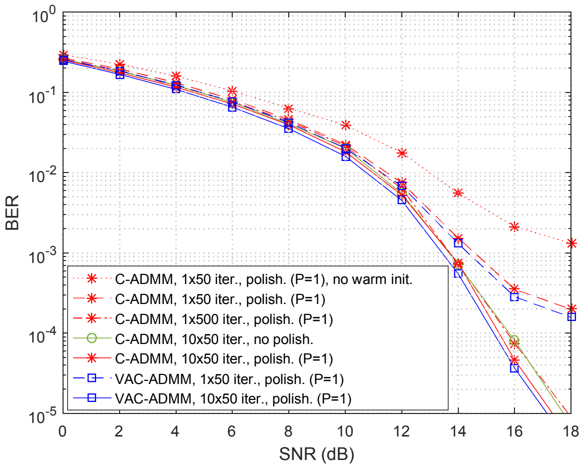

Fig. 1. BER performance of the proposed detector with N_t=64, N_r=16, N_a=3 and 64-QAM.

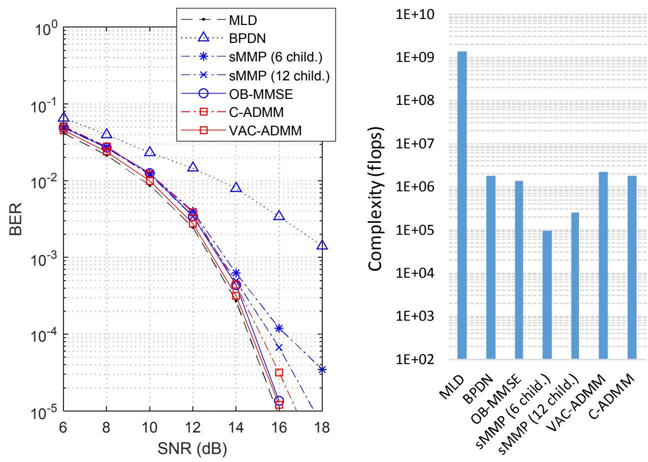

Fig. 2. BER performance and complexity of different GSM-MIMO detectors for N_t=64, N_r=16, N_a=2 and 64-QAM.

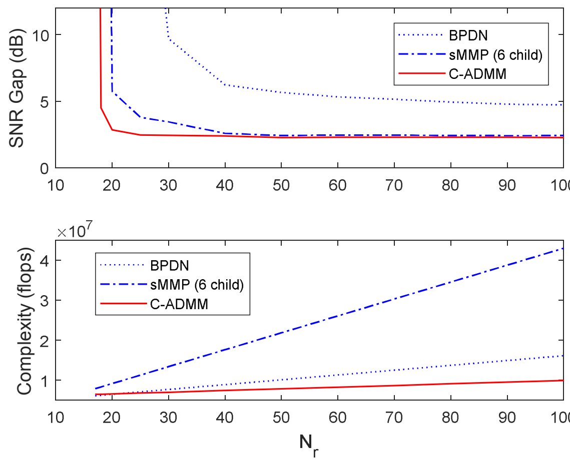

Fig. 3. SNR gap to the Oracle SD and complexity of different GSM-MIMO detectors for N_t=100, N_a=4, 64-QAM, assuming a BER target of 10^{-4}.

multiple initializations the complexity is roughly the same as increasing the number of iterations Q and polishing steps P, since some operations (such as step 3) do not need to be recomputed. According to Table I, while the complexity of OB-MMSE and VAC-ADMM does not grow exponentially with the signal constellation size, M, like in the case of MLD, it still depends on $N_{comb}=2^{\left\lfloor \log_2 \binom{N_t}{N_a} \right\rfloor}$, which can restrict their use when a large number of bits are conveyed on antenna indices. The other detectors do not have this dependency but the complexity of sMMP grows exponentially with the number of active antennas for T larger than 1, which is usually required in order to have competitive performance, as will be shown in Section IV.

