Article:

Liu, W and Zhang, W (2014) Joint Transmission and Reception Diversity Smoothing for Direction Finding of Coherent Targets in MIMO Radar. IEEE Journal of Selected Topics in Signal Processing, 8 (1). pp. 115-124. ISSN 1932-4553

https://doi.org/10.1109/JSTSP.2013.2285520

Joint Transmission and Reception Diversity Smoothing for Direction Finding of Coherent Targets in MIMO Radar

Wei Zhang[a,b], Wei Liu[b], Ju Wang[a], and Siliang Wu[a]

[a] School of Information and Electronics
Beijing Institute of Technology, Beijing 100081, China
{huben,wangju,siliangw}@bit.edu.cn

[b] Communications Research Group
Dept. of Electronic & Electrical Engineering
University of Sheffield, Sheffield S1 3JD, UK
w.liu@sheffield.ac.uk

***Abstract*—The direction estimation problem of coherent targets in multiple-input multiple-output (MIMO) radar systems is studied and a scheme with joint transmission and reception diversity smoothing is proposed. When both the transmitting and receiving antenna arrays are located closely in space, the new approach leads to much more available covariance matrices for spatial smoothing to decorrelate the coherent signals. As a result, a better estimation performance is achieved compared to the existing transmission diversity smoothing (TDS) method. It can also identify more coherent targets when sparse antenna arrays are employed. On the other hand, the proposed approach can be applied to joint direction of arrival (DOA) and direction of departure (DOD) estimation using existing direction estimation algorithms when the transmit and receive arrays are separated far away from each other (i.e. the bistatic case). Two specific methods are proposed under the scheme, one is based on forward-only (FO) spatial smoothing and one is based on forward-backward (FB) processing. Due to the increased number of covariance matrices for spatial smoothing, a further improved performance is achieved by the FB-based one.**

***Index Terms*—MIMO radar, DOA/DOD estimation, coherent targets, transmission-reception diversity smoothing.**

I. Introduction

Unlike the standard phased-array radar, MIMO radar employs multiple transmit antennas for emitting orthogonal waveforms and multiple receive antennas for receiving the echoes reflected by the targets [1]–[3]. Two types of MIMO radar have been investigated, namely, widely separated antennas [4] and colocated antennas [5]. In this paper, we will formulate the problem based on MIMO radar with colocated antennas, where the transmitting side and the receiving side can be located either at the same site or far away from each other. MIMO radar can exploit the waveform diversity to form a virtual array with increased degrees of freedom (DOFs) and a larger aperture compared to the traditional phased-array radar. It has been shown that MIMO radar can provide enhanced spatial resolution, achieve better target detection performance, and significantly improve the system's parameter identifiability [3], [5]–[7].

Many techniques have been proposed (see [8]–[29] for details) for angle estimation in MIMO radar by assuming that all targets are uncorrelated with each other, so that the traditional eigenspace-based algorithms, such as MUSIC [30] and ESPRIT [31], can be employed for multiple-target localization. However, in many radar applications, the received echo signals from different targets are considered as coherent, which implies that the eigenspace-based methods cannot be directly used for angle estimation due to the ill conditioning problem of the covariance matrix [32], [33]. Spatial smoothing is a classic method to decorrelate the signals in the data covariance matrix [34]–[38]. The drawback with it is the decrease of the array aperture and the number of DOFs, resulting in lower resolution and accuracy.

To overcome the coherent-target localization problem in MIMO radar, a preprocessing technique referred to as TDS is used to spatially smooth the signal covariance matrix in order to enable the use of eigenspace-based angle estimation methods [32]. The basic idea of the TDS method is to form a new covariance matrix with decorrelated signal subspace by summing the covariance matrices corresponding to the transmit antennas together. Unlike the traditional spatial-smoothing technique, the TDS method does not decrease the physical array aperture and can be used for any array geometry. However, the maximum number of coherent targets which can be identified by the TDS method is $M-1$, where M is the number of transmit antennas. Therefore, compared to the original MIMO array, the TDS method also significantly reduces the effective array aperture length and the number of DOFs. Additionally, the TDS method is designed for MIMO radar systems where both the transmit and receive arrays are located closely in space. It is not suitable for joint DOA and DOD estimation in bistatic MIMO radar. On the other hand, due to the different phase shifts associated with the different propagation paths from the transmit antennas to targets, these independent waveforms are linearly combined at the targets with different phase factors, leading to linearly independent signal waveforms reflected from different targets. Therefore, the covariance matrix computed from the received data directly without matched filtering can also be used for the application of adaptive array algorithms [33]. This method has the same decorrelation performance as the TDS method [39]. Like the TDS method, however, its application is also limited by the aforementioned drawbacks.

In this work, we propose a class of improved methods to deal with multiple coherent targets in MIMO radar based on uniform or symmetric arrays. Since linearly independent waveforms are transmitted simultaneously via multiple antennas, we can obtain a data matrix based on a set of virtual antennas. A $K_r \times K_t$ receiving-transmitting window is then utilized to slide over this data matrix, where K_t and K_r represent the transmitting and receiving dimensions of the sliding window. Due to the existence of the phase-shift factor between the sliding sub-block data, the corresponding covariance matrices can be employed to perform spatial smoothing for reconstructing the full-rank signal covariance matrix, supported by a detailed analysis of its decorrelation effect. Since both transmission and reception diversity smoothing is utilized, the proposed method has more covariance matrices for the smoothing operation, and therefore can achieve a better estimation result than the TDS method and localize much more coherent targets when sparse arrays are employed [40]. More importantly, it is also suitable for joint DOA and DOD estimation in bistatic MIMO radar by employing the existing joint DOA and DOD estimation algorithms directly due to the use of joint transmission and reception diversity smoothing. Moreover, given the generalized conjugate symmetric property of the effective steering vectors of the array, a forward-backward based smoothing method is proposed to further improve the performance of the system.

This paper is organized as follows. In Sec. II, the signal model for MIMO radar is provided. The proposed spatial smoothing method with a detailed analysis of its decorrelation effect is introduced in Sec. III, where both the FO and the FB based smoothing processes are investigated. Simulation results are presented in Sec. IV and conclusions are drawn in Sec. V.

II. Signal Model for MIMO Radar

Consider a narrowband MIMO radar system with a uniform linear array (ULA) of M antennas for transmitting and a ULA of N antennas for receiving. The M transmit antennas are used to transmit M orthogonal waveforms. Assume that K coherent targets are present and the targets in a coherent processing interval (CPI) do not have range walking across range cells, i.e., they are located at the same range cell of received pulses. Consequently, the output of the matched filters at the receiver at the lth snapshot can be expressed as [3], [9], [12]

$$\begin{aligned}\mathbf{x}[l] &= \big[x_{1,1}[l], x_{2,1}[l], \cdots, x_{N,1}[l], x_{1,2}[l], x_{2,2}[l], \cdots, \\ &\quad x_{N,2}[l], \cdots, x_{1,M}[l], x_{2,M}[l], \cdots, x_{N,M}[l]\big]^T \\ &= \sum_{k=1}^{K} \mathbf{a}_t(\varphi_k) \otimes \mathbf{a}_r(\theta_k) b_k[l] + \mathbf{z}[l] \\ &= [\mathbf{a}_t(\varphi_1) \otimes \mathbf{a}_r(\theta_1), \mathbf{a}_t(\varphi_2) \otimes \mathbf{a}_r(\theta_2), \cdots, \\ &\quad \mathbf{a}_t(\varphi_K) \otimes \mathbf{a}_r(\theta_K)]\mathbf{b}[l] + \mathbf{z}[l] \qquad (1)\end{aligned}$$

where $x_{n,m}[l]$ is the received data at the nth receive antenna associated with the mth transmit antenna, $[\cdot]^T$ denotes the transpose operation, θ_k and φ_k are the DOA and DOD of the kth target, $\otimes$ stands for the Kronecker product operator, and $b_k[l] = \gamma_k e^{j2\pi f_k l}$, with γ_k being the complex-valued reflection coefficient of the kth target and f_k being the Doppler frequency;

$$\begin{aligned}\mathbf{b}[l] &= \big[b_1[l], b_2[l], \cdots, b_K[l]\big]^T, \qquad (2)\\ \mathbf{a}_t(\varphi_k) &= [1, \alpha_k, \cdots, \alpha_k^{M-1}]^T \qquad (3)\\ \mathbf{a}_r(\theta_k) &= [1, \beta_k, \cdots, \beta_k^{N-1}]^T \qquad (4)\end{aligned}$$

are the transmit and receive steering vectors, with $\alpha_k = e^{-j2\pi d_t \sin(\varphi_k)/\lambda}, \beta_k = e^{-j2\pi d_r \sin(\theta_k)/\lambda}$, where d_t and d_r, respectively, are the adjacent antenna spacing for the transmit and receive arrays, and λ denoting the wavelength; $\mathbf{z}[l]$ denotes the received zero-mean complex-valued white noise with a power σ^2.

