Monograph:

Fleming, P.J. and Pashkevich, M.A. (2005) Optimal Advertising Campaign Generation For Multiple Brands Using Multi-Objective Genetic Algorithm. Research Report. ACSE Research Report 876 . Department of Automatic Control and Systems Engineering

OPTIMAL ADVERTISING CAMPAIGN GENERATION FOR MULTIPLE BRANDS USING MULTI-OBJECTIVE GENETIC ALGORITHM

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Abstract. The paper proposes new modified multi-objective genetic algorithm for the problem of optimal TV advertising campaign generation for multiple products. This *NP*-hard combinatorial optimization problem with numerous constraints is one of the key issues for an advertising agency when producing the optimal television mediaplan. The classical approach to the solution of this problem is the greedy heuristic, which relies on the strength of the preceding commercial breaks when selecting the next break to add to the campaign. While the greedy heuristic is capable of generating only a group of solutions that are closely related in the objective space, the proposed modified multi-objective genetic algorithm produces a Pareto-optimal set of chromosomes that (i) outperform the greedy heuristic; and (ii) let the mediaplanner choose from a variety of uniformly distributed trade-off solutions. To achieve these results, the special problem-specific solution encoding, genetic operators, and original local optimization routine were developed for the algorithm. These techniques allow the algorithm manipulating with only feasible individuals, thus significantly improving its performance that is complicated by the problem constraints. The efficiency of the developed optimization method is verified using the real data sets from the Canadian advertising industry.

Key words: multi-objective; combinatorial optimization; genetic algorithms; mediaplanning.

1. Introduction

Every year, an enormous amount of money is spent on *advertising*. In 2003, the total cost of advertising the United States was about 200 billion dollars, and about 50% of this money was spent on TV commercials. Since the advertising involves "big money", media planners are responsible for making the media plans as effective as possible, and increasing the efficiency of the mediaplanning results in huge profits for advertising agencies and television networks. For example, Bollapragada et al. (2002) describe the case where their effective optimization of the sales processes for the US National Broadcasting Network (NBC) resulted in increase of revenues by over 15 million dollars annually.

TV mediaplanning involves two major participants that are television networks (stations) and advertising agencies, and can be briefly outlined as follows. After announcing program schedules, the TV networks finalize their rating forecasts, estimate market demand and set the rate cards for the available advertising breaks. The rate cards contain one-second price and expected rating of a spot in a particular TV show. Mediaplanning agencies buy advertising time for each of their clients from the networks, and then produce advertising campaigns (media plans) for the products of clients.

Both networks and agencies face a number of time-consuming *mathematical problems* during this planning process. The *networks*, on one hand, have to develop optimal program schedule, accurately predict ratings of the programs and expected demand for the commercial breaks in the shows. Besides, they are faced with a problem of optimal advertising breaks distribution between the agencies subject to the required by the agencies constraints and limited advertising inventory restrictions. A simplified flow chart of the TV network planning process is given in Figure 1. In real life, advertising agencies can buy commercial breaks by parts and negotiate with the network on the percentage of the spots that will be aired in the fist and in the last positions of the break. This creates additional cumbersome problem of re-scheduling commercials during the last week before they are broadcasted (Bollapragada and Garbiras, 2004).

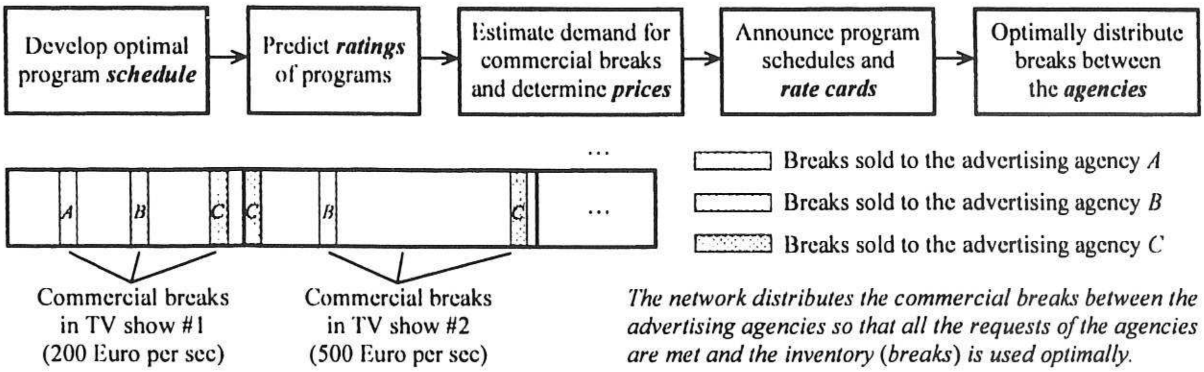

Figure 1. Simplified flow chart of the mediaplanning process for a TV network

The *advertising agencies*, on the other hand, develop their own buying strategies and intend to bring their customers the most efficient media plans possible. They deal with a number of clients, with each client having a set of products to be advertised subject to brand-specific restrictions. A simplified flow chart of the advertising agency planning process is presented in Figure 2. The first important problem for the agency is efficient purchasing of advertising time. Besides, since major advertisers (such as Proctor and Gamble, for example) buy hundreds of commercial breaks and decide on the actual distribution of the breaks between the advertising products later, the agency meets a problem of optimal assigning breaks in pool purchased for a client to the client's brands, subject to budget, minimum impact and other constraints. This problem includes two sub-problems, both of them being quite non-trivial. The first difficulty is to develop a model that would allow accurately forecast impact efficiency for the future advertising campaigns. The second issue (that is a subject of this paper) is generating optimal media plans (advertising schedules) for the client brands that maximize the impact on the TV viewers while satisfying all the required restrictions.

Forecasting the efficiency of future advertising campaigns is a statistical problem that usually involves longitudinal (panel) data analysis techniques. A number of papers were published on this topic, a good review of the approaches proposed can be found in (Sissors and Lincoln 2002). Besides, Weber (2002) published some encouraging results on application of neural networks to forecasting of viewing patterns based on German telemetric viewing data for specific target audiences. Recently, Pashkevich and Kharin (2004) proposed a robust version of the beta-binomial model that was successfully applied for increasing forecasting accuracy in case when the past exposure data was available in binary form (a real data set from the German advertising market).

Optimization of advertising campaign efficiency is a *NP*-hard combinatorial multi-objective optimization problem that involves a number of complicated constraints. A classical approach to the solution of this problem is the greedy heuristic that relies on the strength of the preceding breaks when selecting the next break to add to the campaign. Literature review presented in the following section indicates that very little research was done on this topic. Besides, the proposed optimization algorithms were developed either for generating campaigns for one advertising product or were based on reducing the multi-objective optimization problem to a single-objective by the weighted sum approach (a common way to select weights is based on budgets of products being advertised) (Rust, 1986). This usually leads to discriminating the brands with smaller budgets, that is undesirable from the media-planners point of view. Hence, there is a need for a true *multi-objective optimization algorithm* that would provide the planner with a set of Pareto-optimal solutions and let him decide which one should be used as a final solution, based on his expertise and experience.

In this paper, we propose to use the *multi-objective genetic approach* (Fonseca and Fleming, 1993) to generate a set of Pareto-optimal solutions for the problem of the advertising campaign efficiency optimization.

