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Reliability Equivalence in Public Transport Contexts

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Abstract

This work provides initial investigation into whether equivalence between the mean-variance and scheduling approaches to transport reliability can be applied in the context high frequency public transport services. Each of these approaches is briefly outlined and the current work is framed by previous research attempting to demonstrate equivalence: both theoretically and empirically. The basic assumptions underpinning the theoretical approach to equivalence are explored and then re-formulated based upon which variables are likely to be known. The concept of headway is introduced to the theoretical approach using notation from previous research in order to represent public transport services. An empirical illustration of the method is undertaken using smart card data obtained from the London Underground metro system. The data are combined with timetable data and a previously developed method for estimating passenger preferred arrival times, which in turn allows the theoretical equivalence between mean-variance and scheduling approaches to be tested empirically. This is initially performed for a single origin-destination (OD) pair and then for 23 other ODs of varying headways. The example using a single OD demonstrates that even for a high frequency metro service, application of the theoretical equivalence is problematic, with variable parameters substantially affected. In the case of many ODs, a linear relationship is observed between the ratio of public transport to standard scheduling parameters and headway, suggesting the theoretical equivalence becomes less viable as headway increases. At the lowest values of headway, it is concluded that the equivalence remains problematic and further work is required before equivalence between the mean variance and scheduling approaches can be implemented in the public transport context.

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Keywords: Reliability; Public transport; Headway; Travel time variation; Smart card; Expected Utility

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1. Introduction

This paper is concerned with the reliability of transport trips, an area of research which has received a significant amount of attention due to recognition that reliability can constitute a significant part of the generalised cost incurred by travellers (Eddington, 2006). Despite increasing awareness of the subject across the transport planning community, the concept is often not fully integrated with national guidance on scheme appraisal. Initially part of the problem has been of complexity, comprehension and ease of applicability. Such problems have resulted in efforts to formalise the topic in the academic literature; not only do many studies now refer to the subject as 'travel time variation' (TTV) rather than reliability, but more importantly valuation frameworks have emerged which seek to value these travel time variations in differing circumstances. This paper will devote attention to the two of the principal such frameworks.

Historically, the first of these frameworks to emerge was the mean-variance approach, the origins of which can be found in portfolio theory within the field of finance. This approach considers the cost of insuring against risk; see Markowitz (1999) for a useful account of the early history. The second key framework, this time emanating from the transport literature, is the scheduling approach. This approach considers the costs incurred by the traveller assuming they arrive early or late in relation to an ideal arrival time at the end of their trip. Researchers have attempted to compare and bring together these two approaches as they are essentially trying to explain the same phenomenon (Hollander, 2005; Li et al., 2011; Batley and Ibanez, 2009; Fosgerau and Karlstrom, 2011), although there appears to be a degree of disagreement as to how closely the TTV frameworks approximate to one another.

In the work that follows, the reader is provided with a short background to the mean-variance and scheduling frameworks in turn, and the debate around their apparent similarities is briefly explored with reference to the aforementioned research papers, with particular emphasis on the work of Fosgerau and Karlstom (2011), henceforth referred to as F&K. The contribution of the present paper is to adjust the F&K equivalence methodology to account for high frequency scheduled public transport, thereby addressing an accepted weakness in the methodology (Bates et al., 2001). This modified equivalence methodology will be demonstrated using smart card data from the London Underground metro system.

2. Reliability Frameworks

2.1. *Mean-variance*

Approaching TTV in terms of risky outcomes and risk attitudes has historically allowed a link to be made between transport and the more developed field of risk in finance, namely portfolio theory (Jackson and Jucker, 1982). The early work in a transport context consisted primarily of empirical research to demonstrate the usefulness of a variance term in modelling travellers' decisions. The model broadly adhered to the following specification:

$$\mathrm{E}(U) = \eta\mu + \rho\sigma \tag{1}$$

where E(U) is the traveller's expected utility, μ and σ are mean and standard deviation of travel time respectively, and η and ρ are parameters associated with each of these measures.

