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On Multi-Objective Stochastic User Equilibrium

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Abstract

There is extensive empirical evidence that travellers consider many 'qualities' (travel time, tolls, reliability, etc.) when choosing between alternative routes. Two main approaches exist to deal with this in network assignment models: Combine all qualities into a single (linear) utility function, or solve a multi-objective problem. The former has the advantages of a unique solution and efficient algorithms; the latter, however, is more general, but leads to many solutions and is difficult to implement in larger systems. In the present paper we present three alternative approaches for combining the principles of multi-objective decision-making with a stochastic user equilibrium model based on random utility theory. The aim is to deduce a tractable, analytic method. The three methods are compared both in terms of their theoretical principles, and in terms of the implied trade-offs, illustrated through simple numerical examples.

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1. Introduction

It has long been known that there are many qualities, other than travel time, that motivate travellers in their choice of route, such as trip length, tolls and travel time unreliability. For example, from a route choice survey, Abdel-Aty et al. (1995) identified the three most important qualities to be: (1) shorter travel time (ranked as the first reason by 40% of respondents); (2) travel time reliability (32%); and (3) shorter distance (31%). Note that some people chose to indicate more than one quality as most important, which explains the sum being bigger than 100%. In the present paper we are interested in ways in which such multiple qualities may be accounted for in general in a predictive network model, with a specific focus (given its timeliness) on the way in which travellers deal with the potentially competing objectives of choosing a route to minimise their mean travel time and choosing one to minimise travel time unreliability.

Presently there exist two main ways of dealing with multiple qualities in a (deterministic) network user equilibrium (UE) context. The first (single objective) approach is to combine them into a single measure of generalised cost for each route and compute traffic flows that satisfy the Wardrop (1952) user equilibrium condition, which is attained if no user can improve their cost by unilaterally changing their route. A common approach to incorporate several

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route choice qualities is to consider a generalised cost function, which is the sum of monetary cost (such as tolls and vehicle operating costs, which are closely related to distance) and travel time multiplied by a value of time, see e.g. Dial (1979); Leurent (1993); Florian (2006); Chen et al. (2010). Regarding travel time reliability, Lo et al. (2006) formulated a multiple user-class equilibrium model considering travel time and travel time reliability, combined in a single objective as minimising travel time budget, which is defined as the expected travel time plus a travel time margin (or buffer time), with the travel time margin being dependent on the level of risk aversion of each user class. Watling (2006) proposed a late arrival penalised UE (LAPUE) which assumes users minimise a composite path disutility, incorporating the generalised cost plus a late arrival penalty. A few researchers, such as Larsson et al. (2002) have also considered nonlinear generalised cost functions.

The second approach, which has been the subject of more recent research, is to treat the qualities separately and to aim for a multi-objective equilibrium. This approach follows the principle of Pareto optimality or non-dominance commonly applied in multi-objective optimisation: A multi-objective equilibrium is attained if no user can improve any of the route choice qualities without deteriorating at least one other. Wang et al. (2010) showed that this approach is more general than approaches based on (additive) generalised cost functions, even if the latter consider a distribution of the value of time, as proposed by Leurent (1993) or Dial (1996). In fact, there are multi-objective equilibrium solutions that are based on rational route choices, that generalised cost approaches will miss. Wang and Ehrgott (2013) proposed a bi-objective approach considering the qualities travel time and toll, whereas Wang et al. (2014) consider travel time and travel time reliability (measured as standard deviation of travel time) as route choice qualities, and Wang and Ehrgott (2014) propose a multi-objective equilibrium model with travel time, travel time reliability and toll as objectives users aim to minimise.

In Table 1 we summarise other existing approaches from the literature that deal with multiple criteria network user equilibrium models. For each reference, we distinguish between the route choice criteria that have been considered and the path cost objective used in the models. We also state whether the model follows the UE or stochastic user equilibrium (SUE) principle (SO means social optimum) and what source of heterogeneity is considered.

Table 1. Other multiple criteria user equilibrium models.

Reference	Criteria	Objective	UE vs. SUE	Heterogeneity
Jaber and O'Mahoney (2009)	Service charge, time, toll	Generalised cost	SUE	Multiclass VOT, multigroup information
Leurent (1996)	time, cost	Generalised time	UE	VOT distribution
Nagurney (2000)	time, cost	Generalised cost	UE	Multiclass VOT
Nagurney and Dong (2002)	time, cost	Generalised cost	UE	Multiclass VOT
Tzeng and Chen (1993)	time, air polllution, distance	Generalised cost	UE	Discrete set of weights
Yang and Huang (2004)	time, cost	Generalised cost	UE, SO	Multiclass VOT

The single-objective approach has the advantage of typically providing a unique solution, see e.g. Florian and Hearn (1995) and Gabriel and Bernstein (1997), for the case of additive and non-additive path costs, respectively. This is extremely useful for planners when assessing proposed future policies using the network user equilibrium model. Also, efficient computational methods have been proposed for implementing it in large-scale systems (Dial, 2006; Florian et al., 2009; Bar-Gera, 2010; Gentile, 2014). However, the difficulty in specifying or estimating any general form of utility function means that almost always a constant linear form must be assumed, whereas it is not clear that travellers really perceive or trade off qualities in this way. On the other hand, the multi-objective approach has the advantage that it does not need to pre-suppose any relationship between the qualities (it is invariant to a monotone transformation of the qualities). However, its purpose is to generate a whole set of candidate solutions, which is difficult for planners to use in evaluating policy measures, and also gives rise to computational difficulties for identifying such solution sets for anything more than small-scale systems.

