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When sociable workers pay off: can firms internalize social capital externalities?*

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Abstract

We use an endogenous growth model to contrast the socially optimal allocation of human capital with the decentralized solution, in a context where workers make the choices that determine social capital accumulation. As social capital is expected to increase productivity but is not traded in markets, a positive social capital externality is identified. We discuss the possibility that, in response to this externality, firms subsidize social capital accumulation activities, incurring into additional costs that are recouped through productivity gains. This reaction by firms may be seen as a justification for some Corporate Social Responsibility actions targeted at workers, although a full internalization of the externality does not look achievable in practice.

JEL Classification: M14, O15, O41, Z13.

Keywords: Corporate Social Responsibility, Social Capital, Human Capital, Economic Growth.

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1 Introduction

Worker productivity is crucial for economic growth. Traditional economic models consider the role of physical capital and labour in production, whereas the importance of human capital accumulation, in a wider sense, has been the focus of endogenous growth theory. Social capital is a fairly recent addition to growth models, where it can represent the impact of trust and social networks on productivity and hence on growth. Since there is no specific market for social capital, its decentralized accumulation will generally not be optimal, which means that a social capital externality exists. In fact, in a static world without capital accumulation, the firm could adjust wages so as to induce workers to choose their time allocation optimally. Thus, the social capital externality is essentially a dynamic phenomenon, in which firms and households benefit from an intangible investment for which a market does not exist. Roseta-Palma et al [28] develop an endogenous growth model with natural and social capital where the interaction between these types of capital is studied. In this paper, on the other hand, we investigate whether the social capital externality can act as an incentive for firms to increase their corporate social responsibility (CSR) activities in response to the market failure. In this sense our work fits Heal's [17] suggestion that one of the roles of CSR is as "an institution that has evolved in response to market failures, a Coasian solution to some problems associated with social costs". Therefore, it is welfare-enhancing in all those sectors of economic activity where private and social costs are misaligned. In particular, the author specifically refers to "improved human relations and employee productivity" as one of the multiple benefits of CSR. This is the aspect we explore in the present work.

It should be stressed that ours is not the first attempt to link social capital and corporate social responsibility. Sacconi and Degli Antoni [29] provide a conceptual discussion of this relationship, described as a "virtuous circle that creates favorable conditions for socio-economic development". Furthermore, these are multifaceted concepts and we do not aim to provide a complete framework that could fully explain the relationship between them. For that, the reader is well-advised to head to a recent book, edited by the same authors [31], whose chapters vividly illustrate both the possible interconnections between the two notions and the range of methodological approaches that can be brought to bear on the analysis. We do, however, offer a short primer on the potential role of social capital in the context of an endogenous growth model as well as a brief discussion of previous work that introduces various aspects of CSR into economic literature.

Although there are many ways to define social capital, the one that best adapts to economic models, as noted by Sobel [33], is Bourdieu's 1986 explanation: "Social capital is an attribute of an individual in a social context. One can acquire social capital through purposeful actions and can transform social capital into conventional economic gains. The ability to do so, however, depends on the nature of the social obligations, connections, and networks available to you". A similar

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idea is expressed in Lin's notion of "investment in social relations with expected returns" [22]. We interpret the concept as something that individuals must build up through their choices and whose accumulation increases their utility as well as their productivity, thereby benefiting firms. For example, people might invest time in building up or maintaining a network of people (e.g. by organizing a conference or participating in a team-building corporate activity). In our model, social capital is a single asset which enters consumer utility in a dynamic, representative-agent economy, and we introduce the assumption that in order to accumulate this asset agents incur an opportunity cost in terms of human capital use. In contrast, Sacconi and Degli Antoni [29] use a game-theoretic approach to discuss the various dimensions of social capital, distinguishing cognitive elements, namely beliefs and dispositions, from structural elements that are cooperative linkages between agents. They propose that the general level of trust and cooperation, often associated with the concept of social capital at least since Putnam et al [27] and Knack and Keefer [21], should actually be seen as an effect of other elements. Alternative approaches to social capital include networks of relations, as in Lippert and Spagnolo [23], where interactions between agents are modelled as repeated games with different channels of information between players, and linked social-exchange and economic-exchange games, as in Aoki [1], which proposes that social norms are a result of the joint evolution of social networks and specific patterns of economic exchanges.

Corporate social responsibility (CSR), despite its prominence in management theory and business practice, is also a difficult concept to define. For instance, Lockett et al [24] suggest that even within the management literature CSR knowledge can be described as "a field without a dominant paradigm". Moreover, the use of a given definition for CSR implies a normative choice regarding the goals of corporations. Many economists, famously led by Milton Friedman [16], have held the view that corporations in competitive markets should only focus on profit-maximization, as that is their purpose, enabling them to serve consumers as well as to provide jobs. Linked to a traditional belief in the rationality of economic agents, so that any profitable actions are expected to be identified and implemented by competitive firms, this view minimizes the potential role of CSR and even deems it a harmful endeavour. Likewise, Jensen [19] defends maximization of the long-run value of the firm, equivalent to the discounted value of the future profit stream, as the single-valued objective that should be pursued, albeit taking into account the impact on all the firm's constituencies (stakeholders) so as to promote better management decisions. He does recognize, however, that externalities hamper the maximization of social welfare and recommends that Coasian solutions be found to correct for these deviations. [1] Our model fits in this framework of analysis.

There has been much less development of CSR in the economic literature than

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[1]Other popular views of CSR are stakeholder theory (see Freeman et al [14]) and the contractarian approach (which is described, among other approaches, in Sacconi et al [30]). A summarized review of the competing theories can be found in Melé [25].

in management, and most of what there is is microeconomic in focus. A few papers model CSR as an instance of private provision of public goods. Besley and Ghatak [7] introduce "caring" consumers who value the public good, whereas Baron [4] adds manager and shareholder preferences as well, in a principal-agent framework. Brekke and Nyborg [9] show that the existence of morally motivated workers (who prefer to work at a "green" firm and will have a higher productivity when they do) can justify firms' efforts to be socially responsible. There are also papers which focus on the relationship between CSR and economic performance as measured by existing productivity indicators (see Paul and Siegel, [26] and other articles in the same issue of the Journal of Productivity Analysis). In this paper we explore the role of CSR in growth models, a relatively neglected angle of analysis. The only other works that we are aware of are Dam [12] and Dam and Heijdra [13], which look at the implications for growth of socially motivated investors. The first paper shows that stock markets can deal with intergenerational environmental externalities, while the second finds that the presence of socially responsible investors lower the positive effects on environmental quality of public abatement activities. We, on the other hand, choose to focus on a different externality: the worker productivity effect. In particular, we assess whether CSR actions, undertaken by firms, can be an incentive to social capital accumulation performed at the workers' level. Such a link has been previously mentioned in the management literature, which relates CSR activities by the firm with socially and professionally motivated employees: for example, Turban and Greening [35] focus on employee attraction, while Heslin and Ochoa [18] describe the positive impact of CSR thus: "Besides enabling organizations to attract employees who are driven by socially conscious values, CSR can also keep them engaged and eager to become increasingly valuable to the organization."