2.2. Scheduling

The central proposition of the scheduling approach is that there is a cost to interrupting a prior activity or being late for a subsequent one – arriving after the preferred arrival time (PAT). This proposition is outlined by Gaver (1968), whose focus is the cost incurred by allowing a 'headstart' at the beginning of a trip which balances a dislike of early interruption of a prior activity and late arrival at a subsequent one. This early work also considers unpredictable travel times, thus introducing risk and uncertainty to the approach. The approach is formalised by the work of Small (1982) and Noland and Small (1995), where the expected utility of departing at time t is given by

$$\mathrm{E}(U(t)) = \alpha\mathrm{E}(T) + \beta\mathrm{E}(\mathrm{SDE}) + \gamma\mathrm{E}(\mathrm{SDL}) + \theta P_L \tag{2}$$

where T is the travel time, SDE is the amount of earliness relative to PAT, SDL is the amount of lateness relative to PAT, α represents the marginal utility of travel time, β and γ represent the marginal utilities of earliness and lateness respectively, P_L is the probability of lateness (taking a value of 0 if SDL = 0, or 1 if SDL > 0), and θ is a fixed penalty for lateness.

2.3. Comparison of frameworks

A question that follows is whether the approaches in section 2.1 and 2.2 can be treated as interchangeable. If the assumption is made that the value of travel time is equal across approaches, then the motivating question can be formalised as

$$\rho\sigma = f(\beta.SDE + \gamma.SDL) \tag{3}$$

Some studies have attempted to quantify the nature of the relationship empirically using stated preference surveys (Hollander, 2006; Li et al., 2012) or by modelling the ability of the scheduling approach to fully account for TTV (Lam and Small, 2001; Batley and Ibanez, 2011). The studies of public transport would tend to reject Eq. (3), whereas those with a focus on private car trips would tend to find empirical evidence of equivalence.

A theoretical mechanism to clarify the question of equivalence is provided in the work by F&K (2010). This work assumes that there is no penalty for lateness, that the travel time distribution is independent of travel times (Noland and Small, 1995), and that departure times are optimal and continuous. Under these restrictions, F&K (2010) therefore show that the value of standard deviation, ρ, can be estimated by $H(\beta+\gamma)$, where H is a 'conversion' factor.

3. Equivalence in public transport contexts

3.1. Foundation of equivalence

Despite the stated requirement of continuous departures for equivalence by F&K, intuition suggests that the method can be applied in the case of discrete departures (i.e. scheduled public transport services) in circumstances where the space between such departures (service headway) is small. In this section, we provide a simple modification to the F&K methodology which allows this question to be investigated. The method is subsequently applied using a sample of smart card data.

In the case of continuous departures it has been shown that the individuals optimal departure time, D, is determined by a combination of travel time, standard deviation of travel time, the distribution of travel time, as well as the scheduling parameters for the value of earliness (β) and the value of lateness (γ). D is a positive number of minutes before the PAT and is broadly given by

$$D = f(\mu, \sigma, \Phi^{-1}, \beta, \gamma) \tag{4}$$

where Φ^{-1} represents the inverse cumulative distribution of travel times.

In the work that follows, we assume that an individual's D is revealed or can be estimated through an external data source, as can μ, σ and Φ^{-1}. Furthermore, we also assume that travellers are aware of these parameters and variables. It is possible to re-arrange F&K's equation for D so that the unknown earliness and lateness parameters become the subject:

$$\frac{\beta}{\beta+\gamma} = 1 - \left(\Phi\left(\frac{D-\mu}{\sigma}\right)\right) \tag{5}$$

Readers may recognise that the term on the left corresponds to the previously derived probability of lateness (P_L) under the assumptions of Noland and Small (1995):

$$P_L=\frac{\beta}{\beta+\gamma} \tag{6}$$

The work to this point has been an exposition of some of the insights provided by F&K, which are applicable to modes where departures are continuous. We now turn our attention to the case of high frequency public transport, by introducing passengers' expected waiting time.

3.2. The introduction of headway

In this section we assume that the value of a passengers' D (and the associated PAT) persists, but that the only information relating to the timetable known by the passenger is the headway of the service in minutes, $2h$, where h represents half the headway in minutes. We take the latter to be a reasonable expectation of waiting time if the traveller is ignorant of the published time table but aware only of the value of $2h$.More formally h_{ijk} is a non-negative real number which will vary based upon station i, for line j and service pattern k. In the work that follows, we will dispense with the subscript, and focus instead on a specific case of each of i, j and k.

