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# Eco-driving: Energy Saving based on driver behavior

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### Abstract

The number of vehicles has grown in recent years. As a result, it has increased the fuel consumption and the emission of gaseous pollutants. The emission of gaseous pollutants causes more deaths than traffic accidents. On the other hand, the energy resources are limited and the increase in demand causes them even more expensive. In addition, the percentage of old vehicles is very high. Eco-driving is a good solution in order to minimize the fuel consumption because it is independent of the vehicle age. In this paper, a driving assistant is presented. This solution allows the user acquires knowledge about eco-driving. Unlike other solutions, our proposal adapts the recommendations to the user profile. It also provides information in advance such as: optimal average speed, anomalous events, deceleration pattern, and so on. These recommendations prevent that the user performs inefficient actions. In these type of systems, motivation is very important. Drivers lose the interest over time. To solve this problem, we employ gamification techniques that contribute to avoid drivers coming back to their previous driving habits

#### 1 Introduction

The growth in the number of vehicles in circulation has experienced a strong increase in the last 20 years. Figure 1 captures the number of vehicles since 1970 in US, Australia, and OCDE. The widespread use of the automobile has had very positive effects on the economy of the countries. However, they have also led to major problems due to the pollution produced and the amount of energy required. On the other hand, most of the vehicles employ hydrocarbons, which are not available in all regions, causing energy dependencies between countries. In addition, its extraction has a very large impact on the environment.

The vehicles have become an important problem for governments and residents who suffer from respiratory diseases caused by greenhouses emissions [Caiazzo et al., 2013]. In order to avoid these problems, governments have developed regulations to control the emissions from vehicles. The drivers have also begun to require efficient vehicles due to the increase in fuel prices. Currently, the fuel consumption is a very important factor to buy a vehicle. All this has caused manufacturers to introduce improvements in vehicles in order to minimize the fuel consumption such as: engine optimization, vehicle weight reduction, hybrid engines, and aerodynamic improvements. However, these measures are insufficient because the percentage of older vehicles is still very high.

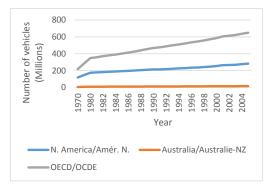


Figure 1. Number of vehicles since 1970. Source: OCDE.

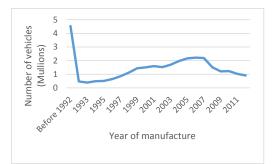
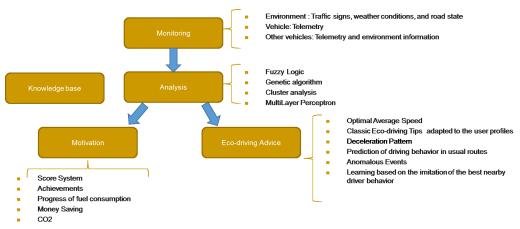
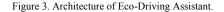


Figure 2. Age of vehicles in Spain. Source: DGT.





Fuel consumption depends on the vehicle, the environment and the behavior of the driver. Recently, a method called "Eco-Driving" has gained popularity. This driving technique allows us to save fuel. It is based on the optimization of the parameters that the user controls such as: speed, acceleration, deceleration and gear. Fuel saving is achieved by minimizing the energy losses. This method to improve fuel consumption and reduce the emission of greenhouse gases is very useful because it is independent of the vehicle technology. However, the driver needs to know about efficient driving rules.

This paper is focused on the need to learn how to drive efficiently following a set of rules, based on physics, which reduce unnecessary energy demands. In this work, we propose the use of an assistant who evaluates the driver behavior from the point of view of efficiency, and recommends improvements in order to save fuel. Furthermore, unlike other proposals, the system provides information about the nearby environment. Therefore, the user can take decisions in advance. This information is very important because the key to "eco-driving" is the driver's ability to predict the road state in the near future. The other main objective is to develop methods to motivate the user for driving efficiently, since many previous studies demonstrates that the driver tends to return to their previous driving habits, even after having received training on "eco-driving".

#### 2 State of the art

Eco-driving is a driving technique that saves fuel regardless of vehicle technology, and which is based on the control of the variables such as: speed, gear, acceleration, and deceleration. We can save up to 25% [Barbe and Boy, 2006] applying this driving technique. Although, this percentage depends on the driver skill, the vehicle type and the environment. For example, in hybrid vehicles the percentage of saving is less because an amount of the wasted energy is used in order to recharge the battery of the vehicle [Dardanelly et al., 2012].