In the present paper we aim to take the best elements of each of these approaches. We adopt the basic philosophy of a multi-objective approach, but then aim to derive probability measures which distribute travellers to particular routes, thus aiming for a unique solution. The methods we shall propose extend and/or generalise the well-known single objective stochastic user equilibrium (SUE) model (Daganzo and Sheffi, 1977). In doing so, therefore, they also provide a future pathway to extending efficient algorithms developed for SUE to our new formulations, so that large-scale systems may be solved. The purpose of the present paper is to set out several alternative candidate formulations of our multi-objective model. Through simple illustrative examples, we demonstrate the features of the new approach(es), and compare them with the existing single-objective SUE approach. In particular, since our ultimate desire is to lead the pioneering work on small-scale multi-objective network problems towards methods that may be scaleable, we shall aim for an efficient analytic formulation of the problem.

2. Multi-objective Route Choice and Stochastic User Equlibrium

The main focus of the present section will be to set out several alternative behavioural principles that might be adopted for individual decision-making in a multi-objective setting under uncertainty, from which new notions of multi-objective SUE are defined. We first set out the well-known principle of random utility theory underlying single-objective SUE, in Section 2.1. We then propose a first model that extends this principle, of computing the probability that a particular route is "best", to the case when multiple route qualities are considered, i.e., we consider the probability of a particular route being the best in *one* of the qualities (Section 2.2). While this model is a natural generalisation of SUE, it retains important features of it, in particular the property that it allows a closed form solution for the choice probabilities of the alternatives. On the other hand, we demonstrate that it does not comply with the principle of Pareto optimality or non-dominance implemented in the multi-objective deterministic user equilibrium (DUE) models reviewed in Section 1.

In Section 2.3, we propose an alternative multi-objective generalisation of SUE. We show that this model complies with the non-dominance principle, i.e., the model is based on probabilities that a certain route is dominated by another route in the sense that there exists an alternative route that is not worse in all qualities and strictly better in at least one of them. This model does, however, require the computation of conditional probabilities, which makes it computationally expensive.

Finally, we present a model that is computationally tractable and also implements the non-dominance principle, in Section 2.4. This model is based on describing the attractiveness of an alternative by means of the differences of the utilities of alternatives (routes) in the different qualities, which are modelled as the sum of a deterministic term plus a random error. While this model allows closed form solutions, it entails the loss of transitivity of the evaluation of quality values for alternatives (it is possible that events of the following kind may have positive probability of simultaneous occurrence with respect to a given quality: Alternative i is more attractive than j, j is more attractive than l, yet l is more attractive than i). We note that while this may seem an undesirable property from a theoretical point of view, it is nevertheless a phenomenon that is observed in real-life decision-making, see e.g. Tversky (1969); Fishburn (1991); Cavagnaro and Davis-Stober (2014) for discussion of non-transitivity of preferences.

We will test the models in Section 3. We shall use these tests to see whether the proposed models comply with the non-dominance principle of multi-objective optimisation. In particular we expect to find (1) that alternatives which are non-dominated (there is no other alternative which is not worse in all qualities, and strictly better in at least one) to all have significantly bigger probabilities of being chosen than dominated ones; (2) that the relationship between the qualities of alternatives is not necessarily linear (this is because the multi-objective paradigm of non-dominance does not postulate any particular functional form of this relationship, or trade-off between alternatives). This second property is also in line with the observation from multi-objective user equilibrium models, that generalised cost models omit certain rational route choices as mentioned in Section 1.

Throughout the paper we will restrict attention to the case of a network with a single origin-destination movement with fixed demand. The reason is only to avoid unwieldy notation; the models presented are readily extended in the obvious way to a general network containing many origin-destination movements, with the relevant choice models applied to the fixed demands for each such movement.

2.1. The conventional SUE formulation

We assume travellers are choosing between n discrete alternatives (routes). The utility U_i of alternative i is assumed to have both a deterministic and a random component. The deterministic component of alternative i is formed from a linear combination of m qualities combined using a linear transformation into a single utility measure

$$U_i = \sum_{k=1}^{m} \theta_k V_{ik} + \epsilon_i \, (i = 1, 2, \ldots, n) , \tag{1}$$

where θ_k ($k = 1, 2, ...m$) are parameters, and $\{\epsilon_1, \epsilon_2, \ldots, \epsilon_n\}$ are continuous random components following some given joint probability distribution. The probability to choose any alternative i is then given by the probability that it is seen as being the best alternative in the sense of having highest utility U_i among all the alternatives,

$$Pr\left(U_i \geq \max\left\{U_j : j \neq i, j = 1, 2, \ldots, n\right\}\right). \tag{2}$$

In order to incorporate this in a formulation for SUE, we then suppose that the qualities (such as mean or standard deviation in travel time) depend on the choices made by travellers, through the flows on the routes of the network. Let the n-vector $\mathbf{f}$ denote the flows on the routes of the network, and let $\mathbf{V}(\mathbf{f})$ denote the $n \times m$ matrix of qualities across all route alternatives as a given function of the flow vector $\mathbf{f}$. Let $\mathbf{P}(\mathbf{V})$ denote the choice probability function, mapping from a given matrix of qualities $\mathbf{V}$ to an n-vector of choice probabilities, through the combination of Eqn. (1) and Eqn. (2). If d denotes the demand on the single origin-destination movement, then a flow vector $\mathbf{f}$ is an SUE if and only if it satisfies the fixed point condition

$$\mathbf{f} = d\mathbf{P}(\mathbf{V}(\mathbf{f})). \tag{3}$$

This is the conventional approach for using models such as SUE for addressing problems where travellers have multiple qualities that motivate their choice. In the special case in which we assume the error terms follow independent Gumbel distributions for the n (route) alternatives, it is well-known that we can derive the probability of alternative i having the highest utility in closed form, based on a multinomial logit model as

$$Pr\left(U_i \geq \max\left\{U_j : j \neq i, j = 1, 2, \ldots, n\right\}\right) = \frac{e^{\beta \sum_{k=1}^{m} \theta_k V_{ik}}}{\sum_{j=1}^{n} e^{\beta \sum_{k=1}^{m} \theta_k V_{jk}}}. \tag{4}$$

Note that β is introduced here as a sensitivity modelling parameter for our later numerical experiments.