III. Proposed Method

A. Construction of full-rank signal covariance matrix

First, we form an $N \times M$ matrix $\mathbf{Y}[l]$ directly from $\mathbf{x}[l]$. The mth column of $\mathbf{Y}[l]$ is the received data at the N receive antennas associated with the mth transmit antenna, and $\mathbf{Y}[l]$ is then given by

$$\begin{aligned}\mathbf{Y}[l] &= \begin{bmatrix} x_{1,1}[l] & x_{1,2}[l] & \cdots & x_{1,M}[l] \\ x_{2,1}[l] & x_{2,2}[l] & \cdots & x_{2,M}[l] \\ \vdots & \vdots & \cdots & \vdots \\ x_{N,1}[l] & x_{N,2}[l] & \cdots & x_{N,M}[l] \end{bmatrix} \\ &= \mathbf{A}_r \Xi \mathbf{A}_t^T + \mathbf{Z}[l] \qquad (5)\end{aligned}$$

where

$$\begin{aligned}\mathbf{A}_t &= [\mathbf{a}_t(\varphi_1), \mathbf{a}_t(\varphi_2), \cdots, \mathbf{a}_t(\varphi_K)], \qquad (6)\\ \mathbf{A}_r &= [\mathbf{a}_r(\theta_1), \mathbf{a}_r(\theta_2), \cdots, \mathbf{a}_r(\theta_K)], \qquad (7)\\ \Xi &= \mathrm{diag}\big[b_1[l], b_2[l], \cdots, b_K[l]\big], \qquad (8)\end{aligned}$$

and $\mathbf{Z}[l]$ denotes the $N \times M$ noise matrix.

Define a $K_r \times K_t$ matrix $\mathbf{Y}_{j,i}[l]\big(1 \le j \le N - K_r + 1, 1 \le i \le M - K_t + 1\big)$, which is the received data from the jth to the $(j + K_r - 1)$th rows of $\mathbf{Y}[l]$ and from the ith to the $(i + K_t - 1)$th columns of $\mathbf{Y}[l]$.

With the notation $\mathrm{Vec}(\cdot)$ for a matrix operation that stacks the columns of a matrix to form a new column vector, we form the following vectors:

$$\begin{aligned}\mathbf{y}_{j,i}[l] &= \mathrm{Vec}\big(\mathbf{Y}_{j,i}[l]\big) \\ &= \sum_{k=1}^{K} \big(\mathbf{a}_t^{(K_t)}(\varphi_k) \otimes \mathbf{a}_r^{(K_r)}(\theta_k)\big)\alpha_k^{i-1}\beta_k^{j-1} b_k[l] \\ &\quad + \mathbf{z}_{j,i}[l] \\ &= \mathbf{A}\boldsymbol{\phi}_t^{i-1}\boldsymbol{\phi}_r^{j-1}\mathbf{b}[l] + \mathbf{z}_{j,i}[l], \\ i &= 1, \cdots, M - K_t + 1, \\ j &= 1, \cdots, N - K_r + 1. \qquad (9)\end{aligned}$$

where $\mathbf{a}_r^{(K_r)}(\theta_k)$ and $\mathbf{a}_t^{(K_t)}(\varphi_k)$ are the $K_r \times 1$ and $K_t \times 1$ truncated versions of the steering vectors $\mathbf{a}_r(\theta_k)$ and $\mathbf{a}_t(\varphi_k)$, respectively,

$$\begin{aligned}\mathbf{A} &= [\mathbf{a}_t^{(K_t)}(\varphi_1) \otimes \mathbf{a}_r^{(K_r)}(\theta_1), \cdots, \\ &\quad \mathbf{a}_t^{(K_t)}(\varphi_K) \otimes \mathbf{a}_r^{(K_r)}(\theta_K)], \qquad (10)\\ \boldsymbol{\phi}_t &= \mathrm{diag}\{\alpha_1, \cdots, \alpha_K\}, \qquad (11)\\ \boldsymbol{\phi}_r &= \mathrm{diag}\{\beta_1, \cdots, \beta_K\}. \qquad (12)\end{aligned}$$

The covariance matrix corresponding to $\mathbf{y}_{j,i}[l]$ is given by

$$\begin{aligned}\mathbf{R}_{j,i} &= E\big[\mathbf{y}_{j,i}[l]\mathbf{y}_{j,i}^H[l]\big] \\ &= \mathbf{A}\boldsymbol{\phi}_t^{i-1}\boldsymbol{\phi}_r^{j-1}\mathbf{S}(\boldsymbol{\phi}_r^{j-1})^H(\boldsymbol{\phi}_t^{i-1})^H\mathbf{A}^H + \sigma^2\mathbf{I} \qquad (13)\end{aligned}$$

where $E[\cdot]$ denotes the expectation operation, $[\cdot]^H$ represents the Hermitian transpose, and $\mathbf{S} = E[\mathbf{b}[l]\mathbf{b}[l]^H]$ is the signal covariance matrix. Like the classic forward only (FO) spatial smoothing technique [34], we can sum all the $\mathbf{R}_{j,i}$ together to spatially smooth the signal covariance matrix:

$$\mathbf{R}_{fo} = \frac{\sum_{i=1}^{(M-K_t+1)}\sum_{j=1}^{(N-K_r+1)}\mathbf{R}_{j,i}}{(M-K_t+1)(N-K_r+1)}. \qquad (14)$$

In practice, the sample covariance matrix of (13)

$$\hat{\mathbf{R}}_{j,i} = \frac{1}{L}\sum_{l=1}^{L}\mathbf{y}_{j,i}[l]\mathbf{y}_{j,i}^H[l] \qquad (15)$$

is used, where L is the number of snapshots.

B. Analysis of the decorrelation effect of the proposed method

Now we study the decorrelation effect of the proposed joint transmission and reception diversity smoothing by extending the results of [41]. Combining (13) and (14), we obtain

$$\mathbf{R}_{fo} = \mathbf{A}\bar{\mathbf{S}}\mathbf{A}^H + \sigma^2\mathbf{I} \tag{16}$$

where $\bar{\mathbf{S}}$ is the $K \times K$ smoothed covariance matrix with

$$\bar{\mathbf{S}} = \frac{\sum_{i=1}^{(M-K_t+1)}\sum_{j=1}^{(N-K_r+1)} \boldsymbol{\phi}_t^{i-1}\boldsymbol{\phi}_r^{j-1}\mathbf{S}(\boldsymbol{\phi}_r^{j-1})^H(\boldsymbol{\phi}_t^{i-1})^H}{(M-K_t+1)(N-K_r+1)}. \tag{17}$$

We now study how progressive joint transmission and reception diversity smoothing reduces the correlation between all the impinging signals. From (17), we have

$$[\bar{\mathbf{S}}]_{p,q} = [\mathbf{S}]_{p,q} \times \frac{\sum\limits_{i=1}^{(M-K_t+1)}\sum\limits_{j=1}^{(N-K_r+1)} \alpha_p^{i-1}\beta_p^{j-1}(\beta_q^{j-1})^*(\alpha_q^{i-1})^*}{(M-K_t+1)(N-K_r+1)}, \tag{18}$$

where $[\cdot]_{p,q}$ denotes the (p,q)th element of the matrix and $(\cdot)^*$ represents the conjugate operation. Clearly for $p = q$, we have $[\bar{\mathbf{S}}]_{p,p} = [\mathbf{S}]_{p,p}$. But for $p \neq q$, we have

$$\begin{aligned}
&\frac{\sum_{i=1}^{(M-K_t+1)}\sum_{j=1}^{(N-K_r+1)} \alpha_p^{i-1}\beta_p^{j-1}(\beta_q^{j-1})^*(\alpha_q^{i-1})^*}{(M-K_t+1)(N-K_r+1)} \\
&= \frac{\sum_{i=1}^{(M-K_t+1)} \alpha_p^{i-1}(\alpha_q^{i-1})^*}{(M-K_t+1)} \frac{\sum_{j=1}^{(N-K_r+1)} \beta_p^{j-1}(\beta_q^{j-1})^*}{(N-K_r+1)} \\
&= \frac{\sum_{i=1}^{(M-K_t+1)} (\alpha_p\alpha_q^*)^{i-1}}{(M-K_t+1)} \frac{\sum_{j=1}^{(N-K_r+1)} (\beta_p\beta_q^*)^{j-1}}{(N-K_r+1)}.
\end{aligned} \tag{19}$$