In the literature, there are many works which demonstrate that this driving technique reduces fuel consumption and the emission of gaseous pollutants considerably. In [Van Mierlo et al., 2004] the authors analyzed the influence of the driving habits and the traffic on the fuel consumption and the emission of gaseous pollutants. Their conclusions were that drivers can save between 5% and 25% fuel following the ecodriving tips.

This driving technique has also positive effects on safety. In [Haworth and Symmons, 2001] the results revealed that learning programs about efficient driving can decrease traffic accidents around 35%, and the emission of polluting gases between 25% and 50%. In another study about the effectiveness of the eco-driving courses for vans [Hedges and Moss, 1996], a decrease of traffic accidents by 40% and an increase on the fuel efficiency by 50% was obtained.

The benefit of efficient driving training courses has been demonstrated on numerous occasions. However, several authors have observed that the positive effects of this learning are lost over time [af Wåhlberg, 2007] [Johansson et al., 2003].

Drivers tend to return to their previous driving habits when there is not motivation method. In the literature, there are many proposals to provide eco-driving tips and several types of feedback [Isler et al., 2010]. It has revealed that these solutions can influence positively the current driver behavior and long term [Van der Voort et al., 2001].

In the eco-driving, there are many research topics that can be classified into five groups:

- Identification of variables that affect fuel consumption
  - Models to estimate the fuel consumption
  - Control models
  - Methods to motivate the driver
  - User interfaces

# 2.1 Fuel Estimation Models

An essential part of efficient driving is to identify what factors influence on fuel consumption [Hiraoka and Terakado, 2009]. These variables are those considered in the laboratory to estimate the fuel consumption. One of the first proposals is still used to certify the vehicles. Afterwards, other authors such as [André, 1996] [Forunung et al., 1999] have increased the number of variables.

Following this research line, [Ericsson, 2001] determines that we must avoid sudden accelerations and high power demand in order to minimize fuel consumption. Similarly, [Johansson et al., 2003 concluded that there are certain characteristics of behavior while driving that are strongly correlated with fuel consumption. According to this author, drivers must avoid unnecessary stops, slow abruptly and driving at high speed in order to decrease energy demand. Also, they highlighted that it is important to not change gear frequently due to energy losses, and that has to recover then accelerating.

On the other hand, it is also important to know what variables affect safety. [Haworth and Symmons, 2001] identified these factors. The authors concluded that the reduction of vehicle speed, the choice of an optimal route, and the smoothly driving have positive effects on both: fuel consumption and safety. However, eco-driving tips such as driving at constant speed can cause traffic accidents.

#### 2.2 Control Models

These proposals obtain the optimal values of the control variables (throttle, brake and gear) from the point of view of energy efficiency. These algorithms are based on the prediction of near future. In the literature we found a large number of proposals in this research area. Its main advantage is that they allow to save fuel regardless of the driver's ability. However, users are reluctant to use them as we can see in [Van Der Laan et al., 1997].

Manufacturers are already introducing automated solutions to optimize fuel consumption. An example are the startstop systems which automatically turn off the engine when the vehicle stops and turn on it when the clutch is depressed. In addition, the engine is adapted, so there is no wear during this operation. In several studies, authors recommend turn off the engine when the vehicle is stopped for more than one minute. However, they also observed that this recommendation can have negative effect on the engine longevity if it is not modified to support this solution [Bishop et al., 2007]. This solution is especially useful in urban road. The fuel economy is from 5% to 10%.

Another example of control system is Nissan Eco-Pedal. The solution consists of installing a servo to the throttle pedal. Servo adds strength to the throttle pedal, depending on the intensity with which the conductor is pressing it. The strength appears when the driver accelerates with an intensity exceeding a threshold, from which the system estimated that fuel consumption will be high. The savings achieved by this solution is from 5% to 10% of fuel. However, many drivers do not find comfortable this system and switched it off [CNET, 2014].

