Condition-based diagnosis of mechatronic systems using a fractional calculus approach

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ABSTRACT

While fractional calculus (FC) is as old as integer calculus, its application has been mainly restricted to mathematics. However, many real systems are better described using FC equations than with integer models. FC is a suitable tool for describing systems characterised by their fractal nature, long-term memory and chaotic behaviour. It is a promising methodology for failure analysis and modelling, since the behaviour of a failing system depends on factors that increase the model's complexity. This paper explores the proficiency of FC in modelling complex behaviour by tuning only a few parameters. This work proposes a novel two-step strategy for diagnosis, first modelling common failure conditions and, second, by comparing these models with real machine signals and using the difference to feed a computational classifier. Our proposal is validated using an electrical motor coupled with a mechanical gear reducer.

Keywords: intelligent maintenance; intelligent diagnostics; application of fractional calculus; identification of fractional order systems

1. Introduction

Competing in the global market requires the production of high-quality goods with low development and manufac- turing periods. New strategies that result in a faster qual- ity control of manufactured products, whilst minimising downtime due to equipment maintenance, are thus essen- tial (Gonc¸alves, 2011; Liu & Makis, 2008; Shikari & Sadi- wala, 2004). A short product development time and the integration of several technologies require highly trained personnel to carry out traditional inspections, quality con- trols and fault diagnoses. Recently, there has been the de- mand for analytical techniques in signal-issued sensors, to describe the behaviour of devices with various compo- nents interacting. This corresponds to a demand for a more highly skilled workforce with a deep knowledge of differ- ent technologies to be used, supporting their diagnoses with computer-recommended systems.

A typical condition-based diagnosis system requires a set of signals with information concerning the current state of the machine, reflecting various phenomena such as vi- bration, noise, temperature and lubrication, amongst others (Funk & Jackson, 2005; Jayaswal, Wadhwani, & Mulchan- dani, 2008). Each signal needs to be treated while discarding irrelevant information accordingly to the type of failure to be isolated. The resulting signals are analysed by means of one or more processing techniques, in order to simplify the failure detection process. That set of signals must contain sufficient information to identify the machine's condition, allowing an expert to diagnose the device and plan a main- tenance action (Bengtsson, Olsson, & Funk, 2004).

A common computer-aided technique used in fault diagnosis identifies the dynamic system of a machine using ordinary differential equations (Duvar, Eldem, & Saravanan, 1990). In the presence of a fault, the system leads to the variation of specific parameters, useful not only to diagnose the problem, but also to estimate the state of the failure. However, this strategy is only useful when the de- vice is simple and its model can be satisfactorily identified adopting a reduced number of parameters (Ljung, 1987). In this case, the space of parameters is small enough to neglect the problem of dimensionality (Kantardzic, 2003). Unfor- tunately, this is not a common situation as real systems typically contain a large number of interactive components, as well as phenomena that are difficult to model (Wang, Wang, & Han, 2010).

Fractional calculus (FC) has been applied by researchers from different areas, due of its ability to describe complex phenomena using a smaller number of parameters, than its integer counterpart, so as to say, taking advantage of the additional degree of freedom given by the arbitrary order (Espindola, Bavastri, & Lopes, 2008; Gutie´rrez-Carvajal, Rosa´rio, & Machado, 2010; Hartley & Lorenzo, 2003).

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However, its use has been restricted to some engineering problems (Santos, Silva, & Suetake, 2012), mainly due to the lack of a simple geometric and physical interpretation (Machado, 2013; Podlubny, 1994). In practice, the solution of a dynamic model of fractional order approximates com-

plex behaviours emerging from systems with multiple interactions (Vinagre, 2007). Consequently, many real systems can be better approximated using compact fractional order equations (Petras, 2006). This is a desirable approach to an automatic system for failure identification, since automatic classifiers require a balance between informative inputs and the amount of entries (Kantardzic, 2011). Therefore, iden-
tifying a system using FC results in a set of indicators

of the machine's condition associated with each parameter. This work proposes a new methodology based on intelligent

maintenance that continually assesses the condition of the system, restricting the amount of required parameters in the identification process. The identified model is used in a classification algorithm that allows the device to be diagnosed.

