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Department of Finance

# Credit VaR and VaR in Credit Default Swaps

Sofia Bernardo Rodrigues

A Thesis presented in partial fulfillment of the Requirements for the Degree of Doctor

Supervisor:
João Pedro Pereira, Assistant Professor,
ISCTE - University Institute of Lisbon,

December, 2013

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Jury:
Doctor Maria Isabel Fraga Alves, Associate Professor, Faculdade de Ciências - Universidade de Lisboa
Doctor Paulo Manuel Marques Rodrigues, Adjunct Associate Professor, Nova School of Business and Economics - Universidade Nova de Lisboa
Doctor Jorge Barros Luís, Adjunct Assistant Professor, ISEG - Lisbon School of Economics and Management - Universidade de Lisboa
Doctor Luís Filipe Sousa Martins, Assistant Professor, ISCTE - University Institute of Lisbon
Doctor João Pedro Pereira, Assistant Professor, ISCTE - University Institute of Lisbon

December, 2013

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# Resumo

Nesta tese são apresentadas duas aplicações de estimação de Value at Risk (VaR): VaR de Crédito e VaR em Credit Default Swaps (CDS).

O VaR de crédito foi estimado com base em pressupostos de correlação diferentes, utilizando as cópulas Gaussiana e *t*, e comparado com a perda observada numa carteira de crédito de uma instituição financeira Portuguesa, num total de 72 observações mensais no período entre 2004 e 2009. Concluo que existe evidência empírica de que algumas das hipóteses assumidas pelas agências de rating para avaliar CDOs são desadequadas em situações de stress, como a crise financeira observada em 2008. As estimativas de VaR de crédito foram comparadas usando procedimentos de backtesting. O modelo que melhor se adequa ao portfólio em análise baseia-se no estimador empírico de correlação proposto por De Servigny e Renault (2002a), considerando a cópula *t* com 8 graus de liberdade.

Relativamente à aplicação de modelos de VaR a CDS, o VaR foi estimado usando vários métodos: Regressão de Quantis, Simulação Histórica, Simulação Histórica Filtrada, Teoria dos Valores Extremos e vários modelos GARCH. A análise baseia-se em 242 entidades, no período entre setembro 2001 e abril 2011. As estimativas de VaR em CDS foram comparadas usando procedimentos de backtesting. Concluo que a Regressão de Quantis proporciona melhores resultados na estimação de VaR que os restantes métodos e que os rácios financeiros propostos por Campbell *et al* (2008) para determinar o risco de falência contribuem para explicar o preço do CDS.

Palavras-Chave: Value at Risk, Cópulas, Correlação, Regressão de Quantis
Classificação JEL: C01, C02

# Abstract

This thesis presents two applications of Value at Risk (VaR) estimation: Credit VaR and VaR in Credit Default Swaps (CDS).

I compare Credit VaR estimates based on different correlation assumptions, using Gaussian and *t* copulas, with the observed loss in a credit portfolio of a Portuguese financial institution, for a time series of 72 monthly observations, covering the period between 2004 and 2009. I provide empirical evidence that some of the assumptions made by rating agencies to evaluate CDOs are inadequate in stress situations like the financial crisis observed in 2008. All Credit VaR estimates were compared using backtesting procedures. I find that the most accurate Credit VaR model for this portfolio is based on asset correlation given by the empirical estimator proposed by De Servigny and Renault (2002a) and assuming a dependence structure given by the *t* copula with 8 degrees of freedom.

Regarding the application of VaR models to CDS, I estimate VaR using several methods: Quantile Regression, Historical Simulation, Filtered Historical Simulation, Extreme Value Theory and GARCH-based models. The analysis of the determinants of CDS spreads is based on 242 reference entities and the time period ranges from September 2001 to April 2011. All VaR models were compared using backtesting procedures. I find that Quantile Regression provides better results than the other models tested and that the financial ratios proposed by Campbell *et al* (2008) to determine the risk of bankruptcy contribute to explain the determinants of the price of CDS.

Keywords: Value at Risk, Copulas, Correlation, Quantile Regression
JEL Classification: C01, C02

# Executive Summary

This thesis presents two applications of Value at Risk (VaR) estimation: Credit VaR and VaR in Credit Default Swaps (CDS).

Value at Risk estimates of credit portfolios depend on default probability, recovery rate and asset correlation. Previous literature has pointed asset correlation as one of the major weaknesses of VaR estimates and a factor that played a major role in the financial crisis observed in 2008. Rating agencies faced heavy criticism regarding the assumptions used to evaluate Collateralized Debt Obligations but there is few empirical evidence to support that criticism. One of the goals of this study is to compare different approaches to calculate credit VaR with the loss observed in a financial institution portfolio and analyze the sensitivity of VaR estimates to different assumptions regarding asset correlation.

I compare Credit VaR estimates based on different correlation assumptions, using Gaussian and *t* copulas, with the observed loss in a credit portfolio of a Portuguese financial institution, for a time series of 72 monthly observations, covering the period between 2004 and 2009. I find that credit VaR estimates differ substantially, depending on the assumptions regarding asset correlation and dependence structure. This finding reinforces the crucial role that the assumption regarding correlation plays in credit VaR estimation. I also provide empirical evidence that some of the assumptions made by major rating agencies to evaluate CDOs are inadequate in stress situations like the financial crisis observed in 2008. I find that the more accurate VaR model for the portfolio used in this study is based on asset correlation given by the empirical estimator of De Servigny and Renault (2002a) and assuming *t* copula with 8 degrees of freedom.

Credit Default Swaps were at the forefront of the recent financial crisis of 2007-2009 and many observers have blamed CDS as one of the lead causes of the crisis. However, a more careful analysis, as done in Stulz (2009), suggests that CDS did not trigger the crisis and that in fact they allowed some institutions to limit their losses and, for this reason, CDS are certain to remain a crucial financial instrument, even though under tighter regulation and more control.

Regarding the application of VaR models to CDS, the goal of this study is to estimate VaR in CDS using Quantile Regression, covering the period of the recent financial crisis, and perform a thorough evaluation of VaR estimates and compare them with alternative methods, namely Historical Simulation, Filtered Historical Simulation, Extreme Value Theory and GARCH-based models, through backtesting methodologies. The analysis of the determinants of CDS spreads is based on 242 reference entities and the time period ranges from September 2001 to April 2011. I find that Quantile Regression provides better results than the other models tested and that the financial ratios proposed by Campbell *et al* (2008) to determine the risk of bankruptcy contribute to explain the determinants of the price of CDS.

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# List of Acronyms

**CAViaR** Conditional Autoregressive model

**CDO** Collateralized Debt Obligation

**CDS** Credit Default Swap

**CVaR** Credit Value at Risk

**EGARCH** Exponential General Autoregressive Conditional Heteroskedastic model

**EVT** Extreme Value Theory

**FHS** Filtered Historical Simulation

**GARCH** Generalized Autoregressive Conditional Heteroskedastic model

**GJR** Glosten, Jagannathan and Runkle model

**HS** Historical Simulation

**QR** Quantile Regression

**VaR** Value at Risk

# Introduction

This thesis presents two applications of Value at Risk (VaR) estimation: Credit VaR and VaR in Credit Default Swaps, and is divided into chapters accordingly.

Value at Risk estimates of credit portfolios depend on default probability, recovery rate and asset correlation. Previous literature has pointed asset correlation as one of the major weaknesses of VaR estimates and a factor that played a major role in the financial crisis observed in 2008. Rating agencies faced heavy criticism regarding the assumptions used to evaluate Collateralized Debt Obligations (CDO) but there is few empirical evidence to support that criticism. One of the goals of this study is to compare different approaches to calculate credit VaR with the loss observed in a financial institution portfolio and analyze the sensitivity of VaR estimates to different assumptions regarding asset correlation.

I compare credit Value at Risk estimates based on different correlation assumptions, using Gaussian and *t* copulas, with the observed loss in a credit portfolio, for a time series of 72 monthly observations, covering the period between 2004 and 2009. I also compare the results obtained with stochastic and deterministic recovery rate.

The portfolio used in the first chapter of this study is from a Portuguese financial institution and comprises all companies whose total credit is over 50 thousand euros, covering 12,736 firms. Correlation assumptions are inspired in previous studies, rating agencies methodologies to evaluate CDOs and Basel III Accord:

- empirical correlation estimator proposed by De Servigny and Renault (2002a)
- correlation values used by Moody's for homogeneous portfolios, as detailed in Meissner *et al* (2008)

- correlation values used by Standard & Poors in their CDO Evaluator model, as detailed in Meissner *et al* (2008)
- Standard & Poors' old values (prior to 2005), as presented in Kiff (2004)
- approximation to Fitch's asset correlation, following Fender and Kiff (2004)
- Basel III Accord's maximum asset correlation
- Basel III Accord's minimum asset correlation

Using Monte Carlo simulation technique and copula functions, I simulate portfolio value distribution and compute credit VaR. Repeating this process for each monthly observation, I obtain the VaR time series, which I then compare with the time series of observed loss in the portfolio using the tests proposed by Kupiec (1995) and Christoffersen (1998), the Loss Function method proposed by Lopez (1998) and the Average Quantile Loss Function proposed by Koenker and Bassett (1978). Regarding VaR backtesting, I also employ a measure of VaR over-conservativeness.

I find that credit VaR estimates differ substantially, depending on the assumptions regarding asset correlation and dependence structure. This finding reinforces the crucial role that the assumption regarding correlation plays in credit VaR estimation. I also provide empirical evidence that some of the assumptions made by major rating agencies to evaluate CDOs are inadequate in stress situations like the financial crisis observed in 2008. I find that the more accurate VaR model for the portfolio used in this study is based on asset correlation given by the empirical estimator of De Servigny and Renault (2002a) and assuming $t$ copula with 8 degrees of freedom. All of the conclusions of this study are invariant to the assumption of deterministic instead of stochastic recovery rate, which suggests that it is possible to significantly reduce computation time with low impact on the final results.

Credit Default Swaps (CDS) were at the forefront of the recent financial crisis of 2007-2009. CDS are essentially insurance contracts that protect against default of an underlying company (the reference entity or name) and thus the CDS price is a measure of the credit risk of the underlying obligor. During the crisis, CDS prices increased by factors of 10, signalling high risk of default and potentially very large losses for the financial institutions working as insurance companies in the CDS market. Therefore, many observers have blamed CDS as one of the lead causes of the crisis. However, a more careful analysis, as done in Stulz (2009), suggests that CDS did not trigger the crisis and that in fact they allowed some institutions to limit their losses and, for this reason, CDS are certain to remain a crucial financial instrument, even though under tighter regulation and more control.

The financial crisis has raised more concern about risk prediction, which now has a more important role in banking and finance. Banks rely on Value at Risk (VaR) measures to control their risk exposure. There are several competing approaches to estimate VaR, including Historical Simulation using past data, parametric models describing the full distribution of interest, Extreme Value Theory and Quantile Regression to model a specific quantile rather than the whole distribution. The goal of this study is to estimate VaR in CDS using Quantile Regression, covering the period of the recent financial crisis, and perform a thorough evaluation of VaR estimates and compare them with alternative VaR estimation methods through backtesting methodologies such as the tests proposed by Kupiec (1995) and Christoffersen (1998), the Average Quantile Loss Function proposed by Koenker and Bassett (1978), the Conditional Tail Expectation proposed by Artzner *et al* (1999) and the Dynamic Quantile test presented by Engle and Manganelli (2004). Quantile regression is potentially useful for estimating VaR in new products with a short history. Furthermore, by incorporating current market expectations embedded in the market prices of the explanatory variables, Quantile Regression has the potential to outperform other methods when market conditions become very different from the past.

I find that Quantile Regression provides better results in the estimation of VaR in CDS than Historical Simulation, Filtered Historical Simulation, Extreme Value Theory and all GARCH-based models tested in this study, especially for CDS names with long history when the forecast horizon of VaR estimates is 30 days and for CDS names with short history when the forecast horizon of VaR estimates is 1 day. I also find that the financial ratios proposed by Campbell *et al* (2008) to determine the risk of bankruptcy and failure contribute to explain the determinants of the price of CDS. Recent studies have shown that Filtered Historical Simulation and Extreme Valued Theory are the most accurate VaR models. However, the empirical evidence provided in this study does not support the extension of this finding to VaR estimation in CDS.

# Chapter 1

# Credit VaR

## 1.1 Literature Review

Recent literature has shown that one of the main weaknesses in Credit Value at Risk estimates is the assumption about correlation.The subprime crisis has shown that this weakness is real.

Niethammer and Overbeck (2008) analyze the effect of estimation errors on risk figures, causing model risk. They point out that estimating correlation is of major importance for banks and find empirical evidence that the obtained values of correlation strongly depend on the method used in the estimation. Crouhy *et al* (2000) show that credit VaR is quite sensitive to estimates of correlations. In this paper, I provide empirical evidence of the sensitivity of credit VaR estimates to the assumptions regarding correlation by calculating credit VaR for a real portfolio considering several correlation assumptions.

According to Duffie (2008), even specialists in CDOs are ill equipped to measure the risk of tranches that are sensitive to default correlation and this is the weakest link in credit risk transfer markets, which could suffer a dramatic loss of liquidity if a surprise cluster of defaults suddenly emerges. Moreover, he argues that correlation parameters used in rating methodologies tend to be based on rudimentary assumptions. Picone (2002) states that the main question in evaluating CDOs has become how to measure the level of diversification in the portfolio, *i.e.*, default correlations. Fender and Kiff (2004) illustrate that incorrect assumptions about default correlation can cause the rating agencies to significantly under or overestimate the risk in a credit portfolio. A comparative analysis of Fitch, Moody's and Standard and Poor's CDO rating approaches is provided by Meissner *et al* (2008). They conclude that at the end of 2007 the main rating agencies were all applying the Merton Structural Model and deriving asset values with Gaussian copula model. The differences between methodologies existed in the way the rating agencies derived the core input parameters, namely default probability, recovery rate and asset correlation. Asset correlation was pointed out as the most critical input parameter, due to its significant impact on default distribution. In this paper I estimate credit VaR considering the asset correlation assumptions used by the major rating agencies to evaluate CDOs, for the period between 2004 and 2009, and compare these estimates with the observed loss in a real portfolio using VaR backtesting methodologies. Due to the fact that the time period considered in this study covers the subprime crisis and the portfolio considered in the analysis could have been securitized, this study provides interesting insights about the accuracy of rating agencies methodologies to evaluate CDOs and also provides empirical evidence to the recent criticism faced by rating agencies.

Several authors criticize the use of the Gaussian copula as market practice to estimate VaR and evaluate CDOs. Previous research has shown that the assumption of the same correlation parameters under different copulas may lead to hazardous understatement of risk. According to Dorey *et al* (2005), the use of Gaussian copula seems to be justified for modeling convenience rather than for theoretical reasons and this methodology significantly underestimates the frequency of multiple extreme defaults. Frey *et al* (2001) indicate that asset correlations are not enough to describe dependence between defaults because they do not fully specify the copula of the latent variables. As a consequence, the assumption of a Gaussian copula may not adequately model the potential extreme risk in the portfolio. They also indicate that models allowing for tail dependence, such as the multivariate *t* copula, give evidence that more worrying scenarios are possible. Mashal and Zeevi (2002) perform a sensitivity analysis that strongly suggests that the Gaussian dependence structure should be rejected in all data sets, when tested against the alternative *t* dependence structure. According to Crouhy *et al* (2000), it is not legitimate to assume normality of the portfolio changes for credit returns which are by nature highly skewed and fat-tailed.The percentile levels of the distribution can not be estimated from the variance only, the calculation of credit VaR requires drawing the full distribution of changes in the portfolio. On the other hand, Hamerle and Rösch (2005) show that misspecification of the distribution and the dependence structure of asset returns does not necessarily produce misleading forecasts of the loss distribution. In order to evaluate the impact of the choice of copula on the final results, I estimate VaR with Gaussian and *t* copula, considering the correlation assumptions used by major rating agencies and the Basel III Accord, and compare these estimates with the observed loss.

## 1.2 Data

The data set used in this study is from one of the 10 largest Portuguese financial institutions. The sample used in this study comprises all companies whose total credit is over 50 thousand euros, covering 12,736 firms in the period between January 2004 and December 2009. Table 1.1 presents the distribution of observations per year and industry.

Table 1.1: Distribution of observations per year and industry

Distribution of the observations used in the empirical analysis, per year and industry, for the period from January 2004 to December 2009.

| | Year | | | | | |
|---|---|---|---|---|---|---|
| **Industry** | **2004** | **2005** | **2006** | **2007** | **2008** | **2009** |
| Banking and Finance | 33 | 34 | 36 | 44 | 61 | 73 |
| Broadcasting/Media/Cable | 15 | 16 | 18 | 23 | 25 | 27 |
| Building, Materials and Real Estate | 2,712 | 2,866 | 3,051 | 3,222 | 3,269 | 3,161 |
| Business Services | 184 | 200 | 242 | 271 | 343 | 429 |
| Materials and Utilities | 40 | 48 | 50 | 60 | 69 | 91 |
| Computers and Electronics | 20 | 22 | 25 | 39 | 40 | 57 |
| Consumer Products | 604 | 611 | 704 | 863 | 1.052 | 1.262 |
| Food, Beverage and Tobacco | 63 | 72 | 88 | 122 | 146 | 170 |
| Gaming, Leisure and Entertainment | 171 | 180 | 201 | 213 | 228 | 275 |
| Health Care and Pharmaceutical | 73 | 90 | 100 | 114 | 132 | 148 |
| Industrial/Manufacturing | 204 | 219 | 257 | 330 | 394 | 489 |
| Lodging and Restaurants | 183 | 211 | 235 | 293 | 376 | 459 |
| Retail | 171 | 193 | 213 | 247 | 301 | 380 |
| Supermarkets and Drugstores | 482 | 516 | 584 | 658 | 756 | 979 |
| Textiles and Furniture | 93 | 100 | 109 | 124 | 156 | 202 |
| Transportation | 32 | 32 | 45 | 69 | 109 | 147 |
| Others | 47 | 42 | 49 | 49 | 71 | 81 |

Portfolio distribution per type of loan and guarantee are presented in tables 1.2 and 1.3. More than half of the total credit is guaranteed by real estate collateral, partly due to the high weight of construction loans, namely 43.10%. Approximately 9% of the portfolio has financial collateral and only 13.37% has no guarantee.

Table 1.2: Portfolio distribution per type of loan

Distribution of the observations used in the empirical analysis, per type of loan, for the period from January 2004 to December 2009.

| **Type of Loan** | **Distribution** (%) |
|---|---|
| Construction Loan | 43.10 |
| Working Capital Loan | 23.93 |
| Investment Loan | 19.57 |
| Line of Credit | 11.38 |
| Leasing | 2.02 |

Table 1.3: Portfolio distribution per type of guarantee

Distribution of the observations used in the empirical analysis, per type of guarantee, for the period from January 2004 to December 2009.

| **Guarantee** | **Distribution** (%) |
|---|---|
| Financial Collateral | 9.05 |
| Real Estate Collateral | 54.21 |
| Other Collateral | 1.78 |
| Personal Guarantee | 21.59 |
| No Guarantee | 13.37 |

## 1.3 Methodology

In this section I explain the procedure used to estimate Credit VaR. I start with the presentation of the Merton model framework and copula functions. Then I explain how correlation is imposed in the estimation procedure and present all asset correlation assumptions tested in this study. Finally, I present the VaR estimation method.

Portfolio value at time t depends on the loan value of each obligor and this, in turn, is a function of the debt amount, the occurrence of default, and the recovery rate, when default occurs. Let the random variable $I_{i,t}$ be the default indicator for obligor *i* at time *t*, taking values in $\{0,1\}$ (we interpret the value 1 as default and 0 as non-default), let $Debt_{i,t}$ be the outstanding value at time *t* of the loan granted to firm *i* and let RR be the recovery rate. Loan value and portfolio value are given by:

$$Loan\ Value_{i,t} = \begin{cases} Debt_{i,t} \times RR_{i,t} & if\ I_{i,t} = 1, \\ \\ Debt_{i,t} & if\ I_{i,t} = 0, \end{cases} \tag{1.1}$$

$$Portfolio\,Value_t = \sum_{i=1}^{N} Loan\,Value_{i,t} \tag{1.2}$$

According to the option pricing approach to the valuation of corporate securities initially developed by Merton (1974), the firm's asset value, $V_t$, follows a geometric Brownian motion

$$dV_t/V_t = \mu dt + \sigma dW_t$$

where $W_t$ is a standard Brownian motion, and $\mu$ and $\sigma^2$ are respectively the mean and variance of the instantaneous rate of return on the assets of the firm, $dV_t/V_t$.

The value V at any future time $t$ is given by:

$$V_t = V_0 exp\Big\{\Big(\mu - \frac{\sigma^2}{2}\Big)t + \sigma W_t\Big\} \tag{1.3}$$

with $W_t$ being normally distributed with zero mean and variance equal to $t$, $V_t$ is log-normally distributed with expected value $E(V_t)=V_0\exp\{\mu t\}$ and $\sqrt{t}Z_t \equiv W_t - W_0$, with $Z_t \sim \mathbf{N}(0,1)$.

Merton's model assumes that a firm has a very simple capital structure, as it is financed only by equity, $S_t$, and a single zero-coupon debt instrument maturing at time T, with face value F, and current market value $B_t$. In this framework, default occurs at maturity of the debt obligation when the value of assets is less than the debt value, F, to the bond holders. The probability of an obligor defaulting, $p_{Def}$, is given by:

$$p_{Def} = \Pr[V_t \leq F]$$

Replacing $V_t$ by equation 1.3 and $W_t$ by $\sqrt{t}Z_t$ it follows:

$$\begin{aligned} p_{Def} &= Pr\Big[V_0 exp\Big\{\Big(\mu - \frac{\sigma^2}{2}\Big)t + \sigma\sqrt{t}Z_t\Big\} \leq F\Big] \\ &= Pr\Big[ln(V_0) + \Big(\mu - \frac{\sigma^2}{2}\Big)t + \sigma\sqrt{t}Z_t \leq ln(F)\Big] \\ &= Pr\Big[\Big(\mu - \frac{\sigma^2}{2}\Big)t + \sigma\sqrt{t}Z_t \leq ln\Big(\frac{F}{V_0}\Big)\Big] \\ &= Pr\Big[Z_t \leq \frac{ln(F/V_0) - (\mu - (\sigma^2/2))t}{\sigma\sqrt{t}} \equiv z\Big] \\ &= \Phi(z) \end{aligned} \tag{1.4}$$

z is simply the threshold point in the standard normal distribution corresponding to a cumulative probability of $p_{Def}$ and is called distance to default.

According to Merton's model framework, the critical value z may be calculated for each obligor in the portfolio considering its specific parameters $V_0$, $\mu$ and $\sigma$. The approach I follow in this paper is the calculation of the critical value z for each industry k such that

$$z^k = \Phi^{-1}(p^k_{Def}) \tag{1.5}$$

where $p^k_{Def}$ is the average default probability observed in the data set for industry k considering the time period between 2004 and 2009. All obligors in industry k will have the same critical value $z^k$.

I simulate the asset value of every obligor in Merton's model framework and compare it to the critical value $z^k$ previously calculated. If the asset value is below the critical point, the obligor defaults. Every simulation run must have embedded the correlation coefficients, in order to generate correlated random numbers that will be used as proxy of the asset value of each obligor. For this purpose, the use of copulas is extremely useful. Copulas are simply the joint distribution of random vectors with standard uniform marginal distributions. Their value is that they provide a way of understanding how marginal distributions of single risks are coupled together to form joint distributions, that is, they provide a way of understanding the idea of statistical dependence and this is the essence of Sklar's theorem.

**Theorem 1** *(Sklar, 1959) Let F be a joint distribution function with continuous marginals $F_1$,...,$F_m$. Then there exists a unique copula $C : [0,1]^m \to [0,1]$ such that*

$$F(x_1, ..., x_m) = C\Big(F_1(x_1), ..., F_m(x_m)\Big) \tag{1.6}$$

*holds. Conversely, if C is a copula and $F_1$,...,$F_m$ are distribution functions, then the function F given by equation 1.6 is a joint distribution function with marginals $F_1$,...,$F_m$.*

For a proof and extensions to discontinuous marginal distributions refer to Schweizer and Sklar (1983).

A unique copula C is extracted from a multivariate distribution function F with continuous marginals $F_1$,...,$F_m$ by calculating

$$C(\mu_1, ..., \mu_m) = F\Big(F_1^{-1}(\mu_1), ..., F_m^{-1}(\mu_m)\Big),$$

where $F_1^{-1}$,...,$F_m^{-1}$ are inverses of $F_1$,...,$F_m$. We call C the copula of F.

If I assume multivariate Gaussian distribution with correlation matrix *R* then the copula may be represented by

$$C_R^{Ga}(\mu_1, ..., \mu_m) = \mathbf{\Phi}_R\Big(\Phi^{-1}(\mu_1), ..., \Phi^{-1}(\mu_m)\Big),$$

where $\mathbf{\Phi}_{\boldsymbol{R}}$ denotes the joint distribution function of a standard *m*-dimensional normal random vector with correlation matrix R, and $\Phi$ is the distribution function of univariate standard normal.

The Gaussian copula with Gaussian marginals is defined as

$$C_R^{Ga}(\mu_1, ..., \mu_m) = \int_{-\infty}^{\Phi^{-1}(\mu_1)} ... \int_{-\infty}^{\Phi^{-1}(\mu_m)} \frac{1}{(2\pi)^{\frac{m}{2}}|R|^{\frac{1}{2}}} exp\Big(-\tfrac{1}{2}\mathbf{x}^T R^{-1}\mathbf{x}\Big) \mathrm{d}x_1 ... \mathrm{d}x_m$$

where $|R|$ is the determinant of R. From the definition of the Gaussian copula we can determine the corresponding density. Using the canonical representation, we have:

$$\frac{1}{(2\pi)^{\frac{m}{2}}|R|^{\frac{1}{2}}} exp\Big(-\tfrac{1}{2}\mathbf{x}^T R^{-1}\mathbf{x}\Big) = C_R^{Ga}(\Phi(x_1), ..., \Phi(x_m)) \times \prod_{j=1}^{m}\Big(\frac{1}{\sqrt{2\pi}} exp\Big(-\frac{1}{2}x_j^2\Big)\Big)$$

Suppose instead that the copula is a Student's *t*. In this case the copula may be represented by

$$T_{R,\upsilon}(\mu_1, ..., \mu_m) = \int_{-\infty}^{t_\upsilon^{-1}(\mu_1)} ... \int_{-\infty}^{t_\upsilon^{-1}(\mu_m)} \frac{\Gamma\left(\frac{\upsilon+m}{2}\right)|R|^{-\frac{1}{2}}}{\Gamma\left(\frac{\upsilon}{2}\right)(\upsilon\pi)^{\frac{m}{2}}} \left(1 + \frac{1}{\upsilon}\mathbf{x}^T R^{-1}\mathbf{x}\right)^{-\frac{\upsilon+m}{2}} \mathrm{d}x_1...\mathrm{d}x_m$$

where $\upsilon$ is the number of degrees of freedom. Using the canonical representation, the copula density for the multivariate student's *t* copula is:

$$C_{R,\upsilon}(\mu_1, ...\mu_m) = |R|^{-\frac{1}{2}} \frac{\Gamma\left(\frac{\upsilon+m}{2}\right)}{\Gamma\left(\frac{\upsilon}{2}\right)} \left(\frac{\Gamma\left(\frac{\upsilon}{2}\right)}{\Gamma\left(\frac{\upsilon+1}{2}\right)}\right)^m \frac{\left(1+\frac{1}{\upsilon}\varsigma^T R^{-1}\varsigma\right)^{-\frac{\upsilon+m}{2}}}{\prod_{j=1}^{m}\left(1+\frac{\varsigma_j^2}{\upsilon}\right)^{-\frac{\upsilon+1}{2}}}$$

where $\varsigma_j = t_\upsilon^{-1}(\mu_j)$.