2.2. A generalised SUE model derived from non-compensatory, multi-objective considerations, NCSUE

The conventional approach to dealing with multiple qualities, as described in Section 2.1, is based on the key tenet of compensatory choice, namely that travellers will trade-off the different qualities through a linear utility function with constant weights. However, it loses a central element of multi-objective decision-making theory, in which individuals consider the best alternative(s) they can choose with respect to each individual quality. In other words, individuals may prefer an alternative that they perceive as performing best in one of the m qualities, regardless of its performance in the other quality. Such an alternative may be assigned a low probability by the multinomial logit model of Eqn. (4). In the present section, we propose an extension to the SUE decision model which aims to retain the spirit of such non-compensatory behaviour, while still providing a tractable formulation.

Assume that travellers must choose between n discrete alternatives. Now instead of summing the utilities of an alternative with respect to m qualities as in Eqn. (1), the attractiveness of each alternative is measured with respect to the m different qualities *separately*, so that the utility U_{ik} of an alternative i with respect to a quality k has both a deterministic and a random component,

$$U_{ik} = \theta_k V_{ik} + \epsilon_{ik} \, (i = 1, 2, \ldots, n; k = 1, 2, \ldots, m), \tag{5}$$

where θ_k ($k = 1, 2, \ldots, m$) are parameters, V_{ik} is the measured/deterministic element of utility for alternative i with respect to quality k, and $\{\epsilon_{ik} : i = 1, 2, \ldots, n; k = 1, 2, \ldots, m\}$ are continuous random components following some given joint probability distribution.

For simplicity let us assume that the random components are independent between qualities. Then we aim to calculate the probability Q_i that for every quality ($k = 1, 2, \ldots, m$), there will be some alternative other than i that will be seen as better than i . This probability will (by the above-made assumption of independence) simply be the product over the qualities that some other alternative exists that betters i with respect to that quality, i.e.,

$$Q_i = \prod_{k=1}^{m} Pr\left(U_{ik} < \max\left\{U_{jk} : j \neq i, j = 1, 2, \ldots, n\right\}\right) (i = 1, 2, \ldots, n). \tag{6}$$

The component probabilities in this product can be calculated according to the usual, single objective random utility model as

$$Pr\left(U_{ik} < \max\left\{U_{jk} : j \neq i, j = 1, 2, \ldots, n\right\}\right) = 1 - Pr\left(U_{ik} \geq \max\left\{U_{jk} : j \neq i, j = 1, 2, \ldots, n\right\}\right). \tag{7}$$

Then we can calculate the complement of the probabilities Q_i above, namely for each alternative i the probability that it is the best alternative with respect to at least one quality is

$$P_i = 1 - Q_i \,(i = 1, 2, \ldots, n). \tag{8}$$

The final element in the choice model is to then propose that travellers choose alternatives according to the odds

$$O_i = \frac{P_i}{\sum_{j=1}^{n} P_j} \,(i = 1, 2, \ldots, n). \tag{9}$$

We may then integrate such a model of probabilistic choice as a way of choosing routes within a congested network assignment model. As for SUE, we suppose that the qualities $\mathbf{V}(\mathbf{f})$ depend on the route flow vector $\mathbf{f}$. Now, however, we let $\mathbf{O}(\mathbf{V})$ denote the odds function, mapping from a given matrix of qualities $\mathbf{V}$ to an n-vector of odds, through the combination of equations Eqn. (5), Eqn. (6), Eqn. (7), Eqn. (8) and Eqn. (9). With d denoting the demand, then we refer to a flow vector $\mathbf{f}$ as an NCSUE (Non-Compensatory SUE) if and only if it satisfies the fixed point condition

$$\mathbf{f} = d\mathbf{O}(\mathbf{V}(\mathbf{f})). \tag{10}$$

In the special case of $m = 1$ quality, the NCSUE model coincides with the conventional SUE model. For $m > 1$ the NCSUE model has an attractive feature that it assigns a unique choice probability to each alternative, and that these are expressible in closed form. However, as we explain in the following section, it does so by making a compromise in terms of expressing 'dominance' in the conventional multi-objective sense. That is to say, in Eqn. (6) it compares the probability of the given alternative with all other alternatives, and does not consider whether there is a single alternative that exists that betters the current one. In the limit, as the θ_k tend to infinity (i.e. as the model approaches deterministic choice) this certainly does not satisfy the standard definition of dominance. Effectively, in the limit case, it assumes that travellers become 'extremists' who do not really trade off. The model is therefore not expected to be so useful in such limit cases. However, if the model is calibrated away from the limit, then trade-offs will occur due to the random error terms.

2.3. Multi-objective stochastic decision-making based on dominance, MSUE

The central element in the model of Section 2.2 is Eqn. (6). Here, due to the assumed independence of the random components between qualities, the probabilities that alternative i is not the best with respect to quality k for $k = 1, \ldots, m$ are multiplied, in other words, Q_i is the probability that alternative i is not the best in any of the m qualities. Naturally, this is true if, for each quality k, there exists an alternative that is better than i. However, this could possibly be a *different* alternative for each quality. In multi-objective optimisation, on the other hand, the principle of non-dominance postulates that there be no single alternative that is at least as good or better than i for all qualities k. Therefore, the NCSUE model proposed does not, in the limit as deterministic choice is approached, satisfy the multi-objective principle of non-dominance. In the present section, as an alternative, we consider a model formulation that does indeed satisfy such a property in the limit.