The paper is organised as follows: Section 2 introduces the fundamentals of FC; Section 3 demonstrates the ex- perimental workbench configuration; Section 4 presents FC implementation and the system validation diagnostic; Section 5 illustrates some experimental results and Sec- tion 7 outlines the main conclusions of this work.

2. Fundamentals of fractional calculus

FC represents the generalisation of integer calculus to real or complex order (Adams, Hartley, & Lorenzo, 2006). One of the reasons why derivatives and integrals of fractional order are still relatively unknown in engineering is that the calculation of fractional order has multiple definitions (Ortigueira, Machado, & da Costa, 2005), making it diffi- cult to interpret geometrically (Machado, 2003; Moshrefi- Torbaty & Hammond, 1998; Podlubny, 2002). However, many phenomena are described by formulations of frac- tional order, as it has the ability to express the past behaviour by means of a limited number of coefficients (Magin & Ovadia, 2008; Machado, Galhano, & Trujillo, 2014). In this work, we use the Riemman–Liouville formulation, whose integral (J_C) and derivative (D^α) definitions are introduced in the following equations, respectively (Cafagna, 2007):

J^α t

$$J_c^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_c^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau, \qquad (1)$$

$$D^\alpha f(t) = \frac{d^m}{dt^m}\left[\frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha+1-m}} d\tau\right], \qquad (2)$$

with $m \in Z^+$ and $m - 1 < \alpha \leq m$. The symbol r stands for gamma function (Gorenflo & Mainardi, 2008; Vale´rio,
Trujillo, Rivero, Machado, & Baleanu, 2013), defined as

$$\Gamma(x) \equiv \int_0^\infty y^{x-1} e^{-y} dy, \qquad (3)$$

or

$$\Gamma(x) \equiv \lim_{N\to\infty}\left[\frac{N!N^x}{x(x+1)(x+2)\ldots(x+N)}\right], \qquad (4)$$

$\forall x \notin \mathbb{Z}^- \cup 0.$

An advantage of using this definition is that, unlike other definitions, it has a strict definition of the Laplace transformation, which facilitates identification algorithms.
It is formally written as

$$\mathcal{L}\{{}_0D_t^\alpha f(t)\} = s^\alpha F(s) - \sum_{j=0}^{n-1} s^j \left[{}_0D^{\alpha-j-1} f(0)\right], \qquad (5)$$

$L\{_0D^\alpha$ α

with $n - 1 < q < n$ and $n \in \mathbb{Z}$ (Ma & Hori, 2004; Valério et al., 2013).

3. Experimental workbench configuration

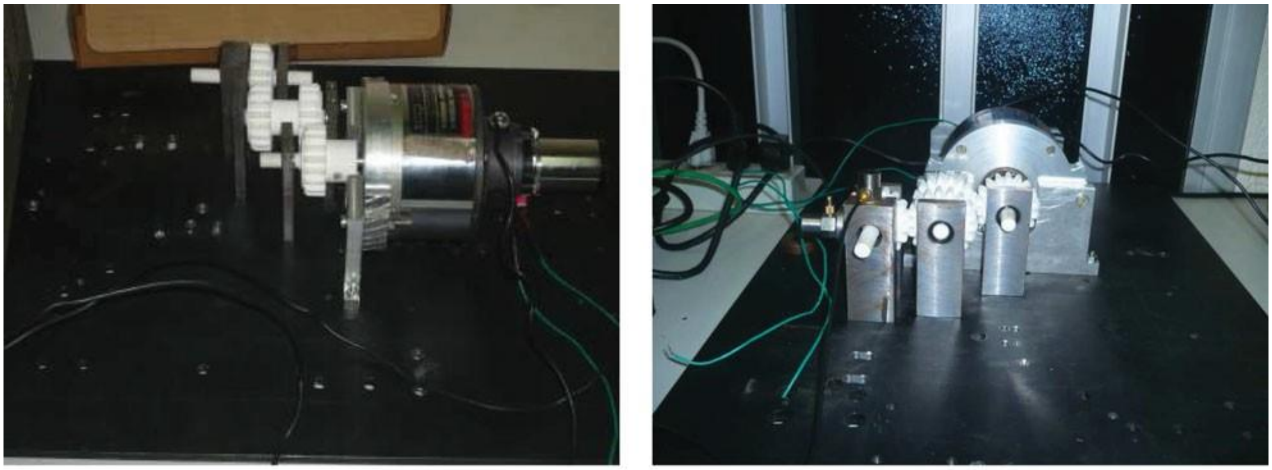
This section defines the experimental workbench proposed for testing and validation. Our experimental bench for test- ing the fault-detection algorithm proposed herein consists of transmitting power through a gear reduction with mul- tiple stages driven by a DC motor, as shown in Figure 1. An accurate model of this machine is difficult to obtain analytically, due to the large number of interactive compo- nents such as gears and bearings. Moreover, the interaction between two gears is still not a well-known phenomenon, with many parameters difficult to measure or to estimate, since they depend on imperfections in the surface of the teeth, the shape of the profile, the contact time between teeth, temperature, friction and others.