The copula implicit in the multivariate *t* is very different from the Gaussian copula because it has the property of tail dependence, so that it tends to generate simultaneous extreme events with higher probabilities than the Gaussian copula. This fact is crucial in the context of Value at Risk, as it leads to higher probabilities of joint defaults.

The linear correlation coefficient fully characterizes statistical dependence only in the class of elliptical distributions, the most important example being the multivariate Normal distribution. One particular shortcoming of this measure concerns the adequacy of correlation as an indicator of potential extreme co-movements in the underlying variables. Correlation is a measure of central tendency involving only first and second moment information but tail dependence is a more representative measure that is used to summarize the potential of extreme co-movements. The concept of tail dependence reflects the tendency of two r.v.s, say X and Y, to "move together", giving the asymptotic indication of how frequently we should expect to observe joint extreme values. For these reasons, I will estimate VaR with *t* and Gaussian copulas and compare the results.

Correlation is imposed in the simulation procedure through a matrix containing all pairwise asset correlations for all obligors in the portfolio. In order to capture the specific nature of each industry, I calculate correlation coefficients between industries and generalize those coefficients to all obligors. Consider for example obligors *a* and *b*, operating in industries *x* and *y*, respectively. The asset correlation coefficient between *a* and $b$, $\rho^A_{ab}$, will be given by the asset correlation between industries $x$ and $y$.

Table 1.4 presents the correlation assumptions tested in this study and the respective source. Sections A and B provide a detailed explanation of these asset correlation assumptions.

Table 1.4: Correlation Assumptions

Correlation coefficients tested in this study and the respective source.

| **Asset Correlation Coefficients** | | |
|---|---|---|
| **Intra-sector** | **Inter-sector** | **Source** |
| Empirical estimator | | De Servigny and Renault (2002) |
| 15% | 3% | Moody's |
| 15% | 5% | Standard & Poors |
| 30% | 0% | Standard & Poors' old values (prior to 2005) |
| 30% | 20% | Approximation to Fitch's values proposed by Fender and Kiff (2004) |
| 24% | 24% | Basel III Accord's maximum value |
| 8% | 8% | Basel III Accord's minimum value |

**A) Constant correlation coefficients** The assumption of constant correlation coefficients is market practice, due to its simplicity. The asset correlation coefficients I test in this study have been used by rating agencies to evaluate CDOs and are implicit in the methodology prescribed by the Basel III Accord.

**B) Empirical estimator of correlation**

Asset values are not observable for most firms operating in the market and, for this reason, asset correlation can not be calculated within my data sample. However, asset correlation affects the joint default probability, as I will show below, meaning that an assumption on joint asset movement like the copula approach allows us to back out the implied asset correlation from joint default probability.

Let $p_i$ and $p_j$ denote the marginal default probability of obligor *i* and *j*, respectively. The joint probability that obligor *i* and *j* both default by some time horizon T is denoted as $p_{ij}$. Let $V_i$,$V_j$ represent the asset values for obligors *i* and *j*, and $z_i$, $z_j$ the respective default threshold. In Merton's model framework we have,

$$p_{ij} = P(V_i \leq z_i, V_j \leq z_j) \tag{1.7}$$

Let $\rho_{ij}^A$ denote the asset correlation between asset values $V_i$ and $V_j$, and let $f_\gamma(u,v)$ denote a density function with correlation coefficient $\gamma$. The joint default probability of obligors *i* and *j* is defined as

$$p_{ij} = \int_{-\infty}^{z_i} \int_{-\infty}^{z_j} f_{\rho_{ij}^A}(u, v)\, du\, dv \tag{1.8}$$

From equation 1.8 we see that asset correlation affects joint default probability. I will now focus on the joint default probability and, once I have estimates for the joint default probability, I will derive the implied asset correlation.

All calculations regarding probability of default will be performed by industry, *i.e.*, there is an implicit assumption that the obligors of each industry are homogeneous groups and that defaults are conditionally independent given a set of common economic factors affecting all obligors. According to Frey and McNeil (2001) the concept of exchangeable vectors is the correct way to mathematically formalize the notion of homogeneous groups that will be used in practice and the concept of mixture models presents the appropriate setup for conditional independence. [1]

Following Akhavein *et al* (2005), I employ static pool methodology to calculate joint default probabilities. Pools are formed by grouping obligors according to their industry classification and each pool is followed forward for one year, resulting in a cohort. Let $D_{k,t}$ denote the number of defaults which have occurred in industry *k* and cohort *t* and $N_{k,t}$ denote the total number of obligors in the same industry and cohort. The marginal default probability of industry *k* and cohort *t* is given by:

$$p_{k,t} = \frac{D_{k,t}}{N_{k,t}} \tag{1.9}$$

[1] Please see appendix A for more details regarding the mathematical framework of exchangeable vectors and mixture models.

The sample from 2004 to 2009 enables me to calculate 6 default probabilities for each industry.

Once yearly default probabilities are calculated, I aggregate them to an average probability over the observation period, assuming that each year is an independent data set. I weight each year by its relative size, that is, by the number of firms present in the sample each year. The marginal default probability of industry *k* aggregated across all cohorts is given by:

$$p_k = \sum_{t=1}^{T} \frac{N_{k,t}}{\left(\sum_{t=1}^{T} N_{k,t}\right)} \frac{D_{k,t}}{N_{k,t}} \tag{1.10}$$

The first term on the right side of the equation corresponds to the weight attributed to each year and the second term is the marginal default probability of industry *k* and cohort *t*, according to equation 1.9. Table 1.5 presents the estimates of marginal default probabilities.

Recall that from a given group with N elements, one can create N(N-1)/2 different pairs. Therefore, if D denotes the number of defaulting obligors, one method to extract the joint default probability for a given year *t* corresponds to drawing pairs of firms without replacement, given by the following equation:

$$\frac{D_t}{N_t} \frac{D_t - 1}{N_t - 1}$$

This is the estimator used by Lucas (1995) and Nagpal & Bahar (2001). In a similar way, based on the framework presented in the appendix A, Frey and McNeil (2001) propose the use of the joint default probability estimator given by:

$$\hat{\pi}_j = \frac{1}{T} \sum_{t=1}^{T} \frac{\binom{D_t}{j}}{\binom{N_t}{j}} = \frac{1}{T} \sum_{t=1}^{T} \frac{D_t(D_t-1)...(D_t-j+1)}{N_t(N_t-1)...(N_t-j+1)}, \quad 1 \leq j \leq min\{N_1, ..., N_n\} \tag{1.11}$$

According to De Servigny and Renault (2002a), this estimator has the drawback that it can generate spurious negative correlation. The one period joint default probability calculated above is always smaller than $(D/N)^2$, which is the square of the univariate probability. Thus, the estimated joint default probability is always lower than that ob-

Table 1.5: Marginal default probabilities

Empirical marginal default probability of each industry aggregated across all cohorts.

| **Industry** | **Marginal default probability (%)** |
|---|---|
| Undefined | 3.46 |
| Banking and Finance | 2.07 |
| Broadcasting/Media/Cable | 5.64 |
| Building, Materials and Real Estate | 4.88 |
| Business Services | 2.89 |
| Materials and Utilities | 3.80 |
| Computers and Electronics | 2.01 |
| Consumer Products | 3.61 |
| Food, Beverage and Tobacco | 3.49 |
| Gaming, Leisure and Entertainment | 3.57 |
| Health Care and Pharmaceutical | 1.80 |
| Industrial/Manufacturing | 3.75 |
| Lodging and Restaurants | 4.71 |
| Retail | 2.89 |
| Supermarkets and Drugstores | 3.13 |
| Textiles and Furniture | 5.90 |
| Transportation | 2.93 |
| Others | 2.72 |

tained under the assumption of independence, which implies negative correlation. For this reason, I calculate the joint default probability for industry *k* using the estimator proposed by De Servigny and Renault (2002a), defined as:

$$p_{kk} = \sum_{t=1}^{T} \frac{N_{k,t}}{\sum_{t=1}^{T} N_{k,t}} \frac{D_{k,t}D_{k,t}}{N_{k,t}N_{k,t}} \tag{1.12}$$

The first term on the right side of the equation corresponds to the weight attributed to each year and the second term is the joint default probability of industry *k* and cohort *t*, calculated assuming the draw of pairs of firms with replacement.

In the case of obligors operating in different industries, the formula becomes:

$$p_{kj} = \sum_{t=1}^{T} \frac{N_{k,t} + N_{j,t}}{\sum_{t=1}^{T} N_{k,t} + N_{j,t}} \frac{D_{k,t}D_{j,t}}{N_{k,t}N_{j,t}} \tag{1.13}$$

Despite the fact that both estimators would yield very similar results in very large samples, in samples of the size of a typical credit portfolio the difference may be substantial. For details regarding the performance of these estimators, see De Servigny and Renault (2003). Table 1.6 presents the obtained estimates of joint default probabilities.

Once I have the joint default probabilities, I calculate the asset correlations implicit in equation 1.8 using a numerical method. For this purpose, I assume multivariate Gaussian distribution with Gaussian marginals. Table 1.7 presents the asset correlations obtained imposing the restriction $|\hat{p}_{kj}\text{-}p_{kj}| \leq 5 \times 10^{-6}$. The average intra sector asset correlation is $12.99\%$ and the average inter sector asset correlation is $0.14\%$, corresponding to an average default correlation of $2.68\%$ and $0.05\%$, respectively.

## Table 1.6: Joint Default Frequency (%)

Empirical joint default frequency between all industries.

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) | (16) | (17) | (18) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (1) Undefined | 0.212 | 0.065 | 0.204 | 0.180 | 0.103 | 0.143 | 0.073 | 0.130 | 0.117 | 0.127 | 0.063 | 0.130 | 0.165 | 0.103 | 0.114 | 0.212 | 0.108 | 0.115 |
| (2) Banking and Finance | 0.065 | 0.124 | 0.096 | 0.099 | 0.061 | 0.073 | 0.046 | 0.070 | 0.067 | 0.065 | 0.048 | 0.077 | 0.107 | 0.061 | 0.063 | 0.124 | 0.055 | 0.043 |
| (3) Broadcasting/Media/Cable | 0.204 | 0.096 | 0.354 | 0.272 | 0.159 | 0.231 | 0.117 | 0.205 | 0.212 | 0.208 | 0.096 | 0.211 | 0.261 | 0.163 | 0.177 | 0.331 | 0.165 | 0.150 |
| (4) Building, Materials and Real Estate | 0.180 | 0.099 | 0.272 | 0.303 | 0.147 | 0.194 | 0.091 | 0.188 | 0.181 | 0.185 | 0.087 | 0.197 | 0.243 | 0.141 | 0.164 | 0.303 | 0.157 | 0.132 |
| (5) Business Services | 0.103 | 0.061 | 0.159 | 0.147 | 0.174 | 0.109 | 0.057 | 0.106 | 0.103 | 0.106 | 0.052 | 0.112 | 0.137 | 0.083 | 0.092 | 0.174 | 0.087 | 0.073 |
| (6) Materials and Utilities | 0.143 | 0.073 | 0.231 | 0.194 | 0.109 | 0.238 | 0.073 | 0.140 | 0.145 | 0.144 | 0.067 | 0.147 | 0.186 | 0.110 | 0.123 | 0.238 | 0.124 | 0.115 |
| (7) Computers and Electronics | 0.073 | 0.046 | 0.117 | 0.091 | 0.057 | 0.073 | 0.117 | 0.071 | 0.068 | 0.069 | 0.039 | 0.073 | 0.091 | 0.060 | 0.061 | 0.114 | 0.050 | 0.043 |
| (8) Consumer Products | 0.130 | 0.070 | 0.205 | 0.188 | 0.106 | 0.140 | 0.071 | 0.217 | 0.131 | 0.134 | 0.063 | 0.141 | 0.172 | 0.104 | 0.116 | 0.217 | 0.110 | 0.094 |
| (9) Food, Beverage and Tobacco | 0.117 | 0.067 | 0.212 | 0.181 | 0.103 | 0.145 | 0.068 | 0.131 | 0.212 | 0.132 | 0.061 | 0.140 | 0.180 | 0.104 | 0.116 | 0.212 | 0.109 | 0.079 |
| (10) Gaming, Leisure and Entertainment | 0.127 | 0.065 | 0.208 | 0.185 | 0.106 | 0.144 | 0.069 | 0.134 | 0.132 | 0.219 | 0.061 | 0.140 | 0.172 | 0.103 | 0.116 | 0.219 | 0.113 | 0.101 |
| (11) Health Care and Pharmaceuticals | 0.063 | 0.048 | 0.096 | 0.087 | 0.052 | 0.067 | 0.039 | 0.063 | 0.061 | 0.061 | 0.108 | 0.067 | 0.088 | 0.053 | 0.056 | 0.108 | 0.050 | 0.044 |
| (12) Industrial/Manufacturing | 0.130 | 0.077 | 0.211 | 0.197 | 0.112 | 0.147 | 0.073 | 0.141 | 0.140 | 0.140 | 0.067 | 0.230 | 0.185 | 0.108 | 0.122 | 0.230 | 0.116 | 0.095 |
| (13) Lodging and Restaurants | 0.165 | 0.107 | 0.261 | 0.243 | 0.137 | 0.186 | 0.091 | 0.172 | 0.180 | 0.172 | 0.088 | 0.185 | 0.284 | 0.137 | 0.152 | 0.284 | 0.146 | 0.111 |
| (14) Retail | 0.103 | 0.061 | 0.163 | 0.141 | 0.083 | 0.110 | 0.060 | 0.104 | 0.104 | 0.103 | 0.053 | 0.108 | 0.137 | 0.167 | 0.091 | 0.167 | 0.083 | 0.067 |
| (15) Supermarkets and Drugstores | 0.114 | 0.063 | 0.177 | 0.164 | 0.092 | 0.123 | 0.061 | 0.116 | 0.116 | 0.116 | 0.056 | 0.122 | 0.152 | 0.091 | 0.187 | 0.187 | 0.097 | 0.079 |
| (16) Textiles and Furnitures | 0.212 | 0.124 | 0.331 | 0.303 | 0.174 | 0.238 | 0.114 | 0.217 | 0.212 | 0.219 | 0.108 | 0.230 | 0.284 | 0.167 | 0.187 | 0.372 | 0.188 | 0.174 |
| (17) Transportation | 0.108 | 0.055 | 0.165 | 0.157 | 0.087 | 0.124 | 0.050 | 0.110 | 0.109 | 0.113 | 0.050 | 0.116 | 0.146 | 0.083 | 0.097 | 0.188 | 0.188 | 0.097 |
| (18) Others | 0.115 | 0.043 | 0.150 | 0.132 | 0.073 | 0.115 | 0.043 | 0.094 | 0.079 | 0.101 | 0.044 | 0.095 | 0.111 | 0.067 | 0.079 | 0.174 | 0.097 | 0.174 |

Table 1.7: Asset Correlation (%)

Empirical asset correlation between all industries.

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) | (16) | (17) | (18) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (1) Undefined | 12.801 | -1.989 | 0.921 | 1.291 | 0.471 | 1.701 | 0.791 | 0.761 | -0.839 | 0.491 | 0.151 | -0.049 | 0.181 | 0.601 | 0.861 | 0.831 | 1.151 | 3.851 |
| (2) Banking and Finance | -1.989 | 21.351 | -4.009 | -0.549 | 0.041 | -1.539 | 1.371 | -1.339 | -1.589 | -2.539 | 4.201 | -0.269 | 1.791 | 0.321 | -0.799 | 0.251 | -1.789 | -4.799 |
| (3) Broadcasting/Media/Cable | 0.921 | -4.009 | 2.651 | -0.379 | -0.609 | 1.621 | 0.621 | 0.081 | 1.641 | 0.631 | -1.239 | -0.049 | -0.409 | 0.021 | 0.001 | -0.149 | -0.149 | -0.569 |
| (4) Building, Materials and R.E. | 1.291 | -0.549 | -0.379 | 5.781 | 0.801 | 0.871 | -1.519 | 1.391 | 1.311 | 1.281 | -0.269 | 1.621 | 1.291 | -0.069 | 1.451 | 1.171 | 1.981 | -0.169 |
| (5) Business Services | 0.471 | 0.041 | -0.609 | 0.801 | 15.691 | -0.219 | -0.469 | 0.231 | 0.261 | 0.351 | -0.129 | 0.581 | 0.121 | -0.279 | 0.111 | 0.331 | 0.311 | -1.679 |
| (6) Materials and Utilities | 1.701 | -1.539 | 1.621 | 0.871 | -0.219 | 11.431 | -0.959 | 0.391 | 1.811 | 1.111 | -0.369 | 0.561 | 0.821 | -0.089 | 0.541 | 1.341 | 2.041 | 2.061 |
| (7) Computers and Electronics | 0.791 | 1.371 | 0.621 | -1.519 | -0.469 | -0.959 | 21.181 | -0.529 | -0.699 | -0.859 | 1.111 | -0.669 | -0.969 | 0.431 | -0.849 | -0.879 | -3.139 | -4.399 |
| (8) Consumer Products | 0.761 | -1.339 | 0.081 | 1.391 | 0.231 | 0.391 | -0.529 | 11.531 | 0.741 | 0.751 | -0.549 | 0.781 | 0.221 | -0.149 | 0.441 | 0.411 | 0.731 | -1.039 |
| (9) Food, Beverage and Tobacco | -0.839 | -1.589 | 1.641 | 1.311 | 0.261 | 1.811 | -0.699 | 0.741 | 12.391 | 1.051 | -0.739 | 1.301 | 1.941 | 0.431 | 1.141 | 0.551 | 1.191 | -3.569 |
| (10) Gaming, Leisure and Entert. | 0.491 | -2.539 | 0.631 | 1.281 | 0.351 | 1.111 | -0.859 | 0.751 | 1.051 | 12.201 | -0.929 | 0.851 | 0.471 | -0.129 | 0.671 | 0.851 | 1.431 | 0.691 |
| (11) Health Care and Pharmac. | 0.151 | 4.201 | -1.239 | -0.269 | -0.129 | -0.369 | 1.111 | -0.549 | -0.739 | -0.929 | 23.591 | -0.149 | 0.651 | 0.221 | -0.379 | 0.261 | -0.919 | -2.109 |
| (12) Industrial/Manufacturing | -0.049 | -0.269 | -0.049 | 1.621 | 0.581 | 0.561 | -0.669 | 0.781 | 1.301 | 0.851 | -0.149 | 11.241 | 1.051 | -0.009 | 0.791 | 0.861 | 1.001 | -1.529 |
| (13) Lodging and Restaurants | 0.181 | 1.791 | -0.409 | 1.291 | 0.121 | 0.821 | -0.969 | 0.221 | 1.941 | 0.471 | 0.651 | 1.051 | 5.971 | 0.121 | 0.641 | 0.541 | 1.141 | -3.069 |
| (14) Retail | 0.601 | 0.321 | 0.021 | -0.069 | -0.279 | -0.089 | 0.431 | -0.149 | 0.431 | -0.129 | 0.221 | -0.009 | 0.121 | 14.861 | -0.039 | -0.479 | -0.559 | -3.099 |
| (15) Supermarkets and Drugst. | 0.861 | -0.799 | 0.001 | 1.451 | 0.111 | 0.541 | -0.849 | 0.441 | 1.141 | 0.671 | -0.379 | 0.791 | 0.641 | -0.039 | 14.121 | 0.251 | 0.931 | -1.559 |
| (16) Textiles and Furnitures | 0.831 | 0.251 | -0.149 | 1.171 | 0.331 | 1.341 | -0.879 | 0.411 | 0.551 | 0.851 | 0.261 | 0.861 | 0.541 | -0.479 | 0.251 | 1.681 | 1.801 | 1.721 |
| (17) Transportation | 1.151 | -1.789 | -0.149 | 1.981 | 0.311 | 2.041 | -3.139 | 0.731 | 1.191 | 1.431 | -0.919 | 1.001 | 1.141 | -0.559 | 0.931 | 1.801 | 17.031 | 3.661 |
| (18) Others | 3.851 | -4.799 | -0.569 | -0.169 | -1.679 | 2.061 | -4.399 | -1.039 | -3.569 | 0.691 | -2.109 | -1.529 | -3.069 | -3.099 | -1.559 | 1.721 | 3.661 | 18.311 |

### 1.3.1 VaR Estimation[2]

Once I have the asset correlation matrix, I start generating correlated random numbers that will be used as proxy of the asset value of each obligor. Let $\boldsymbol{C}$=($C_1$,...,$C_N$)′ be an *N*-dimensional random vector with continuous marginal distributions representing the asset value of each obligor and let $\boldsymbol{z}$=($z_1$,...,$z_N$)′ be a vector of deterministic cut-off levels obtained within Merton's model framework. The following relationship holds:

$$I_i = 1 \Leftrightarrow C_i \leq z_i \tag{1.14}$$

I follow Monte Carlo simulation technique to draw the portfolio value distribution. The first step is the generation of 10,000 scenarios for the asset value of each obligor *i* in each time period *t*. In the case of Gaussian copula, a possible way of transforming a vector of uncorrelated random variables ($\boldsymbol{U}$) into a vector of correlated random variables ($\boldsymbol{C}$) is the multiplication of $\boldsymbol{U}$ by the Cholesky decomposition of the asset correlation matrix $\Omega_t$. Considering a portfolio with N obligors, the Cholesky decomposition of $\Omega_t$ is the $N \times N$ symmetric positive definite lower triangular matrix $\boldsymbol{A}_t$, such that $\Omega_t$=$\boldsymbol{A}_t\boldsymbol{A}'_t$. In the case of the *t* copula, the vector of correlated random variables is obtained from the application of the appropriate copula function. By doing this, it is possible to have a dependence structure with *t*-student distribution and marginals within the Merton model framework.

After the simulation of the asset value for every obligor, I determine which obligors default in each simulation *s* and time period *t*. When a default occurs, a recovery rate is determined by using Beta distribution sampling. The probability density function of the Beta distribution, for $0 \leq x \leq 1$ and shape parameters $\alpha$, $\beta > 0$, is given by:

$$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} \tag{1.15}$$

[2] VaR estimation was performed with the software Matlab.

where $\Gamma(z)$ is the gamma function. The expected value and variance of x are given by:

$$E[x] = \frac{\alpha}{\alpha+\beta} \qquad var[x] = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \tag{1.16}$$

The historical levels of recovery for this type of loans in the financial institution considered in this study have an average value of $61\%$ and a variance of 0.01668, which are reflected in the following parameters of the Beta distribution: $\alpha$ equal to 8.09 and $\beta$ equal to 5.13. Loan value of obligor *i* in simulation *s* and time *t* is given by equation 1.1.

Portfolio value for simulation *s* in period *t* is represented by:

$$Portfolio\, Value_t^s = \sum_{i=1}^{N} Loan\, Value_{i,t}^s\ ,\ s \in (1,...10.000),\ t \in (1,...72) \tag{1.17}$$

After drawing portfolio value distribution for period $t$, the credit $\text{VaR}_t$ estimate for a $99\%$ confidence level is calculated according to the following equation:

$$CVaR_t^{99\%} = Mean\, portfolio\, value_t - portfolio\, value_t^{1\%} \tag{1.18}$$

I repeat this process for each correlation assumption and each copula, in order to estimate VaR with different methodologies and compare the results with the time series of observed loss. For the *t* dependence structure, it is essentially the degrees of freedom parameter that controls the extent of tail dependence and tendency to exhibit extreme co-movements. Considering previous research performed by Dorey *et al* (2005), Cherubini *et al* (2004) and Abid & Naifar (2008), I calculate VaR considering 2, 8 and 12 degrees of freedom.

Figure 1.1 presents the comparison between VaR estimates considering Gaussian copula and the observed loss. Figures 1.2, 1.3 and 1.4 present the comparison between VaR estimates considering *t* copula and respectively 2, 8 and 12 degrees of freedom, and the observed loss. These figures show the behavior of each VaR model through time and compare it with the observed loss in each time period between 2004 and 2009. Every time the line corresponding to a VaR estimate is below the line corresponding to the observed loss (red line in the figures), a violation occurs. A perfect VaR model would follow closely the observed loss time series and should be below the loss 1% of the time (for a 99% confidence level), meaning that the model would accurately predict market movements and react very fast to its changes.