In the following section we present the basic equations of our dynamic model. In section 3 we compare optimal allocations with the decentralized solution and identify the social capital externality, then (section 4) we propose a way to model the impact of CSR activities on social capital accumulation. Finally, section 5 concludes.

2 Model

In this section we present an endogenous growth model where production depends on the accumulation of several types of capital, namely physical capital, human capital, and social capital. The first two are well-known elements of growth models, but social capital has been introduced fairly recently. Like Chou [10], we propose in our model that social capital contributes directly to the formation of human capital, but we take a wider perspective of the potential benefits generated by this asset: we also assume that it affects firms' productivity by having a multiplicative effect on other inputs in the production function, which is coherent with Coleman [11] ("Just as physical capital and human capital facilitate productive activity, social capital does as well."); and finally, we consider that consumers attribute utility

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to the overall level of social capital, to reflect empirical research that suggests a positive correlation between measures of social capital and well-being (Bjornskov [8], Sarracino [32], Bartolini et al [6]).

Physical capital, K_P, is the physical asset base for production, and it is in turn accumulated through production that is not consumed but rather invested. K_P is, as usual, subject to depreciation:

$$\dot{K}_P = Y - C - \delta_P K_P \tag{1}$$

where Y denotes production of final goods, C is consumption, and δ_P represents depreciation.

Human capital, K_H, which can be interpreted broadly as knowledge, is an essential part of the model since it will be driving growth, as is customary in endogenous growth theory. Given that the stock of knowledge and skills can be applied in different ways, we divide human capital into several components, namely: H_Y, which represents the part of K_H that is used in final good production, H_H, which is dedicated to school attendance, and H_S, which reflects the skills that are relevant for social capital accumulation. Considering that these distinct human capital activities aren't done cumulatively, so that an allocative choice must be made, we have:

$$K_H = H_Y + H_H + H_S. \tag{2}$$

From a dynamic point of view, human capital accumulation will come from schooling, but we assume that the total amount of social capital, Ks, also provides a positive contribution, since it affects the educational outcomes that are achieved, as described extensively by Coleman [11] and Teachman et al [34]. This effect is also considered in Chou [10].

$$\dot{K}_H = \xi H_H + \alpha K_S - \delta_H K_H, \tag{3}$$

where H_H are school hours, $\xi > 0$ is a parameter that measures productivity in schooling, $\alpha \geq 0$ measures the contribution of social capital to human capital accumulation, and $\delta_H \geq 0$ is human capital depreciation. Note that this expression models human and social capital as substitutes in the production of human capital.

On the other hand, as noted above, part of the existing human capital stock will in turn be applied in social capital accumulation. We assume, furthermore, that the current stock of social capital will have an impact, so that:

$$\dot{K}_S = \omega H_S + \Omega K_S, \tag{4}$$

where ω measures the productivity of human capital in social capital production and $\Omega \lessgtr 0$ measures the dynamic effect of social capital on its own accumulation, which may be positive or negative. If $\Omega > 0$ the existing stock of social capital could keep growing without additional human capital, reflecting the notion of virtuous self-reinforcement, whereas if $\Omega < 0$ there is a net depreciation effect similar to those of other capital stocks.

The production of final goods will require all capital stocks in the economy to be used (physical, human, and social capital). Assuming a Cobb-Douglas technology, each exponent is the production elasticity of the associated input. We maintain the traditional assumption that there are constant returns to scale on the typical factors (physical capital and human capital), but then append social capital as an additional input, as a way to reflect the current emphasis on the existence of a positive connection between social capital and growth (see for example Knack and Keefer [21], Whiteley [36]) we use:

$$Y = K_P^\beta H_Y^\eta K_S^\sigma; \qquad \beta + \eta = 1. \tag{5}$$

Note that K_P and K_S fully contribute to final good production, but only the portion of human capital that is specifically dedicated to production is considered, since the remainder is used for different activities, as stated in equation (2).

Finally, as noted above we consider that, given the nature of social capital, consumers will improve their welfare if there is more of it. Note that if this were not the case, the positive externalities associated with social capital would be even larger since households would choose lower investment in social capital. Household preferences in our model thus include social capital as an argument of the intertemporal utility function, which will depend also on consumption:

$$U(C, K_S) = \frac{\tau}{\tau - 1} \int_0^\infty \left(C_t K_{St}^\psi \right)^{\frac{\tau-1}{\tau}} e^{-\rho t} dt \tag{6}$$

where τ is the intertemporal elasticity of substitution, ψ is the preference for social capital, and ρ is the utility discount rate, so that a higher ρ indicates more impatient consumers. The t subscripts are dropped in the remaining sections for ease of notation.

3 Results

3.1 Optimal Growth

Since social capital provides utility, while simultaneously acting as a production input, yet has no market price, the decentralized equilibrium solution will be characterized by market failure, i.e., aggregate welfare will not be maximized. In this section we identify the social capital externality by comparing the optimal solution to that which stems from the decentralized decisions of firms and individuals. We begin by solving a social planner's problem in order to characterize optimal growth. The problem can be stated as the maximization of (6) subject to the production function (5) as well as the transition equations for the different types of capital. It gives rise to the following Hamiltonian function:

$$\mathcal{H} = \frac{\tau}{\tau - 1} \left(C K_S^\psi \right)^{\frac{\tau-1}{\tau}} + \lambda_P \left[K_P^\beta K_S^\sigma H_Y^\eta - C - \delta_P K_P \right] + \tag{7}$$
$$+ \lambda_H \left[\xi (K_H - H_Y - H_S) + \alpha K_S - \delta_H K_H \right] + \lambda_S \left(\omega H_S + \Omega K_S \right)$$

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where the λ_j are the co-state variables for each stock K_j, with $j = P, H, S$. Considering choice variables C, H_Y, and H_S, and equation (2), the first order conditions yield:

$$\frac{\partial U}{\partial C} = \lambda_P; \tag{8}$$

$$\lambda_H = \lambda_P \frac{\eta Y}{H_Y \xi}; \tag{9}$$

$$\lambda_H = \frac{\lambda_S \omega}{\xi}; \tag{10}$$

as well as:

$$\rho \lambda_P - \dot{\lambda}_P = \lambda_P \frac{\beta Y}{K_P} - \lambda_P \delta_P; \tag{11}$$

$$\frac{\dot{\lambda}_H}{\lambda_H} = \rho + \delta_H - \xi; \tag{12}$$

$$\rho \lambda_S - \dot{\lambda}_S = \left(\frac{\partial U}{\partial K_S} + \frac{\lambda_P \sigma Y}{K_S} + \lambda_H \alpha + \lambda_S \Omega \right), \tag{13}$$

and the required transversality conditions. As usual, $\frac{\partial U}{\partial C}$, $\frac{\partial U}{\partial K_S}$, represent the marginal utilities of consumption and social capital, respectively.

