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Retail Competition with Switching Consumers in Electricity Markets

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Abstract

The ongoing transformations of power systems worldwide pose important challenges, both economic and technical, for their appropriate planning and operation. A key approach to improve the efficiency of these systems is through demand-side management, i.e., to promote the active involvement of consumers in the system. In particular, the current trend it to conceive systems where electricity consumers can vary their load according to real-time price incentives, offered by retailing companies. Under this setting, retail competition plays an important role as inadequate prices or services may entail consumers switching to a rival retailer. In this work we consider a game theoretical model where asymmetric retailers compete in prices to increase their profits by accounting for the utility function of consumers. Consumer preferences for retailers are uncertain and distributed within a Hotelling line. We analytically characterize the equilibrium of a retailer duopoly, establishing its existence and uniqueness conditions. Furthermore, sensitivities of the equilibrium prices with respect to relevant model parameters are also provided. The duopoly model is extended to a multiple retailer case for which we perform an empirical analysis via numerical simulations. Results indicate that, depending on the retailer costs, loyalty rewards and initial market shares, the resulting equilibrium can range from complete competition to one in which a retailer have a leading or even a dominant position in the market, decreasing the consumers' utility significantly. Moreover, the retailer network configuration also plays an important role in the competitiveness of the system.

Keywords:* *Elastic consumers, electricity market, hotelling line, market equilibrium, retail competition, switching consumers.

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1. Introduction

Over the last decades, electricity systems worldwide have experienced a set of transformations that have notably modified their structure, operation and efficiency. In general, vertically integrated systems, based on fossil-fuels technologies and with inelastic consumers, have evolved into competitive markets with high penetration of renewable production and price-responsive consumers. These transformations pose both economical and technical challenges to the market and system operators.

From an economical perspective, the recent regularization of the electricity systems worldwide has introduced market mechanisms to incorporate horizontal competition at the generation and distribution levels. In general, generating companies compete between them, at different trading floors, to supply their energy at the highest prices possible to retailing companies or to large consumers that participate in the market. Similarly, retailers compete with each other to buy their energy at the same trading floors in order to supply it to the final consumers.

From a technical perspective, new generation technologies and consumption patterns are being integrated into the electricity industry. Regarding the supply side of the system, it should be noted the important growth of renewable generation, which is becoming more and more profitable as its investment costs have been decreasing steadily over these past years. While the main advantages of these new energy sources are their low generation and CO_2 emission costs, most of them present a stochastic nature so that their available production capacities are difficult to predict on a short-term basis. The current practice in electricity systems is to use conventional fast-response generation units, for instance, combined-cycle units, to compensate unexpected falls in renewable production and to balance the system (total generation must match total demand load). However, this approach presents several disadvantages related to the early wear of these units and the increase in the final market prices.

One important approach to deal with these challenges is by promoting the active involvement of consumers in the system. In particular, demand response, i.e., the altering of the consumer load patterns as a reaction to specific incentives, is considered one of the main components of this new paradigm (Kirschen 2003). An appropriate demand-side management can smooth load peaks, mitigate short-term imbalances and improve the efficiency of the system. Among several options, the most extended form of demand-side management is to create price incentives to the consumers to switch their load. For instance, high prices can be assigned to peak-hours so that the consumption is moved to different hours. In fact, the European Union and the USA distribution system operators are actually investing a high amount of resources to promote pricing programs such as real-time pricing (RTP), critical-price pricing (CPP) and time-of-use tariffs (Mallet et al. 2014, U.S. Department Energy 2006, U.S. Gov. Policy Act 2005).

Thus, it is expected that in the coming years an important number of consumers would move from systems with fixed consumption tariffs to others in which the corresponding retailing company would offer prices that are updated in real time. Under this framework, if several retailers have access to the same area of the electricity system, they will have to compete to increase their consumer market share. In that case, the retail competition in the electricity market becomes similar to that in other industries, such as communications, banking, private health care, insurance, etc. As electricity is a near-homogeneous good, in addition to the offered price, retailers can also provide complementary services to the consumers to differentiate their products, for instance: reliability of service, information technologies, loyalty programs, etc.

Under this new setting, the role consumers play in the electricity market is increased. They can adapt their consumption to changes in supply and demand and, moreover, they can switch to retail companies with better services or conditions. To allow the growth in their customer base and to retain existing customers, retail companies can offer incentives to reward the loyalty of consumers, or implement switching costs if they decide to switch retailers (Farrel and Klemperer 2007).

In this work, we propose to model this setting as a game where retailers compete in prices to increase their revenues in the long-term. Game-theoretical models have shown to be very suitable to represent these types of competition, specially considering that, in most electricity markets, a small number of retailers rule the market.

Specifically, we analyze the interaction and the possible equilibria between retailers that compete to supply their energy to the final consumers in an electricity market. The aim of each retailer is to maximize its profit: revenues from selling its energy to the consumers minus the costs of purchasing it, by selecting an adequate retail price to offer to the consumers. In particular, we assume that retailers have different purchasing costs as they follow their own strategy to buy this energy, for instance, by participating in the electricity day-ahead market or through bilateral contracts with the generating firms. Moreover, we consider that retailers are able to anticipate, in a Stakelberg fashion, the reaction of the final consumers which seek to maximize their utility functions taking into account the possible loyalty premiums that retailers may offer. Uncertainty is incorporated to the problem by assuming that the consumer preferences for each retailer are uniformly distributed on a Hotelling line (Hotelling 1929).

In summary, the main contributions of this work are sixfold:

1. To propose a game-theoretical model to represent the interactions between several retailers and the final consumers in an electricity market. Retailers aim to maximize their profit while consumers aim to maximize their utility functions.

2. To explicitly account for the loyalty incentives, or switching costs, that consumers may have if they remain with their original retailers.

3. To provide an analytical characterization of the market equilibrium for the case of a retailer duopoly.

4. To establish the existence and uniqueness conditions for a duopoly equilibrium that are valid for a wide class of consumer utility functions.

5. To provide a sensitivity analysis of the equilibrium prices with respect to relevant problem parameters.

6. To generalize this setting for the case of more that two retailers and perform an empirical analysis via numerical simulations.

The article is structured as follows. In Section 2 we provide a literature review of retail competition in electricity markets and an economic analysis of loyalty premiums. Section 3 is dedicated to study and characterize the equilibrium solutions for the duopoly case. Section 4 generalizes this setting to the multiple retailers case. The empirical analysis is carried out in Section 5. Finally, this work's conclusions are drawn in Section 6.

2. Literature Review

Many engineering and operations research papers have analyzed the pricing decision process of a retailer in an electricity market. For instance, a real-time pricing framework to ease peak load hours in a smart grid is introduced in Qian et al. (2013), where a single retailer selects the best pricing strategy anticipating the reaction of consumers. Similarly, Bu et al. (2013) proposes a game theoretical model formulated as a four-stage problem to assist retailers in their pricing strategies in a smart grid. Based in time-of-use tariffs, García-Bertrand (2013) presents a risk-based pricing tool for retailers to decrease the consumers' load in those periods where pool prices are high. With a similar aim, a risk-averse two-stage bilevel model is proposed in Wei et al. (2015) to be used by a retailer to derive its optimal dispatch and retail prices. Specifically, a Stakelberg game is considered to model demand response.

Most liberalized electricity markets include more than one retailing company that compete to supply the final consumers. Thus, agent-based and game-theoretical models are generally used to represent these interactions. This is the case of Müller et al. (2007), which presents an agent-based model of the German retail market to study the interdependencies between retailers' pricing strategies and consumers' participation in the market. Joskow and Tyrole (2006) studies the effects of combining retail competition and different consumer profiles. It is shown that, apart from the absence of real-time meters, transaction costs and joint interruptibility can also hamper consumers to react to real-time prices. A game-theoretical model to represent the equilibrium between multiple retailers is introduced in Oliveira et al. (2013). Retailers buy their energy from generators at different trading floors and sell it to the final consumers. However, the retail price is considered unique and consumers are aggregated into a single demand curve.

Although these works notably contribute to address many of the challenges that are faced by retailing firms, issues such as consumer loyalty incentives or switching costs, and their economical implications, are in general not considered in the context of electricity markets. As explained in the previous section, this is mainly because, until very recently, consumers have only exhibited a passive role in electricity systems.

Nevertheless, there is an extensive marketing and economic literature that has addressed the market implications of loyalty incentives or switching costs in other industries. See Farrel and

Klemperer (2007) for a detailed review on this topic. In general, one of the most important questions that arises with these incentives is whether they lead to higher or lower market prices. Two main approaches have been used to answer this question: first, the consideration of analytical models, which aim to reproduce either static or dynamic competition, and second, the use of empirical analysis, based on simulations or in real-world data.

Regarding analytical models, the most widespread approach to represent product differentiation is via a Hotelling line (Hotelling 1929), where consumer preferences are distributed within a line that connects two selling firms. Even if homogeneous products are considered, which is the case for electricity, product differentiation arises from asymmetric transportation costs (in this contest, the concept "transportation costs" is used to quantify the consumer's difference in preferences for each retailing firm, rather than physical or monetary costs). This model has become an standard model in oligopolistic competition.

Although the original Hotelling model is restricted to the duopoly case, several extensions have been proposed to represent oligopolistic markets. For instance, Chen and Riordan (2007) introduced the spokes model where several oligopolistic firms interact simultaneously in a market with multiple products. In particular, each selling firm competes directly with all other firms, even considering that consumers are only interested in a maximum number of product varieties. An alternative extension of the Hotelling model is presented in Somaini and Einav (2013), where more than two firms can also be considered. The model assumes that each consumer is limited to buy from two asymmetric firms. Due to its similarities with the current electricity retail market, some of these models' features are used in our numerical analysis to extend the duopoly case.

A dynamic duopoly model is presented in Doganoglu (2010) to study competition between firms. Sufficient conditions are provided for the existence of a Markov equilibrium. Results show that the level of switching costs has an important impact on competition. In particular, when switching costs are low, the resulting prices in the steady state can be lower than without switching costs. A similar analysis is conducted in Rhodes (2014) where the conditions under which switching costs may raise or decrease market prices are established for both the short and long-run. Specifically, it is shown that switching costs are more likely to increase prices in the short-run. An extension of these models is proposed in Chen et al. (2014) where firms in a duopoly are allowed to charge different prices to their own consumers (customized pricing). Results indicate that increasing switching costs (or decreasing loyalty rewards) may decrease firms' profits.

Many empirical analyses have been carried out to show the impact of switching costs in different industries (see Farrel and Klemperer (2007) for a complete revision). Related to the electricity industry, Waterson (2003) studies the impact of consumers behavior in the industry performance. Several case studies from different industries are analyzed, including the electricity market in the

UK, to show that consumer switching costs have a relevant impact on market efficiency. In the same vein, an empirical analysis to study the impact of switching costs is presented in Wilson and Price (2010). The analysis of some databases from the UK electricity market shows that consumers' ability to select the best retailer is limited. This suggests that, apart from purely economical reasons, there are other factors that may condition consumer switching costs.

Compared to the above works, the main contributions of the proposed model are directly related to considering electricity as our trading good. In particular, we use a general function for the consumer utility which depends on the purchased quantity of electricity, represented as a continuous variable. Note that most of the references assume a fixed quantity (or number of units) of the good to be purchased and the only decision variable represents the choice of an appropriate retailing firm. Additionally, we assume that the retailers are asymmetric as they follow their own strategy when acquiring their electricity at different trading floors (with different prices). Under this general setting we provide analytical results that characterize the equilibrium of the duopoly and extend this framework numerically to a multiple-retailer case study.