We see that $\frac{\sum_{i=1}^{(M-K_t+1)}(\alpha_p\alpha_q^*)^{i-1}}{(M-K_t+1)}$ (or $\frac{\sum_{j=1}^{(N-K_r+1)}(\beta_p\beta_q^*)^{j-1}}{(N-K_r+1)}$) goes to zero as $M-K_t+1$ (or $N-K_r+1$) goes to infinity. Thus, the coherent signals are increasingly decorrelated. However, the rate for (19) to approach zero depends on the spacing and directions of the signals. Here, we see the effect of small angular separation on decorrelation between the signals. Let φ_p and φ_q correspond to closely spaced signals, and let $\varphi_p = \varphi_q + \Delta$. We then have $\{\sin(\varphi_p) - \sin(\varphi_q) \approx \Delta\cos(\varphi_p)\}$. Consequently, we can write

$$\begin{aligned}
&\sum_{i=1}^{M-K_t+1} (\alpha_p\alpha_q^*)^{i-1} \\
&= \sum_{i=1}^{M-K_t+1} \exp[-j2\pi(i-1)d_t(\sin(\varphi_p) - \sin(\varphi_q))] \\
&\approx \frac{1-\exp[-j2(M-K_t+1)\pi d_t\Delta\cos(\varphi_p)/\lambda]}{1-\exp[-j2\pi d_t\Delta\cos(\varphi_p)/\lambda]}.
\end{aligned} \tag{20}$$

Thus, the minimum value of $(M-K_t+1)$ required for the numerator of (20) to go to zero is given by

$$(M-K_t+1) = \lambda/\big(d_t\Delta\cos(\varphi_p)\big) \tag{21}$$

Similarly, we can write

$$\begin{aligned}
&\sum_{j=1}^{(N-K_r+1)} (\beta_p\beta_q^*)^{j-1} \\
&\approx \frac{1-\exp[-j2(N-K_r+1)\pi d_r\Delta\cos(\theta_p)/\lambda]}{1-\exp[-j2\pi d_r\Delta\cos(\theta_p)/\lambda]},
\end{aligned} \tag{22}$$

and the minimum value of $(N-K_r+1)$ required for the numerator of (22) to go to zero is then given by

$$(N-K_r+1) = \lambda/\big(d_r\Delta\cos(\theta_p)\big). \tag{23}$$

From (21) and (23), we see that the values of $(M-K_t+1)$ and $(N-K_r+1)$ required for decorrelating the pth and qth signals are large when the angular separation between them is small. Moreover, for a fixed small angular separation between the signals the values of $(M-K_t+1)$ and $(N-K_r+1)$ required for decorrelation go up when the signals approach the end-fire direction, i.e., 90°. However, it should be noted from (19), (21) and (23) that for the proposed method, its decorrelation effect will degrade severely only when both DODs and DOAs approach the end-fire direction.

C. Selection of K_t and K_r

From (9) and (14), the effective aperture length and the number of covariance matrices defined in (13) are related to K_t and K_r. In this section, the selection of K_t and K_r is investigated, and two cases of MIMO radar system will be considered. In the first case, both the transmit and receive arrays are assumed to be closely located in space, so that any target located in the far-field can be seen at the same direction by both arrays, that is, $\theta_k = \varphi_k$. The second one is a bistatic MIMO radar system where the transmit and receive arrays are separated far away from each other.

1) The first case with filled ULA for both the transmit and receive arrays: First consider the case both the transmit and receive arrays are filled (i.e., half-wavelength inter-element spacing with $d_t = d_r = \lambda/2$) ULAs [5]. In this case the $K_tK_r \times 1$ vector $\mathbf{a}_t^{(K_t)}(\theta_k) \otimes \mathbf{a}_r^{(K_r)}(\theta_k)$ has only (K_t+K_r-1) distinct elements; in fact, this appears to be the smallest possible number of distinct elements, and there are $(M-K_t+1)(N-K_r+1)$ number of $\mathbf{R}_{j,i}$ defined in (13); nevertheless, only $(M-K_t+1+N-K_r)$ distinct $\mathbf{R}_{j,i}$ actually used for spatial smoothing. Therefore, to identify K coherent targets when the spatially smoothed covariance matrix is used in conjunction with eigenspace-based techniques, K_t and K_r should satisfy

$$K_t + K_r - 1 > K,\ M - K_t + 1 + N - K_r > K. \tag{24}$$

We see that an enhanced spatial resolution will be obtained by increasing the value of K_t or K_r. However, the number of covariance matrices $\mathbf{R}_{j,i}$ will decrease in such a case, leading to decrease of the maximum number of coherent targets that can be identified by the proposed method. Consequently, there is a trade-off between the sub-array aperture and the number of coherent targets identified by the proposed method. In particular, when the following condition

$$K_t + K_r - 1 = M - K_t + 1 + N - K_r \tag{25}$$

is achieved, i.e., $K_t + K_r = \frac{M+N+2}{2}$, the maximum number of coherent targets that can be identified by the proposed method will be obtained. On the other hand, the proposed method will be equivalent to the TDS method when K_t and K_r are set to 1 and N, respectively. So the TDS method can be considered as a special case of the proposed method. In addition, it is flexible for the proposed method to set its effective aperture length, which can be larger than that of the TDS method by setting K_t and K_r properly.

2) The first case with filled ULA for receive array but sparse ULA for transmit array: When the receive array is a filled ULA and the transmit array is a sparse ULA with $M/2$-wavelength inter-element spacing, the virtual aperture of the MIMO radar system is a filled-element ULA with MN distinct elements [5]. The vector $\mathbf{a}_t^{(K_t)}(\theta_k)\otimes\mathbf{a}_r^{(K_r)}(\theta_k)$ for this case has K_tK_r distinct elements, and there are $(M-K_t+1)(N-K_r+1)$ distinct $\mathbf{R}_{j,i}$ defined in (13) actually used for spatial smoothing. Similarly, the following conditions

$$K_tK_r > K,\ (M-K_t+1)(N-K_r+1) > K \tag{26}$$

should be satisfied to identify K coherent targets. In this case the maximum number of coherent targets which can be identified by the proposed method is obtained when $(K_tK_r) = (M-K_t+1)(N-K_r+1)$ is achieved. For simplicity, we set $K_t = \frac{M+1}{2}$ and $K_r = \frac{N+1}{2}$ in our proposed method. Then, the maximum number of coherent targets that can be identified by the proposed method is $\frac{(M+1)(N+1)}{4}-1$. Note that if $N > 3$, the number of coherent targets identified by the proposed method will be larger than $M-1$, the maximum number of identifiable targets by the TDS method.

3) The second case with filled ULA for both transmit and receive arrays: Because $\theta_k \neq \varphi_k$, the vector $\mathbf{a}_t^{(K_t)}(\varphi_k)\otimes\mathbf{a}_r^{(K_r)}(\theta_k)$ for this case has K_tK_r distinct elements, and there are $(M-K_t+1)(N-K_r+1)$ distinct $\mathbf{R}_{j,i}$ defined in (13) actually used for spatial smoothing. Therefore, in order to identify K coherent targets, K_t and K_r should satisfy the following conditions:

$$\begin{aligned} &K_tK_r > K,\ K_t > 1,\ K_r > 1\\ &(M-K_t+1)(N-K_r+1) > K. \end{aligned} \tag{27}$$

From (27), it can be seen that when the transmit and receive arrays are separated far away from each other, we can obtain sufficient DOFs by joint transmission and reception diversity smoothing without increasing the interelement spacing of the transmit array, while for the first case of the system, the inter-element spacing of the transmit array should be increased to obtain considerable DOFs.

D. Forward backward smoothing technique for the proposed method

In (15), only FO processing is used to smooth the signal's covariance matrix. Therefore, it can be considered as an FO-based smoothing method. However, the estimation performance can be improved greatly by FB smoothing compared with those using FO smoothing [42]. Moreover, the FB smoothing technique leads to a significant reduction in the correlation between signals, and therefore less antennas are needed for coherent signal detection compared with the one without it [35]. In this section, we will develop the FB smoothing technique for the proposed method.

Following the proof in [43], it can be shown that the steering vector $\mathbf{a}_t^{(K_t)}(\varphi)$ has the following property:

$$\mathbf{a}_t^{(K_t)}(\varphi) = e^{-j\phi_t(\varphi)}\mathbf{J}_{K_t}\big(\mathbf{a}_t^{(K_t)}(\varphi)\big)^* \tag{28}$$

where $\phi_t(\varphi) = 2\pi(K_t-1)d_t\sin(\varphi)/\lambda$, and $\mathbf{J}_{K_t}$ is the K_t-dimensional exchange matrix

$$\mathbf{J}_{K_t} = \begin{bmatrix} 0 & \cdots & 1\\ \vdots & \ddots & \vdots\\ 1 & \cdots & 0 \end{bmatrix}. \tag{29}$$

Similarly, we have $\mathbf{a}_r^{(K_r)}(\theta) = e^{-j\phi_r(\theta)}\mathbf{J}_{K_r}\big(\mathbf{a}_r^{(K_r)}(\theta)\big)^*$, where $\phi_r(\theta)$ and $\mathbf{J}_{K_r}$ are defined in the same way as $\phi_t(\varphi)$ and $\mathbf{J}_{K_t}$, respectively. Then we have

$$\begin{aligned} &\mathbf{a}_t^{(K_t)}(\varphi)\otimes\mathbf{a}_r^{(K_r)}(\theta)\\ &= [e^{-j\phi_t(\varphi)}\mathbf{J}_{K_t}\mathbf{a}_t^{(K_t)}(\varphi)^*]\otimes[e^{-j\phi_r(\theta)}\mathbf{J}_{K_r}\mathbf{a}_r^{(K_r)}(\theta)^*]\\ &= e^{-j(\phi_r(\theta)+\phi_t(\varphi))}[\mathbf{J}_{K_t}\otimes\mathbf{J}_{K_r}][\mathbf{a}_t^{(K_t)}(\varphi)^*\otimes\mathbf{a}_r^{(K_r)}(\theta)^*]\\ &= e^{-j(\phi_r(\theta)+\phi_t(\varphi))}\mathbf{J}_{K_rK_t}[\mathbf{a}_t^{(K_t)}(\varphi)\otimes\mathbf{a}_r^{(K_r)}(\theta)]^*. \end{aligned} \tag{30}$$

It can be clearly seen from (30) that the virtual steering vector in MIMO array has the generalized conjugate symmetric structure as the steering vector in the traditional ULAs, and therefore FB processing can be applied here.