Conditions (8)-(10) tell us that for each control variable marginal benefits will have to be equated to marginal costs for efficiency to be achieved. For instance, condition (8) balances the marginal utility of consumption with the shadow price of physical capital (since one unit of production that is consumed is no longer available for capital accumulation); likewise, condition (9) equates the shadow price of human capital to its value in production, whereas condition (10) equate the shadow price of human capital to its value in social capital accumulation.

On the other hand, conditions (11)-(13) show the factors influencing the dynamic evolution of the shadow prices for each one of the capital types. Namely, condition (11) reflects that giving up a unit of K_P yields a benefit (from the discount rate and the avoided depreciation) as well as a loss equal to the value of the marginal productivity of physical capital; condition (12) tells a similar story except the loss is in the accumulation of human capital; condition (13) shows that, for each unit of K_S that is relinquished, the value foregone includes the direct impact on utility, the value of the marginal productivity of social capital, and its contribution to both human and social capital accumulation.

3.1.1 Optimal Growth Rates

At the steady state of the model, growth rates must, by definition, be constant, so equation (1) tells us that K_P, Y, and C all grow at the same rate. Furthermore, K_S and K_H will be growing at a different rate, as will be shown below, while respecting equations (3) and (4).

In the steady-state, we can obtain the human capital growth rate as follows. From (9) we find $\frac{\dot{\lambda}_P}{\lambda_P} = g_{K_H} + \frac{\dot{\lambda}_H}{\lambda_H} - g_Y$ and using the result from equation (12), we

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can then replace the previous two equations into $-\frac{1}{\tau}g_Y + \psi\left(1 - \frac{1}{\tau}\right)g_{K_H} = \frac{\dot{\lambda}_P}{\lambda_P}$, which we calculated from (8). The substitution yields:

$$\left(1 - \frac{1}{\tau}\right)g_Y + g_{K_H}\left(\psi\left(1 - \frac{1}{\tau}\right) - 1\right) = \rho + \delta_H - \xi. \tag{14}$$

To simplify the above expression, we log-differentiate equation (5), yielding:

$$g_Y = g_{K_H}\left(\frac{\eta + \sigma}{1 - \beta}\right). \tag{15}$$

Since $\frac{\eta+\sigma}{1-\beta} = \frac{\eta+\sigma}{\eta} > 1$, the growth rate of Y (and thus of C and K_P) will be larger than the growth rate of human capital.

We can substitute the last expression in equation (14) to get the growth rate of human capital:[2]

$$g^*_{K_H} = \frac{\xi - \delta_H - \rho}{1 + \left(\frac{\eta+\sigma}{1-\beta} + \psi\right)\left(\frac{1}{\tau} - 1\right)}. \tag{16}$$

Substituting equation (16) in equation (15) we get the output growth rate:

$$g^*_Y = \frac{(\xi - \delta_H - \rho)\left(\frac{\eta+\sigma}{1-\beta}\right)}{1 + \left(\frac{\eta+\sigma}{1-\beta} + \psi\right)\left(\frac{1}{\tau} - 1\right)}. \tag{17}$$

Economic growth is driven by productivity and preference parameters and does not have a scale effect. This is consistent with previous literature on endogenous growth models with human capital accumulation as in Arnold [2]. It is interesting to note the positive effect of the share of social capital in final good production (σ) and the negative impact of the social capital weight in preferences (ψ), as in this model there is a utility trade-off between consumption and social capital.

3.2 Decentralized Equilibrium

In the decentralized equilibrium both consumers and firms have choices to make. Consumers are assumed to maximize their intertemporal utility function, which was presented in equation (6) above:

$$\frac{\tau}{\tau - 1}\int_0^\infty \left(C_t K^\psi_{St}\right)^{\frac{\tau-1}{\tau}} e^{-\rho t}dt$$

subject to the budget constraint:

$$\dot{a} = (r - \delta_p)a + W_H H_Y - C, \tag{18}$$

where a represents the value of physical assets, r is the gross rate of return on physical capital, and W_H is the market wage. The market price for the consumption good is normalized to 1. Since it is making an intertemporal choice, the consumer

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[2]A superscript $*$ means that we are working with variables at the optimal level.

also takes into account equations (3)-(4), which represent human and social capital accumulation. Moreover, consumers are aware of the partition between different uses of human capital, equation (2), which implies that they take into consideration that choosing social capital accumulation at a given moment comes at the expense of lower work hours or human capital investment. First order conditions for the consumer problem in the decentralized equilibrium can be found in Appendix A.

As for firms, these are assumed to be non-myopic profit maximizers. In particular, the firm's dynamic profit maximization problem gives rise to the following Hamiltonian:[3]

$$\mathcal{H} = K_P^\beta K_S^\sigma H_Y^\eta - W_H H_Y - I + \lambda_P' [I - \delta_P K_P] + \tag{19}$$
$$+ \lambda_H' [\xi H_H + \alpha K_S - \delta_H K_H]$$

where the choice variables of the firm are H_Y and I. The first order conditions are:

$$W_H = \frac{\eta Y}{H_Y}, \tag{20}$$

$$\lambda_P' = 1, \tag{21}$$

as well as:

$$r = \frac{\beta Y}{K_P}, \tag{22}$$

$$-\delta_H \lambda_H' = (W_H - \delta_H)\lambda_H' - \dot{\lambda}_H', \tag{23}$$

where λ_P' and λ_H' are co-state variables for the stocks of physical and human capital, respectively.