Figure 1.1: CVaR *vs* Observed Loss - Gaussian copula

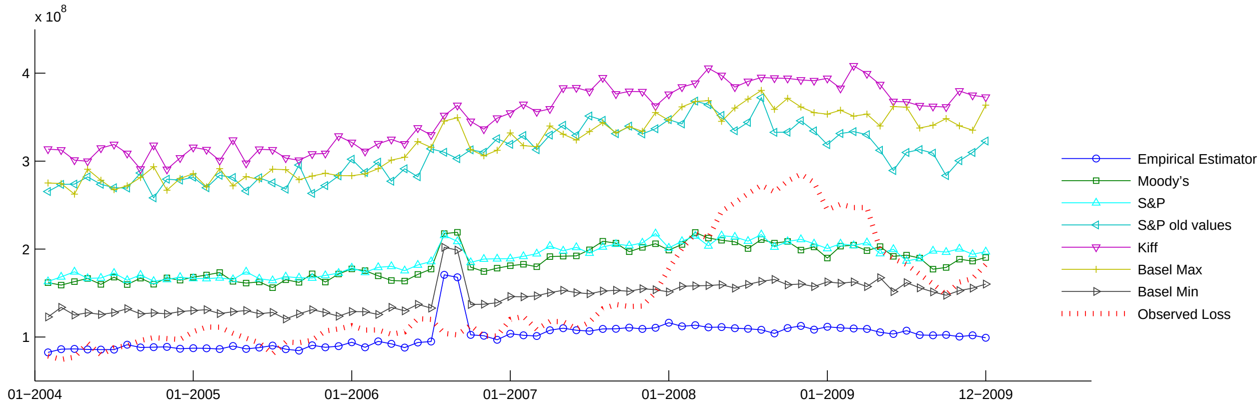



Figure 1.2: CVaR *vs* Observed Loss - *t* copula (DoF=2)

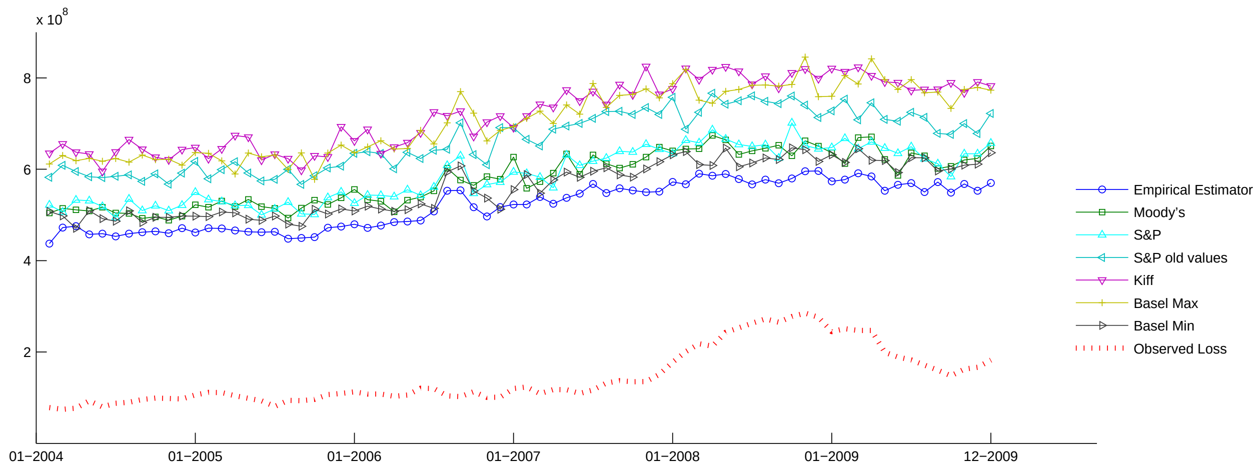


Figure 1.3: CVaR *vs* Observed Loss - *t* copula (DoF=8)

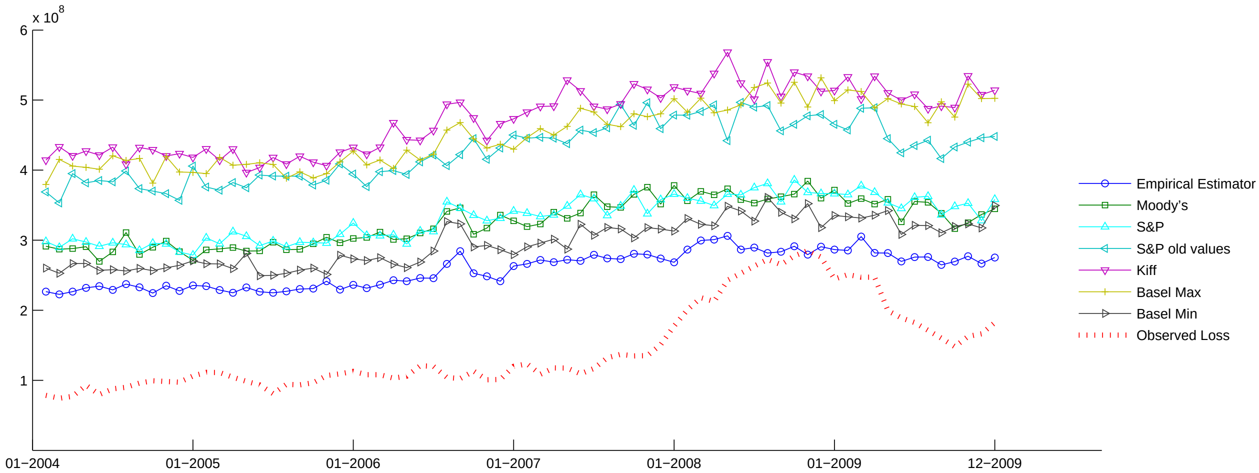
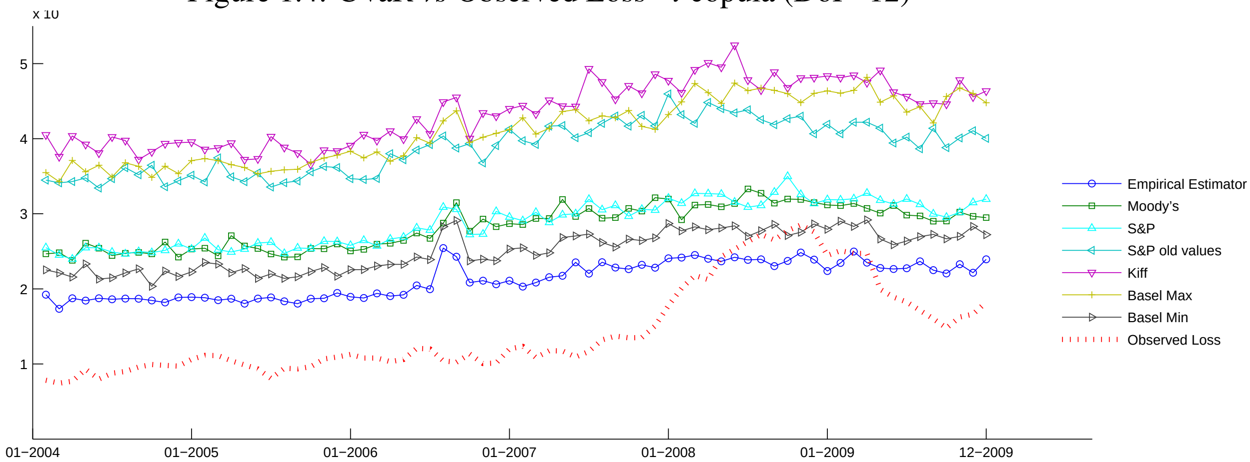



Figure 1.4: CVaR *vs* Observed Loss - *t* copula (DoF=12)




Regarding Gaussian copula, the rate of VaR violations appears to be very high when I assume correlation derived with empirical estimator, Basel minimum value or correlation parameters used by Moody's and S&P. For all other correlation assumptions, Value at Risk is always higher than the observed loss, which suggests overestimation of portfolio risk.

The assumption of *t* copula with 2 degrees of freedom produces very conservative VaR estimates for all correlation assumptions. In the case of *t* copula with 8 degrees of freedom, only the empirical estimator of correlation produces VaR violations. Increasing the degrees of freedom from 8 to 12 apparently increases the number of VaR violations in the case of empirical estimator and Basel minimum value for asset correlation.

## 1.4 Backtesting VaR

The assumptions about correlation and dependence structure tested in this study produced very different VaR estimates. In order to compare the different VaR approaches and identify the most accurate one, I perform backtesting procedures.

The simplest method to verify the accuracy of a VaR model is to record the proportion of times VaR is exceeded in a given sample, the failure rate. Denoting the loss on the portfolio over a fixed time interval as $x_{t,t+1}$ the hit function is given by:

$$I_{t+1}(\theta) = \begin{cases} 1 \quad if\ x_{t,t+1} > VaR_t(\theta) \\ 0 \quad if\ x_{t,t+1} \leq VaR_t(\theta) \end{cases} \tag{1.19}$$

Suppose we have a VaR estimate at the 1 percent left-tail level for a total of T periods. We can count the number of times the actual loss exceeds the previous' period VaR. Defining N as the number of exceptions, it follows that the failure rate ($\pi$) is given by N/T. The goal is to determine whether N is too small or too large under the null hypothesis that $\theta$=0.01 in a sample of size T. The statistical framework for this test is the Bernoulli trials, which means that the number of exceptions x follows a binomial probability distribution:

$$f(x) = \binom{T}{x} \theta^x (1-\theta)^{(T-x)} \tag{1.20}$$

The expected value of x is E[x]=$\theta$T and the variance is Var(x)=$\theta$(1-$\theta$)T. When T is large, we can use the central limit theorem and approximate the binomial distribution by the normal distribution:

$$z = \frac{x - \theta T}{\sqrt{\theta(1-\theta)T}} \approx N(0,1) \tag{1.21}$$

Christoffersen (1998) points out that the problem of determining the accuracy of a VaR model can be reduced to the problem of determining whether the hit sequence satisfies two properties:

**Unconditional coverage property**: The probability of realizing a loss in excess of the estimated VaR should be exactly $\theta$. If losses in excess of the estimated VaR occur more frequently than $\theta$ of the time, then this would suggest that the VaR model systematically understates the portfolio risk. The opposite finding would alternatively provide evidence on an overly conservative VaR model.

**Independence property**: this property places a strong restriction on how VaR exceptions may occur. Intuitively, this condition requires that the previous history of VaR violations must not convey any information about whether a VaR violation will occur in the following period. In general, a clustering of VaR exceptions represents violation of the independence property that provides evidence of a lack of responsiveness in the VaR model, making successive runs of VaR exceptions more likely.

According to Campbell (2005), the unconditional coverage and independence properties of the hit sequence are distinct and must both be satisfied by an accurate VaR model.

**A) Kupiec Test (1995)**

Kupiec (1995) proposes a test to check the unconditional coverage property, based on the number of VaR violations. Kupiec's test examines how many times a VaR is violated over a given span of time. If the number of exceptions differs considerably from $\theta \times 100$, then the accuracy of the VaR model is called into question. The null hypothesis for Kupiec's test is:

$$H_0 : \pi = \theta \tag{1.22}$$

$\hat{\pi}$ is given by:

$$\hat{\pi} = \frac{N}{T} \tag{1.23}$$

The log-likelihood ratio for this test is given by:

$$LR_{uc} = -2ln[(1-\theta)^{T-N}\theta^N] + 2ln\{[1-\hat{\pi}]^{T-N}\hat{\pi}^N\} \tag{1.24}$$

which is asymptotically distributed chi-square with one degree of freedom under the null hypothesis that $\theta$ is the true probability.

**B) Christoffersen Test (1998)**

Christoffersen (1998) developed a test to check the independence property. The test setup is as follows: each period we set a violation indicator to 0 if VaR is not exceeded and to 1 otherwise. We then define $T_{ij}$ as the number of periods in which state j has occurred in one period while it was i the previous period and $\pi_i$ as the probability of observing an exception conditional on state i the previous period. The null hypothesis is:

$$H_0 : \pi_0 = \pi_1 = \pi \tag{1.25}$$

The test statistic is given by:

$$LR_{ind} = -2ln[(1-\hat{\pi})^{(T_{00}+T_{10})}\hat{\pi}^{(T_{01}+T_{11})}] + 2ln[(1-\hat{\pi_0})^{T_{00}}\hat{\pi_0}^{T_{01}}(1-\hat{\pi_1})^{T_{10}}\hat{\pi_1}^{T_{11}}] \tag{1.26}$$

which is asymptotically distributed chi-square with one degree of freedom. $\hat{\pi_0}$ and $\hat{\pi_1}$ are given by:

$$\hat{\pi_1} = \frac{T_{11}}{T_{10}+T_{11}} \qquad \hat{\pi_0} = \frac{T_{01}}{T_{00}+T_{01}}$$

Tables 1.8 and 1.9 present the results obtained for Kupiec and Christoffersen tests for all correlation assumptions considering Gaussian and *t* copulas. The null hypothesis of Kupiec test is rejected in the scenario of Gaussian copula considering asset correlation based on the empirical estimator, Moody's, S&P and Basel Accord minimum value. The same result is obtained in the case of *t* copula with 12 degrees of freedom, considering asset correlation given by the empirical estimator. The null hypothesis of Kupiec test is not rejected in the case of *t* copula with 8 degrees of freedom and correlation based on the empirical estimator and also in the case of *t* copula with 12 degrees of freedom and correlation based on Basel III Accord minimum value. In the remaining correlation assumptions, Kupiec test is inconclusive due to the fact that no exceptions were observed.

The null hypothesis of Christoffersen test is rejected in the case of Gaussian copula with correlation based on Moody's and S&P parameters and in the case of *t* copula with 12 degrees of freedom with correlation based on the empirical estimator. The null hypothesis of this test is not rejected only in the cases of Gaussian copula with correlation based on the empirical estimator and *t* copula with 12 degrees of freedom and correlation prescribed by Basel III Accord minimum value. In all the other cases this test is inconclusive.

Since the unconditional coverage and independence properties of the hit sequence must be both satisfied by an accurate VaR model, at this point I can conclude that some of the models tested in this study are not accurate, namely the models based on Gaussian copula with correlation given by the empirical estimator, Moody's and S&P parameters and Basel III Accord minimum value for correlation and also the model based on *t* copula with 12 degrees of freedom and correlation given by the empirical estimator. I will exclude these VaR models from the remaining backtesting procedures.

At this point an interesting conclusion emerges: since the methodologies applied by Moody's and Standard and Poors to evaluate CDOs are based on the Gaussian copula with the correlation parameters tested in this study and these methodologies produced very high failure rates, leading to the rejection of the null hypothesis of the Kupiec test, I conclude there is empirical evidence that the procedures used by major rating agencies to evaluate CDOs are inadequate in stress situations like the financial crisis observed in 2008.

Some of the correlation assumptions tested in this study produced null failure rate or null results for the statistics $T_{10}, T_{11}$, leading to inconclusive results of Kupiec and Christoffersen tests. These null outcomes might be explained by over conservative VaR models or by the small number of observations, but in either cases it is not possible to draw a conclusion from the performed VaR backtests. The evaluation of the accuracy of these VaR models requires us to find alternative methods.

Table 1.8: Kupiec Test

Results obtained for Kupiec test for all correlation assumptions considering Gaussian and t copulas.

| **Gaussian copula** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min** |
|---|---|---|---|---|---|---|---|
| # exceptions | 63 | 13 | 15 | 0 | 0 | 0 | 25 |
| # observations | 72 | 72 | 72 | 72 | 72 | 72 | 72 |
| Failure Rate (%) | 88 | 18 | 21 | 0 | 0 | 0 | 35 |
| $LR_{uc}$ | 526 | 53 | 66 | na | na | na | 138 |
| Critical Value | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 |
| Test Result | Reject $H_0$ | Reject $H_0$ | Reject $H_0$ | n.a | n.a | n.a | Reject $H_0$ |

| **t copula (DoF=2)** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min** |
|---|---|---|---|---|---|---|---|
| # exceptions | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| # observations | 72 | 72 | 72 | 72 | 72 | 72 | 72 |
| Failure Rate (%) | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $LR_{uc}$ | n.a | n.a | n.a | n.a | n.a | n.a | n.a |
| Critical Value | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 |
| Test Result | n.a | n.a | n.a | n.a | n.a | n.a | n.a |

| **t copula (DoF=8)** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min** |
|---|---|---|---|---|---|---|---|
| # exceptions | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| # observations | 72 | 72 | 72 | 72 | 72 | 72 | 72 |
| Failure Rate (%) | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| $LR_{uc}$ | 0 | n.a | n.a | n.a | n.a | n.a | n.a |
| Critical Value | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 |
| Test Result | Accept $H_0$ | n.a | n.a | n.a | n.a | n.a | n.a |

| **t copula (DoF=12)** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min*** |
|---|---|---|---|---|---|---|---|
| # exceptions | 11 | 0 | 0 | 0 | 0 | 0 | 2 |
| # observations | 72 | 72 | 72 | 72 | 72 | 72 | 72 |
| Failure Rate (%) | 15 | 0 | 0 | 0 | 0 | 0 | 3 |
| $LR_{uc}$ | 41 | n.a | n.a | n.a | n.a | n.a | 2 |
| Critical Value | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 |
| Test Result | Reject $H_0$ | n.a | n.a | n.a | n.a | n.a | Accept $H_0$ |

* Benchmark model

Considering all correlation assumptions that produced valid results for Kupiec and Christoffersen tests, only the hypothesis of asset correlation given by the Basel Accord minimum value (8%) considering a *t* copula with 12 degrees of freedom simultaneously satisfies the unconditional coverage property and the independence property. In order to continue the process of identifying the most accurate VaR model for this particular

Table 1.9: Christoffersen Test

Results obtained for Christoffersen test for all correlation assumptions considering Gaussian and t copulas.

| **Gaussian copula** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min** |
|---|---|---|---|---|---|---|---|
| $\pi_0$ | 0.67 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.02 |
| $\pi_1$ | 0.92 | 0.92 | 0.93 | n.a | n.a | n.a | 1.00 |
| $LR_{ind}$ | 3.88 | 50.45 | 55.84 | n.a | n.a | n.a | n.a |
| Critical Value | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 |
| Test Result | Accept $H_0$ | Reject $H_0$ | Reject $H_0$ | n.a | n.a | n.a | n.a |

| **t copula (DoF=2)** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min** |
|---|---|---|---|---|---|---|---|
| $\pi_0$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $\pi_1$ | n.a | n.a | n.a | n.a | n.a | n.a | n.a |
| $LR_{ind}$ | n.a | n.a | n.a | n.a | n.a | n.a | n.a |
| Critical Value | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 |
| Test Result | n.a | n.a | n.a | n.a | n.a | n.a | n.a |

| **t copula (DoF=8)** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min** |
|---|---|---|---|---|---|---|---|
| $\pi_0$ | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $\pi_1$ | 0,00 | n.a | n.a | n.a | n.a | n.a | n.a |
| $LR_{ind}$ | n.a | n.a | n.a | n.a | n.a | n.a | n.a |
| Critical Value | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 |
| Test Result | n.a | n.a | n.a | n.a | n.a | n.a | n.a |

| **t copula (DoF=12)** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min*** |
|---|---|---|---|---|---|---|---|
| $\pi_0$ | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 |
| $\pi_1$ | 0.82 | n.a | n.a | n.a | n.a | n.a | 0.50 |
| $LR_{ind}$ | 33.26 | n.a | n.a | n.a | n.a | n.a | 5.00 |
| Critical Value | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 | 6.63 |
| Test Result | Reject $H_0$ | n.a | n.a | n.a | n.a | n.a | Accept $H_0$ |

* Benchmark model

portfolio, I will set this model as a benchmark.

### C) Loss Function Estimator

Despite the fact that the hit function plays a major role in the backtesting procedures, the information contained in the hit function is limited, as it ignores, for example, the magnitude of the exceedance of VaR estimates. Lopez (1998) suggests an alternative to the approach that focuses exclusively on the hit series. The loss function suggested by

Lopez (1998) is:

$$L(VaR_t(\theta), x_{t,t+1}) = \begin{cases} 1 + (x_{t,t+1} - VaR_t(\theta))^2 & if\ x_{t,t+1} > VaR_t(\theta) \\ 0 & if\ x_{t,t+1} \leq VaR_t(\theta) \end{cases} \tag{1.27}$$

According to Campbell (2005), a backtest that uses the loss function defined by Lopez (1998) would typically be based on the sample average loss,

$$\hat{L} = \frac{1}{T} \sum_{t=1}^{T} L(VaR_t(\theta), x_{t,t+1}) \tag{1.28}$$

Table 1.10 presents the results for the sample average loss. The average magnitude of the exceedance of VaR estimates considering asset correlation given by the empirical estimator and assuming *t* copula with 8 degrees of freedom is $34 \times 10^{12}$€, approximately half of the average magnitude of the exceedance of VaR estimates considering our benchmark model. For this reason, the former VaR model will also be considered as a reference in the comparison of VaR models.

Table 1.10: Loss Function Estimator ($10^{12}$€)

Results obtained for the Loss Function Estimator for all correlation assumptions considering Gaussian and t copulas.

| Copula | Emp. Est. | Moody's | S&P | S&P (old) | Kiff | Basel Max | Basel Min |
|---|---|---|---|---|---|---|---|
| Gaussian | excluded | excluded | excluded | n.a | n.a | n.a | excluded |
| t (DoF=2) | n.a | n.a | n.a | n.a | n.a | n.a | n.a |
| t (DoF=8) | 34 | n.a | n.a | n.a | n.a | n.a | n.a |
| t (DoF=12) | excluded | n.a | n.a | n.a | n.a | n.a | 67* |

* Benchmark model

**D) Measure of over-conservativeness**

The loss function estimator is only useful when a VaR model has non null failure rate. For this reason, the problem of determining whether inconclusive results of Kupiec and Christoffersen tests were due to over conservative VaR models or to small number of observations remains. In order to measure how conservative a VaR model is, I will define a variant of the loss function proposed by Lopez (1998). While the loss function proposed by Lopez (1998) considers only the time periods in which a violation of VaR occurs (in the remaining periods the function has value 0), this new measure considers only the periods in which the loss is below the estimated VaR (assigning the value 0 when there is violation of the VaR estimate). Thus, we can calculate an average value of over-conservativeness. The advantage of this measure of over-conservativeness is that it provides additional information when the Kupiec and Christoffersen tests are inconclusive.

$$L'(VaR_t(\theta), x_{t,t+1}) = \begin{cases} 1 + (VaR_t(\theta) - x_{t,t+1})^2 & if\ x_{t,t+1} < VaR_t(\theta) \\ 0 & if\ x_{t,t+1} \geq VaR_t(\theta) \end{cases} \tag{1.29}$$

Define N' as the number of periods for which VaR estimates are higher than the actual loss. The sample average is given by:

$$\hat{L}' = \frac{1}{N'} \sum_{t=1}^{T} L'(VaR_t(\theta), x_{t,t+1}) \tag{1.30}$$

Table 1.11 presents the results for this measure of over-conservativeness.

Table 1.11: Measure of over-conservativeness ($10^{12}$€)

Results obtained for the Measure of over-conservativeness for all correlation assumptions considering Gaussian and t copulas.

| Copula | Emp. Est. | Moody's | S&P | S&P (old) | Kiff | Basel Max | Basel Min |
|---|---|---|---|---|---|---|---|
| Gaussian | excluded | excluded | excluded | 28,535 | 43,336 | 32,047 | excluded |
| t (DoF=2) | 142,319 | 189,421 | 196,561 | 270,523 | 332,825 | 316,105 | 173,720 |
| t (DoF=8) | 14,918 | 34,196 | 36,247 | 81,121 | 108,748 | 95,007 | 24,254 |
| t (DoF=12) | excluded | 20,757 | 22,055 | 60,821 | 85,045 | 71,498 | 13,406* |

* Benchmark model

According to table 1.11, all VaR models that produced inconclusive results of Kupiec and Christoffersen tests are more conservative than our benchmark, as $\hat{L}'$ is always higher for these correlation assumptions.

**E) Average Quantile Loss**

Following Koenker and Bassett (1978) I also employ a different loss function, the predictive quantile loss which is based on quantile regression.

$$QL(VaR_t(\theta), x_{t,t+1}) = \begin{cases} \mid x_{t,t+1} - VaR_t(\theta) \mid (\theta) & if\ x_{t,t+1} < VaR_t(\theta) \\ \mid x_{t,t+1} - VaR_t(\theta) \mid (1-\theta) & if\ x_{t,t+1} \geq VaR_t(\theta) \end{cases} \quad (1.31)$$

The economic intuition behind the use of the QL function is that the capital forgone from overpredicting the true VaR should also be taken into account. This function is asymmetric in view of the fact that underestimation and overestimation have diverse consequences, as underprediction of risk might lead to liquidity problems and insolvency, and overprediction implies higher capital charges which reflect the opportunity cost of keeping a high reserve ratio. The best VaR method is the one that generates the lowest average quantile loss (AQL), defined as:

$$AQL = \frac{1}{T}\sum_{t=1}^{T} QL(VaR_t(\theta), x_{t,t+1}) \quad (1.32)$$

Table 1.12 presents the results for the AQL risk measure.

Table 1.12: Average Quantile Loss Function ($10^6$€)

Results obtained for the Average Quantile Loss Function for all correlation assumptions considering Gaussian and t copulas.

| **Copula** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min** |
|---|---|---|---|---|---|---|---|
| Gaussian | excluded | excluded | excluded | 1.63 | 2.05 | 1.75 | excluded |
| t (DoF=2) | 3.76 | 4.34 | 4.42 | 5.19 | 5.75 | 5.61 | 4.15 |
| t (DoF=8) | 1.21 | 1.80 | 1.86 | 2.82 | 3.28 | 3.06 | 1.51 |
| t (DoF=12) | excluded | 1.37 | 1.43 | 2.43 | 2.89 | 2.65 | 1.28* |

* Benchmark model

According to table 1.12, the minimum values for the average quantile loss function are obtained in the case of empirical estimator of correlation with *t* copula with 8 degrees of freedom and in the case of asset correlation given by Basel Accord minimum value and *t* copula with 12 degrees of freedom (the benchmark model).

Considering that our benchmark model satisfies both the unconditional coverage and the independence properties, which means that is an accurate VaR model according to Christoffersen (1998), but has a significantly higher average magnitude of exceedance of VaR estimates according to the loss function estimator and a higher average quantile loss than the VaR model considering asset correlation given by the empirical estimator and assuming *t* copula with 8 degrees of freedom, and that all the other VaR models are either rejected in those tests or over conservative, I conclude that the most accurate VaR model for this portfolio is based on asset correlation given by the empirical estimator and assuming *t* copula with 8 degrees of freedom.

## 1.5 Deterministic *versus* Stochastic Recovery Rate

In the previous sections I presented VaR estimates assuming a recovery rate given by Beta distribution sampling with parameters $\alpha$ equal to 8.09 and $\beta$ equal to 5.13, both estimated with historical information. In this section I present the results obtained assuming that the recovery rate is a constant proportion of the asset value and compare them with the results produced with stochastic recovery rate.

Considering the parameters $\alpha$ and $\beta$ estimated with historical information and the result in equation 1.16, the expected value of the recovery rate is 61%. I will assume that the recovery rate is constant and equal to this value in the simulation procedure.

I repeated the process of VaR estimation described in previous sections, for the same portfolio, time periods and correlation assumptions, assuming a deterministic instead of a stochastic recovery rate. Figures 1.5 to 1.8 present the comparison between all VaR estimates with stochastic and deterministic recovery rates. The analysis of the graphs suggests that VaR estimates considering deterministic recovery rates are very similar to those obtained with stochastic recovery rates.