Using (30) and following the classical FB smoothing technique [35], the proposed FB smoothed covariance matrix can be constructed as

$$\mathbf{R}_{fb} = \frac{\sum_{i=1}^{(M-K_t+1)}\sum_{j=1}^{(N-K_r+1)}\big(\mathbf{R}_{j,i}+\mathbf{J}_{K_rK_t}\mathbf{R}_{j,i}^*\mathbf{J}_{K_rK_t}\big)}{2(M-K_t+1)(N-K_r+1)}. \tag{31}$$

Defining

$$\boldsymbol{\psi} = \mathrm{diag}\{e^{-j(\phi_r(\theta_1)+\phi_t(\varphi_1))},\cdots,e^{-j(\phi_r(\theta_K)+\phi_t(\varphi_K))}\}, \tag{32}$$

and from (30) we have $\mathbf{A}\boldsymbol{\psi}^* = \mathbf{J}_{K_rK_t}\mathbf{A}^*$. $\mathbf{J}_{K_rK_t}\mathbf{R}_{j,i}^*\mathbf{J}_{K_rK_t}$ can then be written as

$$\begin{aligned} &\mathbf{J}_{K_rK_t}\mathbf{R}_{j,i}^*\mathbf{J}_{K_rK_t}\\ &= \mathbf{A}\boldsymbol{\psi}^*(\boldsymbol{\phi}_t^*)^{i-1}(\boldsymbol{\phi}_r^*)^{j-1}\mathbf{S}^*\boldsymbol{\phi}_r^{j-1}\boldsymbol{\phi}_t^{i-1}\boldsymbol{\psi}\mathbf{A}^H+\sigma^2\mathbf{I}. \end{aligned} \tag{33}$$

Thus, the proposed FB smoothed signal covariance matrix can be expressed as

$$\begin{aligned} \bar{\mathbf{S}}_{fb} \;=\; &\frac{1}{2(M-K_t+1)(N-K_r+1)}\sum_{i=1}^{M-K_t+1}\sum_{j=1}^{N-K_r+1}\\ &\big[\boldsymbol{\phi}_t^{i-1}\boldsymbol{\phi}_r^{j-1}\mathbf{S}(\boldsymbol{\phi}_r^{j-1})^H(\boldsymbol{\phi}_t^{i-1})^H+\\ &\boldsymbol{\psi}^*(\boldsymbol{\phi}_t^*)^{i-1}(\boldsymbol{\phi}_r^*)^{j-1}\mathbf{S}^*\boldsymbol{\phi}_r^{j-1}\boldsymbol{\phi}_t^{i-1}\boldsymbol{\psi}\big]. \end{aligned} \tag{34}$$

We can see that the number of smoothing operation for the proposed FB smoothing are twice that for the proposed FO smoothing. As a result, much more coherent targets can be located by the proposed FB smoothing, and it can also be predicted that the proposed FB smoothing has a better estimation performance than the proposed FO smoothing when the angular separation between two signals is small or when the signals approach the end-fire direction.

E. Discussion

Note that the standard spatial smoothing technique is limited to special array geometries such as uniform linear/rectangular arrays. Similarly, our proposed method can not be used directly for arbitrary nonuniform MIMO array systems. However, given a system with an arbitrary geometry, we can employ the array interpolation approach to create one or more virtual arrays having a geometry suitable for the application of the spatial smoothing technique [44]–[46]. Then the DOA estimation problem in nonuniform arrays can be transformed into simpler virtual uniform linear array problems. In addition, another approach, called manifold separation [47]–[51], can be used to model the received wavefield by means of an orthogonal expansion that approximates the true array steering vector of any arbitrary array as the product of a matrix that depends only on the array parameters and a Vandermonde vector depending only on the angle parameter. Therefore, by employing the array interpolation approach, or the manifold separation technique, it is possible to modify the proposed method to deal with angle estimation problems in nonuniform array based MIMO radar systems.

IV. Simulations

In this section, simulations are carried out to investigate the performance of the proposed methods compared with the TDS method. We consider a MIMO array configuration where a ULA of $M = 10$ antennas is used for transmitting and a ULA of $N = 10$ antennas for receiving. Assume that the additive noise is spatially white circularly symmetric complex Gaussian. All simulations are averaged over 500 independent runs. Define the root mean squared error (RMSE) as

$$\frac{1}{K}\sum_{k=1}^{K}\sqrt{\frac{1}{500}\sum_{n=1}^{500}(\vartheta_k - \hat{\vartheta}_{n,k})^2} \tag{35}$$

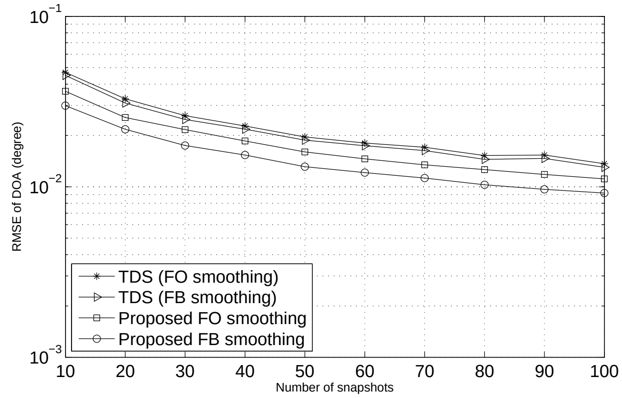
where $\hat{\vartheta}_{n,k}$ is the estimate of DOA/DOD ϑ_k of the nth run.

A. Both the transmit and receive arrays are closely located

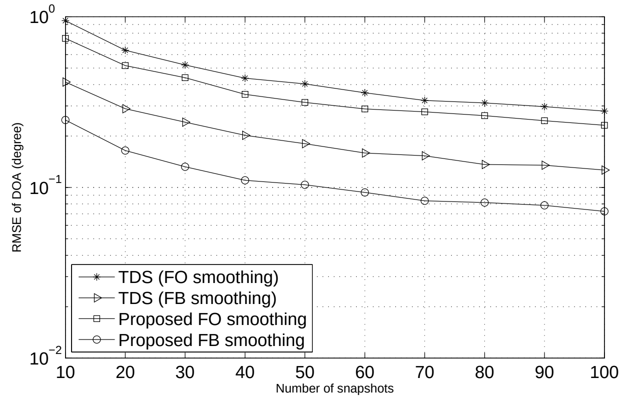
Two scenarios are considered: 1) three coherent targets with the same signal-to-noise ratio (SNR) are located at angles $\theta = 10°$, $20°$ and $30°$; 2) the angles of the three coherent targets change to $\theta = 50°$, $60°$ and $70°$.

1) Both the transmit and receive arrays are filled ULAs: In the first example, both the transmit and receive arrays are arranged with half-wavelength spacing between adjacent antennas. To form the same aperture with the TDS method, the proposed method chooses $K_t = 6$ and $K_r = 5$. The performance of the two methods is investigated using the ESPRIT-based algorithm [10].

Fig. 1 shows the RMSEs of DOA estimation versus the number of snapshots for SNR $= 20$ dB. Fig. 2 shows the RMSEs of DOA estimation as a function of input SNR for $L = 50$. As shown, the proposed method has achieved higher estimation accuracy than the TDS method. The reason is, although there are only 10 distinct covariance matrices defined in (13), the proposed method actually uses $(M-K_t+1)(N-K_r+1) = 30$ covariance matrices for spatial smoothing. Thus, the proposed method has obtained a better conditioned estimate of the covariance matrix than the TDS method, leading to better estimation result. In addition, it can be clearly seen that the performance of the proposed FB smoothing method is much better than the one with FO smoothing, especially for the second scenario where the signals approach the end-fire direction.



(a) Scenario 1 with $\theta = 10°$, $20°$ and $30°$.