The test bench include a voltage source which feeds ac- tuators and instrumentation equipment, a DC motor to drive the gearbox composed of four gears produced by rapid pro- totyping which introduces several types of faults in a simple way. An accelerometer measures the bearing vibration, as shown in Figure 2. Signals generated by the actuator are measured by a resistor (motor current) and a tachometer (speed of the motor shaft). The signals from sensors are sent to a central computer through the data acquisition interface PCI-6221 of National Instruments® , with a CB-68LP card. The set of acquisition has 16-bit precision in reading ana-

logue signals and a maximum sampling frequency of 2.5

MHz. The acquired signals are stored in a database gener- ated by a control programme acquisition implemented in Labview® .

Figure 3 shows the workbench operation diagram. It i not possible to measure a fault directly from the source, due to the transmission path to be physically followed before the signal is measured. Figure 2 demonstrates the trans- mission path between a localised failure in the fourth gear, the accelerometer (vibration sensor) and the resistor (motor
current sensor).



(a) Lateral view (b) Frontal view

Figure 1. Experimental platform.

This work studies four operating conditions, corre- sponding to the transmission path from the failure until the sensor, as follows:

- **Case 1.** Normal operating conditions. The system shows no fault and operates as standard.
- **Case 2.** Broken tooth in gear 2. The second gear is missing one of its nine teeth. The vibration signal (related to the failure) propagates through gears 2, 3 and 4, up to being measured by the accelerometer. This is a failure that least affects the signal obtained by the accelerometer, since the long transmission path reduces its intensity. Moreover, the signal propagates through the resistor in the circuit of the motor current.
- **Case 3.** Broken tooth in gear 3. The pattern of vibration-related failure in gear 3 modulates through