In this case, what we require instead of Eqn. (6) is the probability that alternative i is dominated, i.e. the probability that there is an alternative j that dominates alternative i. This is the product over all qualities $k = 1, \ldots, m$ that some alternative j is better than i in quality k, given that j is already better than i in qualities $k' = 1, \ldots k-1$. This is the product of conditional probabilities

$$Q_i = \prod_{k=1}^{m} Pr\left(U_{ik} < \max\left\{U_{jk} : j \neq i, j = 1, 2, \ldots, n\}|U_{ik'} < \max\{U_{jk'} : j \neq i, j = 1, 2, \ldots, n\right\} \text{ for all } k' < k\right). \tag{11}$$

Thus, from Eqn. (11), and similar to Eqn. (8), the probability that alternative i is non-dominated is

$$P_i = 1 - Q_i \, (i = 1, \ldots, n) \,. \tag{12}$$

The probability of an alternative to be chosen (following Eqn. (9)) is then

$$O_i = \frac{P_i}{\sum_{j=1}^{n} P_j} \, (i = 1, 2, \ldots, n) \,. \tag{13}$$

In the same way as for the NCSUE model, we now define a flow vector $\mathbf{f}$ to be an MSUE (Multi-objective SUE) if and only if it satisfies the fixed point condition

$$\mathbf{f} = d\mathbf{O}(\mathbf{V}(\mathbf{f})), \tag{14}$$

with the difference being that now $\mathbf{O}(\mathbf{V})$ is defined through the combination of equations Eqn. (11), Eqn. (12) and Eqn. (13).

Notice that for the case of $m = 1$, Eqn. (12) gives the same results as Eqn. (2), and hence, just like the NCSUE model of Section 2.2, this model is a proper generalisation of the conventional stochastic user equilibrium model to the multiple objective case. However, the need to consider conditional probabilities in Eqn. (11) incurs a heavy price for modelling the non-dominance principle: We lose the closed form solution available in the single objective case, see Eqn. (4), and in the model of Section 2.2. Thus, it seems that the odds of Eqn. (13) need to be computed via Monte Carlo simulation methods.

2.4. A Multi-Objective Non-Transitive SUE model, MSUE-NT

Assume choosing between n discrete alternatives. The relative attractiveness of an alternative i compared to another alternative j with respect to m different qualities is based on the difference of the utility U_{ik} of an alternative i with respect to a quality k and the utility U_{jk} of alternative j with respect to the same quality k. We assume that this difference has both a deterministic and a random component

$$U_{ik} - U_{jk} = \theta_k \left(V_{ik} - V_{jk} \right) + \epsilon_{ijk} \, (i = 1, 2, \ldots, n; k = 1, 2, \ldots, m) \,, \tag{15}$$

where $\theta_k > 0 \, (k = 1, 2, \ldots, m)$ are parameters, V_{ik} is the measured/deterministic element of utility for alternative i with respect to quality k, V_{jk} is the measured/deterministic element of utility for alternative j with respect to quality k. Most importantly we assume that for each quality k and each pairwise comparison of alternatives (i, j), the random terms ϵ_{ijk} are *independent* between pairs. We suppose that these random terms follow a distribution that is given by the difference of two Gumbel random variables (i.e. a logistic distribution).

Hence, if we consider just a single pair of alternatives, the probability of an alternative j to be *better* than i in terms of quality k would be the same as in the case of a binary logit model as shown in Eqn. (16),

$$\begin{aligned} Q^k_{j,i} &= Pr\left(U_{jk} - U_{ik} > 0 \right) \\ &= Pr\left(U_{jk} > U_{ik} \right) \\ &= \frac{e^{\beta\theta_k V_{jk}}}{e^{\beta\theta_k V_{jk}} + e^{\beta\theta_k V_{ik}}} . \end{aligned} \tag{16}$$

(Note that β is introduced here as a sensitivity modelling parameter as in Eqn. (4).)

The key property that we introduce here is that of independence between the error terms of pairs of alternatives. In order to understand this, imagine there is a single quality and three alternatives from which to choose. A standard multinomial logit model (as underlies SUE) can be effectively implemented by creating random terms $(\epsilon_{12}, \epsilon_{13}, \epsilon_{23})$ for the three pairwise comparisons as above, but the key property is that these terms must be generated by a single set of three independent Gumbel variables $(\epsilon_1, \epsilon_2, \epsilon_3)$, such that $(\epsilon_{12}, \epsilon_{13}, \epsilon_{23}) = (\epsilon_1 - \epsilon_2, \epsilon_1 - \epsilon_3, \epsilon_2 - \epsilon_3)$, and these are certainly not independent. By assuming, on the contrary, that the random terms $(\epsilon_{12}, \epsilon_{13}, \epsilon_{23})$ are independent, it will turn out that we break transitivity of preferences in a probabilistic sense (as we explain below).

Now we apply the concept of non-dominance in multi-objective optimisation. We assume that an individual will consider an alternative as a plausible alternative as long as it is *not* dominated by another alternative. So what we are interested in, as in Section 2.3, is first to find the probability of an alternative being dominated, denoted by Q_i. This is the probability of the union of the events that alternative i is dominated by some of the $n-1$ alternative $j \neq i$. Using the inclusion-exclusion principle, due to the independence of the error terms ϵ_{ijk}, we get

$$Q_i = \sum_{d=1}^{n-1} (-1)^{d+1} \sum_{\substack{j_1, j_2, \ldots, j_d; j_r \neq i \\ 1 \leq j_1 < j_2 < \ldots < j_d}} Q_{j_r, i}, \tag{17}$$

where $Q_{j,i} = \prod_{k=1}^{m} Q_{j,i}^k$ is the probability that alternative i is dominated by alternative j.