## 2.3 Driver Motivation

A lot of works have demonstrated that users tend to return to their previous driving habits, causing a deterioration in efficiency over time [Onoda, 2009]. [Rolim et al., 2003] discuss the impact of the eco-driving lessons. The results showed a decrease in the percentage of sudden acceleration, excessive vehicle speed and fuel consumption (4.8%). However, as other authors said, they conclude that these improvements could be temporary in the case that the user does not receive continuous feedback. New technologies, such as mobile devices and social networks, allow us to develop solutions to encourage the user to drive efficiently [Trommer and Höltl, 2011] [Ryosuke and Yasushide, 2010].

Vehicle manufacturers are also including in their vehicles driving assistants to help the user in order to change the driving habits. For example, Ford Focus 2013 included an assistant that evaluates whether the driver shifts gears properly, and if he or she avoids accelerating and braking sharply. Other similar examples are: Garmin Mechanic [Garmin, 2014], Torque [Torque, 2014] and Honda Eco-Assist [Torque, 2014].

The social networking awareness and behavior can also positively influence the driver's motivation to save fuel. A method for utilizing fuel consumption data in an incentive system for the Tampere City Transport based on sharing individual driver's average fuel consumption in a specific group compared to the average fuel consumption of all drivers in that specific group was proposed in [Tulusan et al., 2011]. The authors achieved savings between 1.4% and 4.6% of fuel using the solution. Also, they highlights the appropriateness of this proposal to keep in time the improvement.

## 2.4 User Interface

When designing an in-vehicle information system it is important to ensure that the recommendations and the method to convey these tips do not negatively affect cognitive processing and driving performance [Peissner et al., 2011]. A very important research topic in eco-driving are communication between the user and the driving assistant.

Many researches analyze what information should be displayed to the user. For example, in [Man et al., 2012], authors conducted a survey online to find out what is the most demanded information according to the user motivation. The conclusions were that user interface should show the amount of money that has been saved in the first place. Drivers also showed interest in the amount of fuel spent on each trip. However, the greenhouse emissions were not useful in order to improve the driving from the point of view of energy consumption.

Other authors try to guess which are the best ways to interact with the user. In several works, they concluded that the best way to communicate with the user is using the voice. However, for this to be true is necessary to ensure compliance with the following requirements [Peissner et al., 2011]:

- The speech synthesizer has to issue a clear voice and humanlike.
- The speech recognizer needs to be precise.

## • Dialogues should not be complex.

The distractions that cause the screens can be mitigated if the driver does not have to take the eyes off the road. Google Glass or Garmin HUB [Google Glass, 2013] [Garmin HUB, 2014] are two proposals that allow the user to receive visual and to pay attention on the road.

#### **3** Eco-driving assistant

The system is continuously monitoring the environment, the vehicle and nearby cars. The information that the system acquires the environment consists of: vertical static traffic signs, traffic, weather conditions, slope and anomalous events. Static traffic signals are obtained using the camera from an Android Mobile Device and the Viola&Jones algorithm adapted for the recognition of traffic signs. The traffic state and weather conditions are achieved through web services, although we could alternatively use the information provided by other vehicles. Anomalous events that affect fuel consumption are detected by vehicles combining fuzzy logic with J48 algorithm. J48 is the implementation of the c4.5 algorithm on Weka. Road slope is estimated using the geographical coordinates (latitude, longitude and height) of two consecutive points.

Then the data are analyzed using different algorithms of artificial intelligence:

- EM Clustering Algorithm: It retrieves information based on a certain criterion [Pumrin and Dailey, 2003]. For example, vehicle telemetry where fuel consumption provided by the manufacturer is 6 l/100 km.
- Fuzzy logic: It is used to evaluate the driving from the point of view of energy efficiency [Zadeh, 1965].
- Genetic Algorithm: It determines the optimal average speed for each segment of a route.
- J48 Algorithm: It detects unusual events such as traffic accidents or traffic jams based on vehicle telemetry.
- Viola & Jones algorithm: It is used to recognize static vertical recognize traffic signs [Viola and Jones, 2011].
- Multilayer Perceptron Neural Network: It allows to estimate the acceleration for a segment of the route taking into account the average speed in combination with the genetic algorithm [Widrow and Hoff, 1969].

The result of data analysis is a set of eco-driving tips and feedback to motivate the user in order to save fuel. Eco-driving advice can be classified into two groups: Preventive tips and post-action tips. Eco-driving advice based on anticipation save a lot of fuel because it prevents the user from making inefficient actions. The key from eco-driving is anticipation to reduce the frequency and intensity of the accelerations (positive and negative). On the other hand, post-action tips warn the user not to make mistakes again in the driving.

