Infinite Servers Queue Systems Busy Period -

A Practical Case on Logistics Problems Solving

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Abstract

In this paper it is exemplified how the busy period of an infinite servers queue is applied to the equipments failures management. With this model it is possible to obtain system performance measures and also to contribute to solve organizing structures' problems, by minimizing the risks of the organizations inoperative structures, with considerable logistics pernicious consequences for companies and often also for the regions where the companies are inserted.

Keywords: Queue Model, Logistics, Equipments Management, Economic Efficiency

1 Introduction

Situations of scarce resources demand very rigorous management rules. Structures management requires strong capabilities and competences. Solving logistics machine failures problems becomes crucial on this context.

In this model with $M|G|\infty$ queuing systems, customers arrive according to a Poisson process at rate λ. Immediately after each one's arrival, it receives a service which length is a positive random variable with distribution function G (.) and mean value α. An important system parameter is the traffic intensity $\rho = \lambda\alpha$. The service of a customer is independent from the other customers' services and from the arrivals process. The busy period of a queuing system begins when a

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customer arrives, finding it empty, and ends when a customer leaves the system letting it empty. During the busy period there is always at least one customer in the system. Formulae that allow the calculation of some of the busy period length parameters for the $M|G|\infty$ queuing system are presented in the next section. These results can be applied in Logistics (see, for instance [4, 5, 6, 10, 13], being the customers the failures occurred in the equipments. The service time in the queue is the time that a machine is idle waiting for reparation or being repaired. See also the operations cases in aircraft, shipping or trucking fleets in [9].

2 Some $M|G|\infty$ Queue System Parameters

In an $M|G|\infty$ queuing system there are neither losses nor waiting. In fact there is no queuing in the common sense. For these systems, to study the population process is not particularly important as for other systems with losses or waiting. Generally it is much more interesting the study of some other processes as, for instance, the busy period. The results related to the busy period length of the $M|G|\infty$ queuing system permit to find performance evaluation measures for the equipments. An illustration will be presented in this study, considering a very simple and short numerical example.

Being B the $M|G|\infty$ queuing system busy period length (see [8]), the mean value of B, whatever is G(.), is given in [15] as

$$\mathrm{E}[B] = \frac{e^\rho - 1}{\lambda} \tag{2.1}.$$

Calling now VAR [B] to the variance of B, it can be seen that it depends largely on the form of B. Anyway in [14] it is stated that:

$$\lambda^{-2} \max\left[e^{2\rho} + e^\rho \ \rho^2 \gamma_s^2 - 2\rho e^\rho - 1; 0 \right] \leq VAR[B] \leq \lambda^{-2} \left(2e^\rho \left(\gamma_s^2 + 1 \right)\left(e^\rho - 1 - \rho \right) - \left(e^\rho - 1 \right)^2 \right)$$
$$\tag{2.2}$$

where γ_s is the variation coefficient of G(.).

Considering now *R(t)* the mean number of busy periods that begin in [0,t] (being *t* = 0 the beginning of a busy period), after [2], it is possible to show that

$$e^{-\rho}(1 + \lambda t) \leq R(t) \leq 1 + \lambda t \tag{2.3},$$

and that, see also [2], if the service time distribution function is $G_1(t) = \frac{e^{-\rho}}{(1-e^{-\rho})e^{-\lambda t}+e^{-\rho}}, t \geq 0,$

$$\text{VAR[B]} = \frac{e^{2\rho}-1}{\lambda^2},$$
$$R(t) = 1 + \lambda e^{-\rho}t \tag{2.4};$$

if it is $G_2(t) = 1 - \dfrac{1}{1-e^{-\rho}+e^{-\rho+\frac{\lambda}{1-e^{-\rho}}t}}, t \geq 0$,

$$\text{VAR[B]} = \frac{(e^{\rho}-1)^2}{\lambda^2},$$
$$\tag{2.5}.$$
$$R(t) = e^{-\rho} + (1-e^{-\rho})^2 + \lambda e^{-\rho}t + e^{-\rho}(1-e^{-\rho})e^{-\frac{\lambda}{1-e^{-\rho}}t}$$

Denote N_B the mean number of the customers served during a busy period in the $M|G|\infty$ queuing systems. Considering the exposed in [3], if G (.) is

$$-Exponential$$
$$N_B^M = e^{\rho}$$
$$-Any\ other\ service\ distribution \tag{2.6}.$$
$$N_B \cong \frac{e^{\left(\rho(\gamma_s^2+1)\right)}(\rho(\gamma_s^2+1)+1)+\rho(\gamma_s^2+1)-1}{2\rho(\gamma_s^2+1)}$$

A busy period is a period, in which there is at least one failure waiting for reparation or being repaired; and an idle period is a period in which there are no failures in the system.

Here some simple expressions - that allow computing the mean and bounds to the variance of the busy period - were given; and also simple bounds to the mean number of busy periods that begin in a certain interval time. Finally, expressions to the mean number of failures that occur in a busy period were also presented.

These formulae are very simple and of evident application. They only require the knowledge of α, λ, ρ and γ_s that are very easy to compute. The only problem is to test the hypothesis that the failures occur according to a Poisson process.