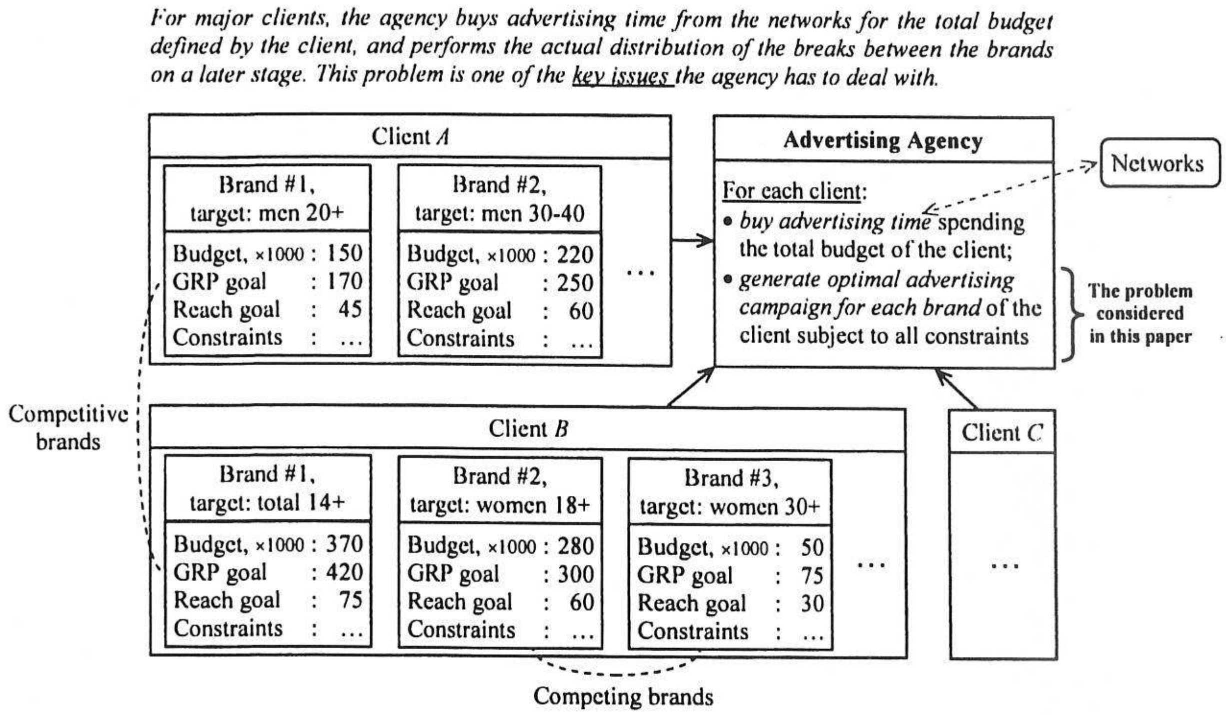

Figure 2. Simplified flow chart of the mediaplanning process for an advertising agency

2. Related Works

A majority of literature on using the optimization techniques in mediaplanning deals with scheduling programs for *television networks* in order to optimize audience ratings. The common method is the "lead-in" strategy that relies on the strength of the preceding programs to boost the ratings of a newly introduced one. This approach was successfully applied in a number of papers, for details see Goodhardt et al. (1975), Headen et al. (1979), Henry and Rinnie (1984), and Webster (1985). Several publications, such as Gensch and Shaman (1980), Rust and Alpert (1984) deal with individual's television viewing choice. A comprehensive review of the viewing choice models can be found in (Rust, 1986; Danaher and Mawhinney, 2001). Rust and Echambadi (1989) developed a heuristic algorithm for scheduling a television network's program to maximize a network's share of audiences. Reddu, Aronson, and Stam (1998) developed an optimal prime time TV program scheduler based on the mixed-integer near-network flow model that was successfully tested using the 1990 data from a US cable TV network. Several authors dealt with advertising scheduling strategies, a review of this approaches is presented in Lilien, Kotler, and Moorthy (1992). Recently, Bollapragada et al. (2002) developed an optimization system for the sales processes of the US National Broadcasting Network (NBC). They used integer and mixed-integer programming techniques to automatically develop schedules of commercials that meet all the requirements (Bollapragada and Garbiras, 2004), and to schedule the commercials evenly throughout the advertising campaign (Bollapragada et al., 2004).

Although the mediaplanning issues for the *advertising agency* and the TV network have much in common, they essentially differ in mathematical formulation that opposes using common optimization tools, including the mentioned above. In contrast to the network mediaplanning, the problem of optimal campaign generation for the advertising agency received very little attention in scientific literature, although a number of papers were published on audience perception forecasting (Cannon et al., 2002; Weber, 2002; Kim and Leckenby, 2001). *Classical approach* to the optimal advertising campaign generation utilizes greedy heuristic that selects the most promising admissible break (for a particular brand campaign, step-by-step) (Sissors and Lincoln, 2002). The breaks are assigned to brands one-by-one and can be ordered in different manner (randomly, depending on a distance to goals, etc.). Within this approach, the multi-objective problem is reduced to a single objective one by the weighted sum technique, and the weights are calculated as normalized brand budgets (Goodrich and Sissors, 2001). Being very simple to implement, this approach is not robust to the local dynamic restrictions, and usually uses some kind of rollbacks to overcome the violated constraints.

Recently, Pashkevich and Kharin (2001) proposed a multi-stage technique based on the hybrid genetic algorithm for generating optimal advertising campaigns for multiple brands. Although this approach proved to be successful in real-world applications, it still relies on the weighted sum technique to improve the solution after all the goals are attained. To our knowledge, there were no results published on applying the multi-objective methodology to the problem of optimal advertising campaign generation for multiple brands.

It should be noted that other advertising problems, that involve newspapers advertising and web-page commercials, were also considered by the optimization research community. Merelo et al. (1997) used the GA for optimal advertisement placement in different media. Naik et al. (1998) utilized the GA for developing the optimal pulsing mediaplans. Van Buer et al. (1999) considered solving the medium newspaper production/distribution problem by means of the GA. Collins and Harris (1999) proposed to use the evolutionary approach to optimal generation of print and multi-media advertising campaigns. Ohkura et al. (2001) employed an extended genetic algorithm for the Japanese newspapers advertisement optimization. Carter and Ragsdale (2002) addressed the problem of scheduling the pre-printed newspaper advertising inserts using the GA. Dawande et al. (2003) proposed special heuristics for optimal advertisement scheduling on a web page. A lot of this research was inspired by the paper (Hurley et al., 1995), which advocated using the genetic algorithms paradigm for solving time-consuming marketing problems.

As follows from the presented above literature review, the single-objective genetic algorithms were efficiently used to solve various mediaplanning optimization problems. Since the theory of multi-objective genetic algorithms was efficiently evolving over the last decade, and had shown to be very valuable for practical applications, the authors propose to rely on the multi-objective evolutionary paradigm to solve the problem considered in this paper.

3. Problem Description

Before presenting a mathematical problem statement, let us give its informal problem description focusing on some practical details. When an advertising agency buys commercial breaks for major advertisers like Proctor & Gamble, Coca-Cola, etc., it sums the budgets of all the brands that the client wants to advertise, and purchases common advertising time from the TV networks. Then, the corresponding set of commercial slots bought, usually referred to as a *pool*, must be *distributed between the brands*, taken into a count a number of specific constraints and goals (as shown in Figure 3.)

The *major constraints* associated with the brand are the maximum budget allowed to spend, the minimum gross rating points (GRP), and the effective reach to be gained from broadcasting. There are also several *minor constraints*, which can be divided into two types. The first of them, the *search space constraints*, can be taken into account prior to the optimization by narrowing the brand search space. Some common examples are the genre of a TV show, its day part and weekday, and the commercial break length (since a brand commercial length must not exceed the break length). Besides, it is prohibited to expose competitive brand commercials in the same TV advertising break (Coca-Cola and Pepsi-Cola, for example). The second type, the *solution space constraints*, highly depends on the mutual positions of the brand commercials in the entire adverting campaign plan. They can be also subdivided into local and global ones, depending on the relationship between the brands. Examples of the first subtype include minimum time interval between the successive brand exposures, and maximum number of the brand commercials in the same TV show. The second