Preventive Tips:

- Optimal Average Speed: The system shows the optimal average speed for each segment of the route. This speed minimizes the frequency and intensity of the accelerations (positive and negative).
- Speed pattern: Solution indicates the recommended speed second-by-second for smoothing the accelerations or informs the user about what is the most efficient close driver in order to he or she imitates their behavior.
- Traffic Sign Detection and Optimal Deceleration: The proposal employs an expert system that, based on the detection or previous knowledge of certain types of traffic signals, proposes a method to reduce fuel consumption by calculating optimal deceleration patterns, minimizing the use of braking.
- Anomalous Events On Road: The fuel consumption increases when there are incidents on the road as traffic jams or weather conditions are adverse. If the driver knows in advance the incident, he may change the route or adapt the vehicle speed in order to avoid having to brake abruptly.
- Inefficient regions: During testing, we have observed that drivers tend to make mistakes at the same places from the point of view of energy efficiency in usual routes. The assistant predicts if the driver is going to make mistakes and notifies him or her if so. The prediction is based on the driver behavior shown close to the inefficient region and the road state.

Post-Actions Tips:

- Real-Time Advice: We can obtain in "real time" the telemetry through the vehicle diagnostic port (OBD) [OBD2 Adapter, 2012] [Godavarty et al., 2000]. Therefore, we can detect when the driver violates any of the eco-driving rules. For example, if the driver brakes sharply. In these cases, the solution alerts the user in order to avoid that her or she makes the same mistake again.
- Eco-driving tips based on other users: the assistant compares the driving with other users who have previously made the same route under similar conditions and extract tips to improve the driving style.

In the eco-driving is essential to encourage and motivate the user to apply the tips and continue using the assistant. Our proposal evaluates the driver when it completes the trip from the point of view of energy consumption and assigns a score to him or her. User scores can be shared with friends and other users establishing a ranking. The gamification techniques are designed based on the goal of obtaining the maximum score. Gamification is the use of game design elements in non-game contexts such as learning environments. The idea is to use concepts from games like: the challenge, the competitiveness and progression in order to motivate the user for improving the driving style.

Score is calculated using a fuzzy logic system. This method allows us to simulate the human knowledge when

carrying out certain tasks such as driving. The objective is to model the behavior of an efficient driver. In the model, a set of input variables is involved and the output is the estimation of energy efficiency of a driver. The output variable is a number between 0 and 10. A high value means that the driver is applying the basic rules of eco-driving thoroughly. The proposed solution is able to evaluate the driver's driving style based on a knowledge database obtained by observing telemetry samples.

On the other hand, we have defined a set of achievements with the purpose of motivate the driver to use the system frequently and in order to allow him to get familiar with ecological challenges. As an example, the user unlocks an achievement when he completes a trip without accelerating sharply. Achievements are a traditional gamification method used to accomplish a certain behavior or to compare the performance of users. Achievements do not normally imply monetary compensation, but they are based on an emotional reward.

# 4 Conclusions

In this paper, we have presented an eco-driving assistant to encourage drivers to change their driving habits. The proposal is continuously monitoring the environment, the vehicle and nearby cars. The captured data are analyzed using different algorithms of artificial intelligence. The result is a set of eco-driving tips and feedback to motivate the user in order to save fuel. The main contribution of this work is that the tips are adapted taking into account the driver profile (aggressive, normal o eco-friendly), the vehicle features, and the environment conditions.

It is important adjust the tips for each type of user in order to avoid the discouragement of the driver. Otherwise, the user may not use the proposal. The thresholds of the eco-driving rules should not be the same for a normal driver than for an aggressive driver.

On the other hand, the impact of accelerations, decelerations, vehicle speed and engine speed depend on the vehicle characteristics. For example, if the vehicle is old, the maximum speed from which the fuel consumption grows exponentially will be lower than in modern vehicles that have aerodynamic improvements.

Finally, the driving style is also influenced by the environment. On an urban road, it is normal that the diver has to increase the frequency and intensity of the accelerations (positive and negative) in comparison with a highway. This increase happens due to traffic signs that force the driver to stop or the presence of other vehicles or pedestrians using the road. In this case, the thresholds of the eco-driving rules should be more permissive.

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