The markets for purchased production factors (human and physical capital) are expected to be competitive. However, the firm cannot buy social capital, as there is, in effect, no market for it and it is hard to imagine direct payments for social capital accumulation (see Section 4.1). Social capital is treated as exogenous by the firm here, although it affects production. Hence, consumer decisions will carry social capital externalities.

From this problem we know that returns on purchased production factors equal marginal productivities, as follows:

$$W_H = \frac{\eta Y}{H_Y}; \tag{24}$$

$$r = \frac{\beta Y}{K_P}. \tag{25}$$

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[3]In this case, maximizing the present value of profits is equivalent to maximizing profit in each period. See Barro and Sala-i-Martin (1999: 67 - 70).

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In Appendix B we have calculated the growth rates of human capital and output in the decentralized equilibrium, and they are shown to be the same as those for the optimal solution. This is a feature of semi-endogenous models of endogenous growth (Jones [20]) and of models of endogenous growth with human capital accumulation and R&D (e.g. Arnold [3]). It results from the fact that in this model, long-run growth is driven by human capital accumulation and that there are no externalities directly associated with the human capital stock. However, in the following section we show that there are human capital allocation distortions in the decentralized equilibrium.

3.3 Distortions

Given the importance of human capital in growth models such as ours, in this section we look at the distortions that arise in human capital allocation. The shares of human capital allocated to the different sectors by the social planner are:

$$u_Y^* = \frac{H_Y}{K_H} = \frac{\left(\xi - \delta_H - \frac{\omega\alpha}{\xi} - \Omega\right)\left(\frac{K_S}{K_H}\right)^*}{\frac{\omega}{\eta}\left(\psi\left(\frac{C}{Y}\right)^* + \sigma\right)}; \tag{26}$$

$$u_S^* = \frac{H_S}{K_H} = \left(\frac{g_{K_S}^* - \Omega}{\omega}\right)\left(\frac{K_S}{K_H}\right)^*; \tag{27}$$

$$u_H^* = \frac{H_H}{K_H} = \frac{1}{\xi}\left(g_{K_H}^* + \delta_H\right) - \frac{\alpha}{\xi}\left(\frac{K_S}{K_H}\right)^*, \tag{28}$$

where an expression for $\left(\frac{C}{Y}\right)^*$ is given in Appendix C. The equations that are presented in this section provide a basis for a complete analysis of all the relationships between the different capital stocks and also a basis for the comparison with the decentralized equilibrium solution. All derivations are in Appendix D.

Using the restriction that $u_Y^* + u_S^* + u_H^* = 1$, we obtain the social to human capital ratio that the social planner would choose:

$$\left(\frac{K_S}{K_H}\right)^* = \frac{1 - \frac{1}{\xi}\left(g_{K_H}^* + \delta_H\right)}{\frac{\left(\xi - \delta_H - \frac{\omega\alpha}{\xi} - \Omega\right)}{\frac{\omega}{\eta}\left(\psi\left(\frac{C}{Y}\right)^* + \sigma\right)} + \left(\frac{g_{K_S}^* - \Omega}{\omega}\right) - \frac{\alpha}{\xi}}. \tag{29}$$

In a simplistic view, the shares are determined by the parameters that also determine the respective growth rates. Thus, a higher growth rate of output driven, e.g., by ξ, would determine a higher allocation to the final good production. Similarly, a higher growth rate of social capital would determine a higher u_S and a higher growth rate of human capital would determine a higher u_H. However a higher social capital to human capital ratio would occur the lower the growth rate of human capital. The combination of all these factors in general equilibrium would determine the above allocations and ratio.

The shares of human capital allocated to the different sectors in the decentralized

equilibrium are:[4]

$$u_Y^{DE} = \frac{H_Y}{K_H} = \frac{\xi - \delta_H - \frac{\omega\alpha}{\xi} - \Omega}{\frac{\omega}{\eta}\left[\psi\left(\frac{C}{Y}\right)^{DE}\right]}\left(\frac{K_S}{K_H}\right)^{DE}; \tag{30}$$

$$u_S^{DE} = \frac{H_S}{K_H} = \left(\frac{g_{K_S}^{DE} - \Omega}{\omega}\right)\left(\frac{K_S}{K_H}\right)^{DE}; \tag{31}$$

$$u_H^{DE} = \frac{H_H}{K_H} = \frac{1}{\xi}\left(g_{K_H}^{DE} + \delta_H\right) - \frac{\alpha}{\xi}\left(\frac{K_S}{K_H}\right)^{DE}, \tag{32}$$

where an expression for $\left(\frac{C}{Y}\right)^{DE}$ is also given in Appendix C and is shown to be equal to $\left(\frac{C}{Y}\right)^*$. Again, we obtain the decentralized equilibrium's social to human capital ratio:

$$\left(\frac{K_S}{K_H}\right)^{DE} = \frac{1 - \frac{1}{\xi}\left(g_{K_H}^{DE} + \delta_H\right)}{\frac{\xi - \delta_H - \frac{\omega\alpha}{\xi} - \Omega}{\frac{\omega}{\eta}\left[\psi\left(\frac{C}{Y}\right)^{DE}\right]} + \frac{g_{K_S}^{DE} - \Omega}{\omega} - \frac{\alpha}{\xi}}. \tag{33}$$

As mentioned above, growth rates are equal in the decentralized equilibrium and in the optimal solution, so that the difference between the two solutions is only seen in the distortion in human capital allocations. Moreover, note that if we build a static model without capital accumulation, the decentralized allocation of a given level of knowledge between work, which is paid a salary, and social interactions, which generate well-being while increasing productivity, there would not be an externality because firms could simply adjust the market salary to ensure the optimal level of social interaction. This highlights the fact that the social capital externality is essentially a dynamic phenomenon.

Proposition 1 *Due to a market failure in social capital, the social planner chooses a higher social-human capital ratio than in the decentralized equilibrium. The socially optimal choice is to allocate more human capital to social capital production and less to final good production and human capital accumulation than the agents do.*

Proof. Comparing equations (29) and (33), we can see that the first expression is larger, given that there is a positive social capital externality. This is due to the beneficial impact of social capital in the production function ($\sigma > 0$), which is not taken into consideration in the decentralized equilibrium because there is no market for social capital. Therefore, decentralized human capital allocation to social capital accumulation (u_S) will be lower than would be desirable, which can also be verified comparing equations (27) and (31). ∎

We give one example to illustrate that the size of the distortion in the allocation of human capital to social capital accumulation can be significant.