Then we can calculate the complement of the probabilities above, namely for each alternative i, the probability P_i that it is *not* dominated by any other alternative as in Eqn. (12),

$$P_i = 1 - Q_i \, (i = 1, 2, \ldots, n) \tag{18}$$

and we choose alternatives according to the odds

$$O_i = \frac{P_i}{\sum_{j=1}^{n} P_j} \, (i = 1, 2, \ldots, n). \tag{19}$$

In the same way as for the NCSUE and MSUE models, we define a flow vector $\mathbf{f}$ to be an MSUE-NT (Multi-objective Non-Transitive SUE) if and only if it satisfies the fixed point condition

$$\mathbf{f} = d\mathbf{O}(\mathbf{V}(\mathbf{f})) \tag{20}$$

with $\mathbf{O}(\mathbf{V})$ defined through the combination of Eqns. (15), (15), (17), (18) and (19).

In the MSUE-NT model, we are thus able to find closed form solutions, by making the assumptions that the error terms of the differences between alternatives are independent, rather than the error terms on the evaluations of alternatives according to qualities, as in Eqns. (1) and (5). So what is it, that we lose in comparison to the conditional probabilities model of Section 2.2? Because of the assumption of independence of the ϵ_{ijk}, it is now possible that $U_{ik} > U_{jk}$, $U_{jk} > U_{lk}$, yet $U_{lk} > U_{ik}$, i.e. we lose transitivity in the comparison of utilities. For example, a traveller may perceive the standard deviation of travel time on Route 1 as smaller than on Route 2, on Route 2 as smaller than on Route 3, yet on Route 3 smaller than on Route 1.

3. Illustration of the Route Choice Models

In this section, we will use a simple illustrative example to compare the conventional SUE model as described in Section 2.1, the NCSUE model described in Section 2.2, and the MSUE-NT model of Section 2.4. Let us assume that we have a single O-D pair with three possible routes, such as depicted in Figure 3. The qualities we are interested in are expected travel time and standard deviation of travel time. Empirical evidence suggests that the standard deviation of travel time has at least two roles in influencing behaviour. The first, and most often used, is the interpretation that higher standard deviation is likely to be associated with arriving late at the destination (see, for example, Watling (2006)). A second alternative is as a measure of inconvenience (Noland et al., 1998). That is to say, while individuals may have flexibility in re-arranging the arrival and departure times of their trips and associated activities, all other things being equal they prefer not to incur the inconvenience of such re-scheduling. Therefore, they would tend to avoid the risk of having to do this wherever possible. For example, it may well be possible to bring forward or delay a meeting in response to travel conditions on the journey to work, but such re-arranging would have a nuisance value that might be avoided. Noland et al. (1998) found that this nuisance effect was something that could be separately identified to the issue of concerns for late arrival.

We first consider the hypothetical case of fixed quality values and use fixed values of $\beta = 0.5$ and $\theta = [3, 3]$. In this case no equilibration is required, and so we can just focus on the probabilities/odds of the alternative routes (we consider the flow-dependent case later).

We consider three cases: In Case 1 all three routes are non-dominated, in Case 2 two routes are non-dominated and the other is dominated and in Case 3 one route is non-dominated, one is weakly non-dominated (i.e. there is no route that is strictly better in all qualities), and the other is dominated. Note that dominance here refers to the deterministic component of the qualities. These cases are illustrated in Figure 1, which plots the values of standard deviation of travel time SDT against expected travel time ET for Route 1 (blue), Route 2 (red), and Route 3 (green).

3.1. Case 1 – All routes are non-dominated

Table 2 shows the values for travel time ET, standard deviation of travel time SDT, and the probabilities assigned to the three routes by the three different models (SUE, NCSUE, and MSUE-NT, respectively). Notice that, because both the expected travel time and standard deviation of travel time are minimised, but all SUE based models work with utilities to be maximised, the corresponding utility values are $-\theta_1$ ET $-\theta_2$ SDT, and $-\theta_1$ ET $-\theta_2$ SDT, respectively. Tables 3 and 4 are analogous for Cases 2 and 3.

The standard SUE model clearly puts almost all probability on Route 1, which has the highest standard deviation, but the lowest expected travel time. Nonetheless its combined utility with the chosen parameter values of β and θ is best. On the other hand, Routes 2 and 3 have very small probabilities of being chosen, despite being rational choices from a multi-objective point of view. On the other hand, the NCSUE model of Section 2.2 distributes probabilities almost equally between Routes 1 and 3, i.e. the two routes that are best for either expected travel time or standard deviation, but shows a very low probability for route 2, which is not the best for any quality, but nevertheless non-dominated. The MSUE-NT model is the only one that assigns significant positive probabilities to all three non-dominated routes.

Table 2. Case 1 – All routes are non-dominated, $\beta = 0.5$, $\theta = [3, 3]$.