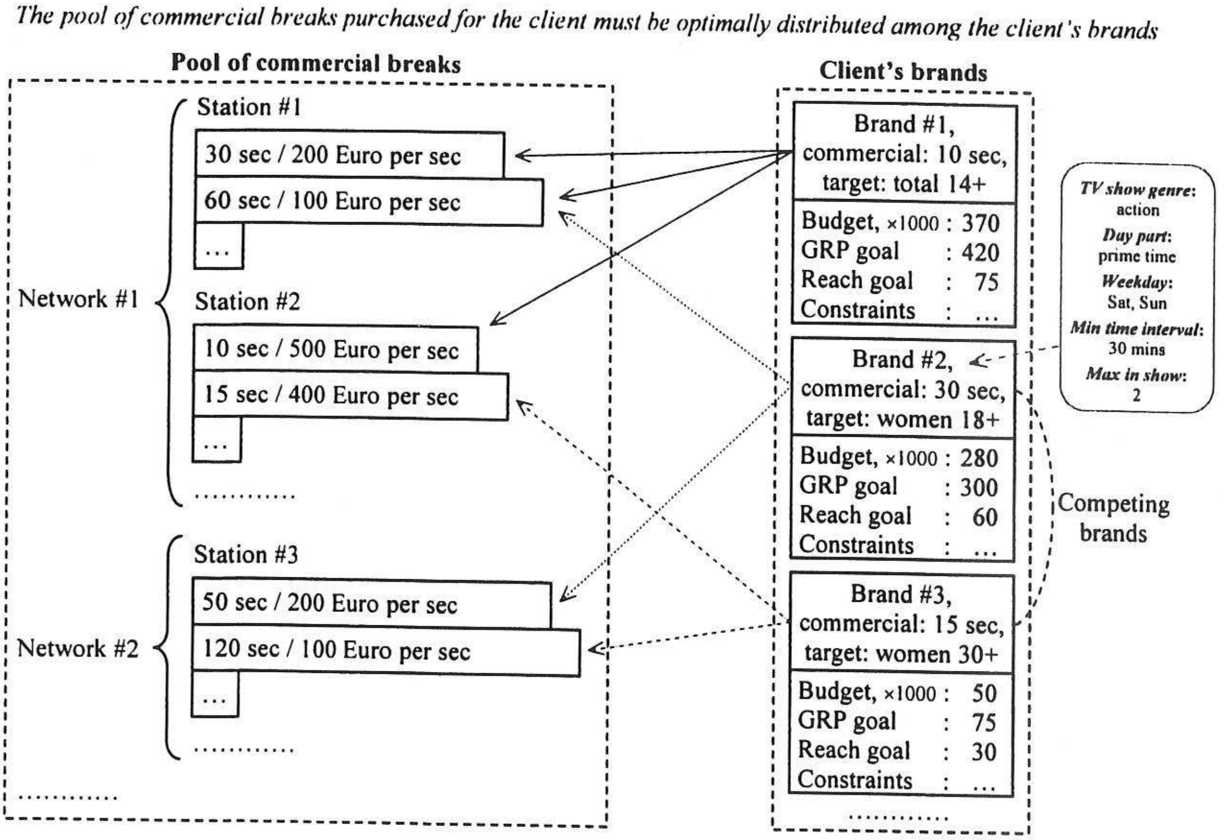

Figure 3. Generating optimal advertising campaigns for multiple brands based on the purchased pool of the commercial breaks

subtype arises when separate advertising campaigns (for single brands) are combined together. Relevant examples comprise maximum sum of the brand commercial lengths within the same TV break, and taboo on exposing competing brand commercials in the same TV advertising break (for instance, washing powders Ariel and Dash of Proctor and Gamble).

For each brand, the *efficiency* of an advertising campaign is measured by two performance indices: (i) gross rating points (*GRP*), and (ii) effective reach (*Reach*). These indices are computed for a particular segment of the viewing audience (*target group*) defined by the agency client. By definition (Sissors and Lincoln, 2002), the GRP is the cumulative sum of audience percentages that watched the brand commercial, which was exposed several times. It is obvious, that this index *may overestimate the commercial impact*, since it duplicates (triplicates, etc.) the percentage of regular viewers, who were covered by all the exposures. In contrast, the Reach index measures the unduplicated audience and is defined as the percentage of the viewers that watched the brand commercial at least once (twice, thrice, etc.). It should be noted that the Reach index saturates up to 100% as the campaign size increases, while the GRP index is additive and may exceed 100%. An example of the client requirements for a brand can be given as follows: (i) target group "Men 35+", i.e. males of age 35 years and older; (ii) contact class "2+", i.e. only target group members who watched the commercial at least twice are included; (iii) minimum Reach 65%; (iv) minimum GRP 240%.

The *primary goals* of the advertising campaigns optimization are achieving the best GRP and Reach for each separate brand, while satisfying the lower bounds on both of them, as defined by the client. Usually, media-planners optimize only one of these criteria (Reach or GRP), and use the second one as a lower bound constraint. Nevertheless, these leads to multi-objective setting with highly competing objectives for different brands and numerous constraints. It should be noted that when assessing the efficiency of the mediaplan, the media-planners often rely on additional criteria that can not be formalized and, therefore, embedded into the optimization algorithm in full scale. This may be caused by rapid changes on the advertising market or some short-term strategical issues that make the planers partially rely on their intuition. For this reason, it seems prudent to propose to the decision maker a set of Pareto-optimal solutions satisfying the formal goals and constraints and let him make final decision relying on his expertise, experience and intuition. This motivates application of the multi-objective genetic algorithms, which are capable to undertake such a problem.

4. Problem Formulation

Basic Notation. The following notation is used to formally introduce the problem.

B	=	the set of commercial breaks, $B = \{b_1, b_2, \ldots, b_m\}$;
m	=	the number of commercial breaks;
i	=	an index of a commercial break;
T_i	=	the length of the commercial break b_i, in seconds;
P	=	the set of products being advertised, $P = \{p_1, p_2, \ldots, p_n\}$;
n	=	the number of products being advertised;
j	=	an index of an advertising product;
q_j	=	the number of commercials for the product p_j;
t_j	=	the set of commercials for the product p_j, $t_j = \{t_{j1}, t_{j2}, \ldots, t_{jq_j}\}$ sorted in ascending order;
t_{jv}	=	the length of the commercial v for the product p_j;
M_j	=	the advertising budget of the product p_j;
h_{jv}	=	the budget share of the commercial v for the product p_j, $\sum_{c=1}^{q_j} h_{jv} = 1$;
Δ_j	=	the minimum time length between two consecutive commercials for the product p_j;
k_j	=	the maximum number of commercials in one show for the product p_j;
G_j	=	the GRP goal for the product p_j;
R_j	=	the Reach goal for the product p_j;
c_{ij}	=	the advertising price of one second in the break b_i for the product p_j;
X	=	the $m \times n$ matrix of the decision variables, $X = \{x_{ij}\}$;
x_{ij}	=	the commercial length if the product p_j is advertised in the break b_i, and 0 otherwise;
X_j	=	the advertising campaign for the product p_j, $X_j = \{x_{1j}, x_{2j}, \ldots, x_{nj}\}$;
D_j	=	the admissible breaks for the product p_j after applying the search space constraints;
d_{ij}	=	1 if the product p_j can be advertised in the break b_i, 0 otherwise, $D_j = (d_{1j}, d_{2j}, \ldots, d_{mj})$;
$R(X_j)$	=	the Reach of the campaign X_j for the product p_j;
$G(X_j)$	=	the GRP of the campaign X_j for the product p_j;
S	=	the set of commercial breaks grouped by TV shows, $S = \{s_1, s_2, \ldots, s_k\}$, $\coprod_{l=1}^{k} s_l = B$;
k	=	the number of distinct TV shows;
l	=	an index of a TV show;
F	=	the $n \times n$ binary symmetric matrix of competing brands constraints, $F = \{f_{j_1 j_2}\}$;
$f_{j_1 j_2}$	=	1 if the products j_1 and j_2 compete, and 0 otherwise;
$t(b_i)$	=	the absolute airing time of the break b_i.

Problem Statement. The mathematical problem formulation may be presented as follows.

Optimize reach for each product

$$R(X_j) \to \max_{I(X_j)\in D_j}, \quad j = 1,2,\ldots,n; \tag{1}$$

subject to the

- budget constraints:

$$\sum_{i=1}^{m} I_{t_{jv}}(x_{ij})\cdot x_{ij}\cdot c_{ij} \le h_{jv}\cdot M_j, \quad j = 1,2,\ldots,n, \; v = 1,2\ldots,q_j; \tag{2}$$

- goal attainment constraints:

$$R(X_j) \ge R_j, \quad G(X_j) \ge G_j, \quad j = 1,2,\ldots,n; \tag{3}$$

- commercial break length constraints:

$$\sum_{j=1}^{n} x_{ij} \le T_i, \quad i = 1,2,\ldots,m; \tag{4}$$

- minimum time length between two consecutive commercials constraints:

$$|t(b_{i_1}) - t(b_{i_2})| \cdot I(x_{i_1 j}x_{i_2 j}) \le \Delta_j, \quad i_1, i_2 = 1,2,\ldots,m, \; i_1 \ne i_2; \tag{5}$$

- maximum number of commercials in one TV show constraints:

$$\sum_{b_i \in s_l} I(x_{ij}) \le k_j, \quad l = 1,2,\ldots,k, \; j = 1,2,\ldots,n; \tag{6}$$

- competing products constraints:

$$\sum_{i=1}^{m}\sum_{j_1=1}^{n}\sum_{j_2}^{n} x_{ij_1}\cdot x_{ij_2}\cdot f_{j_1 j_2} = 0; \tag{7}$$

where $I(x) = \{1 \text{ if } x > 0, \text{ and } 0 \text{ otherwise}\}$, $I_y(x) = \{1 \text{ if } x = y, \text{ and } 0 \text{ otherwise}\}$, and the functions are applied componentwise in the case if x and y are vectors.