3. Electricity market with two retailers

Consider an electricity market with two retailers, R_i and R_j, that sell their energy at prices, p_i and p_j, respectively, to a set of consumers, C. To model the consumer preferences, we assume that the two retailers are located at the ends of a Hotelling line (Hotelling 1929) of length 1, that is, R_i is located at the beginning of the line and R_j at the end. Each consumer is placed at a given position within the Hotelling line $\theta_{ij} \in [0,1]$, so that the closer they are to R_i, i.e., the smaller θ_{ij} is, the higher her preference to buy from R_i. Moreover, consumers are distributed along this line by following a probability distribution, $F(\theta_{ij})$. Additionally, let π_{ij} (π_{ji}) denote the proportion of consumers with retailer R_i (R_j) at the initial state, where $\pi_{ij} + \pi_{ji} = 1$.

For the sake of completeness, we will assume that the consumer utilities include a function u that is a concave increasing function of the purchased energy $q_i \geq 0$, or $q_j \geq 0$, if the energy is bought from R_i or R_j respectively, with continuous second derivatives. We will also assume that it has an invertible first derivative $u'(q) \geq 0$.

The utility maximization problem for a consumer that had bought from R_i at the initial state, and decides to buy again from retailer R_i, is given by

$$\max_{q_i} \; u(q_i) - p_i q_i - k\theta_{ij} + \gamma_i, \tag{1}$$

where the second term represents the payments to R_i. Parameter $k > 0$ denotes the transportation cost per unit of length and is used to quantify the preference levels of consumers with respect to each retailer. In particular, the smaller the value of θ_{ij} the lower the transportation costs $k\theta_{ij}$, yielding an increased preference for retailer R_i for this consumer. This preference asymmetry may be caused by the different services offered by the retailers such as: quality and reliability of supply, information technologies, technical support, flexible payments programs, etc. Note that the transportation cost does not depend on the consumer's actual retailing company. Another parameter, $\gamma_i \geq 0$, is introduced to represent a *loyalty reward or premium*, i.e., the obtained incentive by purchasing again from R_i. Note that $-\gamma_i$ can also be viewed as a switching cost incurred if a consumer decides to switch to R_j.

In particular, if this same consumer decides to switch to R_j, the problem she has to solve is

$$\max_{q_j} \; u(q_j) - p_j q_j - k\theta_{ji}, \tag{2}$$

where $\theta_{ji} = 1 - \theta_{ij}$ is the Hotelling line distance between the consumer and R_j. The switching decision is modeled by letting the term γ_i vanish from the utility function.

Similar problems can be obtained if the consumer initially bought from R_j by interchanging i and j.

Regardless of the initial state, the optimal value of q_k, where $k = \{i, j\}$, is obtained from $u'(q_k) = p_k$ as $q_k^* = v(p_k) \equiv (u')^{-1}(p_k)$, where $v : [\min(c_i, c_j); \alpha] \to \mathbb{R}^+$, and we use the notation $\alpha = u'(0)$. Under our assumptions, v is a nonnegative decreasing function of p_k on its domain; also $v(\alpha) = 0$.

Each retailer R_i or R_j purchases their energy from the spot or futures market, at a cost c_i or c_j and then, they sell the energy to the consumers at prices p_i and p_j, respectively.

Taking into account consumer preferences, retailers compete in prices trying to increase their proportion of consumers from the initial values, π_{ij} and π_{ji}, respectively, by the optimization of their utility functions. The following proposition shows the optimal strategy for each retailer to attain this goal.

Proposition 1. *The optimal strategy for retailer R_i, is defined through the solution of the following problem*

$$\max_{p_i} \quad u_i^r(p_i, p_j) = (p_i - c_i) q_i^*(p_i) \pi_i^*(p_i, p_j) \tag{3}$$

where $q_i^*(p_i) = v(p_i)$ is the optimal consumer energy, and $\pi_i^*(p_i, p_j)$ is the attained proportion of consumers by retailer R_i defined as

$$\pi_i^*(p_i, p_j) = F_{ij}^* \pi_{ij} + \overline{F}_{ji}^* \pi_{ji}, \tag{4}$$

where F_{ij}^* denotes the probability that a consumer who bought from retailer R_i remains with R_i and $\overline{F}_{ji}^*$ the probability that a consumer who bought from retailer R_j switches to R_i. These probabilities are obtained as follows

$$F_{ij}^* = P(\theta_{ij} \leq \theta_{ij}^*), \quad \overline{F}_{ji}^* = P(\theta_{ji} > \theta_{ji}^*) \tag{5}$$

where

$$\theta_{ij}^* = \frac{1}{2k}\left(\varphi(p_i) - \varphi(p_j) + k + \gamma_i\right). \tag{6}$$

and $\varphi(p_i) \equiv u(v(p_i)) - pv(p_i)$ *is a negative and decreasing function of* p_i *that satisfies that* $\varphi'(p_i) = -v(p_i)$.

Proof: From (1) and (2), a consumer who bought from retailer R_i will remain with R_i if it holds that

$$u(q_i) - p_i q_i - k\theta_{ij} + \gamma_i \geq u(q_j) - p_j q_j - k\theta_{ji}, \tag{7}$$

or equivalently if

$$\theta_{ij} \leq \theta_{ij}^* = \frac{1}{2k}\left(\varphi(p_i) - \varphi(p_j) + k + \gamma_i\right), \tag{8}$$

On the other hand, a consumer who bought from retailer R_j will switch to R_i if

$$u(q_i) - p_i q_i - k\theta_{ij} > u(q_j) - p_j q_j - k\theta_{ji} + \gamma_j, \tag{9}$$

or equivalently if

$$\theta_{ji} > \theta_{ji}^* = \frac{1}{2k}\left(\varphi(p_j) - \varphi(p_i) + k + \gamma_j\right). \tag{10}$$

□

Due to the symmetry of the problem, a proposition similar to Proposition 1 can be obtained, by interchanging i and j, to characterize the optimal strategy of retailer R_j.

Retailer R_i's problem (3) and its equivalent for retailer R_j are interrelated as both objective functions depend on prices p_i and p_j, simultaneously. Under this setting, the following game and associated Nash equilibrium can be defined:

Definition 1. *(Nash equilibrium of the duopoly game)* The game between the two retailers R_i and R_j is defined when both players seek to determine their optimal p_i and p_j, respectively, to maximize their profits (3). A Nash equilibrium would be reached if there exists a set of prices p_i^* and p_j^* so that $u_i^r(p_i^*, p_j^*) \geq u_i^r(p_i, p_j^*) \quad \forall p_i \in S_i$ and if $u_j^r(p_i^*, p_j^*) \leq u_j^r(p_i^*, p_j) \quad \forall p_j \in S_j$, where S_i and S_j represent the feasible region for prices p_i and p_j.

3.1. Equilibrium price classification

In this section we classify and characterize the different types of equilibria that could be reached in the market. Some definitions that will be used within this section are introduced below.

Definition 2. We say retailer R_i has a *dominant position* in the Hotelling line if, at equilibrium, $F_{ij}^* = \overline{F}_{ji}^* = 1$. Or equivalently, if $F_{ji}^* = \overline{F}_{ij}^* = 0$.

Note that in this case all consumers remain with or will switch to the dominant retailer, causing a lack of competition in the market.

DEFINITION 3. We say retailer R_i has a *leading position* in the Hotelling line if, at equilibrium prices, $F^*_{ij} = 1$ but $\overline{F}^*_{ji} < 1$. Or equivalently, if $F^*_{ji} < 1$ but $\overline{F}^*_{ij} = 0$. In this latter case we say retailer R_j has a *following position*.

This definition implies all consumers with the leading retailer remain with her, while a proportion of consumers with the follower retailer switch. This causes some competition in the market, with one retailer leading the other one.

DEFINITION 4. We say there is *complete competition* in the Hotelling line if, at equilibrium prices, $0 < F^*_{ij} < 1$ and $0 < F^*_{ji} < 1$.

This definition implies consumers have a certain probability to switch between retailers, allowing more competitive equilibrium prices.

Hereafter, we assume consumer preferences are distributed along the Hotelling line following a uniform distribution, that is, $F_{ij}(\theta_{ij}) = 0$ for $\theta_{ij} < 0$, $F_{ij}(\theta_{ij}) = \theta_{ij}$ for $0 \leq \theta_{ij} \leq 1$, and $F_{ij}(\theta_{ij}) = 1$ for $\theta_{ij} > 1$.

According to the above definitions and depending on the different market settings, we distinguish three types of equilibrium: dominant position, leading positions, and complete competition.

PROPOSITION 2. *If the following condition holds*

$$\varphi(p_i) - \varphi(p_j) \geq k + \gamma_j, \tag{11}$$

then retailer R_i will have a dominant position in the Hotelling line and the optimal price is given by $(p_i - c_i)v'(p_i) + v(p_i) = 0$, while retailer R_j has no demand and gets no income.

The proof for this proposition can be found in the Appendix.

To state the following propositions, we need to define the following function of prices:

$$\Gamma(p; c) = -\frac{(p - c)v(p)\varphi'(p)}{v(p) + (p - c)v'(p)} - \varphi(p) \tag{12}$$

The conditions under which a retailer will exhibit a leading position in the market are provided in the following proposition.

PROPOSITION 3. *If the following condition holds*

$$\varphi(p_i) - \varphi(p_j) \geq k - \gamma_i, \quad k + \gamma_j > \varphi(p_i) - \varphi(p_j) \geq -k + \gamma_j, \tag{13}$$

then retailer R_i will have a leading position in the Hotelling line where the optimal (equilibrium) prices satisfy the following conditions

$$\Gamma(p_i; c_i) = -\varphi(p_j) + \frac{1 + \pi_{ij}}{\pi_{ji}} k - \gamma_j \tag{14}$$

$$\Gamma(p_j; c_j) = -\varphi(p_i) + k + \gamma_j, \tag{15}$$

Table 1 Different values of κ

	κ_i	κ_j
Proposition 3	$\frac{1+\pi_{ij}}{\pi_{ji}}k - \gamma_j$	$k + \gamma_j$
Proposition 4	$k + \pi_{ij}\gamma_i - \pi_{ji}\gamma_j$	$k - \pi_{ij}\gamma_i + \pi_{ji}\gamma_j$

The proof for this proposition can be found in the Appendix.

Similarly, Proposition 4 below characterizes the complete competition case.

PROPOSITION 4. *Assuming that $k \geq (\gamma_i + \gamma_j)/2$, if the following condition holds*

$$k - \gamma_i \geq \varphi(p_i) - \varphi(p_j) \geq -k + \gamma_j, \tag{16}$$

then the Hotelling line works under complete competition, where the optimal (equilibrium) prices satisfy the following conditions

$$\Gamma(p_i; c_i) = -\varphi(p_j) + k + \pi_{ij}\gamma_i - \pi_{ij}\gamma_j \tag{17}$$

$$\Gamma(p_j; c_j) = -\varphi(p_i) + k - \pi_{ij}\gamma_i + \pi_{ji}\gamma_j. \tag{18}$$

The proof for this proposition can be found in the Appendix.

Under the assumption $k \geq (\gamma_i + \gamma_j)/2$, the case where both retailers have a leading position ($F^*_{ij} > 1$ and $F^*_{ji} > 1$ or equivalently $\theta^*_{ij} > 1$ and $\theta^*_{ji} > 1$) is not possible, so it is not considered as a potential equilibrium in this work. Additionally, symmetric equilibria can be characterized from propositions 2 and 3 by interchanging retailer R_i by R_j.