Route	ET	SDT	Probabilities SUE	NCSUE	MSUE-NT
1	10	4	9.9750×10^{-1}	5.0236×10^{-1}	3.6230×10^{-1}
2	15	3	2.4726×10^{-3}	2.3853×10^{-2}	2.9622×10^{-1}
3	20	1	2.7468×10^{-5}	4.7380×10^{-1}	3.4149×10^{-1}

3.2. Case 2 – One route is dominated, the other two are both non-dominated

In this case (see Table 3, Route 2 is dominated, while Routes 1 and 3 are non-dominated. The result for the conventional SUE model is even more extreme, with the probability for choosing Route 1 being 0.99997. The result for the NCSUE model remains almost the same as in Case 1, allocating considerably higher probabilities to the two non-dominated routes (which happen to coincide with the routes optimising the individual qualities). Since the ET and SDT values of Routes 1 and 3 are unchanged compared to Case 1, and Route 2 is not the best in any quality in both cases, this similarity is to be expected. The MSUE-NT model shows a similar solution, with the probabilities for

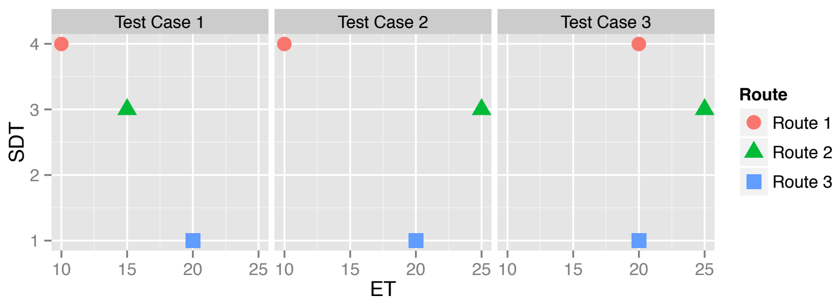

Fig. 1. Expected travel time ET and standard deviation of travel time SDT for three test cases.

Routes 1 and 3 more equal. Notice that the similarity between the NCSUE and MSUE-NT models seen here is due to the fact that there are only two non-dominated routes, as Case 2 illustrates.

Table 3. Case 2 – One route is dominated, the other two are both non-dominated, $\beta = 0.5$, $\theta = [3, 3]$.

Route	ET	SDT	Probabilities SUE	NCSUE	MSUE-NT
1	10	4	9.9997×10^{-1}	5.0263×10^{-1}	4.9305×10^{-1}
2	25	3	7.5824×10^{-10}	2.3588×10^{-2}	1.9330×10^{-2}
3	20	1	2.7536×10^{-5}	4.7378×10^{-1}	4.8762×10^{-1}

3.3. Case 3 - One route is dominated, one route is weakly non-dominated, one route is non-dominated

In the third case, Route 3 is best with respect to both of the qualities, while weakly non-dominated Route 1 is best with respect to travel time but does have higher standard deviation than Route 3, hence we would expect the NCSUE model to assign positive probability to both of these routes, which it does. Notice that the results are similar to those of the MSUE-NT model. On the other hand, the conventional SUE model still puts a very high probability on one of the routes, but now Route 1, which dominates the other two and with the chosen $\theta = (3, 3)$ has the best combined utility. This shows that the SUE model requires careful choice of parameters to avoid such counter-intuitive results. In this case, the MSUE-NT model does assign relatively high odds to non-dominated as well as weakly non-dominated routes, but to different degrees. Since weakly non-dominated routes are best in at least one quality, the NCSUE and MSUE-NT models both compute similar odds in this case.

Table 4. Case 3 – One route is dominated, one route is weakly non-dominated, $\beta = 0.5$, $\theta = (3, 3)$.

Route	ET	SDT	Probabilities SUE	NCSUE	MSUE-NT
1	20	4	1.0987×10^{-2}	3.3152×10^{-1}	3.2832×10^{-1}
2	25	3	2.7233×10^{-5}	3.0975×10^{-2}	2.5478×10^{-2}
3	20	1	9.8899×10^{-1}	6.3750×10^{-1}	6.4621×10^{-1}

In summary, in Cases 2 and 3, where the (weakly) non-dominated routes are the ones that are best in at least one of the qualities, the NCSUE model and the MSUE-NT model give similar results. The difference between the two is illustrated in Case 1, where the NCSUE model is unable to assign a significant probability to Route 2 being chosen, despite its position as a rational compromise between the more extreme choices of Routes 1 and 3. The MSUE-NT model on the other hand assigns similar odds to all three non-dominated routes. In all three cases, the conventional logit model highly favours only one of the non-dominated alternatives, the one which minimises the weighted sum of utilities as in Eqn. (1).

In Figure 2 we show how the odds assigned to Routes 1 (O1) and Route 2 (O2) change with parameter β, which varies between 0.01 and 0.5. Because the probabilities sum to 1, the probability of choosing Route 3 is implicit. The parameter θ remains fixed at [3,3]. In the top row we compare the MSUE-NT model with the standard SUE model, while the bottom row does the same for the NCSUE model. Notice that for $\beta = 0.01$ all models will allocate almost equal probabilities to all three routes in all cases. As β increases, the trajectories of the standard SUE model and our proposed models develop very differently, though. While the SUE model converges towards a solution with probability of almost one on either Route 1 or 3, our models always allocate positive odds to at least two routes. The plots also show that the NCSUE model does in all three cases converge to a solution which assigns significant odds to the routes with the best values for individual qualities. This is not the case for the MSUE-NT model, which assigns close to equal probabilities to all three non-dominated routes in Case 1, no matter what the value of β is.