The objective function (1) simultaneously maximizes Reach index for all advertising products $\{p_1, p_2, \ldots, p_n\}$ that compete over the pool of commercial breaks $\{b_1, b_2, \ldots, b_m\}$. Each product p_j has q_j different commercials $\{t_{j1}, t_{j2}, \ldots, t_{jq_j}\}$ available for it, and each commercial t_{jv} has a budget share assigned (meaning that the sub-budget of this commercial is $h_{jv}\cdot M_j$). Hence, the budget constraints (2) for every product are formulated as a set of inequalities (one for each of the product commercials). Goal attainment constraints (3) ensure that in the generated set of Pareto-optimal solutions, every product gains at least minimum value of Reach and GRP indices. Commercial break length constraints (4) guarantee that the total length of commercials placed into a break does not exceeds its length. Equation (5) describes the constraints for the minimum time length between two consecutive commercials for the same brand, while formula (6) limits the number of commercials of a brand placed to the same break. Competing brands constraints (7) make sure that only one of the competing products can be placed to the same commercial break.

In addition to the hard constraints described above, there are additional soft problem constraints that are not mandatory but are desired to be accomplished. The budget for each commercial length of every advertising product should be spent as completely as possible. This requirement arises from the practical aspects of the problem, since the profit of the media planning agency depends on the advertising budgets of their clients.

The formulated mathematical problem is an *NP*-hard multi-objective optimization problem of high dimension (usual values for n and m are 1500-125000 breaks and 30-75 brands for one month optimization depending on a country). Hence, applying the branch-and-bound or other exact technique does not seem prudent, and the problem is solved by means of metaheuristics. In this paper, a specially designed version of the multi-objective genetic algorithm (MOGA) of Fonseca and Fleming (1993) is applied to solve the problem (1).

5. Multi-Objective Evolutionary Approach

The evolutionary computation employs biology concepts of natural selection and population genetics to solve optimization problems that are hard or impossible to solve using traditional optimization techniques (Michalewicz and Fogel, 2000). The major difference of the evolutionary algorithms (EA) and other heuristical methods is that the EA rely on a population of solutions rather then on a single individual in the decision variable space. This research direction was started by the pioneer work of Rechenberg (1973) who proposed the Evolutionary Strategies to solve complex optimization problems, and was followed by Fogel (1991) with the Evolutionary Programming, and Holland (1975) with the Genetic Algorithms (GA). The theoretical results for the GA obtained by Goldberg (1989), such as Schema Theorem, made them very popular search techniques, that resulted in numerous applications and enhancements of this optimization paradigm (Man et al, 1999).

The genetic algorithms maintain a population of individuals that compete with each other for survival. After evaluation, individuals are given a probability of recombination that depends on their fitness. Offsprings are produced via crossover, where they inherit some features from the ancestors, and via mutation, where some innovative features can appear. At the next iteration, the offsprings compete with each other (and possibly also with their parents), and etc. Population improvement happens due to the repeated selection of the best parents, which are likely to produce better offsprings, and elimination of solutions that have low performance.

The multi-objective genetic algorithms use the GA ideas to solve the multi-criteria optimization problems. Historically, the population-based non-Pareto approaches were used to deal with this problem to start with. The first version of this technique was proposed by Schaffer (1985), whose vector evaluated genetic algorithm (VEGA) had modified selection procedure so that, at each generation, a number of subpopulations is generated according to each objective. Fourman (1985) proposed to use the selection scheme based on the lexicographical ordering of the objectives according to the user priorities. Another version of the Fourman's algorithm consisted of randomly selecting the objective to be used for comparison of individuals in the tournament selection. Kursawe (1991) proposed a multi-objective version of the evolutionary strategies, with each objective used to delete an appropriate fraction of the population during selection. Hajela and Lin (1992) combined the GA with the weighted sum approach by explicitly including the weights into the chromosome, and using the fitness sharing to promote their diversity.

The Pareto-based multi-objective genetic algorithms use the concept of Pareto optimality to rank the individuals in the population. The first version of the Pareto ranking was proposed by Goldberg (1989), and was based on the consecutive computing the dominating sub-populations, thus assigning the individuals the ranks according to the sub-population index. Fonseca and Fleming (1993) proposed an extension of this approach, where a solution's rank corresponds to the number of individuals in the current population by which it is dominated. Therefore, the non-dominating individuals are all assigned the same rank, while the dominated ones are penalized according to the density of the population around them. The calculated ranks are then sorted and mapped into fitness, and the Stochastic Universal Sampling (SUS) is used to perform the selection (Baker, 1987). Besides, Horn and Nafpliotis (1993) proposed a modification of the tournament selection based on the Pareto dominance. Cieniawski (1993) and Ritzel et al. (1994) used the tournament selection that relied on the Goldverg's Pareto-optimal ranking scheme.

The multi-objective genetic algorithm (MOGA) of Fonseca and Fleming (1993) was the first Pareto-based evolutionary technique proposed for the multi-criteria optimization problems. In addition to the special ranking procedure described above, it relies on the niche-formation methods to distribute the solutions uniformly over the Pareto-optimal region, with the fitness sharing performed in the objective function space, and special method for the niche size calculation. Besides, the decision maker (DM) can incorporate the goal attainment information into the ranking procedure. The algorithm extensions proposed in (Fonseca and Fleming, 1998) also allow including to the ranking the DM preferences for the objectives. The MOGA was successfully used to solve a number of applied problems, for example design of a multivariable control system for a gas turbine engine (Chipperfield and Fleming, 1995), multi-objective optimization of ULTIC controller (Tan and Li, 1997), design of a coal burning gasification plant (Griffin et al, 2000), and other applications.

Other well-known versions of the evolutionary multi-objective algorithms include the niched Pareto genetic algorithm (NPGA) of Horn and Nafpliotis (1993), and the non-dominated sorting genetic algorithm (NSGA) of Srinivas and Deb (1994). In the late 1990-s, there was developed a number of new methods for the considered problem, which focused on improving the selection-for-survival aspect, including techniques for population density estimation. The developed techniques include the strength Pareto evolutionary algorithm (SPEA) of Zitzler and Thiele (1998), the Pareto envelope-based selection algorithm (PESA) of Corne, Knowles and Oates's (2000), and the elitist non-dominated sorting genetic algorithm (NSGA-II) of Deb, Pratap, Ararwal and Meyarivan (2002). It should be noted that a more detailed review of the evolutionary approaches to multi-objective optimization problems can be found in (Fonseca and Fleming, 1995; Coello, 2000; Purshouse, 2004).

This paper proposes an application-specific modification of the MOGA for the described above mediaplanning optimization problem. The developed algorithm uses the original MOGA framework, but employs the specially developed encoding procedure and genetic operators, as well as the original local optimization routine. These modifications allow manipulating only with feasible solutions on each algorithm iteration. The efficiency of the developed optimization technique is verified using the real data sets from the Canadian advertising industry.

6. Developed Modified Multi-Objective Genetic Algorithm

The major challenge in developing the efficient MOGA for the problem of optimal advertising campaign generation for multiple brands is effective constraints handling. A modification of the multi-objective genetic algorithm of Fonseca and Fleming (1993) is proposed, which takes into account the problem specificity by using specially developed solution encoding scheme and related genetic operators. Another innovation deals with the local optimization routine, which employs an original approximation procedure for the problem objective functions.

To handle the constraints (2)-(7), they are divided into four groups and are processed in the following way:

(i) The *search space constraints* $\{D_j\}$, the commercial break length constraints (4) and the competing products constraints (7), are taken into account by the solution encoding.

(ii) The *solution space constraints*, i.e. the budget constraints (2), the minimum time length between two consecutive commercials constraints (5) and the maximum number of commercials in one TV show constraints, are taken in consideration via the genetic operators.

(iii) The *goal attainment* constraints are handled via special ranking procedure that is used to calculate the solution fitness.

(iv) The *soft budget constraints* are accomplished by rejecting the solutions that have the commercials with the budget surpluses more a user-defined threshold.

The authors believes that the "death penalty" approach used in (iv) is suitable here, since the large budget overplus in a solution usually means ineffective handling of the commercial break length constraints, and thus is not expected to contribute to the trade-off surface.

The following subsections describe in details the proposed solution encoding, initial population generation, genetic operators, local optimization routine, as well as the fitness and population management used in the developed algorithm.

6.1. Encoding and Decoding

To encode the problem solution, let us introduce the following notation.

G = the chromosome (solution, individual) of the algorithm, $G = \{g_1, g_2, \ldots, g_m\}$;

g_i = the gene that corresponds to the commercial break b_i, $g_i \in \{0, 1, \ldots, r_i\}$;

r_i = the number of possible states for the gene g_i;

W_i = the set of possible states for the gene g_i, $|W_i| = r_i$, $W_i = \{w_i^z \mid z = 0,1,\ldots,r_i\}$;

z = the break state index;

w_i^z = the state number z for the gene g_i, $w_i^z = \{v_{i1}^z, v_{i2}^z, \ldots, v_{in}^z\}$;

v_{ij}^z = 0 if the state z of the gene g_i does not include the product p_j, else index of a commercial.