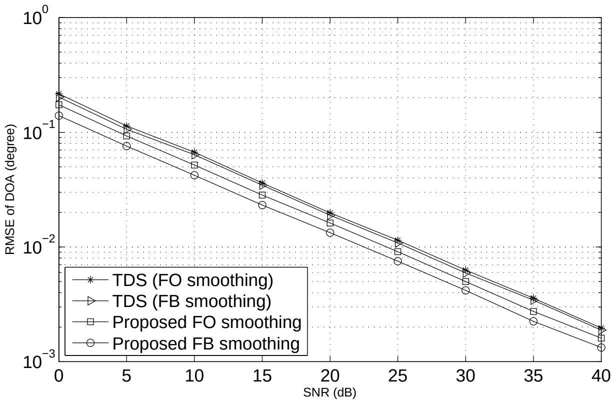


(b) Scenario 2 with $\theta = 50°$, $60°$ and $70°$.

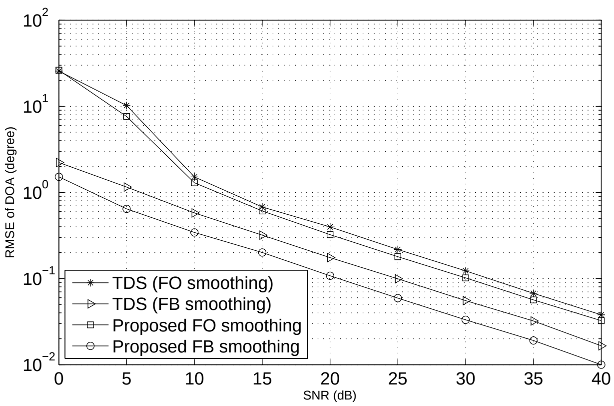
Fig. 1. RMSEs of DOA estimation versus the number of snapshots when both the transmit and receive arrays are filled ULAs and closely located.

To see more clearly the performance of the proposed method, we plot RMSEs against separation angle of two sources in Fig. 3, where $K_t = K_r = 7$, SNR $= 20$ dB, and $L = 50$, respectively. The three sources are assumed to be located at $(10°, 20°, 20° + \Delta)$ and $(50°, 60°, 60° + \Delta)$, respectively, for the first and second scenarios, where Δ varies from $4°$ to $20°$. It is observed that the proposed method has a much better performance than the TDS method for small angular separations because a larger array aperture length is used by the proposed method. As the separation angle Δ increases, their performance becomes very similar to each other.

2) The transmit array is a sparse ULA: In the second example, the receive array is a filled ULA while the transmit array is a sparse ULA. SNR $= 20$ dB and $L = 50$. With $K_t = 6$ and $K_r = 5$, Fig. 4 shows the effect of interelement spacing of the transmit array on the estimation performance for two different signal scenarios as considered in the previous example. From the two figures, we see that again

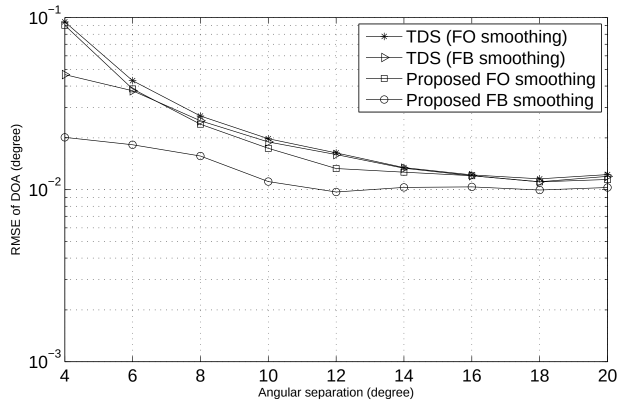


(a) Scenario 1 with $\theta = 10°$, $20°$ and $30°$.

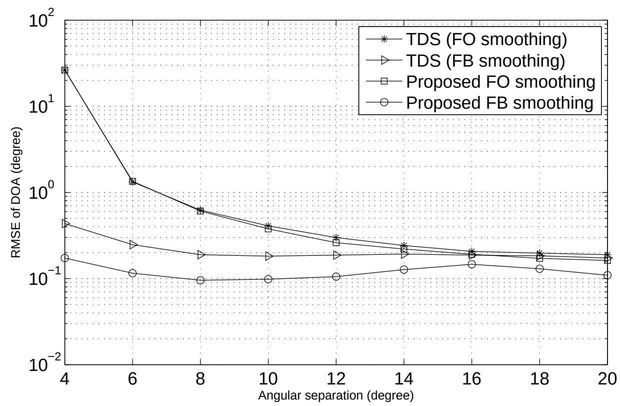


(b) Scenario 2 with $\theta = 50°$, $60°$ and $70°$.

Fig. 2. RMSEs of DOA estimation versus input SNR when both the transmit and receive arrays are filled ULAs and closely located.



(a) Scenario 1 with $\theta = 10°$, $20°$ and $20° + \Delta$.



(b) Scenario 2 with $\theta = 50°$, $60°$ and $60° + \Delta$.

Fig. 3. RMSEs of DOA estimation versus angular separation when both the transmit and receive arrays are filled ULAs and closely located.

the proposed method outperforms the TDS method, especially for the second signal scenario, as the interelement spacing of the transmit array increases.

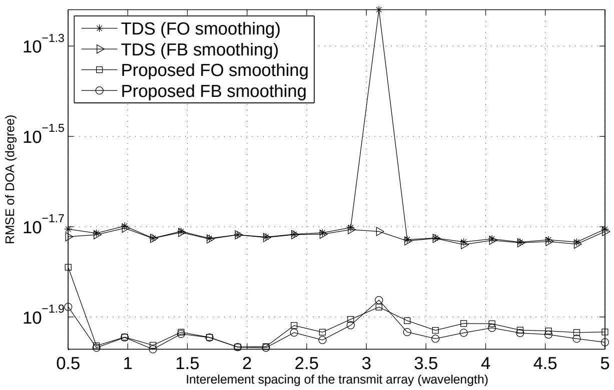
Now assume that 11 coherent targets are located at the angle region $[-80°, 70°]$, with equal angle interval of $15°$. Both K_t and K_r are set to 6 for the proposed method with $d_t = 3\lambda$. In this case, the TDS method fails because the number of coherent targets is larger than the maximum number of coherent targets that can be identified by the TDS method. On the other hand, the proposed method has $K_tK_r = 36$ distinct elements in the vector $\mathbf{a}_t^{(K_t)}(\theta_k) \otimes \mathbf{a}_r^{(K_r)}(\theta_k)$ and has $(M-K_t+1)(N-K_r+1) = 25$ distinct covariance matrices defined in (13) for spatial smoothing. Therefore, the proposed method can localize all the coherent targets. With SNR $= 20$ dB and $L = 50$, the spatial spectrum of the proposed FO smoothing method by applying the classical MUSIC algorithm is shown in Fig. 5 and we can see that the targets have been identified successfully.

B. The transmit and receive arrays are widely separated

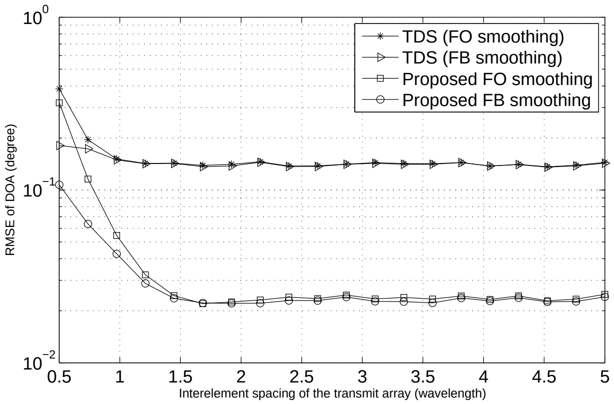
In this example, the transmit and receive arrays are assumed to be separated far away from each other. Here, three scenarios are considered with three coherent targets for each scenario: 1) $(\theta, \varphi) = (10°, 15°)$, $(20°, 25°)$, and $(30°, 35°)$; 2) $(\theta, \varphi) = (50°, 55°)$, $(60°, 65°)$, and $(70°, 75°)$; 3) $(\theta, \varphi) = (10°, 55°)$, $(20°, 65°)$, and $(30°, 75°)$. Both K_t and K_r are set to 5, and $L = 50$.

Fig. 6 shows the RMSEs of joint DOA and DOD estimation results versus input SNR. It can be clearly seen that the proposed FB smoothing has achieved a much better estimation than the proposed FO smoothing when both the DOAs and DODs of signals approach the end-fire direction. Additionally, we see that when only one of them (either DOAs or DODs) approach the end-fire direction, the proposed method still works well due to the benefit of joint transmission and reception diversity smoothing.

We also plot RMSEs against separation angle of two of the three sources in Fig. 7. The three sources are assumed to be located at 1) $(\theta, \varphi) = (10°, 15°)$, $(20°, 25°)$, and $(20° + \Delta, 25° + \Delta)$; 2) $(\theta, \varphi) = (50°, 55°)$, $(60°, 65°)$, and $(60° + \Delta, 65° + \Delta)$; 3) $(\theta, \varphi) = (10°, 55°)$, $(20°, 65°)$, and $(20° + \Delta, 65° + \Delta)$, respectively, for the three considered scenarios, where Δ varies from $4°$ to $20°$. Clearly the proposed FB smoothing has a much better performance than the proposed FO smoothing for small angular separations, especially for the second scenario where both the DOAs and DODs of sources approach the end-fire direction, because the number of smoothing operation used by the FB-based smoothing is larger than that of the FO-based smoothing. In addition, we see from results of the last two scenarios that as the separation angle Δ increases, the DOAs/DODs of sources approach the



(a) Scenario 1 with $\theta = 10°$, $20°$ and $30°$.