Example 1 *Suppose we use a parametrization that considers a per capita economic growth rate of $g_y = 0.0141$, human capital depreciation rate of $\delta_H = 0$, the shares*

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[4]A superscript DE means that we are working with variables at the decentralized equilibrium level.

of physical, human and social capital in final good production $\beta = 0.18$, $\eta = 0.7$ and $\sigma = 0.08$, respectively, the preference for social capital $\psi = 0.2$, the effect of social and human capital on social capital accumulation $\Omega = 0.01$ and $\alpha = 0.01$, respectively, the effect of social capital on human capital accumulation $\omega = 0.01$, the consumption to output ratio $C/Y = 0.89$, the intertemporal elasticity of substitution $\tau = 0.8$, and the discount rate $\rho = 0.01$. [5] *This yields a share of social capital relative to total human capital of $u_S^* = 5.4\%$ for the planner's allocation and $u_S^{DE} = 3.8\%$ for the decentralized allocation. Moreover, if we change the parameters associated to human and social capital in the production function, the distortion can be even more severe: For instance, setting $\sigma = \eta = 0.41$ and leaving all other parameters constant, the model would imply a much larger distortion, with $u_S^* = 18.9\%$ and $u_S^{DE} = 5.9\%$.*

4 Can Firms Induce a Better Social Capital Allocation?

4.1 A New Decentralized Equilibrium

In order to achieve the optimal solution it would be tempting to suggest that firms pay individuals to improve social capital. In particular, a payment of σY would carry the decentralized equilibrium to the optimal solution (see Appendix E). However, it is not realistic to suppose such a payment, because there is no functioning market for social capital and it is not clear that such a market would be feasible or even desirable. Unlike most environmental externalities, which are generally unintended byproducts of consumption or production decisions and for which the attribution of property rights is a route to solving the market failure in the absence of transaction costs, the social capital externality carries a strong moral charge and its commodification could be counterproductive. An instrumental expense by firms that was seen by workers as an attempt to buy trust and commitment would probably crowd out their intrinsic motivations, thus reducing rather than increasing spontaneous trustworthiness between the firm and its workers (see for instance Frey and Jegen, [15]).[6]

Alternatively, we propose that firms can use CSR activities to improve the effectiveness of social capital accumulation, so that even similar choices by individuals would yield higher benefits. The idea is that there is no purchase of social capital as such, but rather an action that facilitates positive collaborations. Examples of such activities are awareness-raising programs that communicate positive company values to employees or community outreach engagements, organized with company resources, where employee involvement is encouraged. These actions require a decision by the firm on the resources it wishes to spend on CSR, but through them the

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[5] This parameter set is obtained from the benchmark exercise in Roseta-Palma *et al.* [28], where each parameter value is justified from empirical evidence.

[6] We thank a referee for this point.

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firm enables workers to build social capital more successfully, by increasing valuable cooperation networks or strengthening workers' levels of trust and sense of belonging in the work environment. In the decentralized equilibrium model, these activities will translate into direct costs for the firms that reduce instantaneous profit, but then bring dynamic benefits through an increase in the H_S effectiveness parameter in the equation for accumulation of social capital:

$$\dot{K}_S = \omega(1 + CSR)H_S + \Omega K_S. \tag{34}$$

The firm's dynamic profit maximization problem now gives rise to the following Hamiltonian:

$$\mathcal{H} = K_P^\beta K_S^\sigma H_Y^\eta - W_H H_Y - I - CSR + \lambda_P''' [I - \delta_P K_P] + \tag{35}$$
$$+ \lambda_S''' [\omega(1 + CSR)H_S + \Omega K_S] + \lambda_H''' [\xi H_H + \alpha K_S - \delta_H K_H]$$

where the choice variables of the firm are H_Y, I, and CSR. The first order conditions are:

$$W_H = \frac{\eta Y}{H_Y},$$
$$\lambda_P''' = 1,$$
$$\omega H_S \lambda_S''' = 1, \tag{36}$$

as well as:

$$r = \frac{\beta Y}{K_P},$$
$$\frac{\sigma Y}{K_S} + \lambda_S''' \Omega + \alpha \lambda_H''' = (\omega + \Omega)\lambda_S''' - \dot{\lambda}_S''', \tag{37}$$
$$-\delta_H \lambda_H''' = (W_H - \delta_H)\lambda_H''' - \dot{\lambda}_H''', \tag{38}$$

where λ_P''', λ_S''', and λ_H''' are the co-state variables for the stocks of physical, social, and human capital, respectively. Condition (36) is new and shows the factors that affect CSR choices, namely by balancing the benefit obtained through an additional unit (given by its impact on K_S accumulation) and its unit cost. Condition (37), also new, is the corresponding co-state equation, which illustrates the multiple roles of social capital, as in condition (13)

Moreover, the first-order condition of the choice variable H_S for the consumer problem, presented in Appendix A, has now changed to reflect the higher value of human capital productivity in social capital production:

$$\lambda_S''' = \frac{\lambda_a W_H}{\omega(1 + CSR)}. \tag{39}$$

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The growth rates are equal to the ones calculated in the previous section, but the shares in the decentralized equilibrium are now different (except for the allocation of human capital dedicated to schools, u_H):[7]

$$u_S^{DE} = \frac{H_S}{K_H} = \left(\frac{g_{K_S}^{DE} - \Omega}{\omega(1 + CSR)}\right) \left(\frac{K_S}{K_H}\right)^{DE}, \tag{40}$$

$$u_Y^{DE} = \frac{H_Y}{K_H} = \frac{\xi - \delta_H - \frac{\omega\alpha(1+CSR)}{\xi} - \Omega}{\frac{\omega}{\eta}\left[\psi(1 + CSR)\left(\frac{C}{Y}\right)^{DE}\right]} \left(\frac{K_S}{K_H}\right)^{DE}. \tag{41}$$

Although it is impossible to work out an expression for the value of CSR that would fully equate the social to human capital ratio obtained in this model to the optimal one derived in Section 3.3, so that a full internalization of the externality is not achievable, we note that corporate social responsibility (CSR) can bring the share of social capital closer to the optimal ratio. In particular, the value for CSR that induces the optimal allocation of human capital to social capital activities, we equate $u_S^* = u_S^{DE}$ and find:

$$CSR = \frac{\xi(\xi - \delta_H - \Omega) - \alpha\omega}{(\eta + \frac{C}{Y})(\sigma + \frac{C}{Y})\alpha\omega}. \tag{42}$$