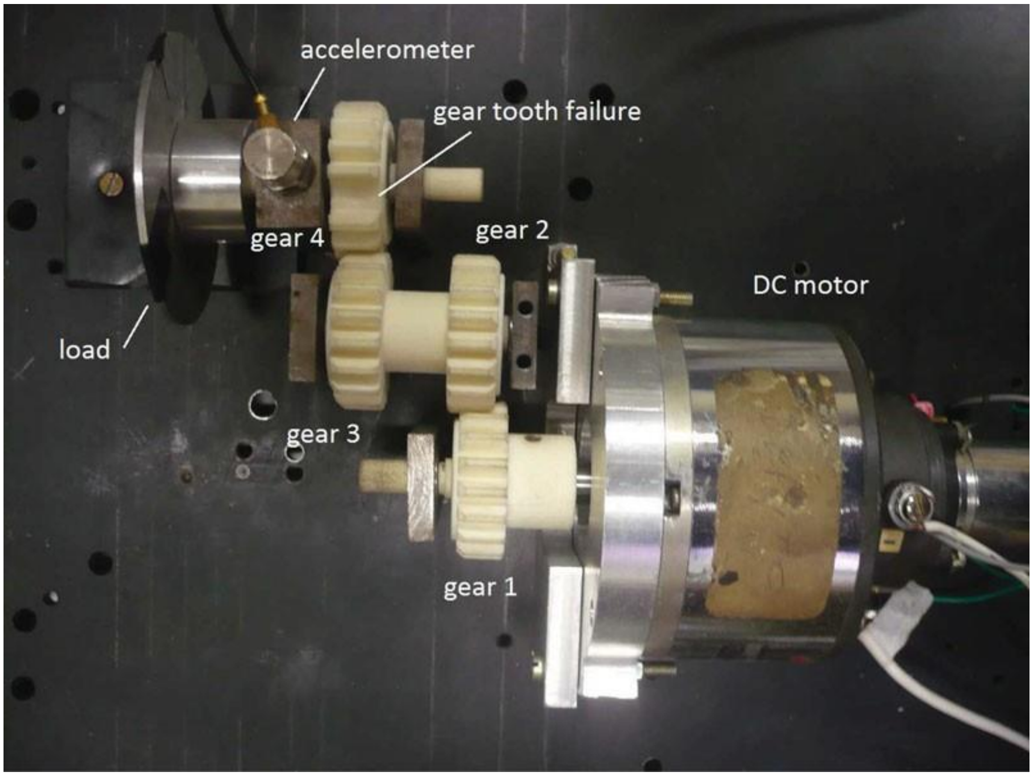


Figure 2. Detail of the gearbox and transmission path associated with failure of one tooth in gear 4.

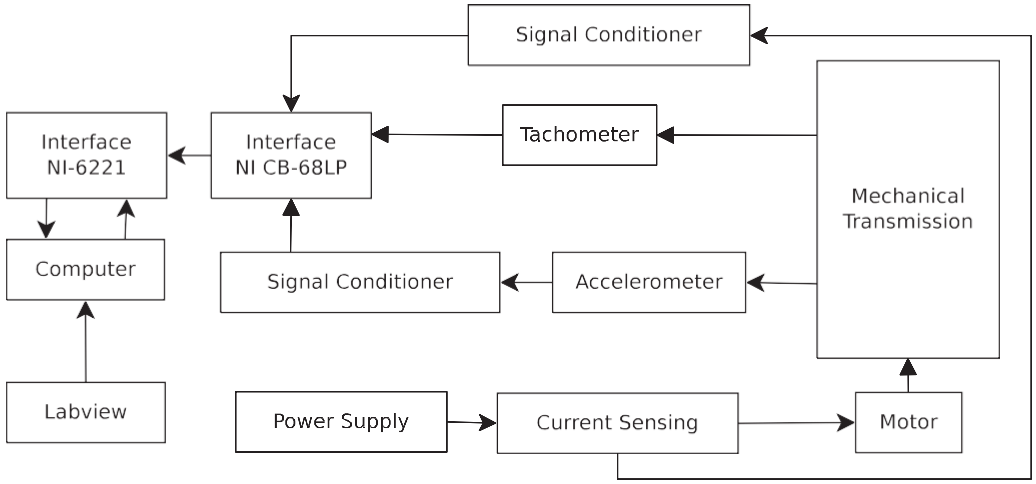


Figure 3. Block diagram of the experimental bench.

gears 3 and 4 (transmission path to the accelerom- eter). The fault signal (which affects the motor cur- rent) propagates through gears 1, 2, 3, and the motor circuit.

- **Case 4.** Broken tooth in gear 4. With the failure of missing a cog, the transmission path of the signal propagates through gear 4 to reach the accelerometer. Moreover, propagation takes place by gears 1, 2, 3, and the motor circuit.

4. FC implementation and system validation diagnostic

The methodology for the development of the proposed strategy is based on the layered model, Open Systems Ar- chitecture for Condition-Based Maintenance (OSA-CBM) (Bengtsson et al., 2004). Initially, different signals of in- terest were acquired to diagnose failures such as vibra- tion, position, electric current and others (Layers 1 and 2, OSA-CBM). The filtering is performed with the compo- nents of the acquired signal and by identifying the model parameters (Layer 3). These operations generate a set of indices, which allow the state of the machine (Layer 4) to be assessed, before being diagnosed by an experienced worker, or by a system with some diagnosis analysis tech- nique (Layer 5).

4.1. Adjusting the system gear model

Identify a model in the following four different steps:

(1) Assume a model structure to be identified.
(2) Obtain and process the experimental data.
(3) Identify the model parameters.
(4) Validate the model by comparing the results to a set of data that was not used to find the parameters.