(b) Scenario 2 with $\theta = 50°$, $60°$ and $70°$.

Fig. 4. RMSEs of DOA estimation versus d_t when both the transmit and receive arrays are closely located and the transmit array is a sparse ULA.

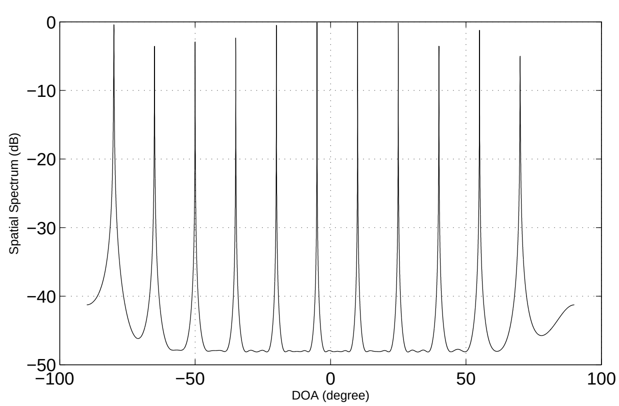
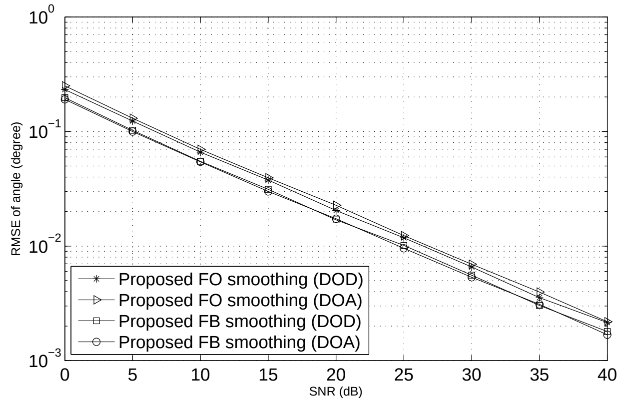
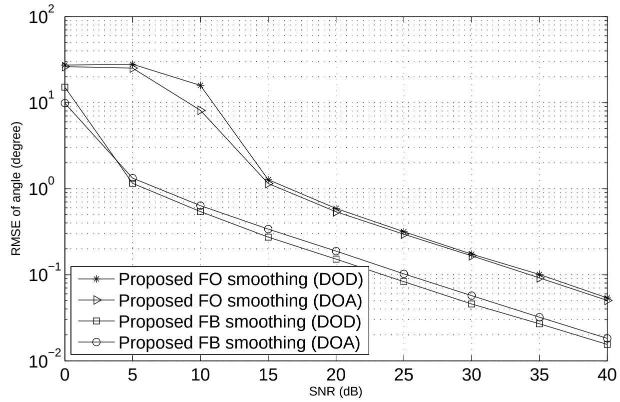


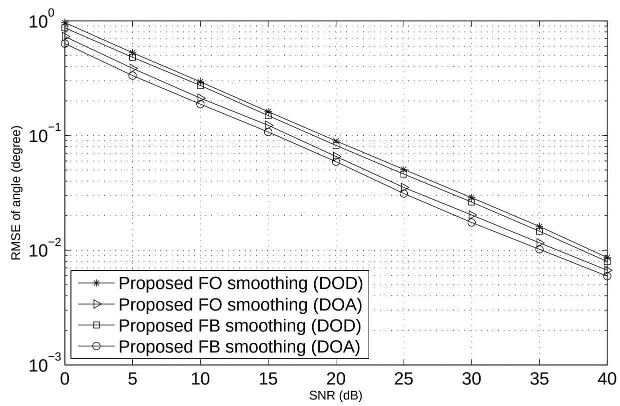
Fig. 5. Spatial spectrum of the proposed FO smoothing method using the MUSIC algorithm with $d_t = 3\lambda$ and 11 targets, when both the transmit and receive arrays are closely located and the transmit array is a sparse ULA.



(a) Scenario 1 with $(\theta, \varphi) = (10°, 15°)$, $(20°, 25°)$, and $(30°, 35°)$.



(b) Scenario 2 with $(\theta, \varphi) = (50°, 55°)$, $(60°, 65°)$, and $(70°, 75°)$.

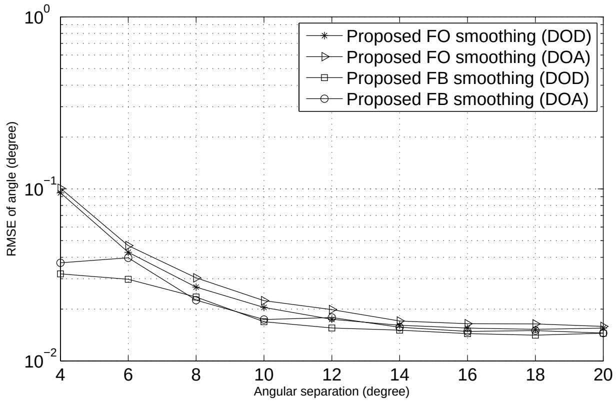


(c) Scenario 3 with $(\theta, \varphi) = (10°, 55°)$, $(20°, 65°)$, and $(30°, 75°)$.

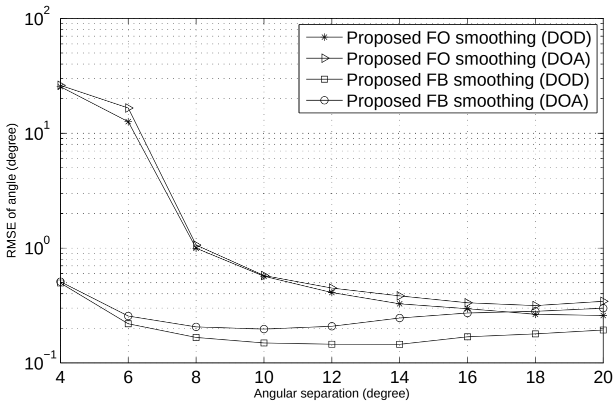
Fig. 6. RMSEs of DOA/DOD estimation versus input SNR when the transmit and receive arrays are widely separated ($K_t = 5$, $K_r = 5$).

end-fire direction, leading to degradation of the estimation performance.

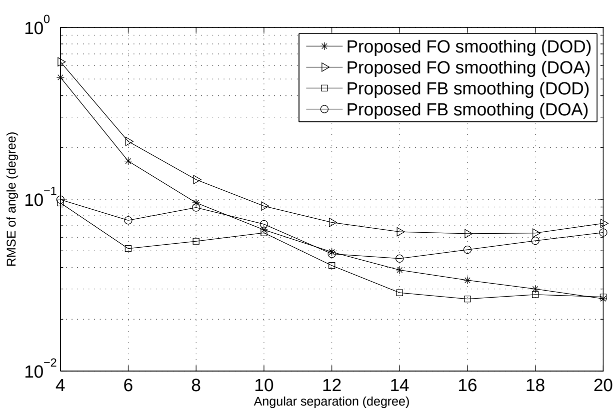
In the last example, K_t and K_r are set to 10 and 9, respectively. With $L = 50$, the result for the first scenario is shown in Fig. 8. In this case, the proposed FB smoothing still works well. However, the proposed FO smoothing fails because there are three coherent targets while only two covariance matrices are available for smoothing. Similar results can be observed for the remaining two scenarios.



(a) Scenario 1 with $(\theta, \varphi) = (10°, 15°)$, $(20°, 25°)$, and $(20°+\Delta, 25°+\Delta)$.



(b) Scenario 2 with $(\theta, \varphi) = (50°, 55°)$, $(60°, 65°)$, and $(60°+\Delta, 65°+\Delta)$.



(c) Scenario 3 with $(\theta, \varphi) = (10°, 55°)$, $(20°, 65°)$, and $(20°+\Delta, 65°+\Delta)$.

Fig. 7. RMSEs of DOA/DOD estimation versus angular separation when the transmit and receive arrays are widely separated.