Example 2 *Using the values set in Example 1, one could calculate CSR=1.125, which mean that CSR expenditures of 1.125 would represent an increase of 112.5% in the productivity of individuals in accumulating social capital and would induce an increase of 1.6% on the allocation of human capital to social capital activities. Despite its benefits, for a per capita GDP of Y=20000, this expenditure would only represent 0.008% of the wage expenditures (ηY). In the alternative setting with $\sigma = \eta = 0.41$, CSR expenditures of 2.038 would represent an increase of 203.8% in the productivity of individuals in accumulating social capital and would induce an increase of 13% on the allocation of human capital to social capital activities. In this case, this expenditure would represent 0.025% of the wage expenditures.*

5   Conclusion

Recent analysis highlights social capital as one of the determinants of both productivity and well-being. In this paper we show that a positive externality can arise from social capital. In particular, we look at the individual firm's decisions in a decentralized model, in a context where workers make the choices that determine social capital accumulation. As social capital is expected to increase productivity but is not traded in markets, there is an externality in the model. We compare the human capital allocation patterns associated with a welfare-maximizing solution to those achieved in a decentralized equilibrium, and find that the decentralized level

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[7]CSR is constant at the steady-state. The ratios $\left(\frac{C}{Y}\right)^*$ and $\left(\frac{C}{Y}\right)^{DE}$ are equal in this model, and are also equal to the value presented in Appendix C.

of social capital relative to human capital is, as expected, too low. Moreover, we use a numerical example to show that the size of this distortion can be fairly large.

We then propose that, given the externality, it may be in the interest of the firms to subsidize social capital accumulation activities, incurring into additional costs (translated in CSR activities) that are recouped through productivity gains. In the title of the paper, we ask whether firms could internalize social capital externalities. Although a full internalization is theoretically possible, as shown in Appendix E, this would require the creation of a market for social capital, an endeavour which is hard to fathom and could actually be counterproductive given the nature of the asset, as direct monetary compensation could well crowd-out intrinsic worker motivation. Instead we model CSR as an expense that yields improvements in the effectiveness of social capital accumulation. This new model provides a way to approach optimal human capital allocations, although not a full internalization of the social capital externality. Our work thus constitutes a first attempt to relate firms' CSR decisions to social capital from the point of view of endogenous growth theory. Further research should be dedicated to the introduction of other facets of CSR and social capital, such as those discussed in [31].

References

[1] Aoki, M., 2010. "Individual" Social Capital, "Social" Networks, and Their Linkages to Economic Game, Annual World Bank Conference on Development Economics.

[2] Arnold, L., 1998. Growth, Welfare and Trade in an Integrated Model of Human-Capital Accumulation and Research. Journal of Macroeconomics 20(1), 81-105.

[3] Arnold, L., 2000. Endogenous growth with physical capital, human capital and product variety: A Comment. European Economic Review 44, 1599-1605.

[4] Baron, D., 2008. Managerial contracting and corporate social responsibility. Journal of Public Economics 92, 268–288.

[5] Barro, R. J., Sala-i-Martin, X., 1999. *Economic Growth*, MIT Press.

[6] Bartolini, S., Bilancini, E., Sarracino, F., 2009. Social Capital Predicts Happiness: World-Wide Evidence from Time Series. Quaderni del Dipartimento di Economia Politica n. 579, Univ. degli Studi di Siena.

[7] Besley, T., Ghatak, M., 2007. Retailing public goods: The economics of corporate social responsibility. Journal of Public Economics 91, 1645–1663.

[8] Bjornskov, C., 2008. Social Capital and Happiness in the United States. Applied Research Quality Life 3, 43–62.

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[9] Brekke, K. A., Nyborg, K., 2008. Attracting responsible employees: Green production as labor market screening. Resource and Energy Economics 30, 509–526.

[10] Chou, Y. K., 2006. Three simple models of social capital and economic growth. The Journal of Socio-Economics 35, 889-912.

[11] Coleman, J., 1988. Social Capital in the Creation of Human Capital. American Journal of Sociology 94, S95-S120.

[12] Dam, L. 2011 Socially Responsible Investment in an Environmental Overlapping Generations Model. Resource and Energy Economics, 33, 1015–1027.

[13] Dam, L., Heijdra, B. J., 2011. The Environmental and Macroeconomic Effects of Socially Responsible Investment. Journal of Economic Dynamics and Control, 35, 1424–1434.

[14] Freeman, R. E., Harrison, J., Wicks, A., Parmar, B., de Colle, S. 2010. Stakeholder Theory: The State of the Art. Cambridge University Press.

[15] Frey, B., Jegen, R., 2001. Motivation Crowding Theory. Journal of Economic Surveys 15, 589-611.

[16] Friedman, M., 1970. The social responsibility of business is to increase its profits. New York Times Magazine, September 13.

[17] Heal, G., 2005. Corporate Social Responsibility – An Economic and Financial Framework. The Geneva Papers on Risk and Insurance 30, 387-409.

[18] Heslin, P. A., Ochoa, J. D., 2008. Understanding and Developing Strategic Corporate Social Responsibility. Organizational Dynamics 37 (2), 125-144.

[19] Jensen, M., 2002. Value Maximization, Stakeholder Theory and the Corporate Objective Function. Business Ethics Quarterly 12, 235-256.

[20] Jones, C., 1995. R&D-based models of endogenous growth. Journal of Political Economy 103, 759-584.

[21] Knack, S., Keefer, P., 1997. Does Social Capital Have an Economic Payoff? A Cross-Country Investigation. Quarterly Journal of Economics 112, 1251-88.

[22] Lin, N., 1999. Building a Network Theory of Social Capital. Connections 22, 28-51.

[23] Lippert, S., Spagnolo, G., 2011. Networks of relations and Word-of-Mouth Communication. Games and Economic Behavior 72, 202–217.

[24] Lockett, A., Moon, J., Visser, W., 2006. Corporate Social Responsibility in Management Research: Focus, Nature, Salience and Sources of Influence. Journal of Management Studies 43, 115-136.

[25] Melé, D., 2008. Corporate Social Responsibility Theories, in Crane, A., McWilliams, A., Matten, D., Moon, J., Siegel, D. (Eds), The Corporate Handbook of Corporate Social Responsibility, Oxford University Press.

[26] Paul, C. J. M., Siegel, D. S., 2006. Corporate social responsibility and economic performance. Journal of Productivity Analysis 26, 207–211.

[27] Putman, R., Leonardi, R., Nanetti, R., 1993. *Making Democracy Work: Civic Traditions in Modern Italy*, Princeton, Princeton University Press.

[28] Roseta-Palma, C., Ferreira-Lopes, A., Sequeira, T. N., 2010. Externalities in an Endogenous Growth Model with Natural and Social Capital. Ecological Economics 69, 608-612.