Tables 1.13 and 1.14 present the results of Kupiec and Christoffersen tests. Regarding Kupiec test, the conclusions are exactly the same that we have previously obtained with stochastic recovery rate. The results obtained in Christoffersen test considering deterministic instead of stochastic recovery rate are different for the case of gaussian copula considering the empirical estimator of correlation and Basel Accord minimum value for asset correlation (the null hypothesis is now rejected for these correlation assumptions). Despite these differences, the conclusions derived from both tests remain unchanged and the benchmark model is also the model based on *t* copula with 12 de-

Figure 1.5: CVaR - Gaussian copula, Stochastic *vs* Deterministic Recovery Rate

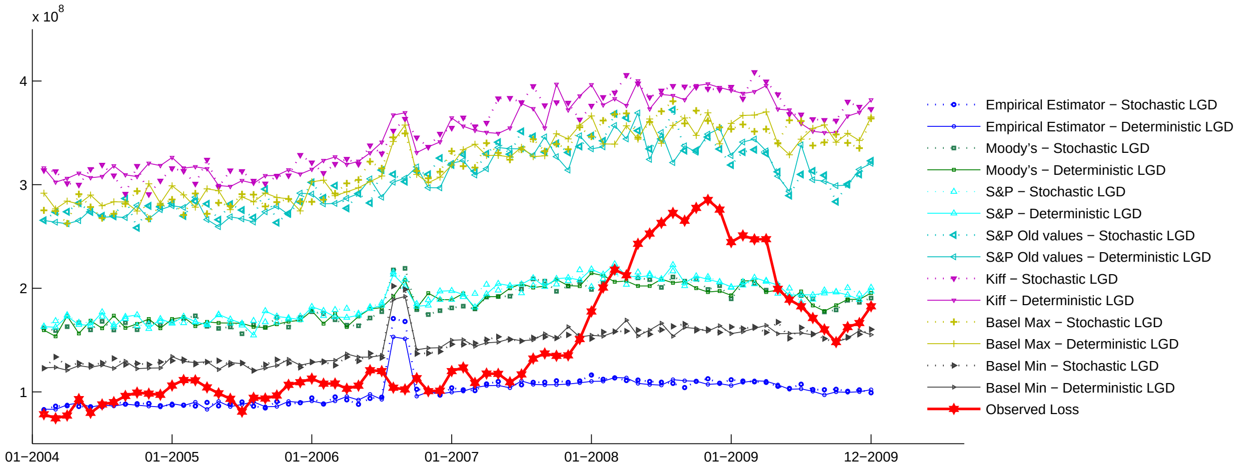
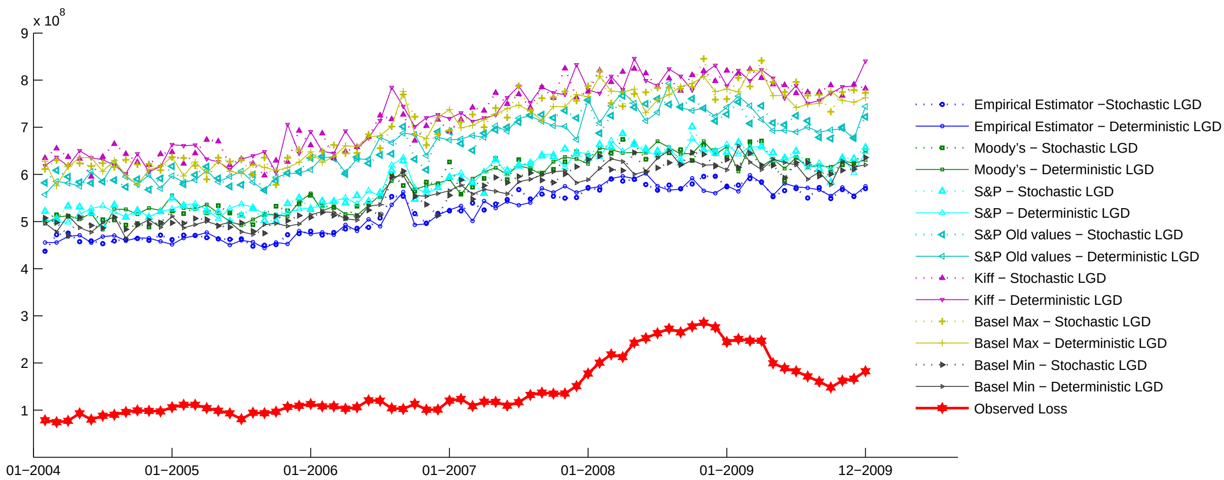



Figure 1.6: CVaR - *t* copula (DoF=2), Stochastic vs Deterministic Recovery Rate




grees of freedom and correlation given by Basel III Accord minimum value.

Tables 1.15, 1.16 and 1.17 present the loss function, over-conservativeness and average quantile loss estimates, respectively. The conclusions we derive from these measures also remain unchanged.

Considering that our benchmark model satisfies both the unconditional coverage and the independence properties, which means that is an accurate VaR model according to

Figure 1.7: CVaR - *t* copula (DoF=8), Stochastic vs Deterministic Recovery Rate

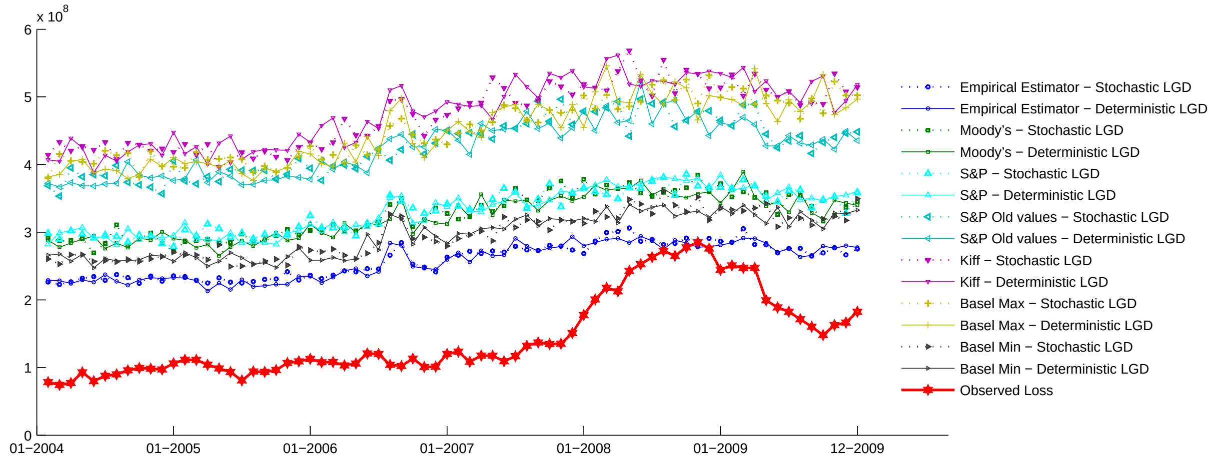
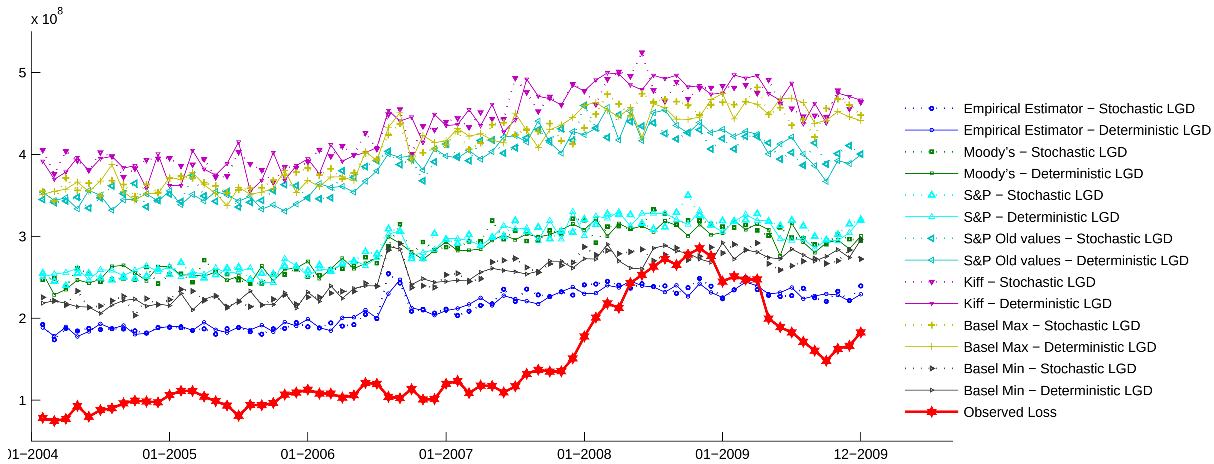



Figure 1.8: CVaR - *t* copula (DoF=12), Stochastic vs Deterministic Recovery Rate




Christoffersen (1998), but has a significantly higher average magnitude of exceedance of VaR estimates and a higher average quantile loss than the VaR model considering asset correlation given by the empirical estimator and assuming *t* copula with 8 degrees of freedom, and that all the other VaR models are either rejected in those tests or over conservative, I conclude that the most accurate VaR model for this portfolio considering deterministic recovery rate is based on asset correlation given by the empirical estimator

Table 1.13: Kupiec Test - Deterministic Recovery Rate

Results obtained for Kupiec test for all correlation assumptions considering Gaussian and t copulas and deterministic Recovery Rate.

| **Gaussian copula** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min** |
|---|---|---|---|---|---|---|---|
| # exceptions | 65 | 16 | 13 | 0 | 0 | 0 | 24 |
| # observations | 72 | 72 | 72 | 72 | 72 | 72 | 72 |
| Failure Rate(%) | 90 | 22 | 18 | 0 | 0 | 0 | 33 |
| $LR_{uc}$ | 553 | 72 | 53 | n.a | n.a | n.a | 130 |
| Critical Value | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 |
| Test Result | Reject $H_0$ | Reject $H_0$ | Reject $H_0$ | n.a | n.a | n.a | Reject $H_0$ |

| **t copula (DoF=2)** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min** |
|---|---|---|---|---|---|---|---|
| # exceptions | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| # observations | 72 | 72 | 72 | 72 | 72 | 72 | 72 |
| Failure Rate (%) | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $LR_{uc}$ | n.a | n.a | n.a | n.a | n.a | n.a | n.a |
| Critical Value | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 |
| Test Result | n.a | n.a | n.a | n.a | n.a | n.a | n.a |

| **t copula (DoF=8)** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min** |
|---|---|---|---|---|---|---|---|
| # exceptions | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| # observations | 72 | 72 | 72 | 72 | 72 | 72 | 72 |
| Failure Rate (%) | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| $LR_{uc}$ | 0 | n.a | n.a | n.a | n.a | n.a | n.a |
| Critical Value | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 |
| Test Result | Accept $H_0$ | n.a | n.a | n.a | n.a | n.a | n.a |

| **t copula (DoF=12)** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min** |
|---|---|---|---|---|---|---|---|
| # exceptions | 11 | 0 | 0 | 0 | 0 | 0 | 2 |
| # observations | 72 | 72 | 72 | 72 | 72 | 72 | 72 |
| Failure Rate (%) | 15 | 0 | 0 | 0 | 0 | 0 | 3 |
| $LR_{uc}$ | 41 | n.a | n.a | n.a | n.a | n.a | 2 |
| Critical Value | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 |
| Test Result | Reject $H_0$ | n.a | n.a | n.a | n.a | n.a | Accept $H_0$ |

and assuming *t* copula with 8 degrees of freedom.

The similarity of the conclusions regarding the accuracy of VaR estimates considering deterministic and stochastic recovery rate suggests that it is possible to save a significant amount of computation time with low impact on the final results by assuming deterministic recovery rate.

## Table 1.14: Christoffersen Test - Deterministic Recovery Rate

Results obtained for Christoffersen test for all correlation assumptions considering Gaussian and t copulas and deterministic Recovery Rate.

| **Gaussian copula** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min** |
|---|---|---|---|---|---|---|---|
| $\pi_0$ | 0.57 | 0.02 | 0.02 | 0.00 | 0.00 | 0.00 | 0.04 |
| $\pi_1$ | 0.95 | 0.94 | 0.92 | n.a | n.a | n.a | 0.96 |
| $LR_{ind}$ | 7.49 | 58.30 | 50.45 | n.a | n.a | n.a | 66 |
| Critical Value | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 |
| Test Result | Reject $H_0$ | Reject $H_0$ | Reject $H_0$ | n.a | n.a | n.a | Reject $H_0$ |

| **t copula (DoF=2)** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min** |
|---|---|---|---|---|---|---|---|
| $\pi_0$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $\pi_1$ | n.a | n.a | n.a | n.a | n.a | n.a | n.a |
| $LR_{ind}$ | n.a | n.a | n.a | n.a | n.a | n.a | n.a |
| Critical Value | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 |
| Test Result | n.a | n.a | n.a | n.a | n.a | n.a | n.a |

| **t copula (DoF=8)** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min** |
|---|---|---|---|---|---|---|---|
| $\pi_0$ | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $\pi_1$ | 0.00 | n.a | n.a | n.a | n.a | n.a | n.a |
| $LR_{ind}$ | n.a | n.a | n.a | n.a | n.a | n.a | n.a |
| Critical Value | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 |
| Test Result | n.a | n.a | n.a | n.a | n.a | n.a | n.a |

| **t copula (DoF=12)** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min** |
|---|---|---|---|---|---|---|---|
| $\pi_0$ | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 |
| $\pi_1$ | 0.91 | n.a | n.a | n.a | n.a | n.a | 0.50 |
| $LR_{ind}$ | 44.35 | n.a | n.a | n.a | n.a | n.a | 5.00 |
| Critical Value | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 | 6.6349 |
| Test Result | Reject $H_0$ | n.a | n.a | n.a | n.a | n.a | Accept $H_0$ |

## Table 1.15: Loss Function Estimator - Deterministic Recovery Rate($10^{12}$€)

Results obtained for the Loss Function Estimator for all correlation assumptions considering Gaussian and t copulas and deterministic Recovery Rate.

| **Copula** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min** |
|---|---|---|---|---|---|---|---|
| Gaussian | excluded | excluded | excluded | n.a | n.a | n.a | excluded |
| t (DoF=2) | n.a | n.a | n.a | n.a | n.a | n.a | n.a |
| t (DoF=8) | 56 | n.a | n.a | n.a | n.a | n.a | n.a |
| t (DoF=12) | excluded | n.a | n.a | n.a | n.a | n.a | 114 |

Table 1.16: Measure of over-conservativeness - Deterministic Recovery Rate($10^{12}$€)

Results obtained for the Measure of over conservativeness for all correlation assumptions considering Gaussian and t copulas and deterministic Recovery Rate.

| **Copula** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min** |
|---|---|---|---|---|---|---|---|
| Gaussian | excluded | excluded | excluded | 27,325 | 42,429 | 32,552 | excluded |
| t (DoF=2) | 142,231 | 190,314 | 195,790 | 271,191 | 334,248 | 309,256 | 172,122 |
| t (DoF=8) | 14,640 | 32,972 | 36,148 | 79,351 | 111,735 | 93,678 | 24,213 |
| t (DoF=12) | excluded | 20,383 | 21,857 | 60,004 | 84,577 | 71,187 | 13,232 |

Table 1.17: Average Quantile Loss Function - Deterministic Recovery Rate ($10^{6}$€)

Results obtained for the Average Quantile Loss Function for all correlation assumptions considering Gaussian and t copulas and deterministic Recovery Rate.

| **Copula** | **Emp. Est.** | **Moody's** | **S&P** | **S&P (old)** | **Kiff** | **Basel Max** | **Basel Min** |
|---|---|---|---|---|---|---|---|
| Gaussian | excluded | excluded | excluded | 1.60 | 2.02 | 1.76 | excluded |
| t (DoF=2) | 3.75 | 4.35 | 4.41 | 5.19 | 5.76 | 5.54 | 4.13 |
| t (DoF=8) | 1.22 | 1.77 | 1.86 | 2.79 | 3.32 | 3.04 | 1.50 |
| t (DoF=12) | excluded | 1.36 | 1.42 | 2.42 | 2.88 | 2.64 | 1.33 |

# Chapter 2

# VaR in Credit Default Swaps

## 2.1 Literature Review

In recent years there have been several studies that propose different methods of calculating VaR. Most of these studies focus on the calculation of VaR for stock returns. The methods that have received most attention fall into four categories: nonparametric, parametric, semi-parametric and hybrid. Among the parametric models, there is the GARCH model proposed by Bollerslev (1986) and all the variants that have been introduced, such as EGARCH proposed by Nelson (1991) and GJR model of Glosten *et al* (1993). Concerning the semi-parametric models, Engle and Manganelli (2004) propose a new approach to VaR estimation, the Conditional Autoregressive model (CAViaR), which includes $VaR_{t-1}$ as an explanatory variable. For a semi-parametric method, Danielsson and De Vries (2000) propose the use of Extreme Value Theory for calculating VaR. Regarding the non parametric models, the most widely used are Historical Simulation and Filtered Historical Simulation. McNeil and Frey (2000) analyze a hybrid model which combines GARCH with Extreme Value Theory.

The diversity of methods available to estimate VaR led to a wide range of studies that provide empirical evidence of the performance of VaR approaches. Kuester *et al* (2006) compare the out of sample performance of several methods of predicting univariate VaR for the NASDAQ index and find that the hybrid method, combining a heavy-tailed GARCH filter with an Extreme Value Theory-based approach, performs best overall, closely followed by a variant on a Filtered Historical Simulation. They also find that none of the CAViaR models tested performs adequately in all tests at any quantile level, showing poor out of sample performance. Regarding the Mixture models, they conclude that some of these models perform worse in smaller window sizes due presumably to their rather large parameterizations and, for this reason, they are generally outperformed by Filtered Historical Simulation and Extreme Value Theory.

Sener *et al* (2012) test and rank twelve different popular VaR methods on the equity indices of eleven emerging and seven developed markets, covering the period of the recent sub-prime mortgage crisis, and find that asymmetric methods, such as EGARCH, generate the best performing VaR forecasts and the methods based in Extreme Value Theory (EVT) approaches perform the worst. Rubia and Sanchis-Marco (2013) consider the return-restricted CAViaR models originally proposed by Engle and Manganelli

(2004) as well as alternative models such as EVT and GARCH and use different variables which are related to measures of trading activity and liquidity. They find that Quantile Regression-based risk models that account for the volatility and other market conditions outperform the other approaches.

Rossignolo *et al* (2012) performs a comparative study of VaR models based on data from emerging and frontier markets and find that Historical Simulation and Filtered Historical Simulation are inaccurate, conditional models represent an improvement, both for GARCH and EGARCH techniques, and heavy tailed distributions, particularly EVT, reveal as the most accurate technique to model market risk. A study performed by Halbleib and Pohlmeier (2012), in which they propose the combination of different VaR approaches to provide robust and precise VaR forecasts and provide empirical evidence of the performance of a wide range of standard VaR approaches, supports the results of Rossignolo *et al* (2012), as they also find that the best VaR approaches are based on EVT.

The forecast of VaR in CDS requires, first, the estimation of the price of CDS in the specific quantile of interest and then the calculation of VaR using a mark-to-market technique. The key issue is the estimation of the price of the CDS in the tail of the price distribution. For this purpose, the methods typically used to calculate VaR for stock returns are applicable.

Considering the overall performance of Filtered Historical Simulation reported by Kuester *et al* (2006) and the empirical evidence of the performance of Extreme Value Theory reported in several of the recent studies presented above, I consider that these are promising methods of estimating VaR and thus I will test them in this study. Following Kuester *et al* (2006), I will also estimate VaR considering the Historical Simulation method and GARCH models. In the analysis of Kuester *et al* (2006), the Mixture models present poor out of sample performance in small samples, which are the situations of most interest in the evaluation of CDS due to their recent history, and for this reason these models are not considered in this study.

There is a growing interest in employing Quantile Regression (QR) in the finance literature. QR was first applied to VaR estimation by Taylor (1999) and Chernozhukov and Umantsev (2000). Taylor (1999) applies QR to estimate multiperiod VaR in the context of exchange rates and compare this new approach with the traditional methods which first estimate the volatility and then assume a probability distribution. Chernozhukov and Umantsev (2000) use QR also to model VaR without, however, examining the performance of this method.

Engle and Manganelli (2004) propose a particular specification for QR, called CAViaR. Instead of modeling the whole distribution of returns, this model allows to concentrate in the quantile directly and specify the evolution of the quantile over time using a special type of autoregressive process. Chen *et al* (2012) propose a new family of VaR models based on QR and find that these models consistently ranked best for VaR forecasting, comparing to the classical approaches based on GARCH models.

Gebka and Wohar (2013) analyze the causality between past trading volumes and index returns in the Pacific Basin countries and find that the QR method reveals strong nonlinear causality even though this relation was not detected by OLS regression. Lee and Li (2012) employ a QR approach and show that the effect of diversification on firm performance is not homogeneous across various quantile levels. Baur (2013) proposes an alternative framework to decompose the dependence using QR and demonstrates that this methodology provides a detailed picture of dependence including asymmetric and non-linear relationships. Allen *et al* (2012) apply QR to measure extreme risk of various European industrial sectors both prior to and during the recent financial crisis and find a highly significant difference in the distance to default between quantiles 50% and 95%.

Pires *et al* (2011) apply QR approach in order to model the distribution of CDS spreads and, through the use of mark-to-market techniques, calculate VaR. Inspired by the results of Pires *et al* (2011), in this study I also employ QR to identify the determinants of CDS spreads in specific quantiles of interest, in order to calculate VaR. However, I extend the results of Pires *et al* (2011) by augmenting the sample to include the recent credit crisis and by testing additional explanatory variables.

The use of QR in the context of forecasting VaR is typically associated with different specifications of the CAViaR model. However, in this study I use QR to identify the determinants of the price of CDS and then calculate VaR, using mark-to-market techniques. I compare the performance of the VaR model based on this specification of the QR with the results obtained with Historical Simulation, Filtered Historical Simulation, GARCH-based models and EVT, through the application of backtesting methodologies. To the best of my knowledge this is the first time that backtesting methodologies are applied to compare different methods of estimating VaR in CDS.

A wide range of macroeconomic and microeconomic factors have been analyzed in the context of determinants of CDS spreads. Das *et al* (2009) compare the explanatory power of market-based and accounting-based models of CDS spreads and find that models including both accounting and market information perform better than separate models. Ericsson *et al* (2009) conclude that variables such as volatility and leverage, which are theoretically implied variables, explain a significant proportion of CDS variations. Zhang *et al* (2005) analyze the impact of equity returns and volatility of the reference entity on the CDS premium and find that CDS spreads can be largely explained by intra-day refined measures of historical volatility and jump probability. Byström (2005) shows that CDS spreads are negatively correlated with stock prices and positively correlated with stock price volatility. Morkoetter *et al* (2012) show that counterparty default risk measures have a negative impact on CDS spreads. Regarding the measure of counterparty default risk, Campbell *et al* (2008) find that financial ratios such as profitability, leverage and liquidity play an important role in explaining the determinants of bankruptcy and failure. Following these studies, I use market-based and accounting-based factors as determinants of CDS spreads, namely stock returns and stock price volatility and also financial ratios such as leverage, return on assets and liquidity.

Tang and Yan (2007) provide evidence that liquidity risk and liquidity level explain a significant proportion of CDS spread variation. CDS liquidity, measured by absolute bid ask spread, is introduced by Bongaerts *et al* (2011) and Pires *et al* (2011) as an explanatory variable of CDS premiums. Pires *et al* (2011) find that CDS spreads significantly increase with absolute bid ask spreads across all conditional quantiles of the CDS distribution. Based on these results, in this study I use absolute bid ask spread as a measure of CDS market liquidity.

## 2.2 CDS Price

### 2.2.1 Estimation Techniques

Banks and regulators are primarily interested in the aggregate VaR across trading activities, raising the question of whether to start by aggregating the data and then apply a univariate VaR model or to start with disaggregate data and then apply multivariate structural portfolio VaR model. In evaluating the portfolio VaR, the multivariate model can have some advantages over the univariate model. According to Bauwens *et al* (2006), one of these advantages is that once we get the covariance matrix by the multivariate approach, we do not need to calculate again the covariance matrix even if the weights of each asset are changed; under the univariate model, we should evaluate the variance of portfolio again whenever the weights of each asset are changed. Another advantage is that a multivariate model may improve the evaluation performance in updating the variances and correlations by considering the individual characteristics of the portfolios components and estimating their linear comovement. However, Berkowitz and O'Brien (2002) show that the aggregation and modeling problems involved in the multivariate approach may lead to poor forecasting accuracy and the simple univariate model can even outperform these complicated structural models. Considering that univariate models are a useful complement of the more complex structural models and may even outperform these models and be sufficient for forecasting portfolio VaR, I restrict attention to the univariate case.

**A) Quantile Regression**

The sensitivities to empirical determinants of CDS spreads may change according to the level of CDS spread itself. Given this, a simple conditional mean regression may not be appropriate to completely describe CDS spreads and in this case a more flexible framework is required, for example the Quantile Regression.

The Quantile Regression was introduced by Koenker and Bassett (1978) and is an extension of the conditional mean regression to a collection of models for different conditional quantile functions.

The $\theta$-th regression quantile is any solution to the Quantile Regression minimization problem:

$$\min_{\beta}\left[\sum_{t|y_t\geq x_t\beta}\theta|y_t-x_t\beta|+\sum_{t|y_t<x_t\beta}(1-\theta)|y_t-x_t\beta|\right] \tag{2.1}$$

where $x_t$ is a row vector of explanatory variables with first element equal to 1 and $\beta$ is a vector of parameters. The usual procedure for building an explanatory model for a variable is to look for a relationship between past observations of that variable and past observations of potential explanatory variables. This is not a feasible procedure for building a model for the quantiles of a variable because past observations of the quantiles are not available (they are unobservable). The attraction of Quantile Regression is that past observations of the quantiles are not required because the variable itself is regressed on explanatory variables to produce a model for the quantile.

An interesting aspect regarding Quantile Regression is the mitigation of some typical empirical problems, such as the presence of outliers, heterogeneity and non-normal errors. The Quantile Regression results are robust to heavy tailed distributions while the standard regression estimators are sensitive to departures from the normality assumption; the Quantile Regression results are invariant to outliers of the dependent variable that tend to $\pm\infty$ according to Coad and Rao (2006), while the standard regression estimators are highly sensitive to outliers; finally, the Quantile Regression approach avoids the assumption that the error terms are identically distributed at all points of the conditional distribution, allowing to acknowledge firm heterogeneity and admit the possibility that slope parameters vary at different quantiles of the conditional distribution of the dependent variable.

**B) GARCH Models**

The data on which the variances of the error terms are not equal, meaning that the error terms may reasonably be expected to be larger for some data points than for others, is said to suffer from conditional heteroskedasticity. ARCH and GARCH models treat heteroskedasticity as a variance to be modeled and as a result a prediction is computed for the variance of each error term.

In some cases, the key issue is the variance of the error terms and this question often arises in financial applications where the variance of the dependent variable (for example returns) represents the risk level of interest. Financial data suggests that some time periods are riskier than others and these riskier times are not scattered randomly across monthly or quarterly data. Instead, there is a degree of autocorrelation in the riskiness of some financial applications. These types of issues are handled by ARCH and GARCH models. For more details on GARCH models, please see Bollerslev (1986).