The equilibrium prices for the preceding competitive cases in Propositions 3 and 4 (we exclude the dominant position from Proposition 2), can be obtained by solving a system of equations of the form

$$\Gamma(p_i; c_i) = -\varphi(p_j) + \kappa_i \tag{19}$$

$$\Gamma(p_j; c_j) = -\varphi(p_i) + \kappa_j, \tag{20}$$

for κ_i and κ_j values depending on k, γ_i, γ_j, π_{ij} and π_{ji} (but independent of p_i and p_j). Table 1 presents the different values of κ.

In the following, we will exploit this mathematical structure to study the existence and uniqueness of the equilibrium prices.

3.2. Existence and uniqueness of equilibrium prices

In this section we characterize the solution of the system (19) and (20), which determines the equilibria solutions for the market configurations where there is some degree of competition, i.e., the cases described within Propositions 3 and 4.

To conduct this study, we will introduce some conditions on the consumers utility function $u(q)$. In what follows, we will make use of the notation $\tilde{c} = \min(c_i, c_j)$.

We define the following conditions:

C.1 We assume that $u(0) = 0$ and there exists $\tilde{q}$ such that $u'(\tilde{q}) = \tilde{c}$.

C.2 We assume that u is three times continuously differentiable on $[0;\tilde{q}]$, that it is nondecreasing and concave ($u''(q) \leq 0$) on $[0;\tilde{q}]$, and that it has an invertible first derivative $u'(q) \geq 0$ on that interval.

C.3 We impose the following condition on u''',

$$\frac{u'''(v(p))}{u''(v(p))^2} \leq \frac{1}{p-\tilde{c}} - \frac{1}{v(p)u''(v(p))}.$$

Conditions C.1 and C.2 are standard. A first interpretation of condition C.3 is that the function u should not change curvature too fast. Note that a sufficient condition for C.3 is $u'''(q) \leq 0$; also, if the function u satisfies condition C.2 on $[0, \infty)$ and the sign of $u'''(q)$ is constant for all large q, then it must hold $u'''(q) \leq 0$ for large q.

Our first result is related to the function Γ defined above, which using $\varphi'(p) = -v(p)$ can be written as

$$\Gamma(p;c) \equiv \frac{(p-c)v(p)^2}{v(p)+(p-c)v'(p)} - \varphi(p). \tag{21}$$

The following two lemmas characterize key analytical properties of $\Gamma(p;c)$ that are needed to prove the existence of equilibrium.

Lemma 1. *For $p \in [c;\alpha]$, $\Gamma(p;\alpha,c)$ defined in (21) has at least one vertical asymptote.*

Proof: This result follows from the continuity of $d(p;c) = v(p) + (p-c)v'(p)$ and the properties that $d(c;c) = v(c) > 0$ and $d(\alpha;c) = v(\alpha) + (\alpha - c)v'(\alpha) = (\alpha - c)v'(\alpha) < 0$, as $v(\alpha) = 0$ and $v'(\alpha) = 1/u''(v(\alpha)) = 1/u''(0) < 0$ from Condition C.2. □

In what follows we will use the notation

$$\tau(c) \equiv \min\{\omega : d(\omega;c) = 0, \omega \in [c;\alpha]\},$$

for the first pole of Γ on $[c;\alpha]$.

An illustration of a plot for a particular realization of function Γ can be seen in Figure 1.

Lemma 2. *For $p \in [c;\tau(c))$, $\Gamma(p;c)$ defined in (21) is a continuous monotonically increasing function taking values in $[-\varphi(c);\infty)$.*

Proof: The derivative of Γ can be written as

$$\Gamma'(p;c) = v + \frac{v(p-c)}{(v+(p-c)v')^2}\left(-vv' + (p-c)(v')^2 - (p-c)vv''\right). \tag{22}$$

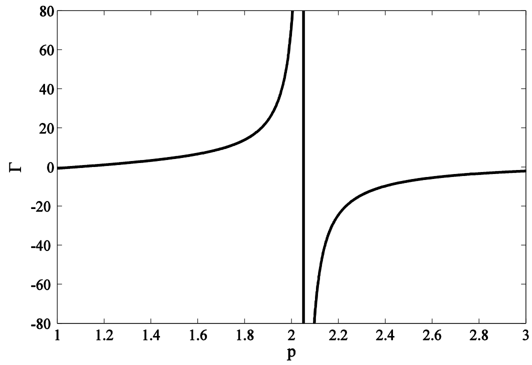


Figure 1 A particular realization of the Γ function.

From Condition C.2, on $[c;\tau)$ we have that v has continuous first and second derivatives, with $v(p) > 0$ and $v'(p) < 0$. Also, as $v'(p) = 1/u''(v(p))$ and $v''(p) = -u'''(v(p))v'(p)/(u''(v(p)))^2$, Condition C.3 implies

$$(p-c)v(p)v''(p) \leq -vv' + (p-c)(v')^2,$$

and as a consequence $\Gamma'(p;c) > 0$ on $[c;\tau)$. $\square$

The following result proves the existence of solutions for the system of equations of interest,

$$\Gamma(p_i;c_i) = -\varphi(p_j) + \kappa_i \tag{23}$$

$$\Gamma(p_j;c_j) = -\varphi(p_i) + \kappa_j, \tag{24}$$

for κ_i and κ_j defined in Table 1, under the following condition:

C.4 The values κ_i satisfy

$$\kappa_i \geq \varphi(c_j) - \varphi(c_i), \quad \kappa_j \geq \varphi(c_i) - \varphi(c_j).$$

In other words, Theorem 1 below proves the existence of equilibrium solutions for the retailers pricing game for two cases: i) when one of the retailers has a leading position in the market (Proposition 3) and ii) when there is complete competition (Proposition 4). This result completes all the possible market configurations, as the dominant retailer case was already characterized in Proposition 2.

Theorem 1. *Under Conditions C.1–C.4, there exists at least a solution for the system (23)–(24). Furthermore, this solution is in* $[c_i;\tau(c_i)) \times [c_j;\tau(c_j))$.

The proof for this theorem can be found in the Appendix.

The next step is to establish the uniqueness of the preceding fixed point. To prove this we need to start by tightening Condition C.3.

Define

$$\rho(p;c) \equiv \frac{p-c}{(v+(p-c)v')^2}\left(-vv' + (p-c)(v')^2 - (p-c)vv''\right). \tag{25}$$

Also, let

$$\bar{c}_i \equiv G(-\varphi(c_i)+\kappa_i; c_i) \geq c_i$$
$$\bar{c}_j \equiv G(-\varphi(c_j)+\kappa_j; c_j) \geq c_j.$$

These values have the property that the iterates generated by the fixed-point equations (EC.7)–(EC.8) lie in $[\bar{c}_k; \tau(c_k))$ for $k=i,j$. We also define $\bar{c} \equiv \min(\bar{c}_i, \bar{c}_j)$ and $\bar{\tau} \equiv \max(\tau(c_i), \tau(c_j))$.

Condition C.3 is replaced with

C.3' The values of u, c_k and κ_k for $k=i,j$ are such that for all $p \in [\bar{c}_k; \tau(c_k))$, $k=i,j$, it holds that

$$1+\rho(p;c_k) > \sqrt{\frac{v(\bar{c})}{v(\bar{\tau})}}.$$

As condition C.3 is equivalent to requiring

$$\rho(p;c_k) \geq 0,$$

and $v(\bar{c}) > v(\bar{\tau})$, condition C.3' can be interpreted as a tightening of C.3.

Another motivation to introduce the $\bar{c}_k$ values is that while it holds that $\rho(c_k;c_k)=0$, we have $\rho(\bar{c}_k;c_k) \geq 0$.

The following theorem establishes the uniqueness of the equilibrium solutions for the retailers pricing game: i) the leading retailer case (Proposition 3) and ii) for the complete competition case (Proposition 4).

THEOREM 2. *Under Conditions C.1, C.2, C.3' and C.4, the solution of (23)–(24) on* $[\bar{c}_i; \tau(c_i)) \times [\bar{c}_j; \tau(c_j))$ *is unique.*

The proof for this theorem can be found in the Appendix.

3.3. Sensitivity analysis

In this section we provide some guidelines on how to compute sensitives of the equilibrium prices with respect to the problem parameters. To this end, consider our fixed-point iteration (23)–(24), written in the form

$$\Gamma(p_i;c_i) + \varphi(p_j) = \kappa_i \tag{26}$$
$$\Gamma(p_j;c_j) + \varphi(p_i) = \kappa_j, \tag{27}$$

for κ_i and κ_j given by (1).

Let ν denote any of the parameters k, γ_i, γ_j, π_{ij}, π_{ji}. The sensitivity of the equilibrium prices with respect to ν can be found from the solution of

$$\begin{aligned} \Gamma'(p_i;c_i)\frac{\partial p_i}{\partial \nu} + \varphi'(p_j)\frac{\partial p_j}{\partial \nu} &= \frac{\partial \kappa_i}{\partial \nu} \\ \Gamma'(p_j;c_j)\frac{\partial p_j}{\partial \nu} + \varphi'(p_i)\frac{\partial p_i}{\partial \nu} &= \frac{\partial \kappa_j}{\partial \nu}. \end{aligned}$$

Using (EC.12), we can rewrite these equations as

$$\begin{aligned} (1+\rho(p_i;c_i))\frac{\partial p_i}{\partial \nu} - \frac{v(p_j)}{v(p_i)}\frac{\partial p_j}{\partial \nu} &= \frac{1}{v(p_i)}\frac{\partial \kappa_i}{\partial \nu} \\ -\frac{v(p_i)}{v(p_j)}\frac{\partial p_i}{\partial \nu} + (1+\rho(p_j;c_j))\frac{\partial p_j}{\partial \nu} &= \frac{1}{v(p_j)}\frac{\partial \kappa_j}{\partial \nu}. \end{aligned}$$

From (22), under Condition C.3 we have that $\rho_k \geq 0$ $(k = i, j)$, and under mild additional conditions (also under Condition C.3', for example), $\rho_k > 0$ and the coefficient matrix in the preceding system is invertible. The values of interest are given by

$$\begin{aligned} v(p_i)\Delta_{ij}\frac{\partial p_i}{\partial \nu} &= \rho(p_j;c_j)\frac{\partial \kappa_i}{\partial \nu} + \frac{\partial \kappa_i}{\partial \nu} + \frac{\partial \kappa_j}{\partial \nu} \\ v(p_j)\Delta_{ij}\frac{\partial p_j}{\partial \nu} &= \rho(p_i;c_i)\frac{\partial \kappa_j}{\partial \nu} + \frac{\partial \kappa_i}{\partial \nu} + \frac{\partial \kappa_j}{\partial \nu}, \end{aligned}$$

where $\Delta_{ij} = \rho(p_i;c_i) + \rho(p_j;c_j) + \rho(p_i;c_i)\rho(p_j;c_j)$.

For example, for $\nu \equiv \pi_{ij}$ and from (1) it will hold that

	$\partial\kappa_i/\partial\nu$	$\partial\kappa_j/\partial\nu$
Proposition 3	$\frac{1}{\pi_j}k$	0
Proposition 4	γ_i	$-\gamma_i$

and for the case in Proposition 4:

$$\begin{aligned} v(p_i)\Delta_{ij}\frac{\partial p_i}{\partial \pi_i} &= \rho(p_j;c_j)\gamma_i \\ v(p_j)\Delta_{ij}\frac{\partial p_j}{\partial \pi_j} &= -\rho(p_i;c_i)\gamma_i, \end{aligned}$$

implying in particular $\partial p_i/\partial \pi_{ij} > 0$ and $\partial p_j/\partial \pi_{ij} < 0$. The higher the initial proportion of consumers that a retailer has, the higher the prices he can set at the equilibrium.