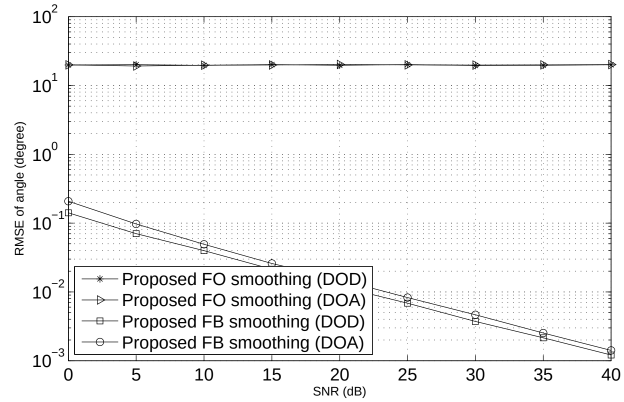


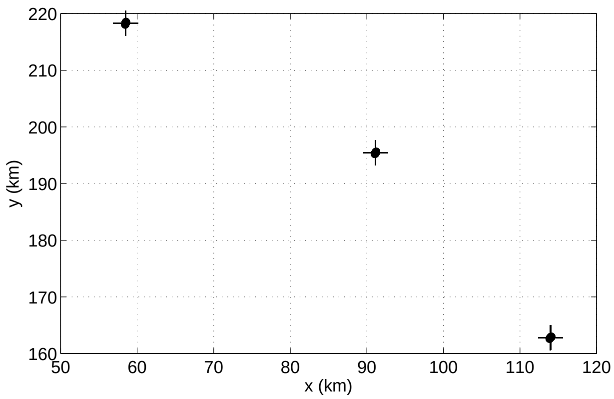
Fig. 8. RMSEs of DOA/DOD estimation versus input SNR when the transmit and receive arrays are widely separated ($K_t = 10$, $K_r = 9$) for the first signal scenario with $(\theta, \varphi) = (10°, 15°)$, $(20°, 25°)$, and $(30°, 35°)$.

C. Target localization

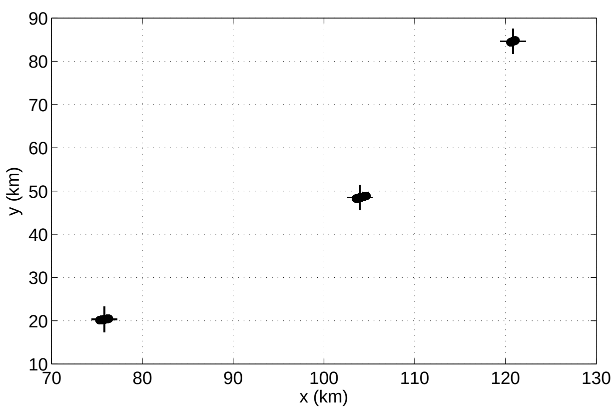
The combined DOA and DOD estimation through a bistatic MIMO array can be used for target localization by estimating their coordinates. In this part, the performance of our proposed method is evaluated for two-dimensional (2-D) target coordinates estimation. Both the transmit and receive arrays are placed along the x-axis and three coherent targets are located on the $x - y$ plane. The transmit array is located at $[0, 0]$ and the receive array at $[20\text{km}, 0]$. Two scenarios are considered, as shown in Fig. 9 with the targets represented by the crosses, which are equivalent to the first two scenarios considered in Sec. IV-B. Both M and N are set to 20, $K_t = K_r = 15$, and $L = 200$. Other parameters are the same as in Sec. IV-B. With SNR $= 20$ dB, Fig. 9 shows the 2-D coordinates estimation results (500 runs) calculated through the DOA and DOD estimates obtained by the proposed FO smoothing method, where the cluster of dots are the estimated locations. We see all the targets have been identified reasonably well with a relatively larger error for the second scenario, as the targets are located at positions closer to the end-fire direction of the arrays in that case.

V. Conclusions

A novel improved DOA estimation technique for coherent targets has been introduced for MIMO radar systems with two methods proposed: the FO-based spatial smoothing method and the FB-based one. Different from the existing method, the proposed ones employ both transmission and reception diversity smoothing to tackle the ill conditioning problem of the covariance matrix. When both the transmit and receive arrays are closely located in space, the FO-based method can achieve a better estimation accuracy than the TDS method since there are more covariance matrices available for spatial smoothing. Moreover, the number of coherent targets which can be identified by the proposed method is much larger than that of the TDS method when the transmit array is a sparse one. On the other hand, the proposed method is suitable for joint DOA and DOD estimation when the transmit and receive arrays are separated far away from each other. Furthermore,



(a) Scenario 1 with $(\theta, \varphi) = (10°, 15°)$, $(20°, 25°)$, and $(30°, 35°)$.



(b) Scenario 2 with $(\theta, \varphi) = (50°, 55°)$, $(60°, 65°)$, and $(70°, 75°)$.

Fig. 9. 2-D coordinates estimation results for the three coherent targets, with crosses denoting the true locations and the cluster of dots denoting the estimated ones.

the FB smoothing method corresponding to the proposed FO-based one has also been developed to improve the performance further. The effectiveness of the proposed method has been demonstrated by extensive simulation results.

References

[1] J. Li and P. Stoica, *MIMO Radar Signal Processing*. New York: Wiley, 2009.

[2] E. Fishler, A. M. Haimovich, R. S. Blum, L. J. Cimini, D. Chizhik, and R. A. Valenzuela, "Spatial diversity in radars-models and detection performance," *IEEE Transactions on Signal Processing*, vol. 54, no. 3, pp. 823–838, March 2006.

[3] A. Hassanien and S. A. Vorobyov, "Phased-MIMO radar: A tradeoff between phased-array and MIMO radars," *IEEE Transactions on Signal Processing*, vol. 58, no. 6, pp. 3137–3151, June 2010.

[4] A. M. Haimovich, R. S. Blum, and L. J. Cimini, "MIMO radar with widely separated antennas," *IEEE Signal Processing Magazine*, vol. 25, no. 1, pp. 116–129, 2008.

[5] J. Li and P. Stoica, "MIMO radar with colocated antennas," *IEEE Signal Processing Magazine*, vol. 24, no. 5, pp. 106–114, Sept. 2007.

[6] J. Li, P. Stoica, L. Xu, and W. Roberts, "On parameter identifiability of MIMO radar," *IEEE Signal Processing Letters*, vol. 14, no. 12, pp. 968–971, Dec. 2007.

[7] G. Frazer, Y. Abramovich, and B. Johnson, "Multiple-input multiple-output over-the-horizon radar: experimental results," *IET Radar, Sonar and Navigation*, vol. 3, no. 4, pp. 290–303, Aug. 2009.

[8] I. Bekkerman and J. Tabrikian, "Target detection and localization using MIMO radars and sonars," *IEEE Transactions on Signal Processing*, vol. 54, no. 10, pp. 3873–3883, Oct. 2006.

[9] X. Zhang, L. Y. Xu, L. Xu, and D. Xu, "Direction of departure (DOD) and direction of arrival (DOA) estimation in MIMO radar with reduced-dimension MUSIC," *IEEE Communications Letters*, vol. 14, no. 12, pp. 1161–1163, Dec. 2010.

[10] D. Chen, B. Chen, and G. Qin, "Angle estimation using ESPRIT in MIMO radar," *Electronics Letters*, vol. 44, no. 12, pp. 770–770, June 2008.

[11] J. He, M. N. S. Swamy, and M. O. Ahmad, "Joint DOD and DOA estimation for MIMO array with velocity receive sensors," *IEEE Signal Processing Letters*, vol. 18, no. 7, pp. 399–402, Dec. 2011.

[12] H. B. Li and B. Himed, "Transmit subaperturing for MIMO radars with co-located antennas," *IEEE Journal of Selected Topics in Signal Processing*, vol. 4, no. 1, pp. 55–65, Feb. 2010.

[13] A. Hassanien and S. A. Vorobyov, "Transmit energy focusing for DOA estimation in MIMO radar with colocated antennas," *IEEE Transactions on Signal Processing*, vol. 59, no. 6, pp. 2669–2682, Jun. 2011.

[14] M. L. Bencheikh, Y. Wang, and H. He, "Polynomial root finding technique for joint DOA DOD estimation in bistatic MIMO radar," *Signal Processing*, vol. 90, no. 9, pp. 2723–2730, Sept. 2010.

[15] X. Liu and G. Liao, "Direction finding and mutual coupling estimation for bistatic MIMO radar," *Signal Processing*, vol. 92, no. 2, pp. 517–522, Feb. 2012.

[16] R. Xie, Z. Liu, and J. Wu, "Direction finding with automatic pairing for bistatic MIMO radar," *Signal Processing*, vol. 92, no. 1, pp. 198–203, Jan. 2012.

[17] X. Zhang, Z. Xu, L. Xu, and D. Xu, "Trilinear decomposition-based transmit angle and receive angle estimation for multiple-input multiple-output radar," *IET Radar, Sonar and Navigation*, vol. 5, no. 6, pp. 626–631, Jul. 2011.

[18] M. L. Bencheikh and Y. Wang, "Joint DOD-DOA estimation using combined ESPRIT-MUSIC approach in MIMO radar," *Electronics Letters*, vol. 46, no. 15, pp. 1081–1082, July 2010.

[19] J. Chen, H. Gu, and W. Su, "A new method for joint DOD and DOA estimation in bistatic MIMO radar," *Signal Processing*, vol. 90, no. 2, pp. 714–718, Feb. 2010.

[20] M. Jin, G. Liao, and J. Li, "Joint DOD and DOA estimation for bistatic MIMO radar," *Signal Processing*, vol. 89, no. 2, pp. 244–251, Feb. 2009.