[29] Sacconi, L. and Degli Antoni, G., 2009. A Theoretical Analysis of the Relationship between Social Capital and Corporate Social Responsibility: Concepts and Definitions, in S. Sacchetti and R. Sugden (Eds), Knowledge in the Development of Economies: Institutional Choices Under Globalisation, Edward Elgar.

[30] Sacconi, L., Blair, M., Freeman, R. E., Vercelli, A. (Eds) 2011. Corporate Social Responsibility and Corporate Governance: The Contribution of Economic Theory and Related Disciplines, IEA Conference Volume no. 149, Palgrave Macmillan.

[31] Sacconi, L., Degli Antoni, G. (Eds) 2011. Social Capital, Corporate Social Responsibility, Economic Behaviour and Performance, Palgrave Macmillan.

[32] Sarracino, F., 2010. Social capital and subjective well-being trends: Comparing 11 western European countries. The Journal of Socio-Economics 39, 482–517.

[33] Sobel, J., 2002. Can we trust social capital? Journal of Economic Literature 40, 139-154.

[34] Teachman, J. D., Paasch, K., Carver, K., 1997. Social Capital and the Generation of Human Capital. Social Forces 754, 1343-1359.

[35] Turban, D. B., Greening, D. W., 1997. Corporate Social Performance and Organizational Attractiveness to Prospective Employees. Academy of Management Journal 40 (3), 658-672.

[36] Whiteley, P., 2000. Economic Growth and Social Capital. Political Studies 483, 443-466.

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A First Order Conditions for the Decentralized Equilibrium

The choice variables for the consumers are C, H_H, and H_S, so the first order conditions for the consumer problem yield:

$$\frac{\partial U}{\partial C} = \lambda_a \tag{A.1}$$

$$\lambda_H'' = \frac{\lambda_a W_H}{\xi} \tag{A.2}$$

$$\lambda_S'' = \frac{\lambda_a W_H}{\omega} \tag{A.3}$$

as well as:

$$\frac{\dot{\lambda}_a}{\lambda_a} = \rho + \delta_P - r \tag{A.4}$$

$$\dot{\lambda}_H'' = \rho \lambda_H'' - (\lambda_a W_H - \lambda_H'' \delta_H) \tag{A.5}$$

$$\dot{\lambda}_S'' = \rho \lambda_S'' - \left(\frac{\partial U}{\partial K_S} + \lambda_H'' \alpha + \lambda_S'' \Omega \right) \tag{A.6}$$

where λ_a is the co-state variable for the budget constraint, and λ_H'' and λ_S'' are co-state variables for the stocks of human and social capital, respectively.

B Growth Rates in the Decentralized Equilibrium

In the steady-state we can obtain the human capital growth rate of the decentralized equilibrium as follows. By using equation (A.5) and replacing it in $\frac{\dot{\lambda}_H''}{\lambda_H''} = \frac{\dot{\lambda}_a}{\lambda_a} + g_W$, which we get by log-differentiating equation (A.2), we find $\frac{\dot{\lambda}_a}{\lambda_a} = \rho + \delta_H - \xi - g_W$. After log-differentiating equation (24) we get $g_W = g_Y - g_{K_H}$. Substituting this last equation in the previous one and introducing both in $-\frac{1}{\tau} g_Y + \psi \left(1 - \frac{1}{\tau}\right) g_{K_H} = \frac{\dot{\lambda}_a}{\lambda_a}$, which we calculated from (A.1), we get:

$$\left(1 - \frac{1}{\tau}\right) g_Y + g_{K_H} \left(\psi \left(1 - \frac{1}{\tau}\right) - 1 \right) = \rho + \delta_H - \xi \tag{B.1}$$

By log-differentiating the production function (5) we get:

$$g_Y = g_{K_H} \left(\frac{\eta + \sigma}{1 - \beta} \right) \tag{B.2}$$

Substituting this last expression into (B.1) we get:

$$g_{K_H}^{DE} = \frac{\xi - \delta_H - \rho}{\left(\frac{\eta + \sigma}{1 - \beta} + \psi \right) \left(\frac{1}{\tau} - 1 \right) + 1} \tag{B.3}$$

By substituting equation (B.3) into (B.2) we find:

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$$g_Y^{DE} = \frac{(\xi - \delta_H - \rho)\left(\frac{\eta+\sigma}{1-\beta}\right)}{\left(\frac{\eta+\sigma}{1-\beta} + \psi\right)\left(\frac{1}{\tau} - 1\right) + 1} \tag{B.4}$$

These growth rates are equal to the ones that we found in the social planner problem, as discussed in the main text.

C   Consumption-Output Ratios

Now we also demonstrate the relationship between the consumption to output ratio in the decentralized equilibrium and in the social planner solution. We use this expression in the calibration of the model.

From equation (1) we get:

$$\frac{\dot{K_P}}{K_P} = \frac{Y}{K_P} - \frac{C}{K_P} - \delta_P \tag{C.1}$$

Also, from this equation we get $g_{K_P} = g_Y = g_C$ because the growth rates have to be constant in steady state. Since we have shown that $(g_{K_P} = g_Y = g_C)^* = (g_{K_P} = g_Y = g_C)^{DE}$ the left hand-side of equation (C.1) is equal in the social planner and in the decentralized equilibrium and δ_P is a constant.

In the social planner problem by transforming equation (8) we obtain - $\frac{\dot{\lambda_P}}{\lambda_P} = -\frac{1}{\tau}g_C + \psi(1 - \frac{1}{\tau})g_{K_S}$. Replacing this last equation in equation (11) we get:

$$\frac{\beta Y}{K_P}^* = \rho + \delta_P + \frac{1}{\tau}g_C - \psi(1 - \frac{1}{\tau})g_{K_S} \tag{C.2}$$

Since we have shown that $g_{K_P}^* = g_{K_P}^{DE}$, hence $\left(\frac{Y}{K_P} - \frac{C}{K_P}\right)^* = \left(\frac{Y}{K_P} - \frac{C}{K_P}\right)^{DE}$. Using the fact that $\frac{C}{K_P}$ can be written as $\frac{C}{Y}\frac{Y}{K_P}$ and putting $\frac{Y}{K_P}$ in evidence, we obtain:

$$\left(\frac{Y}{K_P}\right)^* \left(1 - \frac{C}{Y}\right)^* = \left(\frac{Y}{K_P}\right)^{DE}\left(1 - \frac{C}{Y}\right)^{DE} \tag{C.3}$$