By following a similar reasoning we can compute the signs associated to the rest of price sensitivities, for the two market setting where competition is present:

	$\frac{\partial p_i}{\partial \pi_{ij}}$	$\frac{\partial p_i}{\partial \pi_{ji}}$	$\frac{\partial p_j}{\partial \pi_{ij}}$	$\frac{\partial p_j}{\partial \pi_{ji}}$	$\frac{\partial p_i}{\partial k}$	$\frac{\partial p_j}{\partial k}$	$\frac{\partial p_i}{\partial \gamma_i}$	$\frac{\partial p_i}{\partial \gamma_j}$	$\frac{\partial p_j}{\partial \gamma_i}$	$\frac{\partial p_j}{\partial \gamma_j}$
Proposition 3	>0	<0	>0	<0	>0	>0	$=0$	<0	$=0$	>0
Proposition 4	>0	<0	<0	>0	>0	>0	>0	<0	<0	>0

We obtain that the transportation costs k always have a positive relationship with the equilibrium prices. For the competitive case in Proposition 4, an increase of γ_i leads to an increment on the price offered by retailer R_i while decreasing the price offered by the competitor R_j. However, for the market described in Proposition 3, as R_i is the leader, its γ_i does not have a direct impact on prices. Additionally, it should be noted that, for the case described in Proposition 3, $\partial p_i/\partial \pi_{ij} > 0$ and $\partial p_j/\partial \pi_{ij} > 0$, indicating that an increase in the initial market share of the leader gives rise to simultaneous increases for both the leader's and follower's price, as the later cannot capture any of the leader's consumers (it lacks any incentive to decrease prices).

These and other relevant sensitivities will be further studied in Section 5 for a particular realization of the consumers' utility function $u(q)$.

4. Electricity market with more than two retailers: network

In this section we extend the previous duopoly model to include additional retailers. In particular, we assume that retailers are located at the nodes of a pairwise network while consumer are distributed within the Hotelling lines connecting these nodes. Similar to Somaini and Einav (2013), we consider that each consumer can only buy electricity from the two retailers located at the two ends of her corresponding line. We adopt this simplification for the sake of clarity and to ease the computational burden of the model. Nevertheless, this framework is very similar to the actual retail market in many countries where, despite the existence of many retailing companies, these are distributed within the electricity network so that consumers have only an small set of options to sign contracts with.

We have a network with a set of retailers, $\mathcal{N} = \{1, 2, \ldots, N\}$, and a set of lines, $\mathcal{L} = \{1, 2, \ldots, L\}$. For each line $ij \in \mathcal{L}$ we have different types of consumers with preferences θ_{ij} over the corresponding Hotelling line. Because consumers in the same line are equivalent, we have

$$\theta_{ij} + \theta_{ji} = 1. \tag{28}$$

Each retailer purchases their energy from the spot or futures market, at a cost c_i, for $i = 1, \ldots, N$. Then, they sell the energy to all the consumers at a unique price p_i.

The consumers are shared through the network with initial proportions $\pi^0_{ij} = \pi^0_{ji}$ for each line $ij \in \mathcal{L}$, with $\sum_{i,j>i} {\pi^0}_{ij} = 1$. And for each line, let π_{ij} and $\pi_{ji} = 1 - \pi_{ij}$ denote the initial proportion of consumers within that line with R_i and R_j, respectively.

The following proposition shows the optimal strategy for each retailer to increase its proportion of consumers.

PROPOSITION 5. *The optimal strategy for retailer R_i, for $i = 1, \ldots, N$, is defined through the solution of the following problem*

$$\max_{p_i} \quad u^r_i(p_i, p_{-i}) = (p_i - c_i)\ q^*_i(p_i)\ \pi^*_i(p_i, p_{-i}) \tag{29}$$

where $q_i^*(p_i) = v(p_i)$ *is the optimal consumer energy, and* $\pi_i^*(p_i, p_{-i})$ *is the attained proportion of consumers by retailer* R_i *defined as*

$$\pi_i^*(p_i, p_{-i}) = \sum_{j \in \mathcal{N}_i} \pi_{ij}^*(p_i, p_{-i}) \pi_{ij}^0 \tag{30}$$

where $\mathcal{N}_i$ *denotes the set of retailers adjacent to* R_i *and*

$$\pi_{ij}^*(p_i, p_{-i}) = F_{ij}^* \pi_{ij} + \overline{F}_{ji}^* \pi_{ji} \tag{31}$$

is the proportion of consumers in line ij *that buy from* R_i *where* F_{ij}^* *and* $\overline{F}_{ji}^*$ *are defined in Proposition 1.*

The proof is equivalent to that for Proposition 1.

The analytical characterization of the equilibrium prices of the multiple retailer framework is more complex than in the duopoly case. Hence, we will limit our analysis to a numerical case study. The equilibrium prices will be computed by solving simultaneously the optimality conditions for all the retailers, i.e., we solve the following nonlinear system of equations

$$\frac{\partial u_i^r(p_i, p_{-i})}{\partial p_i} = 0 \qquad \forall i = 1, \ldots, N. \tag{32}$$

To make sure that the above system provides equilibrium prices, an iteration of a diagonalization solution algorithm is performed, where retailers solve iteratively their profit maximization problems assuming that the rival prices are fixed. If no retailer deviates unilaterally from their price strategy, then the resulting set of prices can be considered as a Nash equilibrium. Additionally, for the sake of clarity we will only present results for market settings in which all the retailers have a competitive position in the market (a case analogous to Proposition 4).

5. Empirical analysis

In this section we perform a set of numerical analyses to better understand the impact of different model parameters on the equilibrium market outcomes. First, we focus our analysis on the duopolistic case and then we extend it to the multiple-retailers case.

5.1. Duopoly case

5.1.1. Data In this case study we use a particular realization of the consumers' utility function $u(q)$ in (1). In particular, we have adopted a quadratic formulation, which is an standard assumption within the electricity markets literature Oliveira et al. (2013), Kirschen (2003). Hence, we consider that $u(q) = \alpha q - \frac{1}{2}\beta q^2$. This is a concave function for $\beta \geq 0$ which meets the existence and uniqueness equilibrium conditions discussed in Theorem 1 and, for appropriate values of the parameters, also those of Theorem 2.

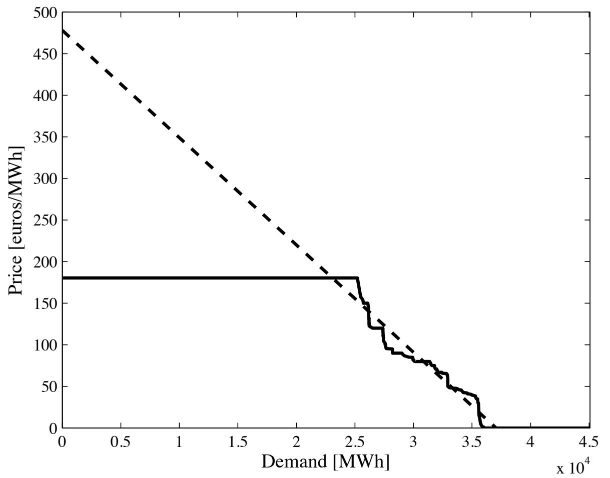


Figure 2 Demand curve Spanish day-ahead electricity market (12:00, 04/08/2015) and its linear approximation

To obtain realistic market outcomes, we have adjusted the demand parameters by using data from a real world electricity market: the Spanish electricity market OMIE (2015). Fig. 2 represents the day-ahead aggregated demand for the Spanish market for a selected day and hour. The demand curve can be identified as the consumers' marginal utility, which for our case is a linear curve, i.e., $P = \frac{\partial u(q)}{\partial q} = \alpha - \beta q$. Hence, we approximate the elastic part of the demand by this marginal utility (dash line) which renders $\alpha = 480$ euros/MWh and $\beta = 0.012$ euros/MWh^2. Observe that to make this approximation we have assumed that the Spanish demand function is directly submitted by the consumers, without intermediate retailers. This is not completely true in the Spanish case as a significant part of the day-ahead energy is traded by retailers in the market. However, retailers are supposed to transfer the final consumers' demand to the market (by including an small revenue margin) so that the main behavior of the consumers demand can be assumed to be that of Fig. 2.

For the base case we assume that retailer R_i is able to purchase its energy in the futures or day-ahead market at a overall cheaper price than retailer R_j so that $c_i = 40$ euros/MWh and $c_j = 47$ euros/MWh. Furthermore, we fix the initial market share of each retailer at 50%, i.e., $\pi_{ij} = 1 - \pi_{ji} = 0.5$.

The selection of adequate values for the transportation costs k and the loyalty rewards γ_i and γ_j is not straightforward, as there is not much real data available for electricity markets (Waterson 2003). Therefore, the values for k, γ_i and γ_j, have been selected so that: a) they have a significant impact on the consumers utility functions (1) and (2) and b) they meet the condition $k \geq (\gamma_i + \gamma_j)/2$, which was required by propositions 3 and 4 to avoid unrealistic equilibria. We assume that $k = 150000$ euros, $\gamma_i = 70000$ euros and $\gamma_j = 30000$ euros. Observe that retailer R_i has a better position in the market as it presents lower costs c_i and higher loyalty premiums γ_i than R_j.

5.1.2. Impact of π_{ij} First we analyze the impact of the initial market share of retailer R_i (π_{ij}) on the market equilibrium outcomes. Results are presented in Fig. 3. For the range $\pi_{ij} \in (0, 0.385)$, R_i has a leading position in the market (Proposition 3) while for $\pi_{ij} \geq 0.385$ there is complete competition (Proposition 4). This is a counterintuitive result that indicates that a retailer may lose market power if it increases its initial market share.

This behavior can be explained by analyzing the prices offered by each retailer. If the prices offered by the leader R_i are too high Fig. 3(a), consumers, initially attached to the leader, will start to consider again the possibility to switch to R_j, opening competition and altering the market equilibrium. In general, as R_i has lower costs, it is able to offer lower prices than R_j for both the leader and complete competition cases. In particular, for the leader case ($\pi_{ij} \in (0, 0.385)$) an increase of π_{ij} increases both p_i and p_j dramatically. However, if the market enters into complete competition, there is a discrete negative jump on both prices and only the leader's price p_i increases again with π_{ij}. Note that these price trends are coherent with the sensitivity analysis performed in Section 3.3.

The resulting market share for each retailer is presented in Fig. 3(b). The appreciable increase on prices in the leader's case entails an small decrease on the leader's market share which is corrected as the market enters complete competition. However, observe that retailer R_i's profit, specially when she acts as a leader, always increases with its initial market share (Fig 3(c)), although there is a discrete decrease when the market enters complete competition. Despite its initial market share decrease, retailer R_j's profit also increases in the leader case, as she indirectly benefits from the rise of prices.

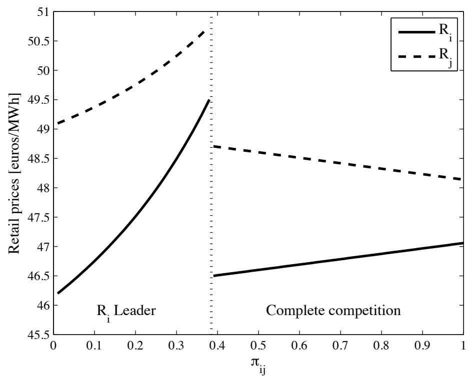
Finally, Fig. 3(d)) presents the consumers utility, which decreases as the leader increases its initial market share but increases again when the market enters complete competition.