In this study I employ three types of GARCH models, the general GARCH, the EGARCH and GJR combined with two distributions of innovations processes, namely Normal and Student's $t$.

i) GARCH(P,Q)

The general GARCH(P,Q) model for the conditional variance of innovations is

$$\sigma_t^2 = k + \sum_{i=1}^{P} G_i \sigma_{t-1}^2 + \sum_{j=1}^{Q} A_j \epsilon_{t-j}^2 \tag{2.2}$$

with constraints

$$\sum_{i=1}^{P} G_i + \sum_{j=1}^{Q} A_j < 1$$
$$k > 0$$
$$G_i \geq 0$$
$$A_j \geq 0$$

The basic GARCH(P,Q) model is a symmetric conditional variance process as it ignores the sign of the disturbance.

ii) EGARCH(P,Q)

The general EGARCH(P,Q) model for the conditional variance of innovations, with leverage terms and an explicit probability distribution assumption is

$$log\ \sigma_t^2 = k + \sum_{i=1}^{P} G_i\ log\ \sigma_{t-1}^2 + \sum_{j=1}^{Q} A_j \left[ \frac{|\epsilon_{t-j}|}{\sigma_{t-j}} - E\left\{ \frac{|\epsilon_{t-j}|}{\sigma_{t-j}} \right\} \right] + \sum_{j=1}^{Q} L_j \left( \frac{\epsilon_{t-j}}{\sigma_{t-j}} \right) \quad (2.3)$$

where

$$E\{|z_{t-j}|\} = E\left\{ \frac{|\epsilon_{t-j}|}{\sigma_{t-j}} \right\} = \sqrt{\frac{2}{\pi}}$$

for the normal distribution, and

$$E\{|z_{t-j}|\} = E\left\{ \frac{|\epsilon_{t-j}|}{\sigma_{t-j}} \right\} = \sqrt{\frac{\nu - 2}{\pi}} \frac{\Gamma(\frac{\nu-1}{2})}{\Gamma(\frac{\nu}{2})}$$

for the Student's $t$ distribution with degrees of freedom $\nu > 2$.

iii) GJR(P,Q)

The general GJR(P,Q) model for the conditional variance of innovations with leverage terms is

$$\sigma_t^2 = k + \sum_{i=1}^{P} G_i \sigma_{t-1}^2 + \sum_{j=1}^{Q} A_j \epsilon_{t-j}^2 + \sum_{j=1}^{Q} L_j S_{t-j} \epsilon_{t-j}^2 \quad (2.4)$$

where $S_{t-j} = 1$ if $\epsilon_{t-j} < 0$ and $S_{t-j} = 0$ otherwise, with constraints

$$\begin{aligned} &\textstyle\sum_{i=1}^{P} G_i + \sum_{j=1}^{Q} A_j + \frac{1}{2} \sum_{j=1}^{Q} L_j < 1 \\ &k \geq 0 \\ &G_i \geq 0 \\ &A_j \geq 0 \\ &A_j + L_j \geq 0 \end{aligned}$$

For GARCH(P,Q) and GJR(P,Q) models, the lag lengths P and Q and the magnitudes of the coefficients $G_i$ and $A_j$ determine the extent to which disturbances persist. In the case of EGARCH models, the persistence is captured by terms $G_i$.

I compute the $\theta$-quantile estimate of the distribution of interest by first fitting a GARCH model to the first order differences of CDS price. Following Engle and Manganelli (1999), the one-period-ahead VaR estimate is calculated with following equation:

$$VaR_{t+1}(\theta) = \hat{\mu}_{t+1} + \hat{\sigma}_{t+1}Q_\theta(\hat{z}) \tag{2.5}$$

where $\hat{\mu}_{t+1}$ and $\hat{\sigma}_{t+1}$ are the estimated conditional mean and conditional standard deviation for $t + 1$, respectively, and $Q_\theta(\hat{z})$ is the empirical $\theta$-quantile of the standardized residuals. In this study, I assume that the expected change in the CDS spread is zero. In terms of equation 2.5, this corresponds to $\hat{\mu}_{t+1} = 0$.

This estimation is a mix of a GARCH and a Historical Simulation applied to the standardized residuals. Thus, whether the error distribution is conditional normal or follows other distribution, it is straightforward to compute the one-step-ahead $\theta$-quantile forecast, since under all distributions we can compute the corresponding quantiles which we then multiply by our conditional standard deviation forecast.

In order to estimate VaR for period $t + h$, I apply a simple scaling rule. Assuming that $\hat{\mu}_{t+h} = 0$, this corresponds to multiplying the one-period-ahead estimated standard deviation by the square root of the number of periods ahead of the forecast ($h$), according to the following equation:

$$VaR_{t+h}(\theta) = \sqrt{h}\hat{\sigma}_{t+1}Q_\theta(\hat{z}) \tag{2.6}$$

**C) Historical Simulation**

The simplest way to estimate the $\theta$-quantile of a distribution is to use the sample quantile estimate based on historical data, which is referred to as Historical Simulation. For Historical Simulation (HS), the $\theta$-quantile estimate for $t+1$ is given by the empirical $\theta$-quantile, $Q_\theta$, of a moving window of $w$ observations up to time $t$.

Despite being a popular way to estimate the $\theta$-quantile of a particular distribution, Historical Simulation has some major flaws. First, this method ignores the possible non-iid nature of the data. Second, the length of the window one chooses must satisfy two contradictory properties: it must be large enough in order to make statistical inference significant and it must not be too large to avoid the risk of taking observations outside of the current volatility cluster. Finally, when the market moves from a period of relatively low volatility to a period of relatively high volatility (or vice versa), $\theta$-quantile estimates based on Historical Simulation will be biased downward (upward).

A variant of the Historical Simulation method presented above is the Filtered Historical Simulation (FHS), which has shown very good results in the study performed by Kuester *et al* (2006). For FHS, a GARCH model is used to prefilter the data and the nonparametric nature of Historical Simulation is retained by bootstrapping (sampling with replacement) from the standardized residuals. These bootstrapped standardized residuals are then used to generate time paths of future CDS prices. One of the appealing features of FHS is its ability to generate relatively large deviations not found in the original time series.

**D) Extreme Value Theory**

Extreme value theory (EVT) focuses on the tails of the distribution of interest. Following Diebold *et al* (1998), in this study I fit a time-varying model to the data and then estimate the tail of the standardized residuals by an EVT model, using the limit result for peaks over threshold (POT). This process first extracts the filtered residuals from each series with a GARCH model, then constructs the sample marginal cumulative distribution function of each asset using a Gaussian kernel estimate for the interior and a generalized Pareto distribution estimate for the tails.

McNeil and Frey (2000) combine an AR(1)-GARCH(1,1) process, assuming normal innovations, with the POT method. The filter with normal innovations is capable of removing the majority of clusterings and for this reason I also assume normal innovations in this study.

The one-day-ahead VaR estimate is given by equation 2.5, where $\hat{\mu}_{t+1}$ and $\hat{\sigma}_{t+1}$ are the estimated conditional mean and conditional standard deviation for $t+1$, respectively, obtained from a AR(1)-GARCH(1,1) process. Moreover, $Q_\theta(\hat{z})$ is the $\theta$-quantile estimate of the standardized residuals, obtained with the POT method. The estimation of VaR for period $t+h$ is given by equation 2.6. I refer to Embrechts *et al* (1997) for more details on EVT.

### 2.2.2 VaR Estimation

The value at risk measure places an upper bound on losses in the sense that these will exceed the VaR threshold with a small target probability, typically chosen between $1\%$ and $5\%$. Conditional on the information given up to time *t*, the VaR for period *t+h* at the confidence level $\theta$ is given by:

$$VaR_{t+h}^{\theta} := Q_{\theta}(L|X_t) = inf\{l \in \mathbb{R} : P(L > l|X_t) \leq 1 - \theta\}, 0 < \theta < 1 \tag{2.7}$$

where $Q_{\theta}(.)$ denotes que quantile function, $L$ is the loss in period $t$ and $X_t$ represents the information available at date $t$. For implementing VaR based measures, one seeks a precise quantile estimate relatively far out in the right tail of the loss distribution for some specified future date.

The estimation methods described in the previous section are used to estimate the price of a CDS in time period $t$, with a certain confidence level, meaning that the output of these estimation methods will be a price for the CDS. The price evolution of CDS allows us to understand the evolution of the risk of the underlying entity and compare the predicted price with the price observed in the market. Thus, by examining the price of CDS over time we can compare different estimation methods and analyze their accuracy, but the price of CDS alone does not provide a loss distribution and a Value at Risk. To transform the distribution of CDS spreads into a distribution of losses I use the reduced-form model described in O'Kane and Turnbull (2003). This transformation allows us to convert basis points into monetary values, thus allowing the calculation of Value at Risk.

Following O'Kane and Turnbull (2003), unlike bonds the gain or loss from a CDS position cannot be computed simply by taking the difference between current market quoted price plus the coupons received and the purchase price. To value a CDS we need to use a term structure of default swap spreads, a recovery rate assumption and a model.

The present value of a position initially traded at time $t_0$ at a contractual spread of $S(t_o, t_N)$ with maturity $t_N$ and which has been offset at valuation time $t_V$ with a position traded at a spread of $S(t_V, t_N)$ is given by:

$$MTM(S(t_o, t_N), \lambda) = \pm[S(t_V, t_N) - S(t_o, t_N)] \times RPV01(t_V, t_N) \tag{2.8}$$

where

$$RPV01 = \sum_{n=1}^{N} Z(t_n) \cdot \Delta t_n \Big[ Q(t_n) + \big( Q(t_{n-1}) - Q(t_n) \big) \cdot \frac{1_{pa}}{2} \Big] \tag{2.9}$$

denotes the Risky present value of 1 bp paid on the premium leg, the indicator $1_{pa}$ equals 1 if the contract specifies premium accrued and 0 otherwise, $\Delta t_n$ is the number of years between payment dates, $Z(t_n)$ is the risk-free discount factor for $t_n$ ($e^{-rt_n}$), $Q(t_n) = e^{-\lambda t_n}$ is the probability of survival until $t_n$ and $\lambda$ is the hazard rate. The positive sign is used for the protection buyer and the negative sign for the protection seller.

Following Pires *et al* (2011), I assume a flat recovery rate (R) of $40\%$ and a flat interest rate (r) of $5\%$. The approximate break-even flat hazard rate is computed as:

$$\lambda = \frac{S}{(1-R)}$$

Let $S_{i,t}$ denote the CDS spread for entity $i$ at time t and $\lambda^{\theta} = \frac{F^{-1}_{S_{i,t}}(\theta|X_{i,t})}{(1-R)}$ denote the hazard rate for the estimated CDS price at a given quantile $\theta$. The value at risk is computed as:

$$VaR_{\theta}(S_{i,t}|X_{i,t}) = MTM(S_{i,t}, \lambda^{\theta})$$

The Value at Risk represents the change in value of a contract initially negotiated at price $S_{i,t}$ due to a change in the hazard rate to $\lambda^{\theta}$. This new hazard rate represents a new CDS spread at a given quantile $\theta$. Different methods of CDS price estimation will lead to potentially different price estimates at a given quantile $\theta$, *i.e.*, each estimation technique will be associated with a specific $\lambda_{\theta}$ for the time $t$ and firm $i$. Assuming that I am a protection seller, I am interested in the risk of the CDS spread increasing. Therefore, I will focus on the forecast of upper quantiles of the price of CDS, namely quantile 99.

## 2.3 Backtesting VaR

The Value at Risk represents the change in value of a contract initially negotiated at price $S_{i,t}$ due to a change in the hazard rate to $\lambda^{\theta}$. However, the real change in the hazard rate observed in the market in a specific time interval might be higher or lower, meaning that the real mark-to-market might be higher or lower than predicted by the VaR model. For this reason, the backtesting will be performed by comparing the estimated VaR with the real mark-to-market considering the implicit hazard rate of the maximum price that the CDS reached in the following 1, 3, 10 and 30 days, allowing us to check the accuracy of VaR estimates with a time horizon of 1, 3, 10 and 30 days.

The backtesting of VaR methods in CDS will be performed based on the tests presented in chapter 1 and two additional tests, namely Conditional Tail Expectation and Dynamic Quantile Test, which I present in the following.

**A) Conditional Tail Expectation**

Artzner *et al* (1999) present Conditional Tail Expectation as a measure of risk defined by:

$$CTE_{\theta}(X) = \mathbb{E}[x_{t,t+1}|x_{t,t+1} > VaR_t(\theta)] \tag{2.10}$$

The CTE measure should be interpreted carefully and should be examined in conjunction with other methods of backtesting, such as the Average Quantile Loss, because the value of the loss observed when there is a violation of VaR gives an idea of the severity of the loss but gives no indication of the closeness between the loss and the VaR estimate, *i.e.*, there may be a violation of VaR and loss observed may be extremely high, and yet, the estimated VaR is very close to the observed loss.

**B) Dynamic Quantile Test**

Engle and Manganelli (2004) propose a test based on the regression of $I_t$ on a regressor matrix X that contains lagged hits, $I_{t-1}$,...,$I_{t-p}$, for example:

$$I_t = \theta_0 + \sum_{i=1}^{p} \beta_i I_{t-i} + \mu_t, \quad (2.11)$$

where, under the null hypothesis, $\theta_0 = \theta$ and $\beta_i = 0, i = 1, ...p$. In vector notation, we have:

$$I - \theta\iota = X\beta + \mu \quad (2.12)$$

where $\iota$ is a vector of ones.

Invoking the central limit theorem yields

$$\hat{\beta}_{LS} = (X'X)^{-1}X'(I - \theta\iota) \overset{asy}{\sim} N\big(0, (X'X)^{-1}\theta(\mathbf{1} - \theta)\big) \quad (2.13)$$

The DQ test consists in testing some linear restrictions in a linear model that links the violations to a set of explanatory variables. Tests such as the proposed by Christoffersen (1998) can detect the presence of serial correlation in the sequence of indicator functions but this is only a necessary but not sufficient condition to assess the performance of a quantile model because, in some situations, the unconditional probabilities of exceeding the quantile are correct and serially uncorrelated but the conditional probabilities given the quantile are not. The tests presented above have no power against this form of inefficiency.

Considering that the empirical application in this study covers CDS with short history and in order to minimize the loss of information,the regressor matrix X contains the constant and two lagged hits.

## 2.4 Empirical Analysis

### 2.4.1 Data Sample

I use the Bloomberg Financial Services database to obtain all the names that belonged to any of the first 16 series of two important CDS indexes for the US market: the CDX North America Investment Grade and the CDX North America High Yield. I restrict the sample to public traded firms by keeping only the names for which I am able to find a matching CUSIP in the CRSP and COMPUSTAT databases. I am left with 242 different names. For these names, I collect daily market data (CDS bid and ask quotes, stock price, and market capitalization) and quarterly accounting data (total assets, total liabilities, total equity, cash holdings, and net income). The sample is from Sep/2001 to Apr/2011.

The original database comprises 330,852 daily observations, corresponding to 242 CDS names. Observations with missing information regarding the bid or ask price of CDS, the stock market variables or accounting data are deleted (25,409 records). Only the daily records with missing information are removed, rather than all daily records relating to Names for which at some point in time there was missing information. The records with insufficient information for calculating the historical volatility are also removed (1,162 daily records). For this purpose, it is considered that it takes at least five days of information in order to calculate the volatility associated with a particular reference entity. The analysis of variable bid ask spread reveals extreme values that indicate unusual events, with potential significant impact on statistical estimation. For this reason, the records in the tails of the distribution of variable bid ask spread are removed, namely the records below percentile 1 and above the percentile 99 (6,109 observations). The final database consists of 298,172 records relating to 227 CDS names. The average number of CDS names per month is presented in Figure 2.1.

The estimation of Quantile Regression is based on the entire database. The estimation of GARCH models, Historical Simulation, Filtered Historical Simulation and Extreme Value Theory is based on CDS names chosen from the available 227 reference entities. In order to have a good control group with a long history I choose 5 CDS names with the largest number of observations (between 2228 and 2181 observations). Additionally, in order to test the adequacy of QR for estimating VaR in products with short history, I also select 5 CDS names with the smallest number of observations (between 65 to 223 observations). None of these CDS names defaulted in the sample period. Table 2.1 presents some relevant summary statistics for these CDS names.

Figure 2.1: Average number of CDS names per month

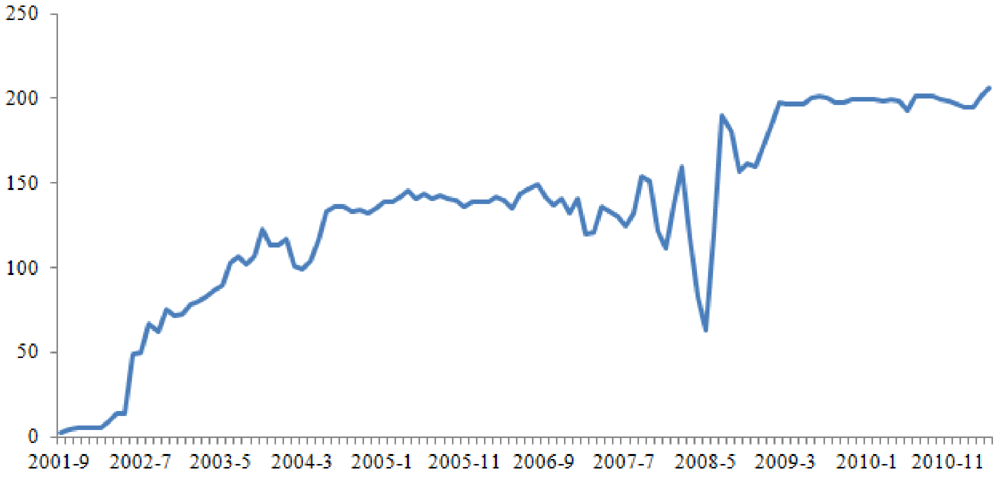



Table 2.1: Summary Statistics - CDS names

Summary statistics of the ten CDS names used in the empirical analysis, for the period from September 2001 to April 2011.

| Id | Company Name | Start Date | End Date | Obs | Mean | Std.Dev. | Skew. | Kurt. | Min | Max |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Int. Bus. Machines C. | 16-10-2001 | 15-04-2011 | 2288 | 35 | 21 | 1,4 | 5,5 | 6 | 136 |
| 2 | Wal-Mart Stores Inc | 16-10-2001 | 15-04-2011 | 2228 | 29 | 23 | 1,8 | 6,2 | 6 | 133 |
| 3 | Dow Chemical Co | 26-03-2002 | 15-04-2011 | 2184 | 96 | 104 | 2,9 | 13,3 | 14 | 675 |
| 4 | Int. Paper Co | 26-03-2002 | 15-04-2011 | 2183 | 122 | 125 | 3,1 | 13,9 | 32 | 851 |
| 5 | Macy's Inc | 26-03-2002 | 15-04-2011 | 2181 | 145 | 162 | 2,5 | 10,0 | 27 | 1037 |
| 6 | Rite Aid C. | 30-06-2006 | 15-04-2011 | 223 | 1019 | 577 | 1,5 | 6,5 | 383 | 3757 |
| 7 | KB Home | 14-07-2006 | 13-07-2009 | 91 | 402 | 158 | -0,4 | 1,9 | 131 | 701 |
| 8 | Forest Oil C. | 06-02-2008 | 15-04-2011 | 74 | 380 | 84 | 0,3 | 1,9 | 238 | 525 |
| 9 | Amkor Technology I. | 06-02-2008 | 15-04-2011 | 67 | 574 | 138 | 1,0 | 4,0 | 394 | 998 |
| 10 | AES Corp. | 07-02-2008 | 15-04-2011 | 65 | 378 | 96 | 3,0 | 10,7 | 326 | 767 |

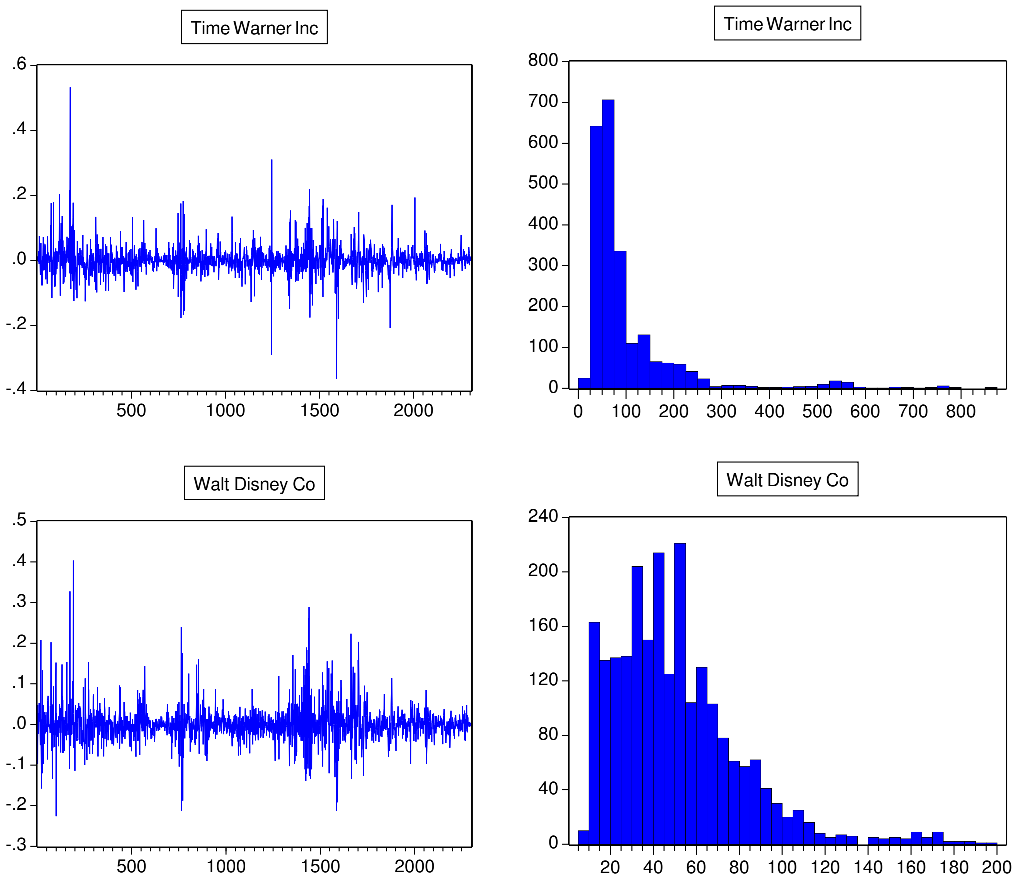
The financial time series usually exhibit special statistical properties, namely volatility clustering, significant kurtosis and some type of skewness. See for example the data in table 2.1 and figure 2.2. As a consequence, methods based on the assumptions of independent and identically distributed observations and normal distribution tend not to suffice and for this reason it is necessary to apply alternative strategies to predict VaR. The methodologies applied in this study take these characteristics into consideration.

### 2.4.2 Variables

#### CDS price

The dependent variable, CDS $price_{i,t}$, is the midpoint between the bid and ask quotes

Figure 2.2: Walt Disney Co and Time Warner CDS Price



for firm $i$ and period $t$.

**Stock return**

Positive equity returns increase the value of equity and therefore diminish the leverage of the firm. Accordingly, it is expected that the CDS quote is negatively impacted by equity returns.

**Volatility of stock returns**

Higher firm-specific equity volatility indicates a higher probability that the firm's value will cross the threshold of default, hence increasing CDS quotes. In this study historical volatility is computed as:

$$Historical\ volatility_t = \sqrt{252 \times \frac{1}{n-1} \times \sum_{i=t-31}^{t-1} r_i^2}, \tag{2.14}$$

Where $r_i$ is the daily stock return.

**Bid ask spread**

The standard measure of liquidity for stocks and bonds is the relative bid ask spread. However, contrary to stock prices, CDS premiums are expressed in comparable units: basis points per annum of the notional amount of the contract. Pires *et al* (2011) provide examples that give intuition that dividing the CDS bid ask spread by the CDS mid quote can bias the comparison of liquidity between different reference entities. Based on this reasoning, absolute rather than relative bid ask spread is used as independent variable.

**Leverage**

According to Merton (1974), a firm defaults if the value of its assets falls below the value of its debt and, hence, the leverage ratio is crucial for determining the distance to default. In other words, an increase in leverage results in an increased probability of default and consequently in an increase of the CDS premiums. Following Campbell *et al* (2008), the leverage ratio of the reference entity is computed considering the book value and also the market value of equity. The ratio is defined as:

$$Leverage_{Market}=\frac{Total\ Book\ Liabilities}{Total\ Market\ Equity}$$

$$Leverage_{Book}=\frac{Total\ Book\ Liabilities}{Total\ Book\ Equity}$$

Whereas total liabilities and total equity are book values being quoted on a quarterly basis, total market equity is defined as the product of the last equity price and the number of shares outstanding at the end of day $t$.

**Return on assets**

Following Campbell *et al* (2008), a standard measure of profitability is constructed: net income relative to total assets. The profitability ratio of the reference entity is computed considering total assets at book value and also considering the equity component of total assets at market value and adding the book value of liabilities:

$$Return\ on\ assets_{Market}=\frac{Net\ Income}{Total\ Market\ Equity+Total\ Book\ Liabilities}$$

$$Return\ on\ assets_{Book}=\frac{Net\ Income}{Total\ Book\ Assets}$$

**Liquidity**

Following Campbell *et al* (2008), liquidity is measured as the ratio of a company's cash and short term assets to its total assets. The ratio is computed considering total assets at book value and also considering the equity component of total assets at market value and adding the book value of liabilities:

$$Liquidity_{Market}=\frac{(Cash+Near\ Cash\ Item)}{Total\ Market\ Equity+Total\ Book\ Liabilities}$$

$$Liquidity_{Book}=\frac{(Cash+Near\ Cash\ Item)}{Total\ Book\ Assets}$$

Table 2.2 presents some relevant summary statistics of the variables considered in the study.

Table 2.2: Summary Statistics - Variables

Summary statistics of the variables available in the empirical analysis considering all CDS names, for the period from September 2001 to April 2011.

| **Variable** | **Obs** | **Mean** | **Std. Dev.** | **Min** | **Max** |
|---|---|---|---|---|---|
| $Bid\ ask\ spread$ | 298,172 | 10.633 | 11.709 | 2.321 | 94.98 |
| $CDS\ Price$ | 298,172 | 154.735 | 243.443 | 5.892 | 60,761.59 |
| $Leverage_{Book}$ | 298,172 | 3.694 | 21.027 | -259.245 | 2,082.333 |
| $Leverage_{Market}$ | 298,171 | 2.579 | 34.754 | .070 | 4,100.762 |
| $Liquidity_{Book}$ | 298,172 | .062 | .069 | 0 | .668 |
| $Liquidity_{Market}$ | 298,171 | .044 | .051 | 0 | .594 |
| $Return\ on\ Assets_{Book}$ | 298,172 | .008 | .029 | -.604 | .259 |
| $Return\ on\ Assets_{Market}$ | 298,171 | .004 | .028 | -.673 | .211 |
| $Stock\ return$ | 298,172 | .0005 | .027 | -.896 | 1.024 |
| $Volatility\ of\ stock\ return$ | 298,172 | .351 | .256 | .015 | 4.367 |

### 2.4.3 CDS Price Estimation[1]

**A) Quantile Regression**

A panel data analysis is performed to determine the impact of cross-sectional and time-series variables on CDS spreads. In order to identify variables which generally contribute most to explain the price of CDS, I start by estimating a linear regression suitable for panel data for the entire database, considering all independent variables available, in particular, information regarding the CDS (bid ask spread), information on the share price of the company (total stock return, stock volatility) and economic and financial information of the company (return on assets, leverage and liquidity).