IV. Numerical Results

In this section, we present BER results obtained using Monte Carlo simulations. The elements of the channel matrix $\mathbf{H}$ were independently drawn according to a complex Gaussian distribution $\mathcal{CN}(0,1)$ while the active AEs transmitted randomly selected 64-QAM symbols with $E\left[\left|s_i\right|^2\right]=1$. The results are plotted as a function of the signal to noise ratio per AE defined as $SNR=10\log_{10}\left(E\left[\left|s_i\right|^2\right]/2\sigma^2\right)$. The values applied for the penalty parameters were ρ_x=ρ_z=2.5 as these were numerically found to result in good recovery performances.

In Fig. 1 we can observe the results for different configurations of the proposed detector as a function of SNR in a scenario with N_t=64, N_r=16 and N_a=3. In most of the tested configurations, the algorithm was run with multiple restarts (the first initialization using the warm procedure and the remainder using the random procedure). For that reason, in the figure we use n_1xn_2 to state that a setting comprising n_1 initializations with Q=n_2 iterations was employed. In the configurations without multiple restarts, the initialization is always through the warm start except in one case (shown as "no warm init." in the legend). According to the results, the adoption of multiple restarts can improve the performance as it increases the probability of converging to the correct solution. This probability also becomes higher when the algorithm starts with the warm procedure and when the polishing step described in section III.B is applied in the last iteration (P=1). Regarding the two versions of the algorithm we can see that, as expected, VAC-ADMM performs better than C-ADMM but the difference is small.

Fig. 2 compares the performance and complexity of the proposed detector against other existing algorithms, namely MLD, BPDN, OB-MMSE and sMMP (with 6 and 12 child candidates per node). The scenario corresponds to a GSM-MIMO system with N_t=64, N_r=16 and N_a=2. Both the VAC-ADMM and C-ADMM employed 4 initializations, Q=50 and a final polishing step (P=1). Although sMMP has the lowest complexity, only OB-MMSE and the proposed detectors can achieve performances close to the MLD, with the VAC-ADMM exhibiting a small gain over OB-MMSE and C-ADMM. Regarding BPDN, while it has a complexity close to the proposed detectors, its performance is rather far from the optimal curve.

The good performance of the proposed detector is even clearer in a larger scenario like the one in Fig. 3 where N_t=100 and N_a=4. This figure shows the complexity and SNR gap to the Oracle SD (for a BER target of 10^{-4}) as a function of N_r. The Oracle SD is an ideal SD detector with prior knowledge of the active antenna combination (therefore solving a conventional overdetermined MIMO scenario with dimensions $N_r \times N_a$). Curves for MLD, OB-MMSE and VAC-ADMM are not included due to their high computational complexity in this scenario (depends on $N_{comb} = 2^{\left\lfloor \log_2 \binom{N_t}{N_a} \right\rfloor}$). C-ADMM was applied with 5 initializations, Q=100 and P=1. It can be seen that this receiver achieves the best performance, especially with a small number of receiver antennas. When N_r increases, both C-ADMM and sMMP benefit from the additional receive diversity and their performances become close. However, sMMP requires a higher complexity in order to achieve those results. The performance of BPDN also improves with the increase of N_r but is always worse than the other two detectors, and the complexity is larger than C-ADMM. The results of Fig. 2 and Fig. 3, combined with the complexity analysis in Table I show that, unlike most of the other detectors, the C-ADMM algorithm version is a very flexible method as it can be applied not only to large antenna configurations but can also cope with an increase of the number of active antennas without an excessive complexity growth.

V. CONCLUSIONS

In this paper a novel iterative detector for GSM-MIMO systems was presented which comprises a sequence of simple processing steps namely, an Euclidean distance minimization, the selection of candidate active antennas and the selection of valid modulated symbols. Two different versions of the algorithm were proposed: one restricts the search of candidate active antennas to valid combinations while the other one applies a simple projection over a set of vectors with restricted cardinality. Simulation results show that compared with other suboptimal decoders, the proposed approach can attain very competitive performance complexity tradeoffs in underdetermined scenarios with large antenna settings.

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