The notation implies that the advertisements for the break b_i are coded in the gene g_i by a break state index z, and there is a bijective mapping between this index and the actual commercials that are aired in the break. The set of possible states W_i for the gene g_i is defined as

$$w_i^z = \{v_{i1}^z, v_{i2}^z, \ldots, v_{in}^z\} \in W_i \quad \Leftrightarrow \quad \begin{cases} v_{ij}^z > 0 \Rightarrow d_{ij} = 1; \\ \sum_{j=1}^{n} I(v_{ij}^z) \cdot t_{jv_{ij}^z} \le T_i; \\ v_{ij_1}^z > 0,\ v_{ij_2}^z > 0 \Rightarrow f_{j_1 j_2} = 0; \end{cases}$$

and the states are numbered from 0 to r_i for implementation convenience. For example, assuming the states lexicographical ordering and two products with all the commercial lengths admissible for a break, the zero break state means that the break is empty, the states $z = 1, \ldots, q_1$ correspond to airing the commercials t_{1v}, $v = 1, \ldots, q_1$, in the break, while the states $z > q_1 + q_2$ stand for airing a mix of commercials for both products in the break. The Pascal-like summary of the routines employed for encoding and decoding is given below, with the first procedure used to prepare the data.

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procedure ***PrepareCodingData***

input: the breaks pool $B = \{b_1, b_2, \ldots, b_m\}$ and the problem constraints;
output: the encoding data $\{W_1, W_2, \ldots, W_m\}$;
begin
for $b_i \in B$ **do**
  begin
    initialize the encoding data for the break b_i: $W_i = \varnothing$, $z = 0$;
    for $\{p_{j_1}, p_{j_2}, \ldots, p_{j_y}\} \subset P$ **such that** $d_{ij_{l_1}} = 1$, $f_{j_{l_1} j_{l_2}} = 0$, $l_1, l_2 = 1,2,\ldots,y$ **do**
      for $t_{j_l v_l} \in t_{j_l}$, $l = 1,2,\ldots,y$ **such that** $\sum_{l=1}^{y} t_{j_l v_l} \le T_i$ **do**
        add new state for b_i: $z \leftarrow z+1$; $w_i^z = \{v_{ij}^z = 0; \exists l, j = j_l \Rightarrow v_{ij}^z = v_l\}$; $W_i \leftarrow W_i \cup w_i^z$;
  end
return the encoding data $\{W_1, W_2, \ldots, W_m\}$;
end.

---

procedure ***EncodeSolution***

input: the matrix of the decision variables $X = \{x_{ij}\}$;
output: the encoded chromosome $G = \{g_1, g_2, \ldots, g_m\}$;
begin
for $i = 1$ to m **do**

2. **begin**
3. *find* $w_i^z \in W_i$ such that $v_{ij}^z > 0 \Leftrightarrow t_{jv_{ij}^z} = x_{ij}, j = 1,2,\ldots,n$;
4. *set* the value of the i-th gene: $g_i \leftarrow z$;
5. **end**
6. **return** the encoded chromosome $G = \{g_1, g_2, \ldots, g_m\}$;

end.

---

procedure *DecodeSolution*

input: the encoded chromosome $G = \{g_1, g_2, \ldots, g_m\}$;
output: the matrix of the decision variables $X = \{x_{ij}\}$;
begin
1. **for** $i = 1$ to m **do**
2. **for** $j = 1$ to n **do**
3. *set* the value of the decision variable x_{ij}: $x_{ij} \leftarrow t_{jv_{ij}^{g_i}}$;
4. **end**
5. **return** the matrix of the decision variables $X = \{x_{ij}\}$;

end.

As mentioned above, the proposed encoding takes into account the search space constraints, the commercial break length constraints (4) and the competing products constraints (7); thus minimizing restrictions to be handled during the algorithm run. Another advantage of the proposed encoding approach is the ability to quickly code and decode solutions, since the sets of the possible states $\{W_i\}$ have to be computed only once before the genetic algorithm iterations start. Having a set of tables of this kind for each break enables fast coding and decoding of the solutions during the algorithm run, as shown in the routines given above.

6.2. Initial Population

To generate the initial population, the classical greedy heuristic is used. In the case when the heuristic is not able to generate a defined number of distinct solutions, multiple mutations of the obtained individuals are use to fill the gap. This approach has shown to be substantially more efficient when compared to various random initial population generation techniques, while still being able to develop even distribution of the Pareto-optimal solutions along the trade-off surface. The idea of the greedy heuristic is to assign the values to the decision variable one by one, making the best available decision at every step (Michalewicz and Fogel, 2000). In the case of the problem considered in this paper, at each greedy algorithm step, the current advertising product picks an admissible commercial break that ensures the minimum cost per one incremental Reach point, and adds it to the campaign. The order in which the products are scheduled to select the breaks is defined by a random permutation at each algorithm round, and thus the heuristic can produce different solutions in different runs. The summary of the greedy heuristic is presented below.

---

procedure *GreedyHeuristic*

input: the breaks pool $B = \{b_1, b_2, \ldots, b_m\}$ and the problem constraints;
output: the matrix of the decision variables $X = \{x_{ij}\}$;
begin
1. *initialize* the advertising campaigns: $x_{ij} \leftarrow 0$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$;
2. *initialize* the status of the products: $a_j \leftarrow 1$, $j = 1, 2, \ldots, n$;

3. **while** $\sum_{j-1}^{n} a_j > 0$ **do**
4. **begin**
5. *generate* random permutation $Y = \{y_1, y_2, \ldots, y_n\}$ of the numbers $\{1, 2, \ldots, n\}$;
6. **for** $u := 1$ to n **such that** $j \leftarrow y_u,\ a_j = 1$ **do**
7. **begin**
8. *find* the current set of admissible breaks A for the product j:

$$b_i \in A \Leftrightarrow \begin{cases} d_{ij}=1,\ x_{ij}=0;\ \sum_{j_1}^{n} x_{ij_1} \cdot f_{j_1 j} = 0;\ \sum_{b_{i_1} \in s_l} I(x_{i_1 j}) + 1 \le k_j,\ b_i \in s_l; \\ \exists v, \sum_{i_1=1}^{m} I_{t_{jv}}(x_{i_1 j}) \cdot x_{i_1 j} \cdot c_{i_1 j} + t_{jv} \cdot c_{ij} \le h_{jv} \cdot M_j,\ \sum_{j_1=1}^{n} x_{ij_1} + t_{jv} \le T_i; \\ |t(b_{i_1}) - t(b_i)| \cdot I(x_{i_1 j}) \le \Delta_j,\ i_1 = 1,2,\ldots,m; \end{cases} \quad (8)$$

9. **for all** $b_i \in A$ **do**
10. **begin (comment:** longer commercials are given priority due to soft constraints)
11. *find* the longest commercial t_{jv} that can be placed to the break b_i:

$$v_m \leftarrow \underset{v=1,2,\ldots,q_j}{\operatorname{argmax}} \left\{ t_{jv} : \sum_{i_1=1}^{m} I_{t_{jv}}(x_{i_1 j}) \cdot x_{i_1 j} \cdot c_{i_1 j} + t_{jv} \cdot c_{ij} \le h_{jv} \cdot M_j,\ \sum_{j_1=1}^{n} x_{ij_1} + t_{jv} \le T \right\};$$

12. *select* the best pair (i^*, v^*) by minimizing the unit-Reach cost:

$$(i^*, v^*) \leftarrow \underset{b_i \in A,\, v_m}{\operatorname{argmin}} \left\{ \frac{t_{jv_m} \cdot c_{ij}}{R(X_j + t_{jv_m} \cdot 1_i) - R(X_j)} \right\}; \quad (9)$$

13. **end**
14. **if** the pair (i^*, v^*) was not found **then** $a_j \leftarrow 0$; *continue*;
15. *add* the commercial t_{jv^*} of the product p_j to the break b_{i^*}: $x_{i^* j} \leftarrow t_{jv^*}$.
16. **end**
17. **end**
18. **return** the matrix of decision variable X;

end.

It should be noted that there exist other versions of the greedy heuristic for the considered optimization problem. For example, each brand can be allowed spending a defined share of the budget (5%, for instance) on each algorithm round. For the case studies considered in this paper, the described above version of the version algorithm has showed to be the most efficient one. However, both versions were implemented, and the preliminary analysis of their efficiency was performed before each computational study.