The passenger of a high frequency public transport service will have an expectation of catching the next scheduled service h minutes after they arrive at the departure point. This will affect the passenger in two ways: firstly they will incur some waiting cost, and secondly h will increase the overall trip duration. Since D (and the related PAT) is fixed in our scenario, an increased trip duration will cause the probability of lateness to increase and this will affect the scheduling parameters in Eq. (6). Against this background, we can adjust F&K's utility function – which applies to continuous departures – for the context of scheduled departures, by introducing D_S as the next schedule departure after D, in minutes before the PAT, as follows:

$$\mathrm{U}(D,D_S,T)=\beta D+\xi(D-D_S)+(\alpha-\beta)T+(\beta+\gamma)(T-D_S)^{+} \tag{7}$$

Where ξ represents the marginal utility of waiting time, and the actual waiting time is defined by the difference between the passenger's arrival time at the departure point (D) and the time the next vehicle departs (D_S). The marginal utility of lateness is given by ($\beta+\gamma$), and we observe that average lateness will increase as $(T\text{-}D) > (T\text{-}D_S)$ in all circumstances except where D_S=D. To formalise the expectation of waiting time in these terms, we have:

$$\mathrm{E}\left(D\text{-}D_S\right)=h \tag{8}$$

We ignore the effect of an increase in generalised cost due to an amount of waiting time, but now introduce h into the F&K methodology.

Taking all other terms in equation 5 as constant, it now becomes:

$$\frac{\beta^*}{\beta^*+\gamma^*}=1\text{-}\left(\Phi\left(\frac{D\text{-}h\text{-}\mu}{\sigma}\right)\right) \tag{9}$$

The introduction of h to equation 9 entails a subtraction from D, representing the additional activity (waiting time) which now must be taken into account when applying the method of equivalence. This change will affect the values of β and γ which have now become β^* and γ^*. Using equation 9 and with reference to the original F&K methodology we now estimate the mean lateness factor, H, using $\frac{\beta^*}{\beta^*+\gamma^*}$.

$$\mathrm{H}=\int_{1-\frac{\beta^*}{\beta^*+\gamma^*}}^{1} \Phi^{-1}(s)ds. \tag{10}$$

To re-cap, the marginal utility of the standard deviation in the mean-variance approach, ρ, can be calculated via H($\beta+\gamma$). Therefore for a given value of ρ, the ($\beta+\gamma$) term increases as $2h$ increases. Using equation 9, we can predict that as headway increases, the value of β estimated will also increase and the value of γ will decrease.

This finding is in line with intuition. Consider the case of two travellers departing at the same time in relation to a preferred arrival time, with the same origin and destination as well as travel time distributions. Traveller 1 leaves by car and therefore can begin the journey at time D. Traveller 2 takes public transport and is likely depart after D when the next scheduled service departs. Traveller 2 in this case will have a higher probability of lateness, but nevertheless has chosen to begin their journey at time D. We would therefore conclude that passenger 2 exhibits a lower aversion to lateness than passenger 1. The method outlined in this section would estimate lower γ and higher β values for passenger 2 than passenger 1. Such a change in marginal valuations of earliness and lateness will have real world consequences by affecting the social cost of reliability (and any planning and investment decisions informed by this social cost). In the case above where the value of γ is diminished, we would find that transport interventions aimed at preventing late arrival at destination would become relatively less attractive for investment.

4. Empirical Illustration

The method outlined above is now tested using smart card data made available from the London Underground metro system. Travel records between a single OD pair during the AM peak hour (weekday trips beginning 08:00-09:00) are isolated – such a restriction of the data supports a simplifying assumption that μ, σ and Φ are identical for all travellers. This simplification is also in line with the common assumption that the travel time distribution is independent of departure time. The OD pair is chosen as it represents a reasonably high frequency peak hour service ($2h$ = 6) and provides passengers with a direct route (no transfers involved) from the origin to the destination.

The dataset contains 152 individual smart card records, with each record representing a single trip. Each individual record contains a start time and end time of the trip, from which we are able to calculate the mean and standard deviation of travel times. The distribution of travel times was positively skewed and resembled a log-normal or log-logistic distribution upon visual inspection. Statistical tests were unable to identify a common distributional form for the data, and so it was decided that a normal distribution of travel times would be assumed, which would meet the purpose of demonstrating the effect outlined in the previous section. Further simplification is made by assuming that the distribution of travel times is related to in-vehicle time. In reality, the empirical distribution contains other components of the trip such as platform access time, but nevertheless is useful for demonstration of the effect of headway outlined so far.