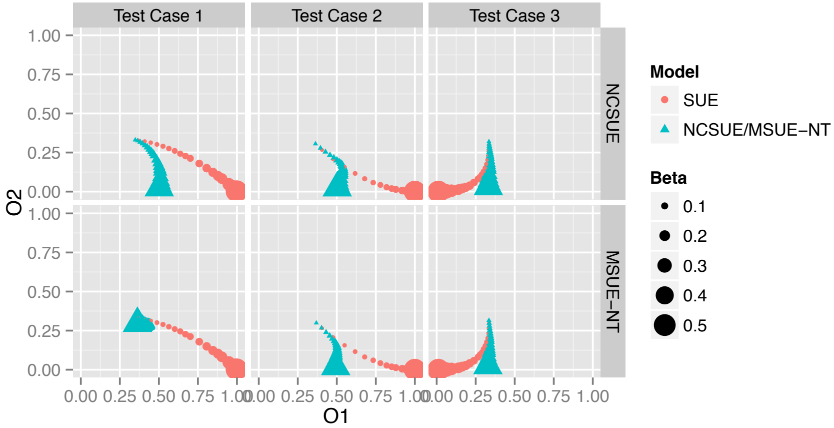

Fig. 2. Odds of choosing routes with the SUE, NCSUE and MSUE-NT models for changing β.

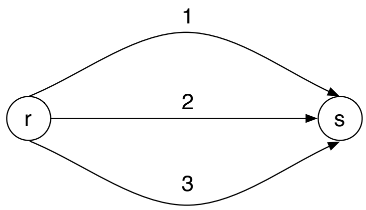

Fig. 3. A three-link example network.

4. A Three-link Example for the Equilibrium Models

In this section, we demonstrate and validate our concepts with a simple three-link example that considers flows and therefore has expected travel time and standard deviation of travel time dependent on link flow. The details for evaluation of travel time and network specifications are given in Section 4.1.

4.1. Network Specification

Our test three-link network is shown in Figure 3, where the link parameters are specified in Table 5. The parameters of the travel time function (21) are $\alpha = 0.15$ and $\gamma = 4$. The total demand is assumed to be fixed at 15,000 vehicles per hour.

Table 5. Route characteristics of the three-link network.

Route a	Free flow travel time (min)	Capacity (veh/hr)	Reliability ϕ_a
1	12	4,000	0.5
2	30	5,400	0.7
3	40	4,800	0.9

Link travel time T_a depends on link flow x_a according to the common BPR function (Bureau of Public Roads, 1964),

$$T_a(x_a, C_a) = t_a^0 \left[1 + \alpha \left(\frac{x_a}{C_a} \right)^{\gamma} \right], \tag{21}$$

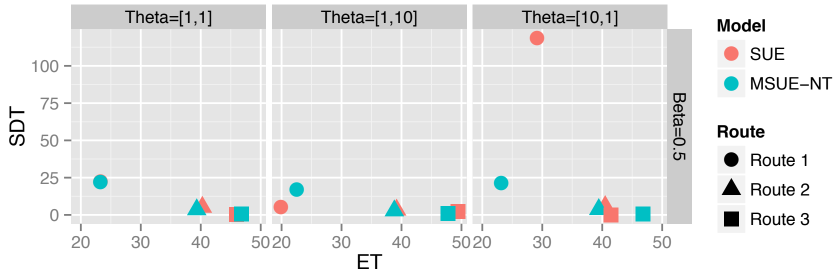

Fig. 4. Standard deviation against expected travel time for the three-link network.

where t_a^0 is free flow travel time, C_a is link capacity, and α and γ are parameters (we chose $\alpha = 0.15$ and $\gamma = 4$).

We assume that link capacity follows a uniform distribution, defined by an upper bound (the design capacity) and a lower bound (the worst-degraded capacity), which is a fraction, ϕ_a, of the design capacity, $\bar{c}_a$, i.e.

$$C_a \sim U(\phi_a \cdot \bar{c}_a, \bar{c}_a)\,. \tag{22}$$

As derived in Lo and Tung (2003), the path travel time T_p is normally distributed, $T_p \sim N\left(E\left(T_p\right), \sigma_{T_p}\right)$ with mean and standard deviation that can be written as

$$E\left(T_p\right) = \sum_a \left[\delta_a^p \cdot E(T_a)\right] \tag{23}$$

$$\sigma_{T_p} = \sqrt{\sum_a \left[\delta_a^p \cdot \mathrm{var}(T_a)\right]}. \tag{24}$$

Here δ_a^p is the usual link-path incidence, i.e. $\delta_a^p = 1$ if link a belongs to path p and 0 otherwise. By applying the assumption of uniformly distributed arc capacity as expressed in Eqn. (22), Lo and Tung (2003) show that the mean and standard deviation of the route travel time distribution are asymptotically

$$E\left(T_p\right) = \sum_a \left\{\delta_a^p \cdot \left[t_a^0 + \alpha t_a^0 x_a^\gamma \frac{1-\phi_a^{1-\gamma}}{\bar{c}_a^\gamma (1-\phi_a)(1-\gamma)}\right]\right\}, \tag{25}$$

$$\sigma_{T_p} = \sqrt{\sum_a \left[\delta_a^p \cdot \alpha^2 \left(t_a^0\right)^2 x_a^{2\gamma} \left\{\frac{1-\phi_a^{1-2\gamma}}{\bar{c}_a^{2\gamma}(1-\phi_a)(1-2\gamma)} - \left[\frac{1-\phi_a^{1-\gamma}}{\bar{c}_a^\gamma (1-\phi_a)(1-\gamma)}\right]^2\right\}\right]}. \tag{26}$$

Note that in Table 5, we specify a travel time reliability parameter of ϕ_a for route a as defined in Eqn. (22). The $\phi-$value for Route 1 is the lowest, meaning that it is the route that could be most degradable although it is the shortest, while Route 3 is assumed to be the most reliable with the highest $\phi-$value.