The mutation procedure, which is used to complete the initial "greedy" population, is described in the following subsection. It is employed only if the classical heuristic fails to generate a defined number of distinct individuals. This may lead to the undesired leak of population diversity during the first iterations of the algorithm, so adaptive tuning of crossover and mutation rates might be needed to overcome this difficulty (see the case study #1).

6.3. Genetic Operators

MUTATION. The mutation of the solution $G = \{g_1, g_2, \ldots, g_m\}$ is performed on componentwise bases, with all the genes having a small fixed probability to be modified. If the gene g_i is selected for the mutation, then its value is randomly changed to the one of the equiprobable states $\{0,1,2\ldots,r_i\}$. In the phenotype terms, it means that different products and commercials that satisfy the break length, search space and competitive brands constraints, are assigned to the break b_i instead of the old ones. After the genes modifications, there exists a possibility of the budget, minimum time interval and maximum number of commercials constraints violation, so the obtained solution may be infeasible. To overcome this problem, the approach of "repairing" the infeasible individuals is utilized by the mutation operator. The repairing is performed by withdrawing some commercials from the breaks, while selecting the ones that lead to the minimum objective function decrease per unit cost. Besides, some brands can have their budgets under-spent after the genes modifications, and it is intuitively appealing that spending the rest of the budget will make the solution more feasible. Hence, a special version of the greedy heuristic is used to optimally distribute the budget surpluses. The summary of the mutation genetic operator is presented below.

---

procedure ***MutationOperator***

input: the original chromosome $G^0 = \{g_1^0, g_2^0, \ldots, g_m^0\}$;

output: the mutated chromosome $G^* = \{g_1^*, g_2^*, \ldots, g_m^*\}$;

parameter: the gene mutation probability Q_m;

begin

initialize the mutated chromosome: $G^* \leftarrow G^0$;

for $b_i \in B$ **such that** $\{\text{random number from } [0, 1]\} \le Q_m$ **do**

  begin (**comment**: modify the gene with the probability Q_m)

    generate a random integer s in the range from 0 to r_i with equal probabilities;

    modify the gene g_i^*: $g_i^* \leftarrow s$;

  end

decode the chromosome G^* to the matrix of the decision variable $X = \{x_{ij}\}$;

for $p_j \in P$ **such that** $\sum_{i=1}^{m} x_{ij} \cdot c_{ij} > M_j$ **do**

  while $\sum_{i=1}^{m} x_{ij} \cdot c_{ij} > M_j$ **do**

    begin (**comment**: repairing the budget constraints)

      select the break b_{i^*} with the minimum objective function decrease per unit cost:

$$i^* \leftarrow \underset{i:\, x_{ij} > 0}{\operatorname{argmin}} \left\{ \frac{R(X_j) - R(X_j - x_{ij_m} \cdot 1_i)}{x_{ij} \cdot c_{ij}} \right\}; \qquad (10)$$

      remove the break b_{i^*} from the campaign for the product p_j: $x_{i^*j} = 0$;

    end

for $p_j \in P$ **do**

  while $\exists b_{i_1}, b_{i_2} \in B : |t(b_{i_1}) - t(b_{i_2})| \cdot I(x_{i_1 j} \cdot x_{i_2 j}) > \Delta_j \vee \exists b_i \in B : \sum_{b_{i_1} \in s_l} I(x_{i_1 j}) > k_j,\ b_i \in s_l$ **do**

    begin (**comment**: repairing the time interval and number of commercials constraints)

      select the break b_{i^*} that satisfies (10) from the conflicting breaks;

      remove the break b_{i^*} from the campaign for the product p_j: $x_{i^*j} = 0$;

    end

for $p_j \in P$ **such that** $\sum_{i=1}^{m} x_{ij} \cdot c_{ij} < M_i$ **do** (**comment**: optimally spent the budget surpluses)

  run ***BudgetFix***(X, j)

code the matrix of the decision variable X to the chromosome $G^* = \{g_1^*, g_2^*, \ldots, g_m^*\}$;

return the mutated solution G^*;

end.

The procedure *BudgetFixed*(X,j) optimally distributes the budget surpluses for the product p_j using a modified greedy approach; it is also used by the crossover operator. Its detailed description is given in the following subsection.

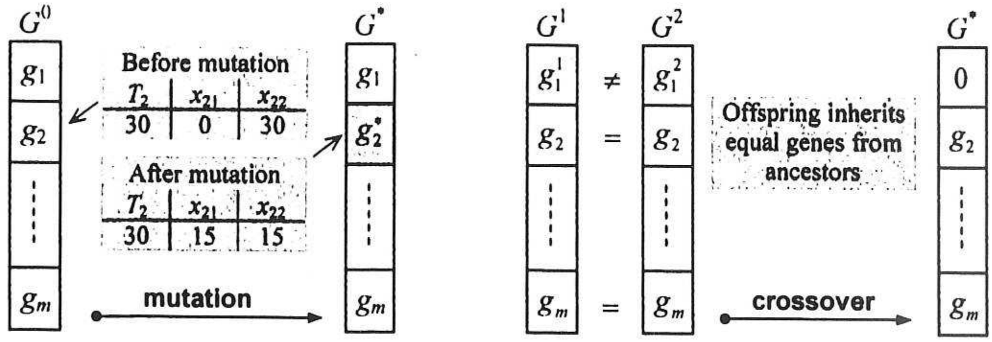

Fig. 4. Mutation and crossover operators (before "repairing")

CROSSOVER. The crossover of the solutions $G^1 = \{g_1^1, g_2^1, \ldots, g_m^1\}$ and $G^2 = \{g_1^2, g_2^2, \ldots, g_m^2\}$ is performed by (i) copying the equal genes of the individuals G^1 and G^2 to the new chromosome; and (ii) optimally distributing the rest of the budget for the offspring solution by means of the greedy heuristic. The summary of the crossover genetic operator is given below.

---

procedure *CrossoverOperator*

---

input: the ancestor chromosomes $G^1 = \{g_1^1, g_2^1, \ldots, g_m^1\}$ and $G^2 = \{g_1^2, g_2^2, \ldots, g_m^2\}$;

output: the offspring chromosome $G^* = \{g_1^*, g_2^*, \ldots, g_m^*\}$;

begin

1. *initialize* the offspring chromosome: $g_i^* = 0,\ \ i = 1,2,\ldots,m$;
2. **for** $b_i \in B$ **such that** $g_i^1 = g_i^2$ **do**
3. **begin** (**comment**: copy the identical genes to the offspring)
4. *set* the offspring gene g_i^* : $g_i^* \leftarrow g_i^1$;
5. **end**
6. *decode* the chromosome G^* to the matrix of the decision variable $X = \{x_{ij}\}$;
7. **for** $p_j \in P$ **such that** $\sum_{i=1}^{m} x_{ij} \cdot c_{ij} < M_i$ **do** (**comment**: optimally spent the budget surpluses)
8. *run* ***BudgetFix***(X,j)
9. *code* the matrix of the decision variable X to the chromosome $G^* = \{g_1^*, g_2^*, \ldots, g_m^*\}$;
10. **return** the offspring solution G^*;

end.

Figure 4 illustrates the ideas of the proposed mutation and crossover operators before the resulted solutions are repaired with respect to violated constraints.

To optimally distribute the budget surpluses for the offspring chromosome, a modification of the greedy heuristic is applied. To handle the soft budget constraints of the problem, longer commercials are given priority when purchasing the advertising time, so the algorithm starts from distributing the rest of the budget for the longest commercial, and proceeds step by step to the shortest. The summary of the heuristic for the optimal budget surpluses spending is presented below.

---

procedure *BudgetFix*

---

input: the matrix of the decision variables $X = \{x_{ij}\}$ and the product index j;

output: the matrix of the decision variables $X = \{x_{ij}\}$ with the optimally distributed budget surplus for the product p_j;

begin

for $v = q_j$ **down to** 1 **do**
  begin (**comment:** longer commercials are given priority due to the soft constraints)
    while $\sum_{i=1}^{m} I_{t_{jv}}(x_{ij}) \cdot x_{ij} \cdot c_{ij} < h_{jv} \cdot M_j$ **do**
      begin
        find the current set of admissible breaks A for the product j according to (8);
        select the best break $b_{i*} \in A$ for the commercial t_{jv} using (9) assuming $v^* = v$;
      end
    if the break b_{i*} was not found **then** *break*;
    add the commercial t_{jv} of the product p_j to the break b_{i^*}: $x_{i^*j} \leftarrow t_{jv}$.
  end
return the matrix of decision variable X;
end.