To obtain efficient results I test whether the variables exhibit autocorrelation by the Lagrange multiplier test (Wooldridge (2002)). The test shows evidence of the presence of autocorrelation, which causes the standard errors of the coefficients to be smaller than they actually are and higher R-squared. Based on a Hausman test, a fixed-effects model is specified for the analysis of the determinants of CDS spreads for the regression model. In order to test for the presence of heteroskedasticity, that is, the error terms $\mu_{i,t}$ do not have constant variance for firm $i$ and time period $t$, a modified Wald test for groupwise heteroskedasticity in fixed effects regression model is performed with the null hypothesis that the variance of one group $j$ equals the overall variance ($\sigma_j^2 = \sigma_{overall}^2$). The null hypothesis is rejected. Considering that there is evidence of the presence of autocorrelation and heteroskedasticity, it is applied a robust estimator of the error variance matrix against both heteroskedasticity and autocorrelation.

To check whether time fixed effects are needed when estimating the fixed-effects model, I include time dummies for each period in the regression model and, based on the F-test, I conclude that time fixed effects are needed. Taking into account these adjustments, the different specifications for the financial ratios based on book values and market values of equity, and the other variables available, several alternative specifications for the regression are tested. The variables that show statistical significance and economic intuition in explaining the determinants of the price of the CDS with the highest explanatory power are: bid ask spread, the stock return, the volatility of stock

[1]Linear Regression and Quantile Regression for panel data were estimated with the software Stata. The remaining models were estimated with the software Matlab.

returns, return on assets and leverage, both based on the book value of equity, and time dummies.The regression model is designed as:

$$\begin{aligned}
CDS\ spread_{i,t} &= \beta_1 bid\ ask\ spread_{i,t} + \beta_2 stock\ return_{i,t} \\
&\quad + \beta_3 stock\ volatility_{i,t} + \beta_4 return\ on\ assets^{Book}_{i,t} \\
&\quad + \beta_7 leverage^{Book}_{i,t} \\
&\quad + a_i + a_t + \mu_{i,t}
\end{aligned}$$

With $a_i$ and $a_t$ representing entity-fixed and time-fixed effects.

Regarding overall goodness of fit, the high F-statistics indicates that all model parameters are different from zero and the $R^2$ value is 53.8%. Table 2.3 presents the results.

Comparing with the results obtained in Campbell *et al* (2008), I conclude that some of the predictor variables of failure are important to explain the CDS prices but there are some differences. In the study performed by Campbell *et al* (2008), the profitability and leverage ratios that perform better are those that measure the equity component of total assets at market value, while in this study the ratios that perform better are based on the book value of equity. Additionally, in Campbell *et al* (2008) the liquidity ratio is statistically significant and has a coefficient according to the economic intuition but in this study these conditions are not verified.

The estimation of linear panel models is a starting point to identify the variables that contribute most to explain the determinants of the price of CDS and understand the characteristics of the data. However, the main interest of this study is to identify the determinants of the price of CDS in a specific quantile and, therefore, I proceed to the estimation of Quantile Regression. Table 2.4 presents the results obtained with Quantile Regression, for the 99 quantile, considering robust standard error estimation based on bootstrap methods.

The Quantile Regression presented in table 2.4 is estimated considering all the available sampling window, *i.e.*, the application of this equation for determining the price of CDS at time period $t$ will be performed based on parameters estimated on a sample that considers periods after $t$. To minimize the problem of estimating the price of CDS based on information that is not yet available at the time, I estimate a new regression in each period considering only the information available at that time. In each subsequent period, the new information available is added to the previous sampling window and a new estimation of Quantile Regression is performed. Taking into account, on the one hand, the importance of having a database sufficiently representative of the determinants of the price of CDS and, second, that since the financial crisis in 2008 there has been a sudden increase in the price of the CDS, the first regression is performed considering the period 2001-2008 and from that date new regressions are estimated for each month until April 2011. The estimation of VaR and its backtesting will be made based on the monthly regressions, whose results are presented in the appendix. Comparing the results obtained in each monthly regression over time, I find that the coefficient of the variable bid ask spread has increased in each monthly regression, opposed to the reduction in the absolute value of the coefficients of the variables stock volatility, return on assets and leverage. This change in the monthly regression coefficients reflects an increase in the contribution of bid ask spread to explain the price of CDS in the 99th percentile.

**B) GARCH Models**

The original time series are not stationary as can be seen by the results of the Dickey-Fuller test presented in table 2.5. In order to transform the original time series into stationary series, I apply first order differences. The time series are modeled with three types of GARCH models, the GARCH(1,1), the EGARCH(1,1) and GJR(1,1), combined with two distributions of innovations processes, namely Normal and Students's $t$.

The estimation of GARCH models considering all the available sampling window followed by the application of these equations for determining the price of CDS at time period $t$ would be performed based on parameters estimated on a sample that considers periods after $t$. For this reason, in line with the procedure followed for Quantile Regres-

Table 2.3: Panel Regression Results

Results from panel regression model for the CDS price. This table shows the estimated parameters and the *t* statistics (in parentheses) using the entire data sample. * refers to p-values smaller than 0.05, ** refers to p-values smaller than 0.01, *** refers to p-values smaller than 0.001. Full results of the regression, including the time dummies are available upon request.

| | CDS spread |
|---|---|
| Panel Regression | Within $R^2$=53.8% |
| $Bid\ ask\ spread$ | 12.85*** |
| | (366.25) |
| $Stock\ total\ return$ | -65.42*** |
| | (-7.62) |
| $Stock\ volatility$ | 161.0*** |
| | (97.90) |
| $Return\ on\ assets_{Book}$ | -265.1*** |
| | (-30.25) |
| $Leverage_{Book}$ | 0.102*** |
| | (8.73) |

Table 2.4: Quantile Regression Results

Results from Quantile Regression model for the CDS price. This table shows the estimated parameters and the *t* statistics (in parentheses) using the entire data sample for the 99 quantile. * refers to p-values smaller than 0.05, ** refers to p-values smaller than 0.01, *** refers to p-values smaller than 0.001. Full results of the regression, including the time dummies are available upon request.

| | CDS spread |
|---|---|
| Quantile 99 | Pseudo $R^2$=72.2% |
| $Bid\ ask\ spread$ | 18.66*** |
| | (70.52) |
| $Stock\ total\ return$ | -160.0** |
| | (-2.60) |
| $Stock\ volatility$ | 389.7*** |
| | (11.94) |
| $Return\ on\ assets_{Book}$ | -605.4*** |
| | (-16.52) |
| $Leverage_{Book}$ | 0.361*** |
| | (11.40) |

Table 2.5: Augmented Dickey-Fuller test

Results from the Augmented Dickey-Fuller test for the ten CDS names used in the empirical analysis. The null hypothesis is that CDS price series has a unit root.

| CDS id | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| p-value | 0.2121 | 0.5072 | 0.2928 | 0.3641 | 0.3186 | 0.5606 | 0.5953 | 0.3169 | 0.4216 | 0.2461 |

sion, I estimate GARCH models in each period for each reference entity considering only the time series available at that time. In each subsequent period, the new information available is added to the previous sampling window and the GARCH models are reestimated. Similar to the Quantile Regression approach, the first GARCH models are estimated considering the period 2001-2008 and from that date onwards GARCH models are reestimated for each month until April 2011. The estimation of VaR and its backtesting will be made based on the monthly GARCH models. Each estimated GARCH model will lead to different percentiles of the series of standardized residuals and to different time series of conditional variance, which are then used to determine the $\theta$-quantile of the distribution of CDS price for each reference entity at each time period $t$. Due to its extent, the results of all GARCH models were not included in the study.

**C) Historical Simulation**

The Historical Simulation methodology is applied considering a moving window of 200, 100, 60 and 30 days observations up to time $t$. Considering the small number of observations available for the CDS names with sort history, in these cases I will present only the results of HS considering a moving window of 30 days.

**D) Extreme Value Theory**

In order to produce results comparable to the previous methods, EVT will also be applied to the 5 CDS names with the largest number of observations and 5 CDS names with the smallest number of observations.

### 2.4.4 Backtesting Empirical Results

Although the performance varies across modeling approaches and distributional assumptions, some patterns emerge. I first discuss the performance of VaR models applied to CDS names with short history and considering a prediction horizon of 1 day, then the results obtained for CDS names with long history and considering a time horizon of 30 days, and finally the performance obtained for the remaining combinations of CDS names and VaR estimation horizons.

As the number of observations for CDS names with short history is small and HS is based on the past observations of VaR, in the case of HS only the results considering the past 30 days are presented. According to table 2.6, at the $1\%$ level, the models that perform well more often with respect to violation frequencies are QR, GARCH(T), GJR(N) and FHS (the criteria is satisfied for 3 CDS names, out of 5). VaR estimates based on HS and EGARCH (N) perform quite poorly, followed by EGARCH (T). I now turn to the information in the sequence of violations, as reflected in the p-values of the LR and DQ test statistic. The models that perform well more often in terms of LR are also QR, GARCH(T), GJR(N) and FHS. However, QR and GJR(N) are the models that satisfy the DQ test more frequently. In general, compared to the other models that verify the criteria of violation frequency and independence, QR provides the best results in terms of CTE or AQL.
Summarizing the results for CDS names with short history and considering a prediction horizon of 1 day: QR is the model that simultaneously satisfies all tests for the larger number of CDS names; HS and EGARCH (N) are the worst models for these reference entities.

Next, I turn to the results obtained for CDS names with long history and considering a time horizon of 30 days, presented in table 2.7. Regarding the violation frequency, in general all models show high violation rates except QR. QR is the best model as it verifies the criteria for all CDS names, followed by EVT, GARCH (T) and GJR (T) which verify the unconditional coverage property only once. All the variations of HS and GARCH-based methods combined with Normal distribution perform very poorly. None of the VaR models verifies the independence criteria and the Dynamic Quantile test. In cases for which, additionally to QR, another model satisfies the violation frequency criteria, QR provides the best results in terms of CTE. In conclusion, QR is also the most accurate VaR model for those CDS names with long history and considering a horizon of 30 days for VaR estimates, compared to the other methods tested in this study.

Regarding CDS names with long history and VaR estimation horizon of 1 day, whose results are presented in table 2.8, all models have very high failure rate compared to the $1\%$ significance level, except QR and GJR(N). The violation frequency of QR is excessively low, hence, the only model that performs well with respect to violation frequencies is GJR(N). None of the models satisfies the DQ test. According to the table 2.11, these findings are invariant to the increase of the prediction horizon from 1 to 3 days, except that the failure rate associated with GJR(N) increases and this method maintains adequacy in terms of violation frequency only for 2 out of 5 CDS names. Increasing the VaR prediction horizon to 10 days leads the failure rate of GJR(N) to significantly overcome the $1\%$ level, as shown in table 2.13, and, hence, the model is considered inadequate in terms of violation frequencies for all CDS names, while the failure rate in QR maintains its low levels. These results combined with the previous findings provide evidence that for CDS with long history QR performs best for higher prediction horizons (as the failure rate for short horizons is very low) and GJR(N) is adequate only when the VaR prediction horizon is 1 day.

The results of VaR models for CDS names with short history and considering a prediction horizon of 30 days are inadequate in almost all cases, as presented in table 2.9. From the 50 cases under analysis (5 CDS names combined with 10 VaR models), only 4 perform well with respect to violation frequencies. In the remaining cases, the failure rate is either $0\%$ or significantly higher than the $1\%$. This result provides empirical evidence to the intuition that none of the VaR models is adequate to estimate VaR in a relatively long time horizon with such a short history.

Kuester *et al* (2006) provide empirical evidence that FHS is one of the best VaR approaches compared to the other models tested in their empirical work. However, in this study FHS performs poorly for all CDS names considering VaR prediction horizons of 1 and 30 days. According to the tables 2.11 and 2.13, in the case of CDS names with long history this finding extends to VaR prediction horizons of 3 and 10 days. According to tables 2.10 and 2.12, a different conclusion emerges in the case of CDS names with short history and considering VaR prediction horizons of 3 and 10 days, for which FHS is the most accurate VaR model in respect to violation frequencies and independence of the violation series.

Recent studies such as Kuester *et al* (2006), Rossignolo *et al* (2012) and Halbleib and Pohlmeier (2012) provide empirical evidence that EVT is one of the most accurate techniques to estimate VaR. However, the empirical evidence in this study does not support the extension of that finding to VaR estimation in CDS, as EVT is not one of the best VaR models in any combination of short/long history and prediction horizon of 1/3/10/30 days.

## Table 2.6: VaR backtesting for CDS names with short history (1 day)

Backtest results for the CDS names with short history and considering a VaR prediction horizon of 1 day. Entries in parenthesis in column Model refer to the distribution used, namely Normal or Student's $t$. Percentage rate of violations for VaR at $\theta$=1%. When the % Viol is 0, the results for $LR_{uc}$ , $LR_{ind}$, DQ and CTE cannot be computed and are identified with "-". The cases where the GARCH estimation failed are identified with "n.a.". Entries in columns $LR_{uc}$, $LR_{ind}$ and DQ are the significance levels (p-values) of the respective tests. Bold type entries indicate p-values greater than 0.01, meaning that the null hypothesis cannot be rejected at the 1% significance level.

| CDS ID | Model | % Viol. | $LR_{uc}$ | $LR_{ind}$ | DQ | CTE | AQL |
|---|---|---|---|---|---|---|---|
| 6 | GARCH (N) | 3.37% | **0.08** | **0.06** | 0.00 | -2.21 | 0.48 |
| | GARCH (T) | 2.25% | **0.31** | **0.02** | 0.00 | -2.95 | 0.13 |
| | EGARCH (N) | 6.74% | 0.00 | 0.00 | 0.00 | -1.31 | 1.15 |
| | EGARCH (T) | 2.25% | **0.31** | **0.02** | 0.00 | -2.98 | 4.4E+126 |
| | GJR (N) | n.a. | n.a. | n.a. | n.a. | n.a. | n.a. |
| | GJR (T) | n.a. | n.a. | n.a. | n.a. | n.a. | n.a. |
| | QR | 1.12% | **0.91** | **0.99** | **1.00** | -5.41 | 0.07 |
| | HS30d | 8.20% | 0.00 | 0.00 | 0.00 | -0.49 | 0.12 |
| | FHS | 3.37% | **0.08** | **0.06** | 0.00 | -2.21 | 0.16 |
| | EVT | 5.62% | 0.00 | **0.11** | 0.00 | -1.27 | 0.20 |
| 7 | GARCH (N) | 5.56% | 0.01 | 0.01 | 0.00 | -1.18 | 0.16 |
| | GARCH (T) | 5.56% | 0.01 | 0.01 | 0.00 | -1.18 | 0.18 |
| | EGARCH (N) | 13.89% | 0.00 | **0.15** | 0.00 | -0.54 | 596.70 |
| | EGARCH (T) | 11.11% | 0.00 | **0.03** | 0.00 | -0.66 | 0.19 |
| | GJR (N) | n.a. | n.a. | n.a. | n.a. | n.a. | n.a. |
| | GJR (T) | n.a. | n.a. | n.a. | n.a. | n.a. | n.a. |
| | QR | 0.00% | - | - | - | - | 0.06 |
| | HS30d | 0.00% | - | - | - | - | 0.05 |
| | FHS | 5.56% | 0.01 | 0.01 | 0.00 | -1.18 | 0.17 |
| | EVT | 5.56% | 0.01 | 0.01 | 0.00 | -1.18 | 0.93 |
| 8 | GARCH (N) | 4.62% | **0.03** | **0.64** | **0.05** | -0.64 | 0.09 |
| | GARCH (T) | 7.69% | 0.00 | **0.39** | 0.00 | 0.19 | 0.12 |
| | EGARCH (N) | 12.31% | 0.00 | **0.04** | 0.00 | -0.49 | 0.08 |
| | EGARCH (T) | 10.77% | 0.00 | **0.16** | 0.00 | 0.28 | 0.10 |
| | GJR (N) | 1.54% | **0.69** | **0.96** | **0.99** | -1.23 | 0.05 |
| | GJR (T) | 4.62% | **0.03** | **0.64** | **0.05** | -0.32 | 0.09 |
| | QR | 4.62% | **0.03** | **0.09** | 0.00 | -0.62 | 0.11 |
| | HS30d | 13.51% | 0.00 | 0.00 | 0.00 | -0.73 | 0.30 |
| | FHS | 6.15% | 0.00 | **0.51** | 0.00 | 0.25 | 0.12 |
| | EVT | 6.15% | 0.00 | **0.51** | 0.00 | 0.25 | 0.12 |
| 9 | GARCH (N) | 9.52% | 0.00 | 0.01 | 0.00 | -0.66 | 0.12 |
| | GARCH (T) | 4.76% | **0.03** | **0.63** | **0.04** | -0.78 | 0.13 |
| | EGARCH (N) | 23.81% | 0.00 | 0.00 | 0.00 | -0.20 | 0.10 |
| | EGARCH (T) | 7.94% | 0.00 | **0.03** | 0.00 | -0.07 | 0.05 |
| | GJR (N) | 3.17% | **0.17** | **0.79** | 0.00 | -1.17 | 0.06 |
| | GJR (T) | 6.35% | 0.00 | **0.21** | 0.00 | -0.58 | 0.08 |
| | QR | 22.22% | 0.00 | 0.00 | 0.00 | -0.09 | 0.21 |
| | HS30d | 17.14% | 0.00 | 0.00 | 0.00 | -0.72 | 0.39 |
| | FHS | 1.59% | **0.67** | **0.96** | **0.99** | -0.02 | 0.08 |
| | EVT | 1.59% | **0.67** | **0.96** | **0.99** | -0.02 | 0.08 |
| 10 | GARCH (N) | 4.52% | 0.00 | **0.46** | 0.00 | -1.35 | 0.15 |
| | GARCH (T) | 2.71% | **0.03** | **0.14** | 0.00 | -1.59 | 0.14 |
| | EGARCH (N) | 3.62% | 0.00 | **0.27** | 0.00 | -1.25 | 0.16 |
| | EGARCH (T) | 3.17% | 0.01 | **0.01** | 0.00 | -1.42 | 0.14 |
| | GJR (N) | 1.36% | **0.61** | **0.85** | **0.98** | -4.81 | 0.06 |
| | GJR (T) | 2.71% | **0.03** | **0.14** | 0.00 | -1.59 | 0.14 |
| | QR | 1.81% | **0.28** | **0.77** | **0.79** | -0.24 | 0.16 |
| | HS30d | 24.87% | 0.00 | 0.00 | 0.00 | -0.91 | 0.49 |
| | FHS | 2.71% | **0.03** | **0.14** | 0.00 | -1.59 | 0.19 |
| | EVT | 2.71% | **0.03** | **0.14** | 0.00 | -1.59 | 0.22 |

## Table 2.7: VaR backtesting for CDS names with long history (30 days)

Backtest results for the CDS names with long history and considering a VaR prediction horizon of 30 days. Entries in parenthesis in column Model refer to the distribution used, namely Normal or Student's $t$. Percentage rate of violations for VaR at $\theta$=1%. When the % Viol is 0, the results for $LR_{uc}$ , $LR_{ind}$, DQ and CTE cannot be computed and are identified with "-". The cases where the GARCH estimation failed are identified with "n.a.". Entries in columns $LR_{uc}$, $LR_{ind}$ and DQ are the significance levels (p-values) of the respective tests. Bold type entries indicate p-values greater than 0.01, meaning that the null hypothesis cannot be rejected at the 1% significance level.

| CDS ID | Model | % Viol. | $LR_{uc}$ | $LR_{ind}$ | DQ | CTE | AQL |
|---|---|---|---|---|---|---|---|
| 1 | GARCH (N) | 6.65% | 0.00 | 0.00 | 0.00 | -2.58 | 0.06 |
| | GARCH (T) | 4.37% | 0.00 | 0.00 | 0.00 | -2.72 | 0.04 |
| | EGARCH (N) | 6.26% | 0.00 | 0.00 | 0.00 | -2.60 | 0.05 |
| | EGARCH (T) | 7.92% | 0.00 | 0.00 | 0.00 | -2.54 | 0.07 |
| | GJR (N) | 6.04% | 0.00 | 0.00 | 0.00 | -2.63 | 0.05 |
| | GJR (T) | 4.59% | 0.00 | 0.00 | 0.00 | -2.72 | 0.05 |
| | QR | 0.52% | **0.01** | 0.00 | 0.00 | -1.48 | 0.05 |
| | HS200d | 11.64% | 0.00 | 0.00 | 0.00 | -2.12 | 0.16 |
| | HS100d | 21.02% | 0.00 | 0.00 | 0.00 | -1.59 | 0.22 |
| | HS60d | 25.40% | 0.00 | 0.00 | 0.00 | -1.54 | 0.31 |
| | HS30d | 34.85% | 0.00 | 0.00 | 0.00 | -1.34 | 0.38 |
| | FHS | 3.02% | 0.00 | 0.00 | 0.00 | -2.67 | 0.03 |
| | EVT | 1.09% | **0.66** | 0.00 | 0.00 | -2.83 | 0.03 |
| 2 | GARCH (N) | 2.83% | 0.00 | 0.00 | 0.00 | -2.66 | 0.04 |
| | GARCH (T) | 1.30% | **0.17** | 0.00 | 0.00 | -3.29 | 0.04 |
| | EGARCH (N) | 3.01% | 0.00 | 0.00 | 0.00 | -2.65 | 0.05 |
| | EGARCH (T) | 75.88% | 0.00 | **1.00** | 0.00 | -0.66 | 0.51 |
| | GJR (N) | 3.46% | 0.00 | 0.00 | 0.00 | -2.60 | 0.05 |
| | GJR (T) | 1.30% | **0.17** | 0.00 | 0.00 | -3.29 | 0.04 |
| | QR | 0.67% | **0.10** | 0.00 | 0.00 | -1.65 | 0.05 |
| | HS200d | 16.57% | 0.00 | 0.00 | 0.00 | -1.45 | 0.18 |
| | HS100d | 26.08% | 0.00 | 0.00 | 0.00 | -1.20 | 0.21 |
| | HS60d | 33.03% | 0.00 | 0.00 | 0.00 | -1.11 | 0.25 |
| | HS30d | 41.67% | 0.00 | 0.00 | 0.00 | -0.97 | 0.31 |
| | FHS | 5.48% | 0.00 | 0.00 | 0.00 | -2.44 | 0.06 |
| | EVT | 5.17% | 0.00 | 0.00 | 0.00 | -2.48 | 0.05 |
| 3 | GARCH (N) | 4.17% | 0.00 | 0.00 | 0.00 | -3.13 | 0.05 |
| | GARCH (T) | 4.26% | 0.00 | 0.00 | 0.00 | -3.08 | 0.05 |
| | EGARCH (N) | 4.72% | 0.00 | 0.00 | 0.00 | -3.04 | 0.05 |
| | EGARCH (T) | 34.51% | 0.00 | 0.00 | 0.00 | -1.82 | 0.34 |
| | GJR (N) | 4.58% | 0.00 | 0.00 | 0.00 | -3.09 | 0.05 |
| | GJR (T) | 4.26% | 0.00 | 0.00 | 0.00 | -3.08 | 0.05 |
| | QR | 0.92% | **0.69** | 0.00 | 0.00 | -3.30 | 0.05 |
| | HS200d | 11.74% | 0.00 | 0.00 | 0.00 | -2.76 | 0.29 |
| | HS100d | 19.34% | 0.00 | 0.00 | 0.00 | -2.26 | 0.34 |
| | HS60d | 26.41% | 0.00 | 0.00 | 0.00 | -1.92 | 0.38 |
| | HS30d | 41.13% | 0.00 | 0.00 | 0.00 | -1.61 | 0.47 |
| | FHS | 2.66% | 0.00 | 0.00 | 0.00 | -3.30 | 0.04 |
| | EVT | 3.02% | 0.00 | 0.00 | 0.00 | -3.39 | 0.04 |
| 4 | GARCH (N) | 4.59% | 0.00 | 0.00 | 0.00 | -2.65 | 0.05 |
| | GARCH (T) | 5.69% | 0.00 | 0.00 | 0.00 | -2.58 | 0.06 |
| | EGARCH (N) | 4.45% | 0.00 | 0.00 | 0.00 | -2.65 | 0.05 |
| | EGARCH (T) | 9.08% | 0.00 | 0.00 | 0.00 | -2.58 | 0.08 |
| | GJR (N) | 4.72% | 0.00 | 0.00 | 0.00 | -2.64 | 0.05 |
| | GJR (T) | 5.69% | 0.00 | 0.00 | 0.00 | -2.58 | 0.06 |
| | QR | 0.73% | **0.19** | 0.00 | 0.00 | -2.60 | 0.05 |
| | HS200d | 18.86% | 0.00 | 0.00 | 0.00 | -2.08 | 0.31 |
| | HS100d | 30.20% | 0.00 | 0.00 | 0.00 | -1.59 | 0.34 |
| | HS60d | 36.22% | 0.00 | 0.00 | 0.00 | -1.45 | 0.37 |
| | HS30d | 41.43% | 0.00 | 0.00 | 0.00 | -1.36 | 0.44 |
| | FHS | 5.96% | 0.00 | 0.00 | 0.00 | -3.04 | 0.06 |
| | EVT | 6.37% | 0.00 | 0.00 | 0.00 | -3.03 | 0.06 |
| 5 | GARCH (N) | 5.32% | 0.00 | 0.00 | 0.00 | -3.55 | 0.09 |
| | GARCH (T) | 5.74% | 0.00 | 0.00 | 0.00 | -3.46 | 0.09 |
| | EGARCH (N) | 6.06% | 0.00 | 0.00 | 0.00 | -3.38 | 0.10 |
| | EGARCH (T) | 10.97% | 0.00 | 0.00 | 0.00 | -2.85 | 0.14 |
| | GJR (N) | 5.55% | 0.00 | 0.00 | 0.00 | -3.53 | 0.09 |
| | GJR (T) | 5.78% | 0.00 | 0.00 | 0.00 | -3.46 | 0.10 |
| | QR | 0.78% | **0.28** | 0.00 | 0.00 | -7.35 | 0.06 |
| | HS200d | 15.09% | 0.00 | 0.00 | 0.00 | -2.50 | 0.34 |
| | HS100d | 22.06% | 0.00 | 0.00 | 0.00 | -1.93 | 0.34 |
| | HS60d | 28.29% | 0.00 | 0.00 | 0.00 | -1.78 | 0.43 |
| | HS30d | 42.91% | 0.00 | 0.00 | 0.00 | -1.51 | 0.53 |
| | FHS | 6.98% | 0.00 | 0.00 | 0.00 | -3.90 | 0.10 |
| | EVT | 7.07% | 0.00 | 0.00 | 0.00 | -3.83 | 0.11 |

Table 2.8: VaR backtesting for CDS names with long history (1 day)

Backtest results for the CDS names with long history and considering a VaR prediction horizon of 1 day. Entries in parenthesis in column Model refer to the distribution used, namely Normal or Student's $t$. Percentage rate of violations for VaR at $\theta$=1%. When the % Viol is 0, the results for $LR_{uc}$ , $LR_{ind}$, DQ and CTE cannot be computed and are identified with "-". The cases where the GARCH estimation failed are identified with "n.a.". Entries in columns $LR_{uc}$, $LR_{ind}$ and DQ are the significance levels (p-values) of the respective tests. Bold type entries indicate p-values greater than 0.01, meaning that the null hypothesis cannot be rejected at the 1% significance level.