5.1.3. Impact of γ_i The impact of the loyalty reward γ_i of retailer R_i on the different market outcomes is presented in Fig. 4.

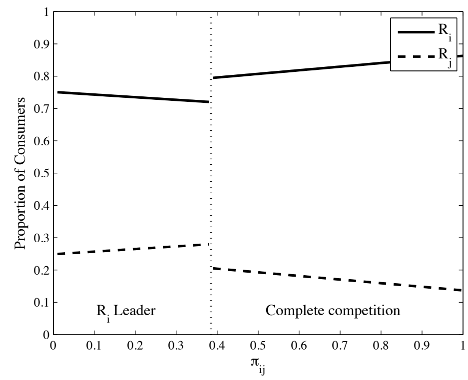
The market is in complete competition if $\gamma_i \in (0, 71000)$ euros, and R_i becomes the leader for values $\gamma_i \geq 71000$ that imply that is able to "lock-in" its consumers. Observe that once $\gamma_i \geq 71000$ the loyalty reward γ_i no longer has an impact on the market.

Regarding the retail prices (Fig. 4(a)), an increase of γ_i allows R_i to charge higher prices to its consumers while R_j needs to reduce its price to be more competitive. Again, once the market leaves complete competition there is a discrete increase for both prices.

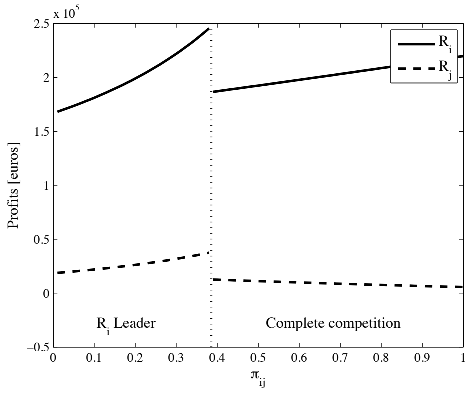
Similarly, for the complete competition case, the market share (Fig. 4(a)) of R_i increases with its loyalty reward while it decreases for R_j. For the leading case R_i's market share drops slightly to 0.7, as the price offered (Fig. 4(a)) for this range of values of γ_i is very high. The overall effect



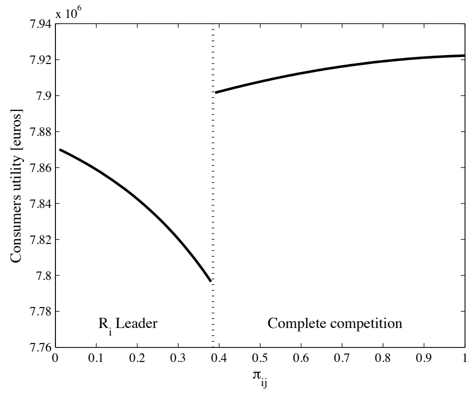
(a) Retail prices



(b) Proportion of consumers



(c) Retailers' Profits



(d) Consumers utility

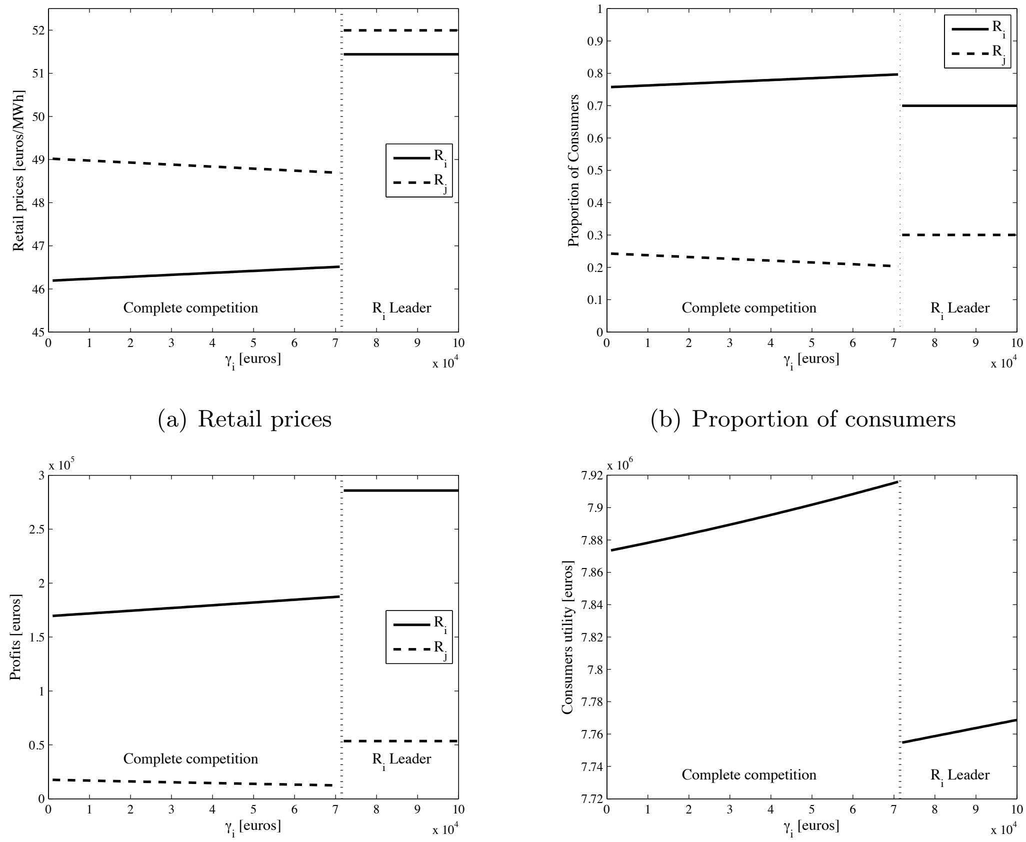
Figure 3 Impact of π_{ij} on the equilibrium market outcomes

is that R_i's profit significantly increases with γ_i (Fig. 4(c)). Observe that R_j also benefits from the overall higher prices in the leading case by increasing its profit.

Finally, consumers' utility increases with γ_i as more consumers stay with R_i, which is the cheaper producer. However, consumers utility drops significantly when the market leaves complete competition.

5.1.4. Impact of c_i Fig. 5 presents the impact of the retailer R_i's cost on the market outcomes, where we assume that both retailers offer the same loyalty rewards: $\gamma_i = \gamma_j = 50000$ euros. When retailer R_i's costs c_i are sufficiently low, $c_i \leq 38.8$ euros/MWh (remember that R_j cost is $c_j = 47$ euros/MWh), then it can act as a leader. On the contrary, when $c_i \geq 38.8$ euros/MWh the market enters complete competition.

Both retail prices increase with c_i, although they drop when the market enters complete competition. When $c_i = c_j = 47$ euros/MWh both retailers offer the same price as their positions are symmetric.



(a) Retail prices (b) Proportion of consumers

(c) Retailers' Profits (d) Consumers utility

Figure 4 Impact of γ_i on the equilibrium market outcomes.

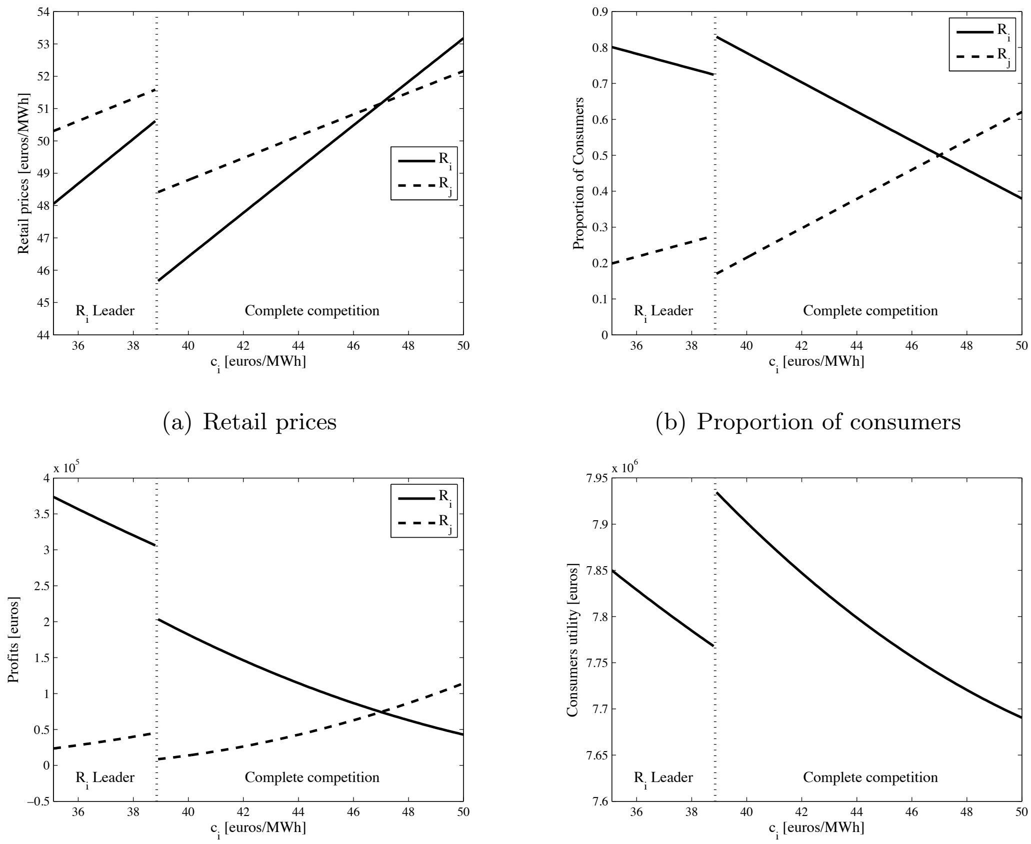
The increase of c_i implies a decrease in its market share (Fig. 5(b)) and its profit (Fig. 5(c)) as it becomes less competitive with respect to R_j, which simultaneously increases its market share and its profit.

Finally, an increase in the retailer costs always decreases the consumers' utility (Fig. 5(d)). However, if there is a leader, an small increase in its costs may have a beneficial effect as the market may enter complete competition.

5.2. Multiple-retailers

In this section we extend the numerical analysis to the multiple retailers case presented in Section 4. In what follows, we will study the impact of the cost c_i and the loyalty incentive γ_i on the market equilibrium outcomes, under different network configurations.

5.2.1. Data For the base case, we consider four retailers (R_1, R_2, R_3 and R_4) that supply energy to the final consumers. We assume that the retailer costs are $c_1 = 40$, $c_2 = 42$, $c_3 = 45$ and



(a) Retail prices (b) Proportion of consumers

(c) Retailers' Profits (d) Consumers utility

Figure 5 Impact of c_i on the equilibrium market outcomes.

$c_4 = 48$ euros/MWh. Furthermore, we assume symmetric loyalty premiums so that $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 50000$ euros, and transportation costs $k = 150000$ euros.

We analyze the market equilibrium under the three network configurations shown in Fig. 6. Note that a connection between R_i and R_j represents a Hotelling line so that consumers located in that line can buy their energy only from R_i or R_j. In the first case depicted in Fig. 6(a), there are connections between all retailers implying that there exists one group of consumers for each combination of any two retailers. The second case considers a linear network (Fig. 6(a)) where there are three groups of consumers that can buy from R_1 or R_2, from R_1 or R_4 and from R_3 or R_4, respectively. Finally, the third case assumes that there are two isolated groups of retailers and consumers. The first group of consumers can select between R_1 and R_2 while the second one selects between R_3 and R_4.