[21] D. Nion and N. Sidiropoulos, "A PARAFAC-based technique for detection and localization of multiple targets in a MIMO radar system," in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing*, Taipei, China, Apr. 2009, pp. 2077–2080.

[22] R. Xie, Z. Liu, and Z. Zhang, "DOA estimation for monostatic MIMO radar using polynomial rooting," *Signal Processing*, vol. 90, no. 12, pp. 3284–3288, Dec. 2010.

[23] X. Zhang, Y. Huang, C. Chen, J. Li, and D. Xu, "Reduced-complexity Capon for direction of arrival estimation in a monostatic multiple-input multiple-output radar," *IET Radar, Sonar and Navigation*, vol. 6, no. 8, pp. 796–801, Oct. 2012.

[24] Y. Cao, Z. Zhang, F. Dai, and R. Xie, "Direction of arrival estimation for monostatic multiple-input multiple-output radar with arbitrary array structures," *IET Radar, Sonar and Navigation*, vol. 6, no. 7, pp. 679–686, Aug. 2012.

[25] X. L. Liu and G. S. Liao, "Multi-target localisation in bistatic MIMO radar," *Electronics Letters*, vol. 46, no. 13, pp. 945–946, 2010.

[26] J. Zhang, L. R. Zhang, Z. W. Yang, and N. Liu, "Signal subspace reconstruction method of MIMO radar," *Electronics Letters*, vol. 46, no. 7, pp. 531–532, April 2010.

[27] X. Zhang and D. Xu, "Low-complexity ESPRIT-based DOA estimation for colocated MIMO radar using reduced-dimension transformation," *Electronics Letters*, vol. 47, no. 4, pp. 283–289, Feb. 2011.

[28] W. Zhang, W. Liu, J. Wang, and S. L. Wu, "Direction-of-arrival estimation in partially calibrated subarray-based MIMO radar," in *Proc. Constantinides International Workshop on Signal Processing*, London, UK, Jan. 2013.

[29] W. Zhang, W. Liu, S. L. Wu, and J. Wang, "Robust DOA estimation for a MIMO array using two calibrated transmit sensors," in *Proc. IET International Radar Conference*, Xi'an, China, Apr. 2013.

[30] R. O. Schmidt, "Multiple emitter location and signal parameter-estimation," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 276–280, Mar. 1986.

[31] R. Roy and T. Kailath, "ESPRIT-estimation of signal parameters via rotational invariance techniques," *IEEE Transactions on Acoustics Speech and Signal Processing*, vol. 37, no. 7, pp. 984–995, Jul. 1989.

[32] J. Tabrikian and I. Bekkerman, "Transmission diversity smoothing for multi-target localization," in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 4, Mar. 2005, pp. iv/1041–iv/1044.

[33] L. Z. Xu, J. Li, and P. Stoica, “Target detection and parameter estimation for MIMO radar systems,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. 44, no. 3, pp. 927–939, July 2008.
[34] T. J. Shan, M. Wax, and T. Kailath, “On spatial smoothing for direction-of-arrival estimation of coherent signals,” *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 33, no. 4, pp. 806–811, Aug. 1985.
[35] S. U. Pillai and B. H. Kwon, “Forward/backward spatial smoothing techniques for coherent signal identification,” *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 37, no. 1, pp. 8–15, Jan. 1989.
[36] A. Lemma, A.-J. van der Veen, and E. Deprettere, “Analysis of joint angle-frequency estimation using ESPRIT,” *IEEE Transactions on Signal Processing*, vol. 51, no. 5, pp. 1264–1283, May 2003.
[37] W. Liu, “Wideband beamforming for multi-path signals based on frequency invariant transformation,” *International Journal of Automation and Computing*, vol. 9, pp. 420–428, August 2012.
[38] L. Zhang and W. Liu, “Robust beamforming for coherent signals based on the spatial-smoothing technique,” *Signal Processing*, vol. 92, pp. 2747–2758, November 2012.
[39] W. Jiang, “Research on key technologies of MIMO radar signal processing,” Ph.D. dissertation, Beijing Institute of Technology, Nov. 2009.
[40] W. Zhang, W. Liu, J. Wang, and S. L. Wu, “DOA estimation of coherent targets in MIMO radar,” in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing*, Vancouver, Canada, pp. 3929–3933, May 2013.
[41] V. Reddy, A. Paulraj, and T. Kailath, “Performance analysis of the optimum beamformer in the presence of correlated sources and its behavior under spatial smoothing,” *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 35, no. 7, pp. 927–936, Jul. 1987.
[42] L. Zhang, W. Liu, and L. Yu, “Performance analysis for finite sample mvdr beamformer with forward backward processing,” *IEEE Transactions on Signal Processing*, vol. 59, no. 5, pp. 2427–2431, May 2011.
[43] L. Zhang, W. Liu, and R. J. Langley, “A class of constrained adaptive beamforming algorithms based on uniform linear arrays,” *IEEE Transactions on Signal Processing*, vol. 58, no. 7, pp. 3916–3922, Jul. 2010.
[44] P. Hyberg, M. Jansson, and B. Ottersten, “Array interpolation and bias reduction,” *IEEE Transactions on Signal Processing*, vol. 52, no. 10, pp. 2711–2720, Oct. 2004.
[45] B. Friedlander and A. Weiss, “Direction finding using spatial smoothing with interpolated arrays,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. 28, no. 2, pp. 574–587, Apr. 1992.
[46] A. Weiss, B. Friedlander, and P. Stoica, “Direction-of-arrival estimation using mode with interpolated arrays,” *IEEE Transactions on Signal Processing*, vol. 43, no. 1, pp. 296–300, Jan. 1995.
[47] M. Doron and E. Doron, “Wavefield modeling and array processing, Part I-spatial sampling,” *IEEE Transactions on Signal Processing*, vol. 42, no. 10, pp. 2549–2559, 1994.
[48] M. Doron and E. Doron, “Wavefield modeling and array processing, Part II-algorithms,” *IEEE Transactions on Signal Processing*, vol. 42, no. 10, pp. 2560–2570, Oct. 1994.
[49] M. Doron and E. Doron, “Wavefield modeling and array processing, Part III-resolution capacity,” *IEEE Transactions on Signal Processing*, vol. 42, no. 10, pp. 2571–2580, Oct. 1994.
[50] F. Belloni, A. Richter, and V. Koivunen, “DOA estimation via manifold separation for arbitrary array structures,” *IEEE Transactions on Signal Processing*, vol. 55, no. 10, pp. 4800–4810, 2007.
[51] M. Rubsamen and A. Gershman, “Direction-of-arrival estimation for nonuniform sensor arrays: From manifold separation to fourier domain MUSIC methods,” *IEEE Transactions on Signal Processing*, vol. 57, no. 2, pp. 588–599, Feb. 2009.

Wei Zhang was born in Changde, China in August 1986. He received his B.Eng. degree in Electrical Engineering in 2008 from the Changsha University of Technology, Changsha, China, and M.Sc. degree in Signal and Information Processing in 2009 from the Beijing Institute of Technology, Beijing, China. He is currently working towards his Ph.D. degree in the School of Information and Electronics, Beijing Institute of Technology. From September 2011 to March 2013, he was a visiting researcher with the Communications Research Group, Department of Electronic and Electrical Engineering, University of Sheffield, UK. His research interests include array and radar signal processing.

Wei Liu (S’01-M’04-SM’10) received his B.Sc. in Space Physics (minor in Electronics) in 1996, L.L.B. in Intellectual Property Law in 1997, both from Peking University, China, M.Phil. from the Department of Electrical and Electronic Engineering, University of Hong Kong, in 2001, and Ph.D. in 2003 from the Communications Research Group, School of Electronics and Computer Science, University of Southampton, UK. He then worked as a postdoc in the same group and later in the Communications and Signal Processing Group, Department of Electrical and Electronic Engineering, Imperial College London. Since September 2005, he has been with the Communications Research Group, Department of Electronic and Electrical Engineering, University of Sheffield, UK, as a lecturer. His research interests are mainly in sensor array signal processing, blind signal processing, multirate signal processing and their various applications in wireless communications, sonar, radar, satellite navigation, speech enhancement and biomedical engineering. He has now authored and co-authored 130 journal and conference publications, and a research monograph about wideband beamforming (“Wideband Beamforming: Concepts and Techniques”, John Wiley & Sons, March 2010).

Ju Wang was born in 1973. She received her Ph.D. degree from Beijing Institute of Technology. She is now an associate professor in Beijing Institute of Technology. Her current research interests include satellite navigation and signal processing.

Siliang Wu received his Ph.D. degree from Harbin Institute of Technology in 1995 and then worked as a post-doctor in Radar Research Institute in Beijing Institute of Technology from 1996 to 1998. He is now a professor and supervisor for doctorate students in Beijing Institute of Technology and a senior member of Chinese Institute of Electronics. His research interests include radar system and theory, satellite navigation, and application of modern signal processing.