To avoid lethal offsprings, mating restrictions were introduced to the crossover process. The similarity measure between two chromosomes was defined as

$$D(G^1, G^2) = m^{-1} \cdot \sum_{i=1}^{m} I_{g_i^1}(g_i^2),$$

and the individuals were allowed to mate only if their similarity measure was above a defined threshold. The empirically defined value of the threshold for both case studies was 0.9.

The developed genetic operators and the solution encoding technique allow having feasible solutions at each iteration of the developed MOGA (for hard problem constraints). The soft problem constraints are taken care of by the "death penalty" approach. In the following subsection, it is presented a local optimization routine that is used to speed-up the performance of the algorithm.

6.4. Local Optimization Routine

To improve the performance of the algorithm, it is hybridized with a specially developed local optimization routine. The basic idea of the proposed technique is to generate a set of close promising feasible solutions for the individual, that can be then used to develop a part of the trade-off surface in the neighbourhood of this individual. Thus, the developed local optimization procedure was subject to the following requirements:

(i) the generation of the neighbourhood solutions process must not be time consuming;

(ii) each solution in the generated set must be feasible;

(iii) from all the solutions close to the individual, the promising ones must be favoured;

(iv) the technique must not be limited to the greedy heuristic philosophy.

The first requirement is to enable the algorithm to apply the local optimization search to all new individuals generated by the genetic operators (memetic approach). The last requirement is meant to overcome the limitation of the mutation and crossover operators that both rely on the *BudgetFix*(.) procedure, with the greedy heuristic ideas in it.

The following approach, that takes into account all the mentioned above properties, is proposed. First, a defined small share of the budget (5% was used for the case studies) is unloaded for each product, using the same approach as for the mutation operator. Then, for each product p_j, the commercial t_{jv}, and the admissible break b_i, Reach per cost RpC_{ijv} is calculated (or set to zero if this combination is not admissible). This value is considered to measure the validity of adding the break b_i to the campaign of the product p_j using the commercial t_{jv}. Afterwards, the Reach per cost is transferred to the "attractiveness" probability for every product using the following function:

$$P_A(i,j,v) = f(i,j,v) \Big/ \sum_{i_1=1}^{m}\sum_{v_1=1}^{q_j} f(i_1,j,v_1), \quad f(i,j,v) = \left(RpC(i,j,v) \Big/ \min_{i_1,v_1} RpC(i_1,j,v_1)\right)^{\alpha}, \quad (11)$$

where $\alpha \geq 1$ defines the importance that is assigned to the breaks with higher Reach per cost ($\alpha = 2$ was used for the case studies). Finally, at each procedure step, the products randomly select breaks from the corresponding distributions (11); with products order being also defined randomly on each step. The summary of the proposed local optimization routine is presented below.

---

procedure ***LocalOptimizationOperator***

---

input: the original chromosome $G^0 = \{g_1^0, g_2^0, \ldots, g_m^0\}$ with small budget surpluses;
output: the neighbourhood chromosomes $\Theta = \{G^z = \{g_1^z, \ldots, g_m^z\},\ z = 1, \ldots, Z_{max}\}$;
parameter: the neighbourhood size Z_{max};
begin
 decode the chromosome G^0 to the matrix of the decision variable $X^0 = \{x_{ij}\}$;
 initialize the set of the neighbourhood chromosomes: $\Theta = \varnothing$;
 for $p_j \in P$, $t_{jv} \in t_j$ **do**
  begin (**comment**: calculate the cost per Reach point for admissible breaks)
   initialize the cost per Reach point: $CpR_{ijv} = 0$, $i = 1, 2, \ldots, m$;
   find the set of admissible breaks A for the pair (p_j, t_{jv}) according to (8);
   for $b_i \in A$ **do**
    begin (**comment**: calculate the efficiency of adding t_{jv} to b_i)
     calculate the cost per Reach point for the break b_i and the commercial t_{jv}:
$$RpC(i,j,v) = \left(R(X_j + t_{jv} \cdot 1_i) - R(X_j)\right) \cdot (t_{jv} \cdot c_{ij})^{-1};$$
    end
 end
 transform the cost per Reach point to "attractiveness" probabilities according to (11);
 for $z = 1$ to Z_{max} **do**
  begin
   initialize the solution: $X \leftarrow X^0$, and the status of the products: $a_j \leftarrow 1, j = 1, 2, \ldots, n$;
   while $\sum_{j=1}^{n} a_j > 0$ **do**
    begin
     generate random permutation $Y = \{y_1, \ldots, y_n\}$ of the numbers $\{1, \ldots, n\}$;
     for $u := 1$ to n **such that** $j \leftarrow y_u$, $a_j = 1$ **do**
      begin
       if $P_A(i,j,v) = 0$ for $\forall\ i, v$ **then** $a_j \leftarrow 0$; *continue*;
       generate the random pair (i, v) from the probability distribution $P_A(.,j,.)$ for p_j;
       add the commercial t_{jv} of the product p_j to the break b_i: $x_{ij} \leftarrow t_{jv}$;
       reset the probability to all pairs (i_1, v_1) that became infeasible: $P_A(i_1, j, v_1) \leftarrow 0$;
       normalize the probability distributions $P_A(.)$ for all the products;
      end
    end
   code the matrix of the decision variable X to the chromosome $G^z = \{g_1^z, g_2^z, \ldots, g_m^z\}$;
  end
 return the set of the neighbourhood chromosomes Θ;
end.

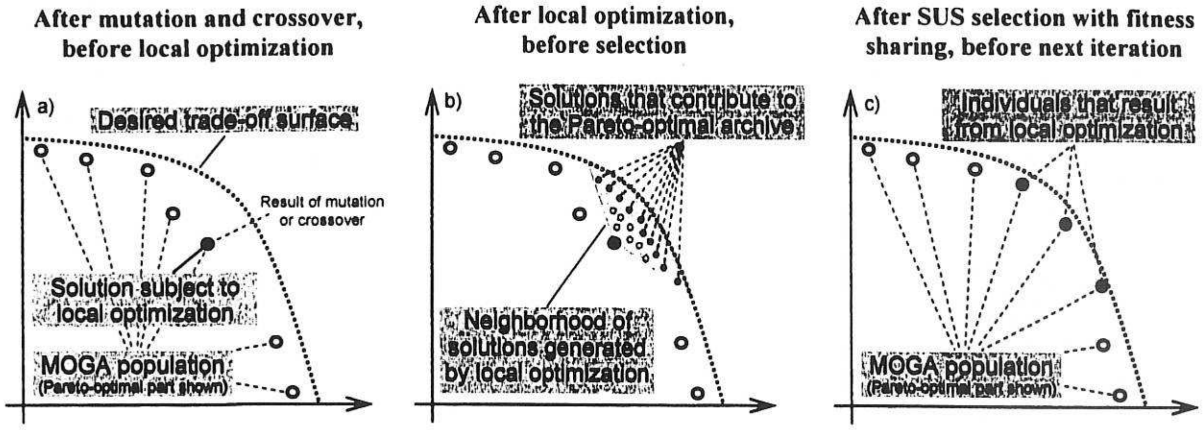

Figure 5. Contribution of the local optimization operator to the developed modified MOGA

The developed local optimization technique contributes both to the improvement of the objective function values, and to the uniform distribution of the solutions in the Pareto-Optimal set. Figure 5 illustrates the key ideas that make this additional genetic operator useful for the considered problem. The local optimization is applied to all new solutions that were generated by the crossover and mutation (Figure 5a). For each of these solutions, the neighbourhood of promising individuals is produced, that results in the new Pareto-optimal chromosomes being added to the archive (Figure 5b). Besides, these Pareto-optimal individuals are uniformly distributed around the solution that was used for local optimization, thus increasing the result of the classical genetic operators (since instead of getting a single point for each new solution, there is also a sub-Pareto-optimal surface piece for this individual). Finally, after applying the selection with the fitness sharing and stochastic universal sampling (SUS), the new generation contains the solutions that tend to distribute evenly along the developed trade-off set (Figure 5c).

6.5. Fitness and Population Management

Following the comparison results reported in (Purshouse, 2004), the combination of the adaptive fitness sharing (Fonseca and Fleming, 1993) and the elitist selection (Eshelman, 1991) is used to manage the population. The performed simulation results has supported this approach, with the adaptive fitness sharing ensuring better individuals distribution along the trade-off surface, and the elitist selection speeding up the algorithm convergence. Besides, the modified multi-objective ranking procedure that allows including the goal attainment information was used when ranking the individuals. A brief overview of the techniques employed is presented below.