In all these three cases, we assume that consumers are initially uniformly distributed along the existing lines and, for each line, the initial proportion of consumers that bought from each retailer is 0.5. More specifically, for the first case: $\pi^0_{ij} = 1/6$ and $\pi_{ij} = 1/2$ for $i = 1,2,3,4$, $j = 1,2,3,4$ and

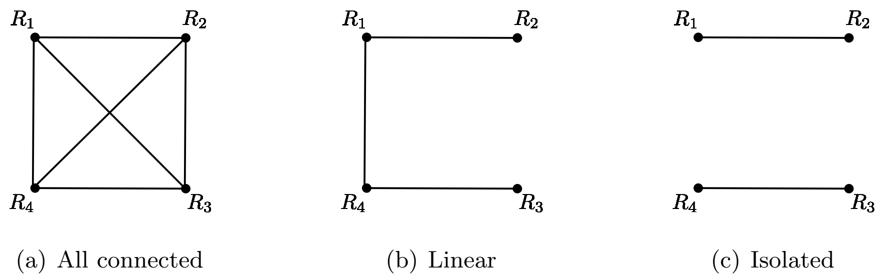


Figure 6 Network configurations

$i \neq j$. For the second case: $\pi_{ij}^0 = 1/3$ and $\pi_{ij} = 1/2$ for $(i,j) = (\{1,2\},\{1,4\},\{3,4\})$. For the third case: $\pi_{ij}^0 = 1/2$ and $\pi_{ij} = 1/2$ for $(i,j) = (\{1,2\},\{3,4\})$.

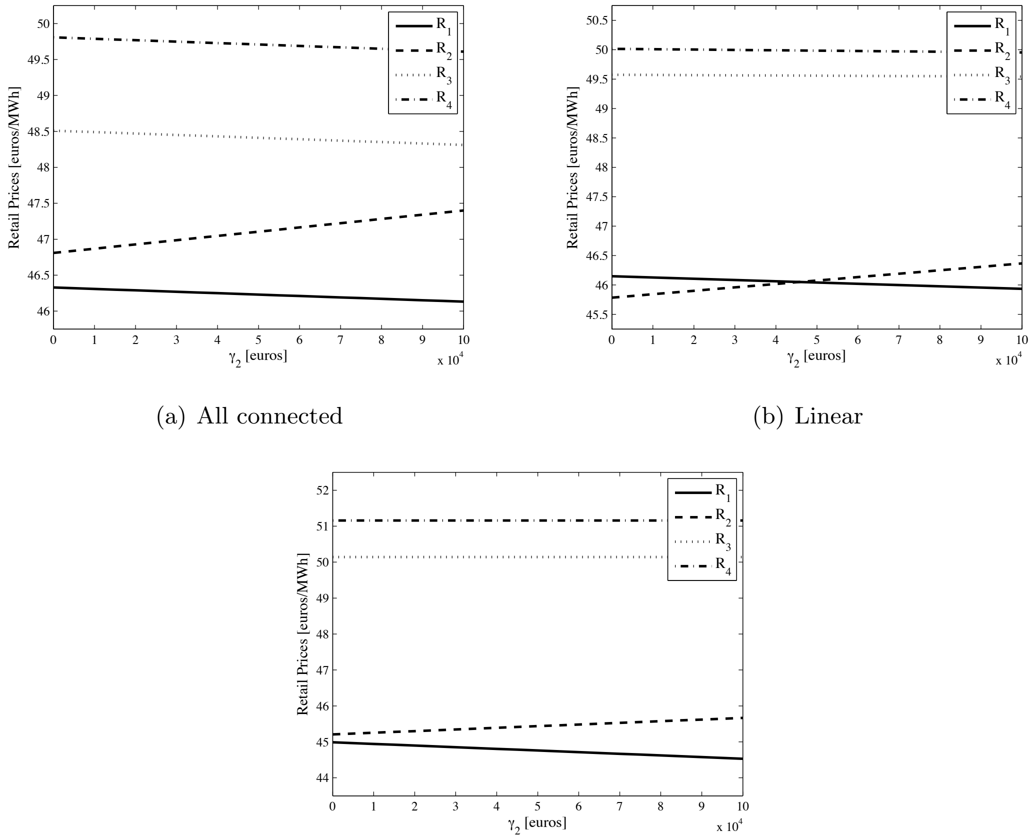
For the sake of clarity, all the solutions presented within the following sections correspond to market equilibria with complete competition, i.e., no retailer acts as a leader in any of the Hotelling lines.

5.2.2. Impact of γ_i In this section we analyze the impact of the value of retailer R_2's loyalty reward γ_2 on some market outcomes. Fig. 7 represents how retail prices evolve with γ_2 for the three network configurations shown in Fig. 6. As observed in the duopoly case, the retail price offered by R_2 increases with its loyalty reward while the prices offered by its rivals decrease. This influence is stronger for the case in which all of the retailers are connected (Fig. 7(a)) and becomes less relevant as the network gets less connected (Fig. 7(c)). Similarly, the difference between the retailer prices (maximum minus minimum price) increases as consumers are more isolated.

A similar behavior is observed when analyzing the market shares for each retailer (Fig. 8). Retailer R_2 increases its market share with γ_2 while decreasing those of its competitors. It is relevant to notice that the linear network configuration is specially favorable to retailer R_1 as it supplies energy to two groups of consumers, while being the cheapest retailer in both Hotelling lines. On the contrary, retailer R_4, that is also present in two Hotelling lines, cannot attain a significant market share as it offers the highest prices.

The profits for each retailer and network configuration are presented in Fig. 9. Retailer R_2's profits increase with greater values of its loyalty reward γ_2. The rest of retailers decrease their profit, although this effect disappears for retailers R_3 and R_4 if they are isolated from retailer R_2 (Fig. 9(c)). Again, we can observe that the linear network configuration is beneficial for retailer R_1 as it achieves significantly greater profits than its rivals.

The consumers' utility is presented in Fig. 10. For the three cases considered, the utility increases with γ_2 where the worst configuration is the isolated network. However, for small values of γ_2 the linear case yields higher utility values, while for bigger values of γ_2 the all-connected case offers better results. This can be explained by noticing that in the linear case, half of the consumers are



(a) All connected

(b) Linear

(c) Isolated

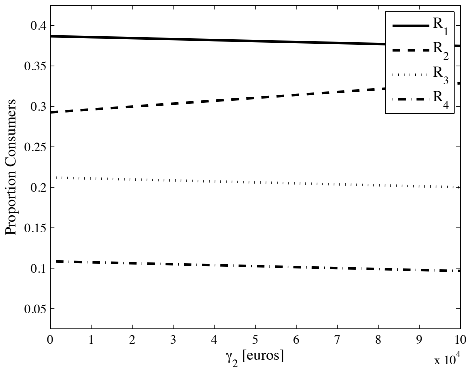
Figure 7 Impact of γ_2 on equilibrium prices.

with retailer R_1, which offers the cheapest prices. Hence, if γ_2 is not sufficiently large, this case would be preferable for consumers. However, R_2 has a greater market share in the all-connected configuration so that further increasing its γ_2 would yield a higher impact on consumers' utilities.

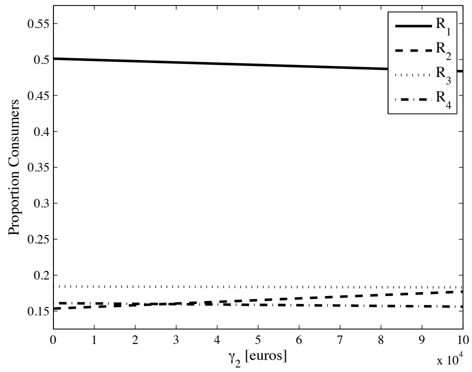
5.2.3. Impact of c_i Now we study how the cost increment of one retailer, in particular retailer R_2's cost c_2, affects prices, market shares and profits.

Fig. 11 depicts the retail prices for different network configurations. An increment in retailer R_2 cost yields higher prices for all retailers, although this increment is bigger for R_2. Again, a more interconnected network (Fig. 11(a)) implies more homogeneous retail prices, but at the same time prices are more sensitive to variations in a single retailer costs. For the isolated network (Fig. 11(c)), retailers R_3 and R_4 are not affected by c_2.

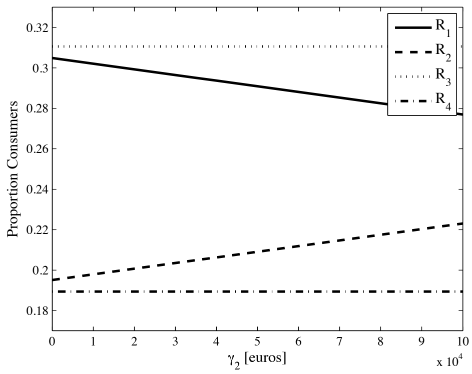
Fig. 12 presents the retailers' market shares. The cost increment of R_2 makes it less competitive, thus reducing its market share while increasing its rivals' ones. This effect is mitigated for retailers R_3 and R_4 for the isolated case (Fig. 12(c)).



(a) All connected



(b) Line



(c) Isolated

Figure 8 Impact of γ_2 on quilibrium market shares.

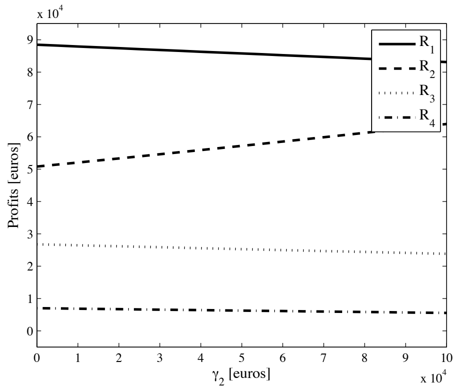
As Fig. 13 shows, the trend for profits is similar to the trend for market shares. Retailer R_2's profits decrease with its costs, while rival retailers benefit from the reduction in R_2's competitiveness. In particular, retailer R_1, which has the lowest costs, also presents the largest profit increases as it is always connected to R_2.

Finally, consumer utilities decrease with the increment of retailer R_2 costs. As it was observed before, the worst network configuration is when R_1 and R_2 are isolated from R_3 and R_4. For small values of c_2, the maximum utility is achieved for the all-connected network. However, as c_2 increases the linear configuration is preferable, since the high prices offered by R_2, which is located at one extreme of the network, find it more difficult to have an effect on the other retailers.

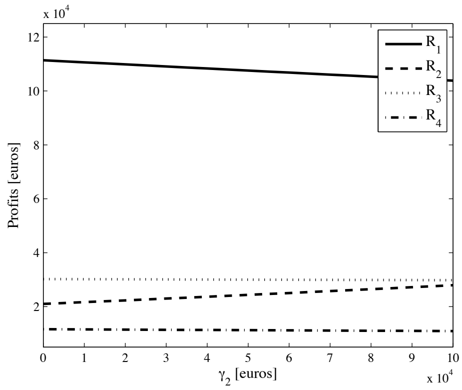
6. Conclusions

We have proposed a game theoretical framework to analyze the interactions between retailers and consumers in an electricity market.