Eq. (5) estimates the scheduling parameters as a function of D. However, the D term is defined in relation to a passenger's preferred arrival time (PAT), which is not directly revealed. For the purposes of this illustration, we assign a PAT value to each of the passengers based upon a random draw from a symmetrical triangular distribution. The possible values for the PAT were bounded by observed travel times and were given in minutes past the observed start time. The minimum value of this distribution is the minimum travel time observed in the dataset, with the maximum as the 95th percentile of travel times observed (chosen to eliminate extreme outliers). The generated PAT for all passengers is normalised to 0 and each individual value of D is in minutes before the PAT. The generation of PATs provides a usable method of modelling heterogeneity in the sample of travellers, in line with the previous work of Batley et al. (2001).

We use this information to calculate scheduling parameters for each observed passenger, using Eq. (5) and Eq. (9) in turn. Eq. (5) represents the trip as if there were no headway between services (i.e. h = 0), whereas h = 3 is assumed in Eq. (9). At this point we observe average values of $P_L = 0.392$ and $P_L^* = 0.512$, representing a substantial increase in probability of lateness for h = 3. From these values it is possible to estimate average values of H and H^* across the sample, which largely resulted in $H^* > H$. Exploiting existing evidence on the value of ρ, specifically 0.67 \$/min from the work of Li et al. (2011), it is possible to calculate representative β and γ values for the two situations.

Beta and Gamma behave as might reasonably be expected in the high frequency public transport scenario, with the effect of increased h being to reduce values of γ and conversely increase β. Previous empirical estimates show in all cases that $\gamma > \beta$, however the introduction of h = 3 results in the median $\beta > \gamma$. The practical implications of such a change are that the public transport routes with a greater propensity for early arrival may now appear more 'unreliable' than those with greater propensity for late arrival. In other words, for a value of $2h$ = 6, the F&K equivalence becomes problematic. A question remains whether $2h$ = 6 can truly be considered 'high frequency', given that some Metro services on the same network are as frequent as $2h$ =2. To address this concern, the methodology is applied to a further 23 similar OD pairs with different advertised headways (a full list of these OD pairs is provided in appendix A of this paper) as follows.

Table 1. Summary of β and γ estimates with introduction of headway to the F&K methodology

	$2h = 0$		$2h = 6$	
	Beta	Gamma	Beta	Gamma
Mean	0.629	1.256	0.780	0.825
Median	0.572	0.995	0.699	0.693
StDev	0.216	0.95	0.298	0.543
Max	1.452	5.781	1.928	3.208
Min	0.331	0.249	0.375	0.169
σ/μ	0.343	0.756	0.382	0.659

Median β and γ values are calculated for each OD pair without taking into account headway. The β^* and γ^* values are also calculated for each OD taking into account the mean peak hour headway as calculated from published timetables. The ratios (β^*: β and γ^*: γ) are plotted for all 23 OD pairs in Fig. 1.

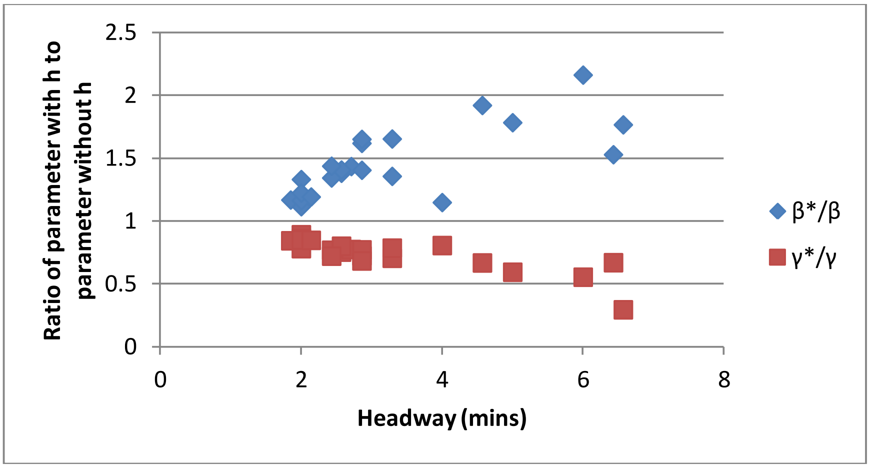

Fig. 1. Ratio of headway-adjusted scheduling variables to unadjusted scheduling variables (by OD pair)