Consequently, identifying a system can be treated as a problem of minimising the error obtained between the model and the actual data. Therefore, one must find the best parameter vector $\hat{\mathbf{p}}$ that minimises the objective function error f_e from the real system $G_r(s)$ and the model $G_m(\hat{\mathbf{p}}, s)$:

$$f_e(\vec{p}) = |G_r(s) - G_m(\vec{p}, s)|. \tag{6}$$

Different approaches may be used to minimise (6) with- out losing generality. In this study, we adopt the *simplex method* (Lagarias, Reeds, Wright, & Wright, 1998), which consists of an iterative algorithm that searches for a candi- date solution by calculating the centroid from three starting points. It subsequently analysed whether the centroid is bet- ter than any of the starting points, and, if so, it replaces the worst of them. The algorithm runs until it converges, or until it reaches a specified number of iterations.

4.2. Objective function

The input signals are obtained from an accelerometer lo- cated over the output bearing, since it must obtain the vibration due to failure. Changes on the vibration signa- tures directly affect the motor torque and hence the motor current (output signal). For each failure mode presented herein, information was acquired from the experimental bench working at different speeds. After obtaining accel- eration records, a filtering operation was applied using a moving average (MA) to reduce the effect of noise. Fur- thermore, the Fourier transformation was calculated using the Hanning window with a duration of one second to re- duce noise introduced during the scanning process. Here the motor current (I) is considered as the system output and the voltage generated by the accelerometer as input (V). We can define the current state of the device with the *empirical estimation of the transfer function* (EETF) (Ljung, 1987) as follows:

$$G_{\mathrm{EETF}_i}(\omega) = \frac{\mathcal{F}\{I_i(t)\}}{\mathcal{F}\{V_i(t)\}} \quad \text{where } i \in \{1,2,3,4\}, \quad (7)$$

where $\mathcal{F}(\cdot)$ is the Fourier transform, i denotes the ith failure and ω is the angular frequency, in the range 100–1000 rad/s, since the beat frequency of the gears' teeth is within that bandwidth.

We propose identifying each EETF$_i$ using an FC model, with a structure having five parameters:

$$G_i(s) = \frac{1}{as^\alpha + bs^\beta + c}, \qquad \{a, b, c, \alpha, \beta\} \in \Re \quad (8)$$

The parameters of this model were adjusted using a set of 20 data-sets for tuning and 10 data-sets for evaluation, with an objective function that minimises the error between G_{EETF_i} and g_i:

$$f_e = |G_{\mathrm{EETF}_i}(\omega) - G_i(\omega, [a, b, c, \alpha, \beta])|. \quad (9)$$

We assume that a particular failure behaves close to a specific condition model. Therefore, we compute the dif- ference between the actual machine system G_{EETF} and each condition model G_i, using the mean square error to each model as a failure index to be assessed by an automated classification system.

4.3. *Failure diagnosis*

The aim is to evaluate the proposed strategy for failure di- agnosis. Our technique enables automated grading testing when a particular device is failing, and allows one to lo- cate the part with the problem and to assess its state. Faults are classified by the *kNN* algorithm (*k-nearest neighbours*). This strategy involves comparing the model identified in the current state of the machine with a database contain- ing known flaws identified with models. The classification completes itself with the categories of k closest systems, by means of a strategy of choice (Cover & Hart, 1967). This method is presented in Algorithm 1.