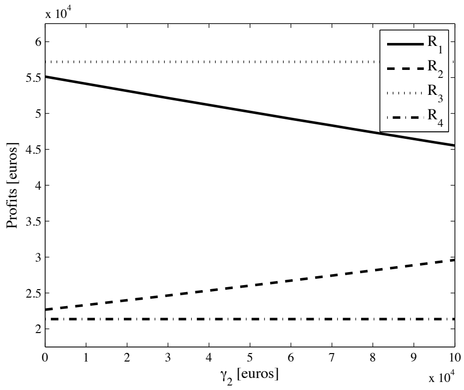
Retailers seek to maximize their profit by selecting the optimal prices to offer to the consumers. This is achieved by anticipating the optimal response of the consumers, which maximize their utility



(a) All connected



(b) Linear



(c) Isolated

Figure 9 Impact of γ_2 on retailer's profits.

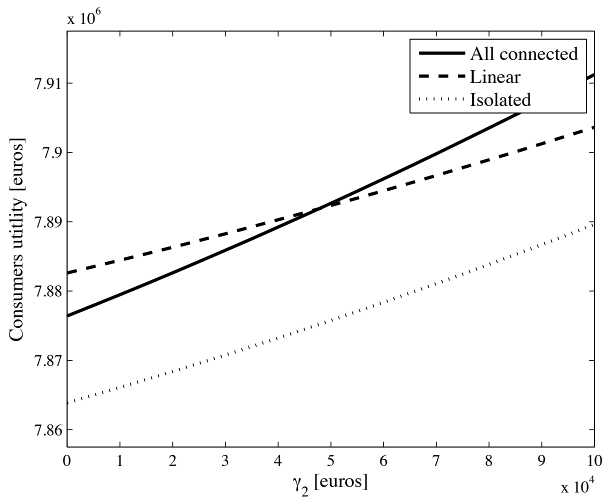
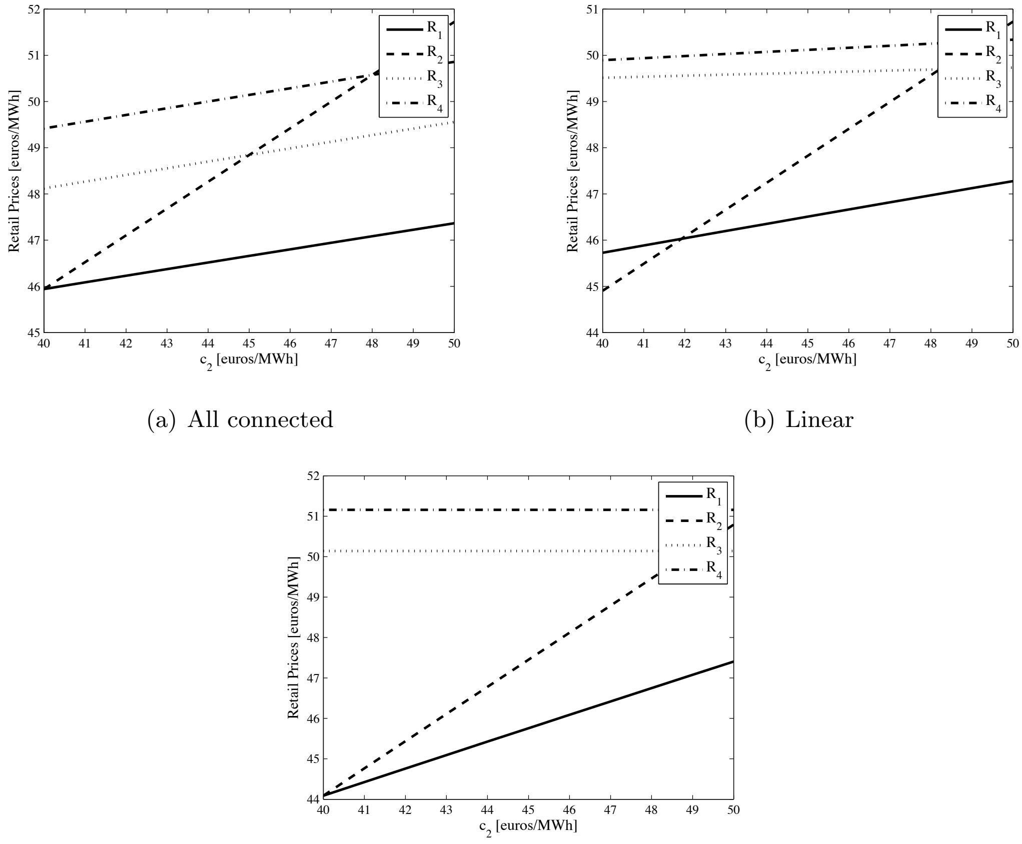


Figure 10 Impact of γ_2 on consumers' utility.



(a) All connected

(b) Linear

(c) Isolated

Figure 11 Impact of c_2 on prices

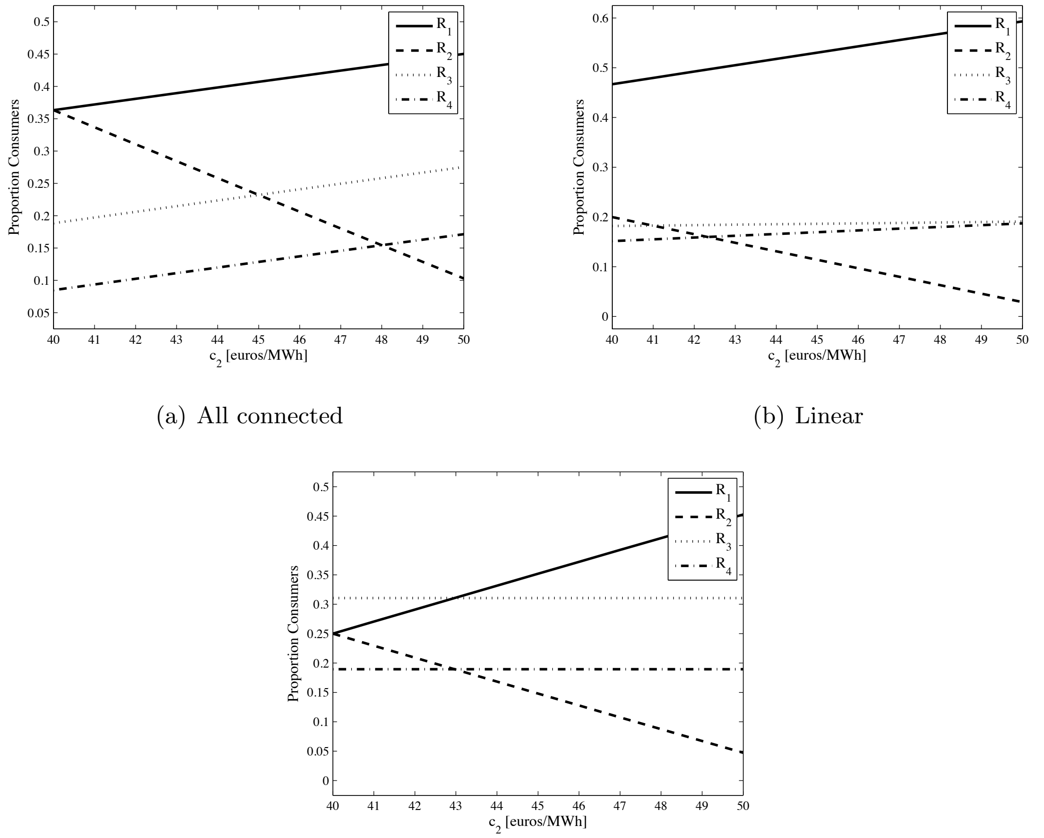
by deciding their optimal load quantity. Additionally, we account for the loyalty incentives that consumers may lose if they decide to switch from their original retailer. Uncertainty is incorporated to our model by assuming that consumer preferences for each retailer lay in a Hotelling line.

For a retailer duopoly, we analytically characterize the equilibrium, providing price sensitivities as well as existence and uniqueness conditions that hold for a wide class of consumer utility functions.

The duopoly is extended to a general case where several retailers located in a network compete simultaneously for groups of consumers.

We have performed an empirical analysis via numerical simulation to show how the different model parameters as well as the different network configurations impact the equilibrium market outcomes.

As highlights for the general duopoly case, and depending on the market conditions, the resulting price equilibrium can be classified into three categories: i) dominant position of a retailer; ii) leading position of a retailer; iii) complete competition; each of which with different market implications. The numerical simulations show how deviating from the complete competition case implies higher



(a) All connected

(b) Linear

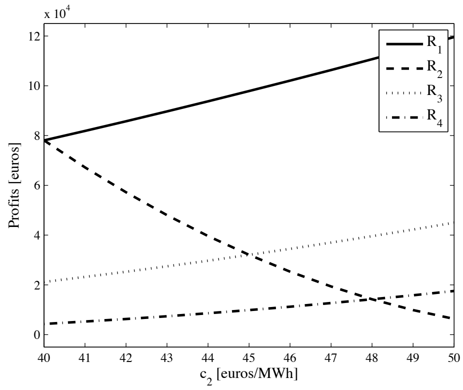
(c) Isolated

Figure 12 Impact of c_2 on market shares

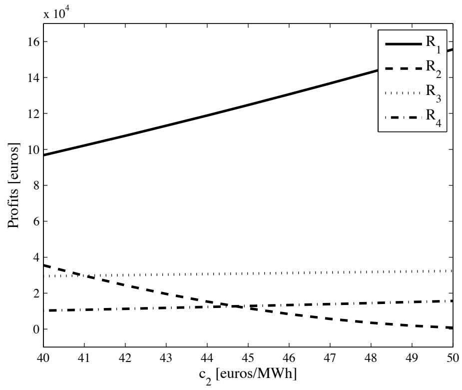
retail prices, higher retailer profits and lower consumer utilities. This deviation can be caused by an increase in a retailer's loyalty reward or by a decrease in a retailer costs. Moreover, results indicate that an increase in the initial market share of a leading retailer may be beneficial for the consumers as the market equilibrium may move to complete competition.

In the multiple-retailers case, a fully connected network, i.e., there exist groups of consumers that can select between any pair of retailers, entails that equilibrium prices are more homogeneous than in an incomplete network.

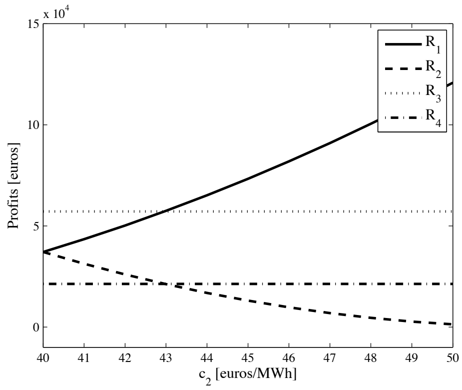
It should be noted that an increase of a retailer's loyalty reward increases the margin to raise its price while forcing its rivals to decrease theirs. Furthermore, if a retailer purchases its energy at higher costs, it is forced to set higher retail prices. However, in this case rival retailers benefit from this competitiveness decrease by also raising their retail prices. These effects are more relevant as the network is more connected. From a consumer's perspective, the worst network configuration is when retailers are isolated from each other. Under this setting, retailers can further exercise their market power over their corresponding consumers.