Fig. 1 supports the effect observed in Table 1 that the ratio of β^* to β is greater than one in all cases, with the ratio of γ^* to γ less than one. A statistically significant positive linear relationship between headway and β^*: β is observed, indicating increasing difficulty in justifying the F&K methodology as headway increases. A significant but negative linear relationship exists between γ^*: γ and headway. These relationships show that the change in β and γ is more acute at higher headways. It should be noted however that at the lowest levels of headway ($2h \sim 2$), the estimates of β^* and γ^* are different from β and γ. We also note an asymmetric relationship between changes to the marginal value of earliness and lateness as headway increases: β^* is unbounded as it increases but γ^* cannot decrease below 0. This asymmetry, taken together with the significant changes in the values of earliness and lateness with headway, leads to the conclusion that application of the F&K method with very frequent public transport services remains problematic. Applying the method would lead to a skewed cost of reliability, by overvaluing passengers' aversion to earliness and undervaluing aversion to lateness.

5. Conclusion

In this paper we have established an approach for applying F&K's equivalence using an empirical data source, and shown the impact of introducing public transport headway into the underpinning utility function.

The mean-variance and scheduling approaches to reliability were initially outlined and existing work to bring them together has been noted. Particular emphasis was placed on the work which introduced the mean lateness factor, H, and the specific assumptions under which it is derived were highlighted. The F&K methodology was then introduced, drawing upon the established linkages provided by the term H.

We assumed that passengers optimise their departure times based upon the methods developed by F&K for continuous departures. By introducing headway to the F&K equivalence, it is suggested that the probability of lateness is affected, which will in turn affect the β and γ estimates based upon a single value of reliability. This method was initially demonstrated for a single OD under the assumption of a normal distribution of travel times. We then went on to introduce a number of similar OD pairs with differing lengths of headway between vehicles.

It is shown that the introduction of headway increases the value of β whilst reducing γ. We suggest that β is more responsive to headway at higher values in comparison to γ, but further empirical testing is required to confirm this preliminary finding. The impact of this finding is that applying the F&K equivalence in the case of high frequency public transport remains problematic, as scheduling parameters are highly, but not equally, affected by the headway of the service. Such attempts are likely to result in skewed and misleading valuations of reliability, with a negative impact on decisions related to improving reliability of public transport services. The conclusion of the work is that the theoretical equivalence between the mean-variance and scheduling approaches to valuing reliability is problematic in the case of scheduled public transport. The difference between the two approaches becomes more apparent as headway increases, but even at low headway values the method should not be applied.

Extensions to this methodology will be to conduct further analysis for different values of headway, and to derive the mathematical properties of the value h. Of benefit to this and other research will be to assess the method of generating PATs and explore possible alternatives. We note that restrictions on the departure time, D is somewhat problematic for the methodology outlined.

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Appendix A

Table A1. Summary of OD pairs used in the study

Origin	Destination	n	Line	Peak Hour Headway (mins)
East Finchley	Old Street	106	Northern	6.43
Walthamstow Central	Warren Street	209	Victoria	3.29
Stratford	Bond Street	181	Central	2
Balham	Moorgate	161	Northern	2.86
Woodford	Liverpool Street	115	Central	4
Baker Street	Canary Wharf	168	Jubilee	1.85
Tooting Broadway	London Bridge	100	Northern	2.57
North Greenwich	Bond Street	218	Jubilee	2.14
Clapham South	Moorgate	137	Northern	2.71
Bounds Green	Holborn	170	Piccadilly	2.43
Ealing Broadway	Marble Arch	110	Central	6
Earls Court	Monument	139	District	2.86
Finchley Road	Liverpool Street	116	Metropolitan	4.57
Balham	Bank	184	Northern	2.86
Walthamstow Central	Euston	123	Victoria	3.29
Stratford	London Bridge	154	Jubilee	2.57
West Hampstead	London Bridge	162	Jubilee	2
South Harrow	Hammersmith D	116	Piccadilly	5
Leyton	Tottenham Court Rd	121	Central	2
East Finchley	Tottenham Court Rd	146	Northern	6.57
Seven Sisters	Victoria	121	Victoria	2
Hammersmith D	Holborn	118	Piccadilly	2.43
North Greenwich	Green Park	163	Jubilee	2.14