From the decentralized equilibrium, using (25) we note that $\left(\frac{\beta Y}{K_P}\right)^{DE} = r$. Using equations (A.1) and (A.4) we get:

$$\left(\frac{\beta Y}{K_P}\right)^{DE} = \rho + \delta_P + \frac{1}{\tau}g_C - \psi(1 - \frac{1}{\tau})g_{K_S} \tag{C.4}$$

Hence

$$\left(\frac{\beta Y}{K_P}\right)^* = \left(\frac{\beta Y}{K_P}\right)^{DE} \tag{C.5}$$

Now substituting (C.5) into (C.3), we get:

$$\left(\frac{C}{Y}\right)^* = \left(\frac{C}{Y}\right)^{DE} \tag{C.6}$$

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We can determine the value of $\left(\frac{C}{Y}\right)$ dependent on known values by substituting equation (C.2) into (C.1) and using $\frac{Y}{K_P} - \frac{C}{K_P} = \left(\frac{Y}{K_P}\right)\left(1 - \frac{C}{Y}\right)$ to obtain:

$$\left(\frac{C}{Y}\right)^* = \left(\frac{C}{Y}\right)^{DE} = \frac{g_{K_H}\left[\left(\frac{1}{\tau\beta}-1\right)\left(\frac{\eta+\sigma}{1-\beta}\right) + \frac{\psi}{\beta}\left(\frac{1}{\tau}-1\right)\right] + \delta_p\left(\frac{1}{\beta}-1\right) + \frac{\rho}{\beta}}{\frac{\rho+\delta_p}{\beta} + g_{K_H}\left[\frac{1}{\tau\beta}\left(\frac{\eta+\sigma}{1-\beta}\right) + \frac{\psi}{\beta}\left(\frac{1}{\tau}-1\right)\right]} \quad (43)$$

D Human Capital Shares

D.1 Social Planner

To obtain the share of human capital allocated to production in the social planner, we first use equation (10) which gives us: $\lambda_H = \frac{\lambda_S \omega}{\xi}$ and by substituting this expression into equation (9) we get:

$$\lambda_P = \frac{H_Y \lambda_S \omega}{\eta Y} \quad (D.1.1)$$

We know that $\frac{\partial U}{\partial C} = \lambda_P$ by equation (8) and we also know that $\frac{\partial U}{\partial K_S} = \psi \frac{\partial U}{\partial C}\frac{C}{K_S}$, hence $\frac{\partial U}{\partial K_S} = \psi \lambda_P \frac{C}{K_S}$. Substituting (D.1.1) in this last expression we get:

$$\frac{\partial U}{\partial K_S} = \psi \frac{H_Y \lambda_S \omega}{\eta Y}\frac{C}{K_S} \quad (D.1.2)$$

By equation (10) we get that $\frac{\dot{\lambda}_H}{\lambda_H} = \frac{\dot{\lambda}_S}{\lambda_S}$, which by equation (12) is equal to $\rho + \delta_H - \xi$. After substituting this last expression and also (D.1.2) into equation (13) and dividing the referred equation by λ_S we finally obtain (26).

We get the share of human capital allocated to school time (32) from equation (3) and the share of human capital allocated to investing in social capital (27) from equation (4).

D.2 Decentralized Equilibrium

To obtain the share of human capital allocated to production in the decentralized equilibrium, we first use equation (A.3) which gives us:

$$\lambda_a = \frac{\lambda_S^{''}\omega}{W_H} \quad (D.2.1)$$

We know that $\frac{\partial U}{\partial C} = \lambda_a$ by (A.1) and we also know that $\frac{\partial U}{\partial K_S} = \psi \frac{\partial U}{\partial C}\frac{C}{K_S}$, hence $\frac{\partial U}{\partial K_S} = \psi \lambda_a \frac{C}{K_S}$. Substituting (D.2.1) in this last expression we get:

$$\frac{\partial U}{\partial K_S} = \psi \frac{\lambda_S^{''}\omega}{W_H}\frac{C}{K_S} \quad (D.2.2)$$

By the substitution of (A.2) into (A.3) and by log-differentiating we get that $\frac{\dot{\lambda}_H^{''}}{\lambda_H^{''}} = \frac{\dot{\lambda}_S^{''}}{\lambda_S^{''}}$, which by equation (A.5) is equal to $\rho + \delta_H - \xi$. After substituting this

last expression and also (D.2.2) into equation (A.6) and dividing this equation by λ_S'' we get:

$$\xi - \delta_H - \frac{\omega\alpha}{\xi} - \Omega = \psi\frac{\omega}{W_H}\frac{C}{K_S} \tag{D.2.3}$$

We know by (24) that $W_H = \frac{\eta Y}{H_Y}$ and after substituting this expression into (D.2.3) we get the expression for the share of human capital allocated to production as in (30) in function of $\frac{K_S}{K_H}$.

The share of human capital allocated to school time (32) and the share of human capital allocated to investing in social capital (31) are equal to the social planner allocations – despite the differences in $\frac{K_S}{K_H}$ – between both solutions and they are obtained in the same way as in the social planner's problem. The equilibrium and optimum levels for $\frac{K_S}{K_H}$ as in (33) is obtained using $1 = u_S + u_H + u_Y$.

E   Social Capital "Wage"

If the firm could pay a wage for social capital, equal to W_S, the budget constraint would be:

$$\dot{a} = (r - \delta_p)a + W_H H_Y + W_S K_S - C \tag{E.1}$$

The modifications in terms of first-order conditions for the decentralized equilibrium problem will be only in equation (A.6):

$$\dot{\lambda}_S'' = \rho\lambda_S'' - \left(\frac{\partial U}{\partial K_S} + \lambda_a W_S + \lambda_H''\alpha + \lambda_S''\Omega\right) \tag{E.2}$$

In the firm problem instantaneous profit would be:

$$\pi = K_P^\beta K_S^\sigma H_Y^\eta - W_H H_Y - rK_P - W_S K_S \tag{E.3}$$

and profit maximization would have one extra condition:

$$W_S = \frac{\sigma Y}{K_S} \tag{E.4}$$

The share of human capital dedicated to the social capital production in the decentralized equilibrium (u_S^{DE}), calculated in a manner equivalent to that of Appendix D.2, would now be equal to equation (27), the share of human capital dedicated to the final good production in the decentralized equilibrium (u_Y^{DE}) would now be equal to equation (26), the share of human capital dedicated to the human capital accumulation in the decentralized equilibrium (u_H^{DE}) would now be equal to equation (28) and the externality would be fully internalized.