Algorithm 1: *k*-nearest neighbours, where the type of failure is estimated from the more representatives in the *k*-neighbourhood

```
Data: system_actual
Result: kind of_failure
i ← 0;
while i < number of condition_models stored in the database do
    d_i ← |condition_model_i ⊖ system_actual|;
    i ← i + 1;
while k-th model is close to system_actual do
    kind of_failure ← Type(system_i) ⊕ Type failure;
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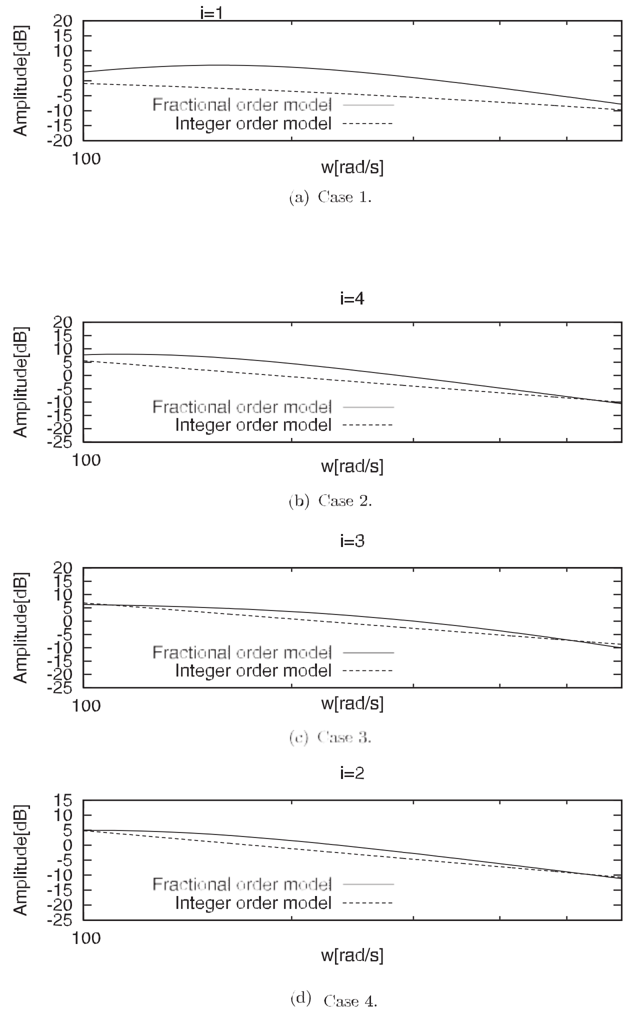

Figure 4. Approximation model (fractional and integer order) for each EETF failure.

5. Results

Following the methodology described, we adopted a set of 240 G_{EETF} data records, not used previously for parameter identification of standard $G_i(s)$, introducing four known fault conditions. Figure 4 demonstrates the identification results using the proposed strategy; the results with the fractional order model (FOM) are also compared with a classical second integer order model (IOM), i.e. having $\alpha = 2$ and $\beta = 1$.

Note that the fractional order approximation is consis- tent with the data and also more accurate than the integer approach. Table 1 depicts the average error and standard deviation, both for the data used for tuning the model and the data used for testing purposes. The FOM fits better for the whole data-sets than the IOM.

5.1. *Failure identification*

In order to test the generality of the technique, we adopted a strategy of rating 10 subsets, using one as a test set and

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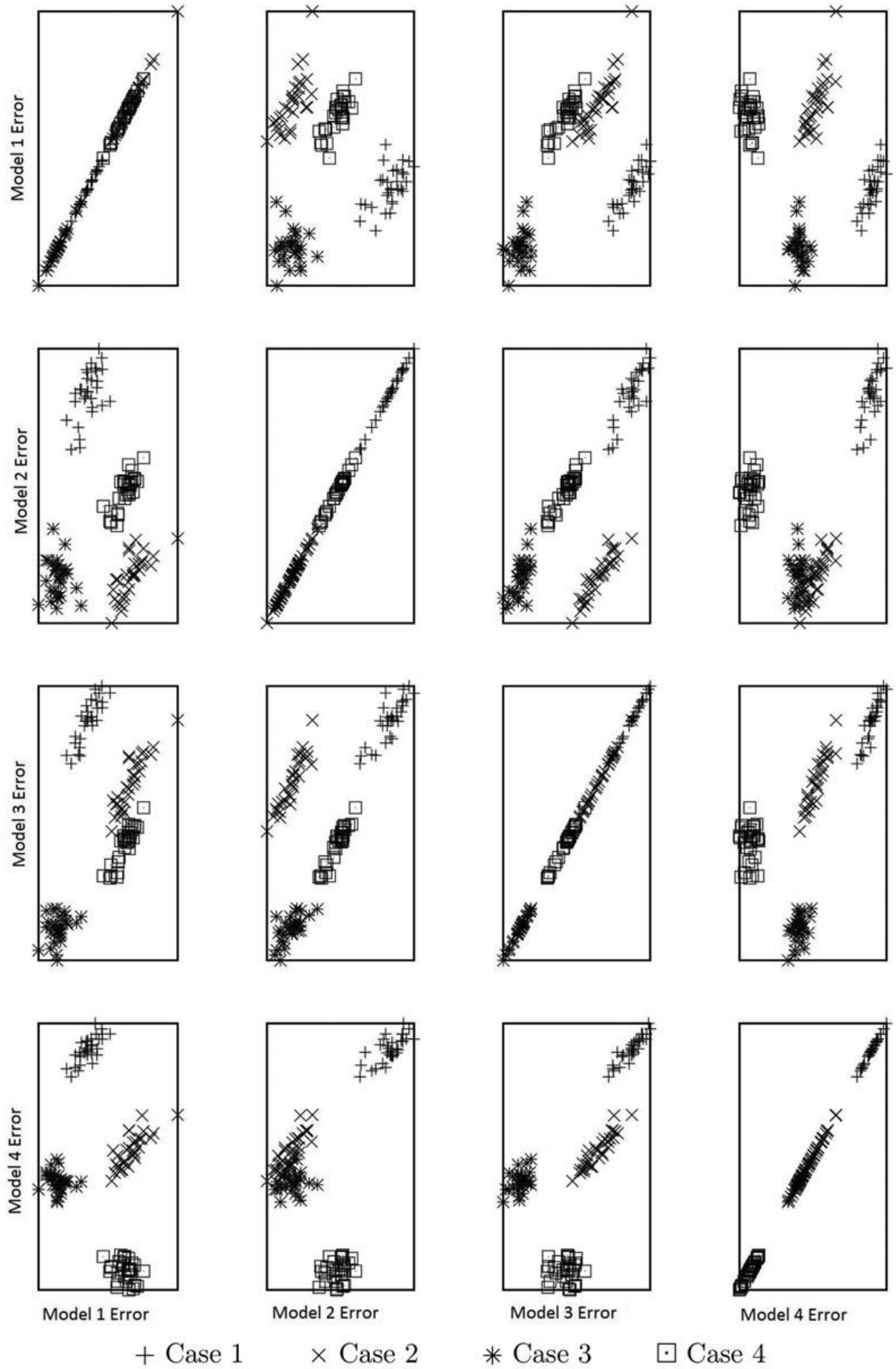


Figure 5. Indices found during failure diagnosis.

Table 1. Train and test mean errors ± standard deviation of the FOM and the IOM when compared with real data.

	Train error ±deviation		Test error ± deviation	
	FOM	IOM	FOM	IOM
Case 1	1.84 ± 0.15	3.23 ± 0.18	2.06 ± 0.23	3.47 ± 0.27
Case 2	1.12 ± 0.05	2.26 ± 0.05	1.05 ± 0.02	2.17 ± 0.03
Case 3	1.44 ± 0.34	2.43 ± 0.38	1.41 ± 0.09	2.37 ± 0.10
Case 4	0.74 ± 0.05	1.36 ± 0.06	0.89 ± 0.07	1.51 ± 0.07

Table 2. Diagnosis estimation yielded by the proposed algorithm.

		Estimated diagnosis			
		Case 1	Case 2	Case 3	Case 4
Actual failure	Case 1	56	4	0	0
	Case 2	0	48	12	0
	Case 3	0	0	60	0
	Case 4	0	4	0	56