(a) All connected



(b) Line



(c) Isolated

Figure 13 Impact of c_2 on retailer's profits

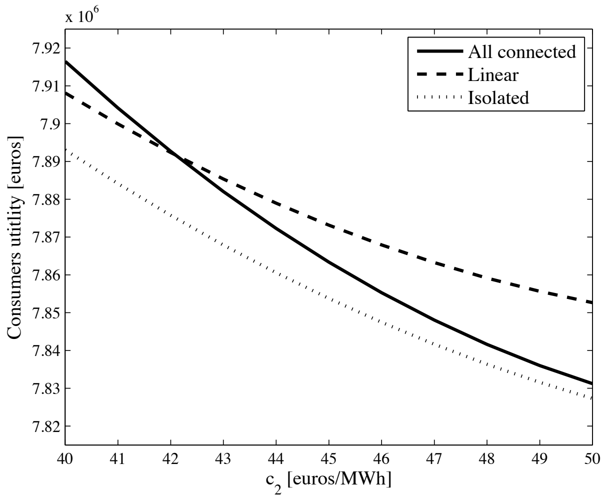


Figure 14 Impact of c_2 on consumers' utility

Acknowledgments

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Proofs of Statements

Proof for PROPOSITION 2:

From (8) and (10) we can check that, under condition (11), it holds that

$$\theta^*_{ij} > 1 \text{ and } \theta^*_{ji} \leq 0,$$

implying $F^*_{ij} = 1$ and $F^*_{ji} = 0$, with satisfies Definition 2 of a dominant position of retailer R_i.

For retailer R_i, from (4) we get $\pi^*_i(p_i, p_j) = 1$ and thus the optimal utility becomes

$$u^r_i(p_i, p_j) = (p_i - c_i)v^*(p_i) \tag{EC.1}$$

and attains its maximum for a price p_i that satisfies

$$(p_i - c_i)v'(p_i) + v(p_i) = 0. \tag{EC.2}$$

For retailer R_j, because $\pi^*_j(p_j, p_i) = 0$, the optimal utility vanishes and thus the profit regardless of the price p_j. □

Proof for PROPOSITION 3:

From (8) and (10), we can check that under condition (13), we have

$$\theta^*_{ij} \geq 1 \text{ and } 0 < \theta^*_{ji} < 1,$$

implying $F^*_{ij} = 1$ and $F^*_{ji} = \theta^*_{ji}$.

Hence, from (4) we get, $\pi^*_i(p_i, p_j) = \pi_{ij} + (1 - \theta^*_{ji})\pi_{ji}$ for retailer R_i and $\pi^*_j(p_1, p_2) = \pi_{ji}\ \theta^*_{ji}$ for retailer R_j. The respective utilities become

$$u^r_i(p_i, p_j) = (p_i - c_i)v^*(p_i)(\pi_{ij} + (1 - \theta^*_{ji})\pi_{ji}) \tag{EC.3}$$

$$u^r_j(p_i, p_j) = (p_j - c_j)v^*(p_j)(\pi_{ji}\theta^*_{ji}). \tag{EC.4}$$

By considering (8) and (10), the first order conditions associated to maximize (EC.3) and (EC.4), with respect to p_i and p_j, respectively, yields

$$(v(p_i) + (p_i - c_i)v'(p_i))\,(2k\pi_{ij} + \pi_{ji}(\varphi(p_i) - \varphi(p_j) + k - \gamma_j)) + \pi_{ji}(p_i - c_i)v(p_i)\varphi'(p_i) = 0$$

$$(v(p_j) + (p_j - c_j)v'(p_j))\,(2k\pi_{ji} - \pi_{ji}(\varphi(p_i) - \varphi(p_j) + k - \gamma_j)) + \pi_{ji}(p_j - c_j)v(p_j)\varphi'(p_j) = 0.$$

Using the definition (12) the above conditions can be rewritten as

$$\Gamma(p_i; c_i) = -\varphi(p_j) + \frac{1 + \pi_{ij}}{\pi_{ji}}k - \gamma_j$$

$$\Gamma(p_j; c_j) = -\varphi(p_i) + k + \gamma_j.$$

□

Proof for PROPOSITION 4:

Under condition (16) we have

$$0 < \theta^*_{ij} < 1 \text{ and } 0 < \theta^*_{ji} < 1,$$

implying $F^*_{ij} = \theta^*_{ij}$ and $F^*_{ji} = \theta^*_{ji}$.

By using (4) we get $\pi^*_i(p_i, p_j) = \theta^*_{ij}\ \pi_{ij} + (1 - \theta^*_{ji})\pi_{ji}$, and $\pi^*_j(p_i, p_j) = \theta^*_{ji}\ \pi_{ji} + (1 - \theta^*_{ij})\pi_{ij}$. Thus, the profit function for each retailer becomes

$$u^r_i(p_i, p_j) = (p_i - c_i)v^*(p_i)(\theta^*_{ij}\ \pi_{ij} + (1 - \theta^*_{ji})\pi_{ji}) \tag{EC.5}$$

$$u^r_j(p_i, p_j) = (p_j - c_j)v^*(p_j)(\theta^*_{ji}\ \pi_{ji} + (1 - \theta^*_{ij})\pi_{ij}). \tag{EC.6}$$

By considering (8) and (10), the first order conditions associated to maximize (EC.5) and (EC.6), with respect to p_i and p_j, respectively, yield

$$(v(p_i) + (p_i - c_i)v'(p_i))\,(\varphi(p_i) - \varphi(p_j) + k + \pi_{ij}\gamma_i - \pi_{ji}\gamma_j) + (p_i - c_i)v(p_i)\varphi'(p_i) = 0$$
$$(v(p_j) + (p_j - c_j)v'(p_j))\,(\varphi(p_j) - \varphi(p_i) + k + \pi_{ji}\gamma_j - \pi_{ji}\gamma_i) + (p_j - c_j)v(p_j)\varphi'(p_j) = 0.$$

Using definition (12) the above optimal conditions can be rewritten as

$$\Gamma(p_i; c_i) = -\varphi(p_j) + k + \pi_{ij}\gamma_i - \pi_{ij}\gamma_j$$
$$\Gamma(p_j; c_j) = -\varphi(p_i) + k - \pi_{ij}\gamma_i + \pi_{ji}\gamma_j.$$

□

Proof for THEOREM 1:

As $\Gamma(c; c) = -\varphi(c)$, under Condition C.4 and from Lemma 1 there always exists a value $p_i \in [c_i; \tau(c_i))$ satisfying (23) and $p_j \in [c_j; \tau(c_j))$ satisfying (24), given any value for p_j or p_i respectively. Furthermore, this value is unique.

For $q \in [-\varphi(c); \infty)$ define

$$G(q; c) \in \Gamma^{-1}(q; c), \quad G(q; c) < \tau(c).$$

Lemma 1 implies G is a continuous and increasing function of q on $[-\varphi(c); \infty)$.

From this definition, a solution of

$$p_i = G(-\varphi(p_j) + \kappa_i; c_i) \tag{EC.7}$$

$$p_j = G(-\varphi(p_i) + \kappa_j; c_j), \tag{EC.8}$$

is also a solution of (23)–(24). Define

$$\omega_i \equiv G(-\varphi(\tau(c_j)) + \kappa_i; c_i), \quad \omega_j \equiv G(-\varphi(\tau(c_i)) + \kappa_j; c^j),$$

and note that for any $p=(p_i,p_j)$ it will hold that $p\in[c_i;\omega_i)\times[c_j;\omega_j)$. Hence, by considering the property that $-\varphi$ is an increasing function, $\omega_k<\tau(c_k)$, for $k=\{i,j\}$, and $\Gamma(c;c)=-\varphi(c)$, we have

$$\begin{aligned}&-\varphi(c_i)\leq-\varphi(c_j)+\kappa_i\leq-\varphi(p_j)+\kappa_i<-\varphi(\tau(c_j))+\kappa_i\\ \Rightarrow\ & c_i\leq G(-\varphi(p_j)+\kappa_i;c_i)<\omega_i,\end{aligned}$$

with an equivalent bound holding for ω_j.

As a consequence, system (EC.7)–(EC.8) defines a fixed-point iteration $p=\Phi(p)$ where $\Phi:[c_i;\omega_i)\times[c_j;\omega_j)\to[c_i;\omega_i)\times[c_j;\omega_j)$ and is continuous on that set. Brouwer's fixed-point theorem then implies the existence of a fixed point for Φ, that will also be a solution for (23)–(24). □

Proof for THEOREM 2:

Consider the fixed-point dynamics for the solution defined by (EC.7)–(EC.8), which we write in compact form as $p=\Phi(p)$, and define the fixed-point iteration

$$p=\Phi_j(\Phi_i(p))\equiv\psi(p), \tag{EC.9}$$

where $\Phi=(\Phi_i\ \Phi_j)^T$ and $\psi:[c_j;\tau(c_j))\to[c_j;\tau(c_j))$. This iteration is equivalent to (EC.7)–(EC.8), as any solution of (EC.9) provides a solution for (EC.7)–(EC.8) by letting $p_j=p$ and $p_i=\Phi_i(p)$, and a solution for (EC.9) can be obtained from (EC.7)–(EC.8) by letting $p=p_j$.

G is differentiable wrt p on $[-\varphi(c_k);\infty)$, $k=i,j$, and we have that

$$G'(-\varphi(p)+\kappa_i;c_i)=\frac{v(p)}{\Gamma'(G(-\varphi(p)+\kappa_i;c_i);c_i)} \tag{EC.10}$$

$$G'(-\varphi(\Phi_i(p))+\kappa_j;c_j)=\frac{v(\Phi_i(p))}{\Gamma'(G(-\varphi(\Phi_i(p))+\kappa_j;c_j);c_j)}. \tag{EC.11}$$

Using the expression for Γ' in (22), we can write

$$\Gamma'(p;c)=v(p)(1+\rho(p;c)), \tag{EC.12}$$

for ρ defined in (25). We can rewrite (EC.10)–(EC.11) as

$$G'(-\varphi(p)+\kappa_i;c_i)=\frac{v(p)}{v(G(-\varphi(p)+\kappa_i;c_i))}\frac{1}{1+\rho(G(-\varphi(p)+\kappa_i;c_i);c_i)} \tag{EC.13}$$

$$\begin{aligned}G'(-\varphi(\Phi_i(p))+\kappa_j;c_j)=&\frac{v(\Phi_i(p))}{v(G(-\varphi(\Phi_i(p))+\kappa_j;c_j))}\\ &\times\frac{1}{1+\rho(G(-\varphi(\Phi_i(p))+\kappa_j;c_j);c_j)}.\end{aligned} \tag{EC.14}$$

From (EC.9) and using (EC.7)–(EC.8), replacing (EC.13)–(EC.14) and $\Phi_i(p)=G(-\varphi(p)+\kappa_i;c_i)$ we have

$$\begin{aligned}\psi'(p)=&\frac{v(p)}{v(G(-\varphi(\Phi_i(p))+\kappa_j;c_j))}\\ &\times\frac{1}{1+\rho(G(-\varphi(\Phi_i(p))+\kappa_j;c_j);c_j)}\frac{1}{1+\rho(G(-\varphi(p)+\kappa_i;c_i);c_i)}.\end{aligned}$$

From Condition C.3', for all $p \in [\bar{c}_k; \tau(c_k))$ and $k = i, j$ it holds that $1 + \rho(p; c_k) > \sqrt{v(\bar{c})/v(\bar{\tau})}$, and as we also have $v(\bar{c}) \geq v(\bar{c}_j) \geq v(p) \geq v(\tau(c_j)) \geq v(\bar{\tau})$, it follows that

$$\psi'(p) < \frac{v(\bar{c}_j)}{v(\tau(c_j))} \frac{v(\bar{\tau})}{v(\bar{c})} < 1,$$

implying $1 - \psi'(p) > 0$.

From this bound, $p - \varphi(p)$ is striclty increasing on $[\bar{c}_j; \tau(c_j))$, there can only be one zero of the function in that interval and the fixed point for (EC.9) is unique on $[\bar{c}_i; \tau(c_i)) \times [\bar{c}_j; \tau(c_j))$. $\square$