Note yet that, calling I(t) the idle period of the $M|G|\infty$ queuing system distribution function, $I(t) = 1 - e^{-\lambda t}$, as it happens for any queue with arrival Poisson process. In this application it gives the probability that the time length with no failures is lesser or equal than t.

3 A Short Numerical Case

Let's suppose a fleet (or any machine's system) where the failures occur at a rate of 20 per year. So $\lambda = 20/year$. Suppose also that the mean time to repair a failure is 4 days ($\alpha = 4$ day $= (4/365)$ year). In consequence $\rho \cong 0.22$.

Consider the possibility of decreasing ρ to 0.11. Either making $\lambda = 10$/year, for instance buying more items and decreasing, in consequence, each one use intensity. Or making $\alpha = 2$ day, for instance increasing the teams affected to the failures repairs.

On the contrary, if nothing is changed, things can get worse and maybe ρ jump to 0.44. The values 0.88 and 1^2 are also considered.

If it is supposed that the repair services times are exponential[3] $\gamma_s = 1$, and after (2.1), (2.2), (2.3) and (2.6), with t=1 year, being $SD[B] = \sqrt{VAR[B]}$,

Table 1 Example for exponential service times

ρ	E [B]	SD [B] (Lower Bound)	SD [B] (Upper Bound)	R (1) (Lower Bound)	R (1) (Upper Bound)	N_B^M
0.11	2.12 day	2.16 day	2.2 day	18	21	1.12
0.22	4.5 day	4.65 day	4.82 day	16	21	1.25
0.44	10 day	10.72 day	11.46 day	13	21	1.60
0.88	26 day	28.5 day	32 day	9	21	2.40
1.00	31 day	35 day	40 day	8	21	2.70

and it is possible to conclude, for these values, that when ρ increases, less busy periods in one year occur, with more failures in each one, of course in mean values. The busy period mean and dispersion length also increase with ρ.

If it is supposed now that the repair service times are constant (D = deterministic), $\gamma_s = 0$, and after (2.1), (2.2)[4], (2.3) and (2.6), with $t = 1$ year.

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[2] A neutral value for which the service rate equals the arrivals rate.
[3] A very frequent supposition assumed for this kind of services.
[4] In this case the lower bound is equal to the upper bound and so the real value of VAR [B] is got.

Table 2 Example for deterministic service times

ρ	E [B]	SD [B]	R (1) (Lower Bound)	R (1) (Upper Bound)	N_B^D
0.11	2.12 day	0.41 day	18	21	1.59
0.22	4.5 day	1.22 day	16	21	1.68
0.44	10 day	3.85 day	13	21	1.90
0.88	26 day	14 day	9	21	2.51
1.00	31 day	18 day	8	21	3.09

E [B] and the R (1) bounds are the same that in the former case, evidently. The behavior of the parameters with the increase of ρ is similar to the one of the exponential situation. But now the busy period length dispersion is much lesser and the mean value of failures in each busy period is greater.

As for the service times with distribution functions $G_1(t)$ and $G_2(t)$, it is not possible to present results for N_B because there is not an efficient formula to calculate γ_s. But $SD[B]$ and $R(1)$ are exactly calculated after (2.4) and (2.5) for $G_1(t)$ and $G_2(t)$, respectively.

Table 3 Example for service times with $G_1(t)$ distribution function

ρ	E [B]	SD [B]	R (1)
0.11	2.12 day	9.1 day	19
0.22	4.5 day	13.57 day	17
0.44	10 day	21.68 day	14
0.88	26 day	40 day	9
1.00	31 day	46.13 day	8

Table 4 Example for service times with $G_2(t)$ distribution function

ρ	E [B]	SD [B]	R (1)
0.11	2.12 day	2.12 day	19
0.22	4.5 day	4.5 day	17
0.44	10 day	10 day	13
0.88	26 day	26 day	9
1.00	31 day	31 day	8

Note that for $G_1(t)$ service time distribution function the busy period is exponentially distributed with an atom at the origin. For $G_2(t)$ service time distribution function the busy period is purely exponential. Anyway, in both cases, for these traffic intensity values, it is possible to conclude that the busy period mean and dispersion length also increase with ρ.

4 Concluding Remarks

When operating a fleet[5] managers are interested in big idle periods and little busy periods. And if these busy periods occur they prefer that they are as rare as possible, with the shortest number of failures possible.

If he/she knows α, λ, ρ and γ_s the manager of the fleet can evaluate the quality of the operation, namely in terms of :

➢　The mean length of a period with failures,

➢　The length dispersion of a period with failures,

➢　The mean number of periods with failures that will occur in a certain length of time,

➢　The mean number of failures that occur in a period with failures.

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[5] Or any company.

As the expressions depend only on a few parameters, very simple to obtain and interpret, they show tracks to improve the operation, although they may be hard to implement depending on the company capabilities.

In the context of recent financial and economic crisis, numerical reliable indicators are very important because they allow defining good managing politics and practices. Besides their simplicity, the ones proposed in this paper own this reliability property.

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