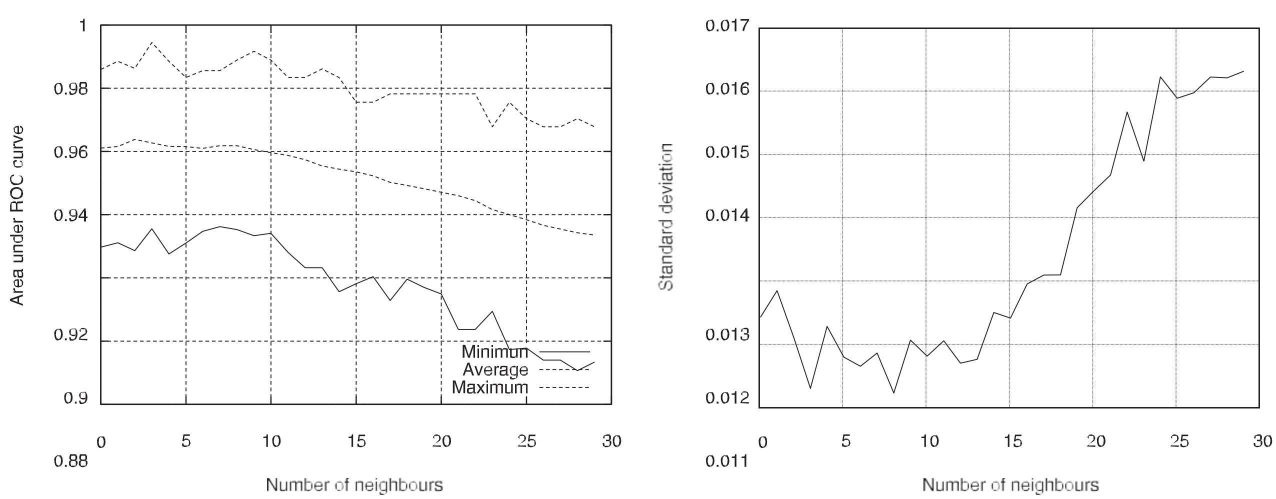
nine as training sets. We employed the use of *kNN* here to estimate automatically a diagnostic, varying the number of neighbours *k*. The next step was to constitute another test set, repeating this operation until all data had been tested. Considering the results obtained during the estimation of the FOM, 60 *G*EETF samples were randomly taken for each case of study, with the aim of testing a classification strategy. The indices obtained are depicted in Figure 5. Note that, as expected, failure types are grouped into different spatial regions making it possible to use a very simple classification technique for equipment failure diagnosis.

On average, the classifier obtained similar performances independently of the neighbourhood size. Nevertheless, the lower data dispersion was achieved using three neighbours, as shown in Figure 6. Table 2 presents the results obtained with three neighbours.

6. Discussion

Nowadays, it is of paramount importance to quickly evalu- ate machinery and product performance in order to improve client services. A large part of failures occur due to wear on specific pieces of machinery. In fact, many maintenance procedures are planned, based on supplier requirements (Endrenyi et al., 2001), due to the existence of a small set of known failures exist that affects machine performance. These failures could occur before or after a maintenance task, unnecessarily stopping the machine in the first case or, eventually causing a fatal failure. Herein, we analysed four particular machine conditions, being the normal per- formance point and three conditions of failure, all affecting different parts of the machine and measured using the same set of sensors. Other cases and degrees of failure could be considered using the same approach without losing gen- erality, that is, considering them as new conditions of the machine. For this set of failures, the system accurately di- agnoses the location of the failure.