| CDS ID | Model | % Viol. | $LR_{uc}$ | $LR_{ind}$ | DQ | CTE | AQL |
|---|---|---|---|---|---|---|---|
| 1 | GARCH (N) | 4.16% | 0.00 | 0.00 | 0.00 | -0.30 | 0.02 |
| | GARCH (T) | 4.37% | 0.00 | 0.00 | 0.00 | -0.30 | 0.02 |
| | EGARCH (N) | 4.07% | 0.00 | 0.00 | 0.00 | -0.30 | 0.02 |
| | EGARCH (T) | 4.16% | 0.00 | 0.00 | 0.00 | -0.30 | 0.02 |
| | GJR (N) | 1.05% | **0.81** | 0.00 | 0.00 | -0.94 | 0.01 |
| | GJR (T) | 4.42% | 0.00 | 0.00 | 0.00 | -0.30 | 0.02 |
| | QR | 0.13% | 0.00 | 0.00 | 0.00 | 0.01 | 0.05 |
| | HS200d | 2.59% | 0.00 | 0.00 | 0.00 | -0.29 | 0.04 |
| | HS100d | 4.89% | 0.00 | 0.00 | 0.00 | -0.23 | 0.04 |
| | HS60d | 6.33% | 0.00 | 0.00 | 0.00 | -0.23 | 0.03 |
| | HS30d | 9.34% | 0.00 | 0.00 | 0.00 | -0.20 | 0.04 |
| | FHS | 3.02% | 0.00 | 0.00 | 0.00 | -0.32 | 0.02 |
| | EVT | 2.19% | 0.00 | 0.00 | 0.00 | -0.38 | 0.01 |
| 2 | GARCH (N) | 4.18% | 0.00 | 0.00 | 0.00 | -0.22 | 0.01 |
| | GARCH (T) | 3.64% | 0.00 | 0.00 | 0.00 | -0.18 | 0.01 |
| | EGARCH (N) | 4.04% | 0.00 | 0.00 | 0.00 | -0.23 | 0.01 |
| | EGARCH (T) | 1.62% | 0.01 | 0.00 | 0.00 | -0.12 | 0.04 |
| | GJR (N) | 1.03% | **0.88** | **0.24** | 0.00 | -0.59 | 0.01 |
| | GJR (T) | 3.41% | 0.00 | 0.00 | 0.00 | -0.17 | 0.01 |
| | QR | 0.09% | 0.00 | **0.89** | 0.00 | 0.06 | 0.05 |
| | HS200d | 4.83% | 0.00 | 0.00 | 0.00 | -0.18 | 0.04 |
| | HS100d | 6.30% | 0.00 | 0.00 | 0.00 | -0.17 | 0.03 |
| | HS60d | 8.03% | 0.00 | 0.00 | 0.00 | -0.17 | 0.03 |
| | HS30d | 9.55% | 0.00 | 0.00 | 0.00 | -0.16 | 0.03 |
| | FHS | 6.87% | 0.00 | 0.00 | 0.00 | -0.19 | 0.02 |
| | EVT | 6.42% | 0.00 | 0.00 | 0.00 | -0.21 | 0.02 |
| 3 | GARCH (N) | 2.25% | 0.00 | 0.00 | 0.00 | -0.60 | 0.02 |
| | GARCH (T) | 2.38% | 0.00 | 0.00 | 0.00 | -0.64 | 0.02 |
| | EGARCH (N) | 2.38% | 0.00 | 0.00 | 0.00 | -0.58 | 0.02 |
| | EGARCH (T) | 2.43% | 0.00 | 0.00 | 0.00 | -0.63 | 0.02 |
| | GJR (N) | 0.92% | **0.69** | 0.00 | 0.00 | -0.94 | 0.01 |
| | GJR (T) | 2.38% | 0.00 | 0.00 | 0.00 | -0.64 | 0.02 |
| | QR | 0.27% | 0.00 | 0.00 | 0.00 | 0.28 | 0.05 |
| | HS200d | 4.74% | 0.00 | 0.00 | 0.00 | -0.25 | 0.06 |
| | HS100d | 5.28% | 0.00 | 0.00 | 0.00 | -0.30 | 0.05 |
| | HS60d | 7.20% | 0.00 | 0.00 | 0.00 | -0.26 | 0.05 |
| | HS30d | 10.82% | 0.00 | 0.00 | 0.00 | -0.23 | 0.05 |
| | FHS | 3.16% | 0.00 | 0.00 | 0.00 | -0.53 | 0.03 |
| | EVT | 3.35% | 0.00 | 0.00 | 0.00 | -0.52 | 0.03 |
| 4 | GARCH (N) | 2.71% | 0.00 | 0.00 | 0.00 | -0.32 | 0.01 |
| | GARCH (T) | 3.21% | 0.00 | 0.00 | 0.00 | -0.33 | 0.01 |
| | EGARCH (N) | 2.89% | 0.00 | 0.00 | 0.00 | -0.33 | 0.01 |
| | EGARCH (T) | 3.03% | 0.00 | 0.00 | 0.00 | -0.31 | 0.01 |
| | GJR (N) | 1.15% | **0.50** | **0.03** | 0.01 | -0.63 | 0.01 |
| | GJR (T) | 3.16% | 0.00 | 0.00 | 0.00 | -0.32 | 0.01 |
| | QR | 0.09% | 0.00 | **0.89** | 0.00 | 0.02 | 0.05 |
| | HS200d | 6.30% | 0.00 | 0.00 | 0.00 | -0.20 | 0.06 |
| | HS100d | 7.92% | 0.00 | 0.00 | 0.00 | -0.20 | 0.05 |
| | HS60d | 9.23% | 0.00 | 0.00 | 0.00 | -0.19 | 0.04 |
| | HS30d | 11.89% | 0.00 | 0.00 | 0.00 | -0.19 | 0.04 |
| | FHS | 3.21% | 0.00 | 0.00 | 0.00 | -0.35 | 0.02 |
| | EVT | 3.67% | 0.00 | 0.00 | 0.00 | -0.36 | 0.02 |
| 5 | GARCH (N) | 3.12% | 0.00 | 0.00 | 0.00 | -0.47 | 0.02 |
| | GARCH (T) | 3.40% | 0.00 | 0.00 | 0.00 | -0.43 | 0.02 |
| | EGARCH (N) | 3.85% | 0.00 | 0.00 | 0.00 | -0.45 | 0.02 |
| | EGARCH (T) | 3.49% | 0.00 | 0.00 | 0.00 | -0.42 | 0.02 |
| | GJR (N) | 1.10% | **0.64** | **0.03** | 0.01 | -0.80 | 0.01 |
| | GJR (T) | 3.40% | 0.00 | 0.00 | 0.00 | -0.44 | 0.02 |
| | QR | 0.00% | - | - | - | - | 0.05 |
| | HS200d | 4.75% | 0.00 | 0.00 | 0.00 | -0.26 | 0.07 |
| | HS100d | 5.19% | 0.00 | 0.00 | 0.00 | -0.26 | 0.05 |
| | HS60d | 8.06% | 0.00 | 0.00 | 0.00 | -0.25 | 0.05 |
| | HS30d | 12.09% | 0.00 | 0.00 | 0.00 | -0.22 | 0.05 |
| | FHS | 4.73% | 0.00 | 0.00 | 0.00 | -0.43 | 0.02 |
| | EVT | 4.73% | 0.00 | 0.00 | 0.00 | -0.41 | 0.02 |

## Table 2.9: VaR backtesting for CDS names with short history (30 days)

Backtest results for the CDS names with short history and considering a VaR prediction horizon of 30 days. Entries in parenthesis in column Model refer to the distribution used, namely Normal or Student's $t$. Percentage rate of violations for VaR at $\theta$=1%. When the % Viol is 0, the results for $LR_{uc}$ , $LR_{ind}$, DQ and CTE cannot be computed and are identified with "-". The cases where the GARCH estimation failed are identified with "n.a.". Entries in columns $LR_{uc}$, $LR_{ind}$ and DQ are the significance levels (p-values) of the respective tests. Bold type entries indicate p-values greater than 0.01, meaning that the null hypothesis cannot be rejected at the 1% significance level.

| CDS ID | Model | % Viol. | $LR_{uc}$ | $LR_{ind}$ | DQ | CTE | AQL |
|---|---|---|---|---|---|---|---|
| 6 | GARCH (N) | 0.00% | - | - | - | - | 1.81E+11 |
| | GARCH (T) | 24.72% | 0.00 | 0.00 | 0.00 | -5.71 | 10.61 |
| | EGARCH (N) | 1.12% | **0.91** | **0.99** | **1.00** | -6.32 | 6.97E+07 |
| | EGARCH (T) | 0.00% | - | - | - | - | 3.96E+214 |
| | GJR (N) | n.a. | n.a. | n.a. | n.a. | n.a. | n.a. |
| | GJR (T) | n.a. | n.a. | n.a. | n.a. | n.a. | n.a. |
| | QR | 22.47% | 0.00 | 0.00 | 0.00 | -5.45 | 0.22 |
| | HS30d | 54.10% | 0.00 | 0.00 | 0.00 | -2.41 | 1.09 |
| | FHS | 3.37% | **0.08** | **0.71** | **0.24** | -5.88 | 1.07 |
| | EVT | 11.24% | 0.00 | 0.00 | 0.00 | -5.71 | 0.30 |
| 7 | GARCH (N) | 5.56% | 0.01 | 0.00 | 0.00 | -3.84 | 0.58 |
| | GARCH (T) | 1.39% | **0.75** | **0.97** | **1.00** | -4.42 | 3.33 |
| | EGARCH (N) | 0.00% | - | - | - | - | 0.15 |
| | EGARCH (T) | 0.00% | - | - | - | - | 0.29 |
| | GJR (N) | n.a. | n.a. | n.a. | n.a. | n.a. | n.a. |
| | GJR (T) | n.a. | n.a. | n.a. | n.a. | n.a. | n.a. |
| | QR | 5.56% | 0.01 | 0.01 | 0.00 | -3.64 | 0.09 |
| | HS30d | 0.00% | - | - | - | - | 0.05 |
| | FHS | 5.56% | 0.01 | 0.00 | 0.00 | -3.84 | 0.39 |
| | EVT | 5.56% | 0.01 | 0.01 | 0.00 | -2.89 | 2.7E+13 |
| 8 | GARCH (N) | 0.00% | - | - | - | - | 0.83 |
| | GARCH (T) | 41.54% | 0.00 | 0.00 | 0.00 | -4.61 | 0.39 |
| | EGARCH (N) | 0.00% | - | - | - | - | 0.38 |
| | EGARCH (T) | 0.00% | - | - | - | - | 0.19 |
| | GJR (N) | 0.00% | - | - | - | - | 2.03 |
| | GJR (T) | 0.00% | - | - | - | - | 0.73 |
| | QR | 47.69% | 0.00 | 0.00 | 0.00 | -4.37 | 1.32 |
| | HS30d | 18.92% | 0.00 | 0.00 | 0.00 | -2.83 | 0.63 |
| | FHS | 0.00% | - | - | - | - | 3.75 |
| | EVT | 0.00% | - | - | - | - | 2.62 |
| 9 | GARCH (N) | 39.68% | 0.00 | 0.00 | 0.00 | -5.36 | 74.27 |
| | GARCH (T) | 42.86% | 0.00 | 0.00 | 0.00 | -4.78 | 8.99 |
| | EGARCH (N) | 41.27% | 0.00 | 0.00 | 0.00 | -5.34 | 2.87 |
| | EGARCH (T) | 28.57% | 0.00 | 0.00 | 0.00 | -5.37 | 3.05 |
| | GJR (N) | 38.10% | 0.00 | 0.00 | 0.00 | -5.37 | 753 |
| | GJR (T) | 34.92% | 0.00 | 0.00 | 0.00 | -5.40 | 1.74E+03 |
| | QR | 55.56% | 0.00 | 0.00 | 0.00 | -4.68 | 2.34 |
| | HS30d | 17.14% | 0.00 | 0.00 | 0.00 | -2.65 | 0.72 |
| | FHS | 0.00% | - | - | - | - | 1.42 |
| | EVT | 0.00% | - | - | - | - | 1.25 |
| 10 | GARCH (N) | 5.43% | 0.00 | 0.00 | 0.00 | -26.73 | 4.71E+03 |
| | GARCH (T) | 32.58% | 0.00 | 0.00 | 0.00 | -15.79 | 2.63E+02 |
| | EGARCH (N) | 3.17% | 0.01 | 0.00 | 0.00 | -31.92 | 1.28E+05 |
| | EGARCH (T) | 46.61% | 0.00 | 0.00 | 0.00 | -12.22 | 3.94 |
| | GJR (N) | 2.71% | **0.03** | 0.00 | 0.00 | -32.14 | 5.10E+03 |
| | GJR (T) | 30.32% | 0.00 | 0.00 | 0.00 | -15.98 | 62.51 |
| | QR | 32.58% | 0.00 | 0.00 | 0.00 | -11.67 | 2.21 |
| | HS30d | 78.24% | 0.00 | 0.00 | 0.00 | -9.14 | 6.41 |
| | FHS | 0.00% | - | - | - | - | 1.81E+05 |
| | EVT | 0.00% | - | - | - | - | 5.80E+04 |

## Table 2.10: VaR backtesting for CDS names with short history (3 days)

Backtest results for the CDS names with short history and considering a VaR prediction horizon of 3 days. Entries in parenthesis in column Model refer to the distribution used, namely Normal or Student's $t$. Percentage rate of violations for VaR at $\theta$=1%. When the % Viol is 0, the results for $LR_{uc}$ , $LR_{ind}$, DQ and CTE cannot be computed and are identified with "-". The cases where the GARCH estimation failed are identified with "n.a.". Entries in columns $LR_{uc}$, $LR_{ind}$ and DQ are the significance levels (p-values) of the respective tests. Bold type entries indicate p-values greater than 0.01, meaning that the null hypothesis cannot be rejected at the 1% significance level.

| CDS ID | Model | % Viol. | $LR_{uc}$ | $LR_{ind}$ | DQ | CTE | AQL |
|---|---|---|---|---|---|---|---|
| 6 | GARCH (N) | 1.12% | **0.91** | **0.99** | **0.99** | -5.92 | 28.55 |
| | GARCH (T) | 8.99% | 0.00 | 0.00 | 0.00 | -2.86 | 0.30 |
| | EGARCH (N) | 2.25% | **0.31** | **0.84** | 0.00 | -5.83 | 2.58 |
| | EGARCH (T) | 0.00% | - | - | - | - | 5.5E+65 |
| | GJR (N) | n.a. | n.a. | n.a. | n.a. | n.a. | n.a. |
| | GJR (T) | n.a. | n.a. | n.a. | n.a. | n.a. | n.a. |
| | QR | 3.37% | **0.08** | 0.00 | 0.00 | -5.88 | 0.09 |
| | HS30d | 14.75% | 0.00 | 0.00 | 0.00 | -0.93 | 0.19 |
| | FHS | 4.49% | **0.02** | 0.00 | 0.00 | -4.53 | 0.22 |
| | EVT | 6.74% | 0.00 | 0.00 | 0.00 | -3.37 | 0.27 |
| 7 | GARCH (N) | 6.94% | 0.00 | 0.00 | 0.00 | -2.72 | 0.23 |
| | GARCH (T) | 4.17% | **0.04** | 0.00 | 0.00 | -3.42 | 0.21 |
| | EGARCH (N) | 6.94% | 0.00 | **0.02** | 0.00 | -2.22 | 0.19 |
| | EGARCH (T) | 8.33% | 0.00 | **0.06** | 0.00 | -1.88 | 0.18 |
| | GJR (N) | n.a. | n.a. | n.a. | n.a. | n.a. | n.a. |
| | GJR (T) | n.a. | n.a. | n.a. | n.a. | n.a. | n.a. |
| | QR | 1.39% | **0.75** | **0.97** | **1.00** | -4.16 | 0.06 |
| | HS30d | 0.00% | - | - | - | - | 0.05 |
| | FHS | 6.94% | 0.00 | 0.00 | 0.00 | -2.72 | 0.25 |
| | EVT | 6.94% | 0.00 | 0.00 | 0.00 | -2.72 | 123 |
| 8 | GARCH (N) | 3.08% | **0.18** | **0.79** | **0.55** | -1.58 | 0.09 |
| | GARCH (T) | 13.85% | 0.00 | 0.00 | 0.00 | -1.40 | 0.21 |
| | EGARCH (N) | 4.62% | **0.03** | **0.64** | **0.05** | -1.39 | 0.10 |
| | EGARCH (T) | 3.08% | **0.18** | **0.79** | **0.55** | -1.58 | 0.08 |
| | GJR (N) | 0.00% | - | - | - | - | 0.08 |
| | GJR (T) | 4.62% | **0.03** | **0.64** | **0.05** | -1.39 | 0.10 |
| | QR | 6.15% | 0.00 | 0.01 | 0.00 | -2.08 | 0.19 |
| | HS30d | 18.92% | 0.00 | 0.00 | 0.00 | -1.91 | 0.46 |
| | FHS | 3.08% | **0.18** | **0.79** | **0.55** | -1.58 | 0.14 |
| | EVT | 3.08% | **0.18** | **0.79** | **0.55** | -1.58 | 0.13 |
| 9 | GARCH (N) | 11.11% | 0.00 | 0.00 | 0.00 | -1.95 | 0.21 |
| | GARCH (T) | 19.05% | 0.00 | 0.00 | 0.00 | -1.19 | 0.20 |
| | EGARCH (N) | 11.11% | 0.00 | 0.00 | 0.00 | -1.95 | 0.23 |
| | EGARCH (T) | 6.35% | 0.00 | 0.01 | 0.00 | -2.08 | 0.17 |
| | GJR (N) | 7.94% | 0.00 | 0.00 | 0.00 | -2.02 | 0.23 |
| | GJR (T) | 9.52% | 0.00 | 0.00 | 0.00 | -2.24 | 0.27 |
| | QR | 30.16% | 0.00 | 0.00 | 0.00 | -0.85 | 0.28 |
| | HS30d | 17.14% | 0.00 | 0.00 | 0.00 | -2.07 | 0.62 |
| | FHS | 0.00% | - | - | - | - | 0.11 |
| | EVT | 0.00% | - | - | - | - | 0.11 |
| 10 | GARCH (N) | 4.98% | 0.00 | 0.00 | 0.00 | -4.67 | 0.27 |
| | GARCH (T) | 11.31% | 0.00 | 0.00 | 0.00 | -4.01 | 0.41 |
| | EGARCH (N) | 4.52% | 0.00 | 0.00 | 0.00 | -4.37 | 0.36 |
| | EGARCH (T) | 24.89% | 0.00 | 0.00 | 0.00 | -2.28 | 0.55 |
| | GJR (N) | 3.17% | 0.01 | 0.00 | 0.00 | -7.42 | 0.17 |
| | GJR (T) | 10.86% | 0.00 | 0.00 | 0.00 | -4.04 | 0.36 |
| | QR | 3.17% | 0.01 | 0.01 | 0.00 | -1.57 | 0.17 |
| | HS30d | 34.72% | 0.00 | 0.00 | 0.00 | -2.12 | 0.92 |
| | FHS | 1.81% | **0.28** | **0.77** | 0.00 | -7.77 | 0.60 |
| | EVT | 0.90% | **0.89** | **0.27** | **0.99** | -8.09 | 0.75 |

## Table 2.11: VaR backtesting for CDS names with long history (3 days)

Backtest results for the CDS names with long history and considering a VaR prediction horizon of 3 days. Entries in parenthesis in column Model refer to the distribution used, namely Normal or Student's $t$. Percentage rate of violations for VaR at $\theta$=1%. When the % Viol is 0, the results for $LR_{uc}$ , $LR_{ind}$, DQ and CTE cannot be computed and are identified with "-". The cases where the GARCH estimation failed are identified with "n.a.". Entries in columns $LR_{uc}$, $LR_{ind}$ and DQ are the significance levels (p-values) of the respective tests. Bold type entries indicate p-values greater than 0.01, meaning that the null hypothesis cannot be rejected at the 1% significance level.

| CDS ID | Model | % Viol. | $LR_{uc}$ | $LR_{ind}$ | DQ | CTE | AQL |
|---|---|---|---|---|---|---|---|
| 1 | GARCH (N) | 3.28% | 0.00 | 0.00 | 0.00 | -0.95 | 0.02 |
| | GARCH (T) | 2.27% | 0.00 | 0.00 | 0.00 | -1.02 | 0.02 |
| | EGARCH (N) | 3.02% | 0.00 | 0.00 | 0.00 | -0.98 | 0.02 |
| | EGARCH (T) | 3.98% | 0.00 | 0.00 | 0.00 | -0.89 | 0.02 |
| | GJR (N) | 1.92% | 0.00 | 0.00 | 0.00 | -1.33 | 0.02 |
| | GJR (T) | 2.27% | 0.00 | 0.00 | 0.00 | -1.02 | 0.02 |
| | QR | 0.13% | 0.00 | 0.00 | 0.00 | -0.01 | 0.05 |
| | HS200d | 3.88% | 0.00 | 0.00 | 0.00 | -0.64 | 0.05 |
| | HS100d | 7.22% | 0.00 | 0.00 | 0.00 | -0.51 | 0.05 |
| | HS60d | 10.05% | 0.00 | 0.00 | 0.00 | -0.48 | 0.06 |
| | HS30d | 14.53% | 0.00 | 0.00 | 0.00 | -0.42 | 0.07 |
| | FHS | 2.32% | 0.00 | 0.00 | 0.00 | -0.98 | 0.02 |
| | EVT | 1.49% | **0.03** | 0.00 | 0.00 | -1.12 | 0.02 |
| 2 | GARCH (N) | 2.20% | 0.00 | 0.00 | 0.00 | -0.70 | 0.01 |
| | GARCH (T) | 0.72% | **0.16** | 0.00 | 0.00 | -0.91 | 0.01 |
| | EGARCH (N) | 2.38% | 0.00 | 0.00 | 0.00 | -0.67 | 0.01 |
| | EGARCH (T) | 55.12% | 0.00 | **1.00** | 0.00 | -0.17 | 0.15 |
| | GJR (N) | 1.66% | 0.00 | 0.00 | 0.00 | -0.89 | 0.01 |
| | GJR (T) | 0.76% | **0.24** | 0.00 | 0.00 | -0.88 | 0.01 |
| | QR | 0.13% | 0.00 | **0.94** | 0.00 | -0.34 | 0.05 |
| | HS200d | 6.41% | 0.00 | 0.00 | 0.00 | -0.45 | 0.05 |
| | HS100d | 9.12% | 0.00 | 0.00 | 0.00 | -0.40 | 0.05 |
| | HS60d | 12.59% | 0.00 | 0.00 | 0.00 | -0.36 | 0.05 |
| | HS30d | 15.20% | 0.00 | 0.00 | 0.00 | -0.33 | 0.05 |
| | FHS | 4.94% | 0.00 | 0.00 | 0.00 | -0.58 | 0.02 |
| | EVT | 4.94% | 0.00 | 0.00 | 0.00 | -0.60 | 0.02 |
| 3 | GARCH (N) | 1.70% | 0.00 | 0.00 | 0.00 | -1.46 | 0.03 |
| | GARCH (T) | 1.65% | 0.01 | 0.00 | 0.00 | -1.44 | 0.03 |
| | EGARCH (N) | 1.88% | 0.00 | 0.00 | 0.00 | -1.40 | 0.03 |
| | EGARCH (T) | 22.46% | 0.00 | 1.00 | 0.00 | -0.49 | 0.10 |
| | GJR (N) | 1.33% | **0.14** | 0.00 | 0.00 | -1.66 | 0.02 |
| | GJR (T) | 1.74% | 0.00 | 0.00 | 0.00 | -1.44 | 0.03 |
| | QR | 0.27% | 0.00 | 0.00 | 0.00 | 0.28 | 0.05 |
| | HS200d | 5.75% | 0.00 | 0.00 | 0.00 | -0.71 | 0.08 |
| | HS100d | 7.15% | 0.00 | 0.00 | 0.00 | -0.76 | 0.07 |
| | HS60d | 10.73% | 0.00 | 0.00 | 0.00 | -0.62 | 0.08 |
| | HS30d | 16.02% | 0.00 | 0.00 | 0.00 | -0.53 | 0.09 |
| | FHS | 2.98% | 0.00 | 0.00 | 0.00 | -1.30 | 0.03 |
| | EVT | 2.89% | 0.00 | 0.00 | 0.00 | -1.31 | 0.03 |
| 4 | GARCH (N) | 2.66% | 0.00 | 0.00 | 0.00 | -0.81 | 0.02 |
| | GARCH (T) | 3.12% | 0.00 | 0.00 | 0.00 | -0.82 | 0.02 |
| | EGARCH (N) | 2.38% | 0.00 | 0.00 | 0.00 | -0.79 | 0.01 |
| | EGARCH (T) | 5.55% | 0.00 | 0.00 | 0.00 | -0.68 | 0.02 |
| | GJR (N) | 1.47% | **0.04** | 0.00 | 0.00 | -1.02 | 0.01 |
| | GJR (T) | 3.12% | 0.00 | 0.00 | 0.00 | -0.82 | 0.02 |
| | QR | 0.14% | 0.00 | **0.94** | 0.00 | -0.42 | 0.05 |
| | HS200d | 8.12% | 0.00 | 0.00 | 0.00 | -0.54 | 0.08 |
| | HS100d | 11.23% | 0.00 | 0.00 | 0.00 | -0.49 | 0.07 |
| | HS60d | 13.47% | 0.00 | 0.00 | 0.00 | -0.44 | 0.07 |
| | HS30d | 17.37% | 0.00 | 0.00 | 0.00 | -0.41 | 0.08 |
| | FHS | 3.76% | 0.00 | 0.00 | 0.00 | -0.91 | 0.02 |
| | EVT | 3.48% | 0.00 | 0.00 | 0.00 | -0.88 | 0.02 |
| 5 | GARCH (N) | 2.75% | 0.00 | 0.00 | 0.00 | -1.19 | 0.02 |
| | GARCH (T) | 3.26% | 0.00 | 0.00 | 0.00 | -1.13 | 0.02 |
| | EGARCH (N) | 3.58% | 0.00 | 0.00 | 0.00 | -1.08 | 0.03 |
| | EGARCH (T) | 5.78% | 0.00 | 0.00 | 0.00 | -0.90 | 0.03 |
| | GJR (N) | 2.29% | 0.00 | 0.00 | 0.00 | -1.33 | 0.02 |
| | GJR (T) | 3.26% | 0.00 | 0.00 | 0.00 | -1.13 | 0.02 |
| | QR | 0.00% | - | - | - | - | 0.05 |
| | HS200d | 6.11% | 0.00 | 0.00 | 0.00 | -0.67 | 0.09 |
| | HS100d | 7.45% | 0.00 | 0.00 | 0.00 | -0.61 | 0.07 |
| | HS60d | 11.83% | 0.00 | 0.00 | 0.00 | -0.54 | 0.08 |
| | HS30d | 18.27% | 0.00 | 0.00 | 0.00 | -0.46 | 0.10 |
| | FHS | 3.99% | 0.00 | 0.00 | 0.00 | -1.09 | 0.03 |
| | EVT | 4.64% | 0.00 | 0.00 | 0.00 | -1.05 | 0.03 |