4.2. Results

The results of the equilibrium models based on the SUE and MSUE-NT formulations are shown in Figures 4 and 5. Figure 4 shows the standard deviation SDT versus the mean travel time ET on the three routes with fixed $\beta = 0.5$ and three values of θ for both the SUE and MSUE-NT models. Figure 5 shows the flows on Route 2 (x2) versus Route 1 (x1) for both the SUE and MSUE-NT models at equilibrium for three fixed θ values and β ranging from 0.01 to 0.5.

4.2.1. Equilibrium route travel time standard deviation versus expected travel time

SUE versus MSUE-NT. In Figure 4, the SUE solutions seem to line up on a straight line. This is similar to what we observed in Wang and Ehrgott (2013) that user equilibrium based on linear generalised cost corresponds to a linear utility function, illustrated by routes with positive flow all lying on a straight line when plotting one quality against the other. This is expected as the utility of each alternative is derived based on a combined utility value, i.e. a linear combination of the systematic components as shown in Eqn. (1). This feature is not evident in the MSUE-NT solutions.

Importance of standard deviation (σ_{T_p}) versus mean travel time ($E(T_p)$). This is modelled by three different combinations of θ values,

1. $E(T_p)$ and σ_{T_p} are equally important, $\theta = [1, 1]$
2. σ_{T_p} is ten times more important than $E(T_p)$, $\theta = [1, 10]$
3. $E(T_p)$ is ten times more important than σ_{T_p}, $\theta = [10, 1]$

When we look at Figure 4, we can observe the range of standard deviation at equilibrium to be very different for the three cases, while the range of expected travel time is quite similar. Note that when mean travel time is ten times more important than reliability, as shown on the right of Figure 4, the range of standard deviation is huge because reliability is relatively unimportant. The interesting thing here is that it seems that the relative importance of standard deviation affects only the range of equilibrium standard deviation values but not that much the mean travel time at all.

4.2.2. *Flow on Route 2 versus Flow on Route 1*

SUE vs MSUE-NT. If we look at Figure 5, the trajectories of the SUE solution generally follow the shape of an ‘S’ curve while the trajectories of the MSUE-NT case would bend as in our hypothetical example where all the three routes are non-dominated. In this case, as shown in Figure 4, all three routes are indeed non-dominated. Therefore, the observations made here are consistent with our hypothetical tests for both the SUE and MSUE-NT models.

Impact of θ values on the trajectories of route flows. It is important to note the characteristics of our three routes. Here Route 1 has the lowest free-flow travel time but has the highest probability of significant capacity reduction caused by traffic incidents, in other words, it is the least reliable. At the other extreme, Route 3 has the longest free-flow travel time but the least variability. Since we consider a fixed demand, the sum of the flows on the three routes is a constant.

Due to the choice of θ values, we would expect that when mean travel time is important, more users would choose Route 1 while when reliability is more important, more users would choose Route 3. Now if we look at Figure 5, the equilibrium flow on Route 1 is indeed higher when expected travel time is more important. On the other hand, when reliability is more important, Route 1 and 2 have lower flows as compared with Route 3.

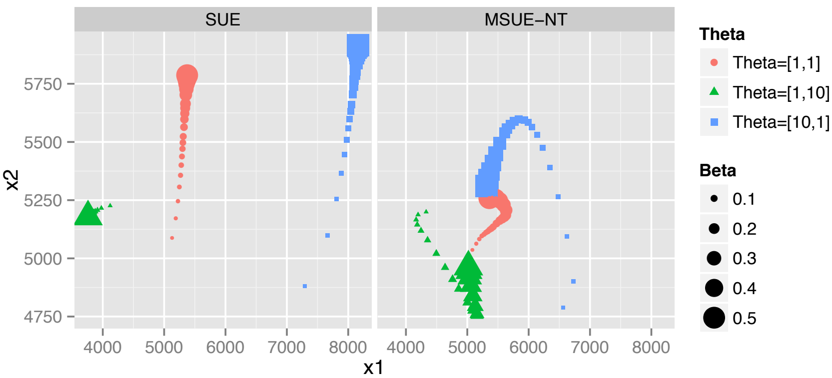

Fig. 5. Equilibrium flows for the three-link network with $0.01 \leq \beta \leq 0.5$.

Influence of sensitivity β and relative importance of qualities θ. Interestingly, when $\theta = [1, 1]$, i.e. when mean travel time is equally important as reliability, both the SUE and MSUE-NT solution will move towards an equal split between the three routes when $\beta = 0.01$, i.e. when users are all insensitive to the differences. Now the biggest difference between the SUE and MSUE-NT models arises when mean travel time becomes very important, i.e. $\theta = [10, 1]$, as shown in Figure 5. In this case, the SUE solution will have much higher flow on Route 1 as compared with the MSUE-NT solution.

In summary, applying the SUE and MSUE-NT models for a simple three link network with congestion effects, highlights the differences between the models, with the MSUE-NT model being in line with the non-dominance principle from multi-objective decision making, whereas the conventional SUE model tends to produce more extreme answers as the difference between the θ values increases.

5. Conclusions

In this paper, we have proposed three model formulations, that extend the conventional SUE model of route choice to the case that travellers consider several qualities for route choice separately. The first, non-compensatory model NCSUE in the limit favours routes that are best in some of the qualities, while the MSUE model and the MSUE-NT model incorporate the principle of non-dominance from multi-objective decision-making. The MSUE model requires the evaluation of conditional probabilities, which requires further research and may turn out to be possibly computationally expensive, the MSUE-NT model allows closed form solution at the expense of not guaranteeing transitivity of comparisons of utilities. It also requires the computation of probabilities according to the inclusion-exclusion principle, which is exponential in the number of alternatives.

In future research, we will further develop the theoretical basis of multi-objective SUE models, and develop algorithms that allow the application solutions of the proposed models for realistic networks systems.

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