The algorithm proposed requires two main conditions: first, a general, but accurate, model of the system and, sec- ond, a known set of frequent failures to identify. In order to meet the first condition, we compared integer and fractional order models. The results reveal that FOM consistently ob- tains a better system than the IOM. It allows the algorithm to finally conform disaggregated groups, as presented in Figure 5, where each group represents a condition of the machine. This improved signal representation is due to the derivative operator that adds additional degrees of freedom. Conforming the groups allows a simple technique, such as



(a) Area under the ROC curve.

(b) Standard deviation of ROC values.

Figure 6. Effect of increase the number of neighbours *k*.

kNN, to accurately diagnose the current condition of the machine. The amount of neighbours taken into account for voting to identify the machine's condition, does affect the classifier's performance, reducing the accuracy whilst in- creasing the uncertainty of the result, as shown in Figure 6.

7. Conclusions

FC is used in several scientific areas, but up to now, there have been no studies on the use of its adoption for fault prediction in the literature. However, fractional order al- gorithms are a promising tool for this type of modelling, since the system behaviour depends on the machine's op- eration history and the wear of the parts. Starting from a model close to the plant, it was possible to extract simple failure rates that are good descriptors of the current state of the device. Due to this fact, it was possible to use simple classification techniques proposed in the literature.

If we consider industrial requirements, FC is an alter- native strategy to obtain the state of a device, by means of identification systems based on the fact that a particular fault recurs quite frequently. These failures vary the opera- tion of the system in a known manner, which could also be identified using a model with few parameters. The proposed strategy generates a failure rate, in the frequency domain, that can be used to diagnose a particular device. The high accuracy of the implemented system in diagnosing failures is basically due to the use of a fractional order structure as the basis of the identification system. In fact, the use of a single canonical structure, using only three coefficients and two orders, was able to sufficiently approximate the device's behaviour for each failure under study.

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