## Table 2.12: VaR backtesting for CDS names with short history (10 days)

Backtest results for the CDS names with short history and considering a VaR prediction horizon of 10 days. Entries in parenthesis in column Model refer to the distribution used, namely Normal or Student's $t$. Percentage rate of violations for VaR at $\theta$=1%. When the % Viol is 0, the results for $LR_{uc}$ , $LR_{ind}$, DQ and CTE cannot be computed and are identified with "-". The cases where the GARCH estimation failed are identified with "n.a.". Entries in columns $LR_{uc}$, $LR_{ind}$ and DQ are the significance levels (p-values) of the respective tests. Bold type entries indicate p-values greater than 0.01, meaning that the null hypothesis cannot be rejected at the 1% significance level.

| CDS ID | Model | % Viol. | $LR_{uc}$ | $LR_{ind}$ | DQ | CTE | AQL |
|---|---|---|---|---|---|---|---|
| 6 | GARCH (N) | 3.37% | **0.08** | 0.00 | 0.00 | -6.03 | 1.55E+05 |
| | GARCH (T) | 20.22% | 0.00 | 0.00 | 0.00 | -4.27 | 0.71 |
| | EGARCH (N) | 6.74% | 0.00 | 0.00 | 0.00 | -5.75 | 1.67E+03 |
| | EGARCH (T) | 0.00% | - | - | - | - | 3.88E+122 |
| | GJR (N) | n.a. | n.a. | n.a. | n.a. | n.a. | n.a. |
| | GJR (T) | n.a. | n.a. | n.a. | n.a. | n.a. | n.a. |
| | QR | 11.24% | 0.00 | 0.00 | 0.00 | -5.75 | 0.16 |
| | HS30d | 34.43% | 0.00 | 0.00 | 0.00 | -1.69 | 0.43 |
| | FHS | 4.49% | **0.02** | 0.00 | 0.00 | -6.02 | 0.25 |
| | EVT | 8.99% | 0.00 | 0.00 | 0.00 | -5.18 | 0.31 |
| 7 | GARCH (N) | 6.94% | 0.00 | 0.00 | 0.00 | -3.75 | 0.28 |
| | GARCH (T) | 5.56% | 0.01 | 0.00 | 0.00 | -3.80 | 0.34 |
| | EGARCH (N) | 5.56% | 0.01 | 0.01 | 0.00 | -2.86 | 0.14 |
| | EGARCH (T) | 5.56% | 0.01 | 0.01 | 0.00 | -2.86 | 0.14 |
| | GJR (N) | n.a. | n.a. | n.a. | n.a. | n.a. | n.a. |
| | GJR (T) | n.a. | n.a. | n.a. | n.a. | n.a. | n.a. |
| | QR | 5.56% | 0.01 | 0.01 | 0.00 | -3.59 | 0.09 |
| | HS30d | 0.00% | - | - | - | - | 0.05 |
| | FHS | 8.33% | 0.00 | 0.00 | 0.00 | -3.23 | 0.29 |
| | EVT | 8.33% | 0.00 | 0.00 | 0.00 | -3.23 | 2.64E+06 |
| 8 | GARCH (N) | 4.62% | **0.03** | 0.00 | 0.00 | -5.11 | 0.16 |
| | GARCH (T) | 16.92% | 0.00 | 0.00 | 0.00 | -3.43 | 0.43 |
| | EGARCH (N) | 10.77% | 0.00 | 0.00 | 0.00 | -4.79 | 0.19 |
| | EGARCH (T) | 4.62% | **0.03** | 0.00 | 0.00 | -5.11 | 0.12 |
| | GJR (N) | 0.00% | - | - | - | - | 0.20 |
| | GJR (T) | 7.69% | 0.00 | 0.00 | 0.00 | -4.74 | 0.19 |
| | QR | 16.92% | 0.00 | 0.00 | 0.00 | -3.89 | 0.51 |
| | HS30d | 18.92% | 0.00 | 0.00 | 0.00 | -2.83 | 0.63 |
| | FHS | 4.62% | **0.03** | 0.00 | 0.00 | -5.11 | 0.41 |
| | EVT | 4.62% | **0.03** | 0.00 | 0.00 | -5.11 | 0.35 |
| 9 | GARCH (N) | 15.87% | 0.00 | 0.00 | 0.00 | -4.66 | 1.33 |
| | GARCH (T) | 23.81% | 0.00 | 0.00 | 0.00 | -3.17 | 0.79 |
| | EGARCH (N) | 15.87% | 0.00 | 0.00 | 0.00 | -4.66 | 0.70 |
| | EGARCH (T) | 15.87% | 0.00 | 0.00 | 0.00 | -4.66 | 0.63 |
| | GJR (N) | 15.87% | 0.00 | 0.00 | 0.00 | -4.66 | 3.40 |
| | GJR (T) | 15.87% | 0.00 | 0.00 | 0.00 | -4.66 | 5.11 |
| | QR | 42.86% | 0.00 | 0.00 | 0.00 | -2.09 | 0.85 |
| | HS30d | 17.14% | 0.00 | 0.00 | 0.00 | -2.65 | 0.72 |
| | FHS | 0.00% | - | - | - | - | 0.30 |
| | EVT | 0.00% | - | - | - | - | 0.27 |
| 10 | GARCH (N) | 6.79% | 0.00 | 0.00 | 0.00 | -13.95 | 5.27 |
| | GARCH (T) | 21.72% | 0.00 | 0.00 | 0.00 | -8.08 | 1.93 |
| | EGARCH (N) | 3.17% | 0.01 | 0.00 | 0.00 | -13.59 | 33.44 |
| | EGARCH (T) | 34.84% | 0.00 | 0.00 | 0.00 | -6.03 | 1.50 |
| | GJR (N) | 0.90% | **0.89** | **0.95** | **1.00** | -15.25 | 5.73 |
| | GJR (T) | 18.55% | 0.00 | 0.00 | 0.00 | -8.89 | 1.27 |
| | QR | 10.41% | 0.00 | 0.00 | 0.00 | -6.02 | 0.36 |
| | HS30d | 53.37% | 0.00 | 0.00 | 0.00 | -4.74 | 2.37 |
| | FHS | 0.45% | **0.36** | **0.94** | **0.95** | -1.67 | 48.56 |
| | EVT | 0.45% | **0.36** | **0.94** | **0.95** | -1.67 | 33.94 |

## Table 2.13: VaR backtesting for CDS names with long history (10 days)

Backtest results for the CDS names with long history and considering a VaR prediction horizon of 10 days. Entries in parenthesis in column Model refer to the distribution used, namely Normal or Student's $t$. Percentage rate of violations for VaR at $\theta$=1%. When the % Viol is 0, the results for $LR_{uc}$ , $LR_{ind}$, DQ and CTE cannot be computed and are identified with "-". The cases where the GARCH estimation failed are identified with "n.a.". Entries in columns $LR_{uc}$, $LR_{ind}$ and DQ are the significance levels (p-values) of the respective tests. Bold type entries indicate p-values greater than 0.01, meaning that the null hypothesis cannot be rejected at the 1% significance level.

| CDS ID | Model | % Viol. | $LR_{uc}$ | $LR_{ind}$ | DQ | CTE | AQL |
|---|---|---|---|---|---|---|---|
| 1 | GARCH (N) | 3.76% | 0.00 | 0.00 | 0.00 | -1.96 | 0.04 |
| | GARCH (T) | 2.93% | 0.00 | 0.00 | 0.00 | -2.10 | 0.03 |
| | EGARCH (N) | 3.41% | 0.00 | 0.00 | 0.00 | -2.04 | 0.04 |
| | EGARCH (T) | 4.42% | 0.00 | 0.00 | 0.00 | -1.87 | 0.04 |
| | GJR (N) | 3.28% | 0.00 | 0.00 | 0.00 | -2.10 | 0.03 |
| | GJR (T) | 2.93% | 0.00 | 0.00 | 0.00 | -2.10 | 0.03 |
| | QR | 0.13% | 0.00 | 0.00 | 0.00 | -0.10 | 0.05 |
| | HS200d | 6.47% | 0.00 | 0.00 | 0.00 | -1.26 | 0.08 |
| | HS100d | 12.11% | 0.00 | 0.00 | 0.00 | -0.96 | 0.10 |
| | HS60d | 16.43% | 0.00 | 0.00 | 0.00 | -0.91 | 0.14 |
| | HS30d | 24.53% | 0.00 | 0.00 | 0.00 | -0.77 | 0.16 |
| | FHS | 2.62% | 0.00 | 0.00 | 0.00 | -1.98 | 0.03 |
| | EVT | 1.40% | **0.07** | 0.00 | 0.00 | -2.32 | 0.03 |
| 2 | GARCH (N) | 2.74% | 0.00 | 0.00 | 0.00 | -1.34 | 0.02 |
| | GARCH (T) | 0.49% | 0.01 | 0.00 | 0.00 | -1.29 | 0.02 |
| | EGARCH (N) | 3.05% | 0.00 | 0.00 | 0.00 | -1.36 | 0.02 |
| | EGARCH (T) | 66.17% | 0.00 | 1.00 | 0.00 | -0.36 | 0.27 |
| | GJR (N) | 3.01% | 0.00 | 0.00 | 0.00 | -1.38 | 0.02 |
| | GJR (T) | 0.67% | **0.10** | 0.00 | 0.00 | -1.26 | 0.02 |
| | QR | 0.18% | 0.00 | **0.98** | 0.00 | -1.18 | 0.05 |
| | HS200d | 10.26% | 0.00 | 0.00 | 0.00 | -0.87 | 0.09 |
| | HS100d | 15.84% | 0.00 | 0.00 | 0.00 | -0.74 | 0.09 |
| | HS60d | 21.59% | 0.00 | 0.00 | 0.00 | -0.66 | 0.11 |
| | HS30d | 26.43% | 0.00 | 0.00 | 0.00 | -0.60 | 0.13 |
| | FHS | 4.85% | 0.00 | 0.00 | 0.00 | -1.38 | 0.03 |
| | EVT | 4.72% | 0.00 | 0.00 | 0.00 | -1.41 | 0.03 |
| 3 | GARCH (N) | 2.43% | 0.00 | 0.00 | 0.00 | -2.38 | 0.04 |
| | GARCH (T) | 2.29% | 0.00 | 0.00 | 0.00 | -2.43 | 0.04 |
| | EGARCH (N) | 2.57% | 0.00 | 0.00 | 0.00 | -2.33 | 0.04 |
| | EGARCH (T) | 26.72% | 0.00 | **1.00** | 0.00 | -1.09 | 0.19 |
| | GJR (N) | 2.34% | 0.00 | 0.00 | 0.00 | -2.46 | 0.03 |
| | GJR (T) | 2.34% | 0.00 | 0.00 | 0.00 | -2.41 | 0.04 |
| | QR | 0.27% | 0.00 | 0.00 | 0.00 | 0.03 | 0.05 |
| | HS200d | 8.22% | 0.00 | 0.00 | 0.00 | -1.57 | 0.14 |
| | HS100d | 12.43% | 0.00 | 0.00 | 0.00 | -1.42 | 0.15 |
| | HS60d | 18.17% | 0.00 | 0.00 | 0.00 | -1.22 | 0.17 |
| | HS30d | 26.74% | 0.00 | 0.00 | 0.00 | -1.04 | 0.21 |
| | FHS | 2.52% | 0.00 | 0.00 | 0.00 | -2.36 | 0.04 |
| | EVT | 2.52% | 0.00 | 0.00 | 0.00 | -2.44 | 0.04 |
| 4 | GARCH (N) | 2.71% | 0.00 | 0.00 | 0.00 | -2.01 | 0.03 |
| | GARCH (T) | 3.21% | 0.00 | 0.00 | 0.00 | -1.91 | 0.03 |
| | EGARCH (N) | 2.61% | 0.00 | 0.00 | 0.00 | -2.02 | 0.03 |
| | EGARCH (T) | 5.82% | 0.00 | 0.00 | 0.00 | -1.57 | 0.04 |
| | GJR (N) | 2.66% | 0.00 | 0.00 | 0.00 | -2.08 | 0.03 |
| | GJR (T) | 3.21% | 0.00 | 0.00 | 0.00 | -1.91 | 0.03 |
| | QR | 0.41% | 0.00 | 0.00 | 0.00 | -1.52 | 0.05 |
| | HS200d | 11.75% | 0.00 | 0.00 | 0.00 | -1.20 | 0.15 |
| | HS100d | 18.96% | 0.00 | 0.00 | 0.00 | -0.97 | 0.15 |
| | HS60d | 22.56% | 0.00 | 0.00 | 0.00 | -0.89 | 0.16 |
| | HS30d | 28.29% | 0.00 | 0.00 | 0.00 | -0.80 | 0.19 |
| | FHS | 4.31% | 0.00 | 0.00 | 0.00 | -1.95 | 0.04 |
| | EVT | 4.13% | 0.00 | 0.00 | 0.00 | -1.95 | 0.04 |
| 5 | GARCH (N) | 4.04% | 0.00 | 0.00 | 0.00 | -2.27 | 0.04 |
| | GARCH (T) | 4.50% | 0.00 | 0.00 | 0.00 | -2.20 | 0.05 |
| | EGARCH (N) | 4.82% | 0.00 | 0.00 | 0.00 | -2.12 | 0.05 |
| | EGARCH (T) | 8.03% | 0.00 | 0.00 | 0.00 | -1.77 | 0.07 |
| | GJR (N) | 4.13% | 0.00 | 0.00 | 0.00 | -2.29 | 0.04 |
| | GJR (T) | 4.64% | 0.00 | 0.00 | 0.00 | -2.16 | 0.05 |
| | QR | 0.09% | 0.00 | **0.89** | 0.00 | -4.50 | 0.05 |
| | HS200d | 9.19% | 0.00 | 0.00 | 0.00 | -1.49 | 0.16 |
| | HS100d | 12.40% | 0.00 | 0.00 | 0.00 | -1.24 | 0.15 |
| | HS60d | 18.53% | 0.00 | 0.00 | 0.00 | -1.08 | 0.18 |
| | HS30d | 29.43% | 0.00 | 0.00 | 0.00 | -0.90 | 0.23 |
| | FHS | 4.50% | 0.00 | 0.00 | 0.00 | -2.43 | 0.05 |
| | EVT | 5.00% | 0.00 | 0.00 | 0.00 | -2.33 | 0.06 |

# Conclusion

This thesis presents two applications of Value at Risk (VaR) estimation: Credit VaR and VaR in Credit Default Swaps.

I compare Credit Value at Risk estimates based on different correlation assumptions, using Gaussian and t copulas, with the observed loss in a credit portfolio of a Portuguese financial institution, for a time series of 72 monthly observations, covering the period between 2004 and 2009. The correlation assumptions tested in the study were inspired in rating agencies methodologies to evaluate Collateralized Debt Obligations, empirical estimator suggested by De Servigny and Renault (2002a) and Basel III Accord. In order to estimate Credit VaR, I simulate portfolio value distribution with Monte Carlo simulation technique, within the Merton model framework. I show that Credit VaR estimates are very sensitive to assumptions regarding asset correlation and dependence structure, reinforcing the crucial role played by correlation in credit loss estimates. I also provide empirical evidence that some of the assumptions made by rating agencies to evaluate CDOs are inadequate in stress situations like the financial crisis observed in 2008.

All Credit VaR estimates were compared using backtesting procedures as Kupiec (1995) and Christoffersen (1998) tests, the Loss Function proposed by Lopez (1998), the Average Quantile Loss proposed by Koenker and Basset (1978) and also a measure of over-conservativeness proposed in this study. I find that the most accurate Credit VaR model for this portfolio is based on asset correlation given by the empirical estimator proposed by De Servigny and Renault (2002a) and assuming a dependence structure given by the *t* copula with 8 degrees of freedom. All conclusions of the study are invariant to the assumption of deterministic instead of stochastic recovery rate.

Regarding the application of VaR models to Credit Default Swaps, I estimate VaR in CDS using several estimation methods: Quantile Regression, Historical Simulation, Filtered Historical Simulation, Extreme Value Theory and several GARCH-based models. I use market-based and accounting-based factors as determinants of CDS spreads, namely stock return and stock price volatility and also financial ratios such as leverage, return on assets and liquidity. The analysis of the determinants of CDS spreads is based on 242 different reference entities and the time period ranges from September 2001 to April 2011, covering the period of the recent financial crisis.

In order to identify the most accurate VaR model I compare the results obtained with Quantile Regression with those obtained with other estimation methods through the application of backtesting methodologies such as the tests proposed by Kupiec (1995) and Christoffersen (1998), the Average Quantile Loss Function proposed by Koenker and Bassett (1978), the Conditional Tail Expectation proposed by Artzner *et al* (1999) and the Dynamic Quantile Test presented by Engle and Manganelli (2004). To the best of my knowledge this is the first time that backtesting methodologies are applied to compare different methods of estimating VaR in CDS.

I find that Quantile Regression provides better results in the estimation of VaR in CDS than Historical Simulation, Filtered Historical Simulation, Extreme Value Theory and all GARCH-based models tested in this study, especially for CDS names with long history when the forecast horizon of VaR estimates is 30 days and for CDS names with short history when the forecast horizon of VaR estimates is 1 day. I also find that the financial ratios proposed by Campbell *et al* (2008) to determine the risk of bankruptcy and failure contribute to explain the determinants of the price of CDS. Recent studies have shown that Filtered Historical Simulation and Extreme Valued Theory are the most accurate VaR models. However, the empirical evidence provided in this study does not support the extension of this finding to VaR estimation in CDS.

# Appendix A

To simplify the analysis I assume that the default indicator $I_{i,t}$ is exchangeable. A random vector **S** is called exchangeable if

$$(S_1, ...S_m) \stackrel{d}{=} (S_{\Pi(1)}, ..., S_{\Pi(M)})$$

for any permutation $(\Pi(1), ...\Pi(m))$ of $(1, ..., m)$. The consequence is that for any $j \in \{1, ..., m-1\}$ all of the $\binom{m}{j}$ possible j-dimensional marginal distributions of **S** are identical. In this situation, default probabilities and joint default probabilities are given by

$$\pi_j := P(I_{i_1} = 1, ..., I_{i_j} = 1), \ \ \{i_1, ..., i_j\} \subset \{1, ..., m\}, 1 \leq j \leq m,$$

$$\pi := \pi_1 = P(I_i = 1), \ \ i \in \{1, ..., m\}$$

The $j$th order joint default probability, $\pi_j$, is the probability that an arbitrarily selected subgroup of j obligors defaults.

I will now introduce the definition of Bernoulli mixture model and explain under which conditions this model is exchangeable. This setup is the mathematical base of the joint default probability estimator. For this purpose, consider a generic exchangeable group of N obligors where D obligors default.

**Definition 1** *(Bernoulli Mixture Model) Given some $p < N$ and a $p-$dimensional random vector $\Psi = (\Psi_1, ..., \Psi_p)$, the random vector $I = (I_1, ..., I_N)'$ follows a Bernoulli mixture model with factor vector $\Psi$ if there are functions $Q_i : R^p \rightarrow [0, 1], 1 \leq i \leq N$, such that conditional on $\Psi$ the default indicator I is a vector of independent Bernoulli random variables with $P(I_i = 1|\Psi) = Q_i(\Psi)$.*

A Bernoulli mixture model is exchangeable if the functions $Q_i$ are all identical and, in that case, the vector I is exchangeable. Considering the random variable $Q := Q_1(\Psi)$ we get for $I = (I_1, ..., I_N)'$ in $\{0, 1\}^N$

$$P(\mathbf{I} = I|\Psi) = Q_1(\Psi)^{\sum_{i=1}^{N} I_i}(1 - Q_1(\Psi))^{N - \sum_{i=1}^{N} I_i} = P(\mathbf{I} = I|Q)$$

and, in particular, $P(I_1 = 1|Q) = Q$. Denote by G(q) the distribution function of Q. The unconditional distributions of I and of the number of defaults are given by

$$p(I) = \int_0^1 q^{\sum_{i=1}^N I_i}(1-q)^{N-\sum_{i=1}^N I_i}\mathrm{d}G(q) \tag{15}$$

$$P(D=j) = \binom{N}{j}\int_0^1 q^j(1-q)^{N-j}\mathrm{d}G(q) \tag{16}$$

Further calculation give

$$\pi_j = P(I_1 = 1, ..., I_j = 1) = E(E(I_1, ..., I_j|Q)) = E(Q^j) \tag{17}$$

which means that unconditional default probabilities of first and higher order can be seen as moments of the mixing distribution. Following Frey and McNeil (2001), the following proposition holds.

**Proposition 1** *Define the random variable*

$$\binom{D}{j} := \binom{D}{j}^{(N)} := \begin{cases} \frac{D!}{j!(D-j)!} & 1 \leq j \leq D, \\ 0 & j > D \end{cases}$$

*to be the number of possible subgroups of j obligors in the D defaulting obligors. Then*

$$E\binom{D}{j} = \binom{N}{j}E(Q^j) = \binom{N}{j}\pi_j, \quad 1 \leq j \leq N,$$

For more details, please see Frey and McNeil (2001).

# Appendix B

## Table 14: Monthly Regressions (1-7)

Results from quantile regression model for the CDS price. This table shows the estimated parameters using monthly data samples for the 99 quantile. * refers to p-values smaller than 0.05, ** refers to p-values smaller than 0.01, *** refers to p-values smaller than 0.001. Full results of the regressions, including the time dummies, are available upon request.

| **CDS Price** | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Regression Id** | | | | | | | |
| **Variable** | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| bid ask spread | 17.87*** | 17.74*** | 17.44*** | 16.92*** | 16.70*** | 16.28*** | 16.32*** |
| stock return | -202.5*** | -172.1*** | -195.7*** | -225.7*** | -223.5*** | -191.4*** | -190.1*** |
| stock volatility | 755.4*** | 719.6*** | 651.5*** | 617.0*** | 590.3*** | 572.1*** | 554.4*** |
| return on assets$_h$ | -1751*** | -1699*** | -1646*** | -1644*** | -1657*** | -1692*** | -1670*** |
| leverage$_h$ | 0.573*** | 0.595*** | 0.634*** | 0.631*** | 0.587*** | 0.530*** | 0.485*** |
| constant | -240.0*** | -229.6*** | -209.4*** | -195.5*** | -185.4*** | -175.0*** | -170.7*** |
| N | 198,959 | 202,177 | 205,899 | 209,499 | 212,800 | 216,766 | 220,740 |

## Table 15: Monthly Regressions (8-14)

Results from quantile regression model for the CDS price. This table shows the estimated parameters using monthly data samples for the 99 quantile. * refers to p-values smaller than 0.05, ** refers to p-values smaller than 0.01, *** refers to p-values smaller than 0.001. Full results of the regressions, including the time dummies, are available upon request.

| **CDS Price** | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Regression Id** | | | | | | | |
| **Variable** | (8) | (9) | (10) | (11) | (12) | (13) | (14) |
| bid ask spread | 16.36*** | 16.36*** | 16.37*** | 16.32*** | 16.43*** | 16.70*** | 17.01*** |
| stock return | -187.9*** | -187.9*** | -164.2*** | -150.3*** | -146.6*** | -161.2*** | -184.6*** |
| stock volatility | 537.4*** | 522.2*** | 504.5*** | 493.1*** | 479.1*** | 483.8*** | 489.9*** |
| return on assets$_h$ | -1622*** | -1595*** | -1563*** | -1538*** | -1507*** | -1010*** | -686*** |
| leverage$_h$ | 0.464*** | 0.460*** | 0.448*** | 0.447*** | 0.438*** | 0.477*** | 0.502*** |
| constant | -167.4*** | -163.8*** | -159.2*** | -156.0*** | -153.7*** | -172.1*** | -186.2*** |
| N | 224,513 | 228,281 | 232,226 | 235,779 | 239,721 | 243,169 | 246,567 |

Table 16: Monthly Regressions (15-21)

Results from quantile regression model for the CDS price. This table shows the estimated parameters using monthly data samples for the 99 quantile. * refers to p-values smaller than 0.05, ** refers to p-values smaller than 0.01, *** refers to p-values smaller than 0.001. Full results of the regressions, including the time dummies, are available upon request.

| **CDS Price** | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Regression Id** | | | | | | | |
| **Variable** | (15) | (16) | (17) | (18) | (19) | (20) | (21) |
| bid ask spread | 17.07*** | 17.09*** | 17.11*** | 17.24*** | 17.49*** | 17.67*** | 17.83*** |
| stock return | -181.2*** | -181.3*** | -175.2*** | -179.4*** | -157.8*** | -152.9*** | -158.3*** |
| stock volatility | 492.1*** | 494.1*** | 487.7*** | 472.1*** | 459.7*** | 442.7*** | 428.7*** |
| return on assets$_h$ | -659.5*** | -663.6*** | -666.7*** | -668.1*** | -658.7*** | -673.4*** | -676.2*** |
| leverage$_h$ | 0.505*** | 0.505*** | 0.498*** | 0.488*** | 0.465*** | 0.443*** | 0.424*** |
| constant | -188.1*** | -188.7*** | -186.8*** | -183.4*** | -181.9*** | -177.9*** | -175.2*** |
| N | 250,693 | 254,481 | 258,045 | 261,888 | 265,685 | 269,679 | 273,500 |

Table 17: Monthly Regressions (22-28)

Results from quantile regression model for the CDS price. This table shows the estimated parameters using monthly data samples for the 99 quantile. * refers to p-values smaller than 0.05, ** refers to p-values smaller than 0.01, *** refers to p-values smaller than 0.001. Full results of the regressions, including the time dummies, are available upon request.

| **CDS Price** | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Regression Id** | | | | | | | |
| **Variable** | (22) | (23) | (24) | (25) | (26) | (27) | (28) |
| bid ask spread | 17.97*** | 18.19*** | 18.36*** | 18.54*** | 18.53*** | 18.66*** | 18.66*** |
| stock return | -156.5*** | -152.6*** | -152.0*** | -141.1*** | -147.5*** | -153.8*** | -160.0*** |
| stock volatility | 420.6*** | 412.1*** | 402.3*** | 392.2*** | 389.2*** | 388.0*** | 389.7*** |
| return on assets$_h$ | -651.0*** | -605.0*** | -594.1*** | -575.0*** | -576.0*** | -606.5*** | -605.4*** |
| leverage$_h$ | 0.418*** | 0.412*** | 0.403*** | 0.365*** | 0.365*** | 0.357*** | 0.361*** |
| constant | -174.7*** | -175.4*** | -174.3*** | -173.2*** | -172.3*** | -172.2*** | -172.9*** |
| N | 277,280 | 281,078 | 284,999 | 288,555 | 291,938 | 296,132 | 298,172 |

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