In the multi-objective ranking of Fonseca and Fleming (1993), an individual is assigned rank based on a number of solutions in the population that dominate it. Consider a chromosome G^i of the generation t that is dominated by $p_i^{(t)}$ solutions in the current population. Then the rank of the individual G^i is defined as

$$rank(G^i,t) = 1 + p_i^{(t)}.$$

According to the given above expression, all the non-dominated individuals will be assigned rank 1, while the dominated ones will be penalized according to the population density of the corresponding region of the trade-off surface. Besides, this ranking technique is extended to the case when each objective is assigned a goal and priority. For example, a dominated solution that satisfies all the goals may be considered more preferable to the non-dominated solution that does not meet all the objectives. In this paper, the described ranking procedure is used to take into account the goal attainment constraints (3). Subsequently, the exponential rank-to-fitness mapping with the selective pressure e is used to calculate the fitness.

The fitness sharing in the MOGA is aimed at providing the uniform sampling of the solutions in the Pareto-optimal set (Goldberg and Richardson, 1987). During the GA iterations, the diversity of the population can be lost due to the effect of the random genetic drift (Goldberg, 1989), where the solutions tend to converge to a single point that represents the optimum solution. While being acceptable for the single-objective unimodal optimization problem, this phenomenon can lead to identifying only a small region of the trade-off surface for the multi-criteria setting. To overcome this difficulty, niche induction techniques were introduced to improve the diversity in the population (Goldberg and Wang, 1998). According to this approach, the solutions tend to distribute themselves around the multiple optima and form regions that are referred to as niches. Fitness sharing is one of the niching techniques that lowers each individual's fitness by an amount that depends on the number of "similar" individuals according to some measure (Sareni and Kraahenbuhl, 1998). For the multi-objective optimization problems, the similarity measure is usually introduced in the objective function space, since the goal is to achieve even distribution of the Pareto-optimal solutions along the trade-off surface (Fonseca and Fleming, 1998).

In the fitness sharing approach, the shared fitness of the individual l is defined as $f_l' = f_l / m_l$, where the niche count m_l measures the approximate number of individuals with whom the fitness f_l is shared:

$$m_l = \sum_{t=1}^{N} sh(d_{lt}).$$

Here N is the population size, d_{lt} is a distance between the individuals l and t, and $sh(.)$ is the function that measures the individuals similarity:

$$sh(d_{lt}) = \begin{cases} 1-(d_{lt}/\sigma_s)^{\alpha}, & \text{if } d_{lt} < \sigma_s; \\ 0, & \text{otherwise.} \end{cases}$$

The parameter α regulates the shape of the sharing function and is commonly set to one, with the resulting sharing function referred to as the triangular function (Goldberg, 1989).

The developed in this paper algorithm relies on the phenotypic sharing that measures the distance d_{lt} in the objective function space. Euclidian measure is employed to compute d_{lt}, and the estimation of the niche size parameter σ_s is performed by solving the equation

$$N \cdot \sigma_s^{n-1} - \frac{\prod_{j=1}^{n}(M_j - m_j + \sigma_s) - \prod_{l=1}^{n}(M_j - m_j)}{\sigma_s} = 0,$$

where M_j and m_j are the maximum and minimum of each objective respectively.

Elitism can be summarized as preserving the high-performance solutions from one generation to the next. This approach have proved to be an powerful tool for improving the efficiency of the evolutionary algorithms [Zitzler et al, 2000; Deb et al, 2001]. The conducted simulation study has shown that this fact also holds for the optimization problem considered in this paper, so one individual with the highest fitness was always kept in the population.

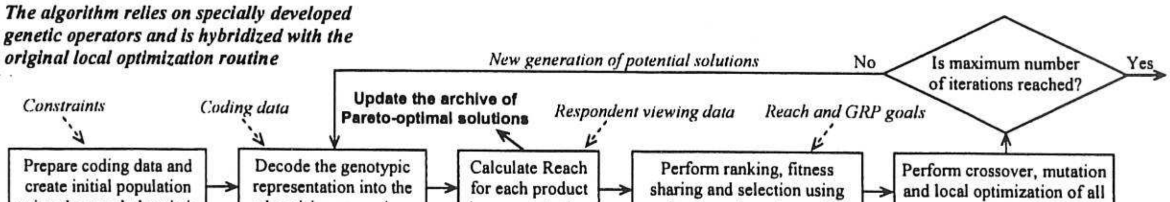

Figure 6. Flow chart of the developed modified multi-objective genetic algorithm

Figure 6 summarizes the work-flow of the developed modified multi-objective genetic algorithm. Using the classification employed in (Ishibuchi and Yoshida, 2002), the proposed routine is based of the generic framework when the local search is applied to all new solutions generated by the multi-objective evolutionary algorithm. The following section presents the application example that confirms the efficiency of the proposed technique.

7. Computational Results

The presented in this paper modified MOGA for the problem of optimal advertising campaign generation for multiple brands was tested using the real data for the Canadian advertising market. There was used a mediaplan for September 2004, which was generated by an advertising agency for one of the major advertisers in Canada. To develop the optimal advertising campaigns, the agency was using the greedy heuristic described in the previous section. The efficiency of the developed in this paper modified multi-objective algorithm is demonstrated below by improving the advertising effectiveness for subsets of products from the original campaign, and thus increasing the overall impact of the mediaplan.

The specificity of the considered optimization problem for the Canadian market with respect to the general problem statement is that all the commercial breaks and brand commercials can be only either 15 or 30 seconds long. This is not always the case in other countries, for example in Germany there is no defined length for the break or brand commercials, the breaks are usually much longer (90-100 seconds), and the commercials can have various length. Although the described specificity of the Canadian version of the problem might seem to simplify it, the trade-off is that the number of breaks significantly increases due to their short length, thus increasing the dimension of the problem dramatically. Another reason for this increase is the different TV broadcasting systems in Canada and Germany. For instance, the usual values for n (the number of breaks) and m (the number of products) are 125000 and 75 for one month optimization for Canada, while only 1500 and 30 for Germany.

In the following subsections, there are considered two case studies that investigate the performance of the developed heuristic algorithm. Both studies are performed for the Ontario region of Canada for the first week of September, 2004 (30.08.2004-05.09.2004). The study #1 was conducted for two low-budget non-competing products, while the study #2 was held for three products with budgets of average size, and two of the products were competing. The idea of both case studies was to demonstrate how the developed multi-objective optimization algorithm could be used to improve the original mediaplan generated by the advertising agency. The obtained results and their detailed analysis follow.

7.1. Case Study #1: Two Low-Budget Products

In the case study #1, there were selected two low-budget non-competing products p_1 and p_2, and the pool of commercial breaks was generated as all the advertising time that was assigned to these two products by the agency in the original mediaplan. Afterwards, this pool was distributed between the products

(i) by means of the greedy heuristic; and

(ii) using the developed modified multi-objective genetic algorithm.

Finally, the trade-off surface generated by the developed modified MOGA was compared to the "greedy" solution. It should be noted that in this case, the multiple runs of the greedy heuristic produced very similar individuals that resulted in low diversity during the first algorithm iterations.

The selected products p_1 and p_2 had the budgets \$3,740 and \$7,332 respectively, and were aimed at the heavily intersecting target groups "Women 18-34" and "Women 18-49 with kids under 12". The first product p_1 had the commercials of length 15 and 30 sec with the corresponding budget shares

20% and 80%, while the second product p_2 had only the commercials of 15 seconds long. The minimum Reach constraints were set at 10% and 13% respectively, while the GRP goals for these products were defined as 15% and 40%. The pool consisted of 112 commercial breaks with about 73% of 15 seconds and 27% of 30 seconds long.

The following parameter values were selected for the developed modified MOGA: the population of 50 individuals, 250 iteration count, adaptive crossover rate $0.1+0.7\cdot y/250$, adaptive mutation rate $0.8-0.7\cdot y/250$, mutation parameter 0.2, where y denotes the iteration number. As follows from the algorithm settings given above, the very high mutation rate that decreases with the iterations, and the very low crossover rate that increases as the algorithm runs, were used for the computational study. Since the usual GA recommendation consider very low mutation rate and high crossover rate, the explanation of this distinction follows.

The developed algorithm is hybridized in a number of ways, in particular, the initial population is constructed by the multiple runs of the classical heuristic (that has some stochastic features and might produce different solutions in various runs). However, the considered in this study two products circumstances can be seen as a very extreme problem case, when the number of the products n is the minimum possible. This fact together with the Canadian problem specificity (15 and 30 seconds breaks and commercials), and the test-specific data leads to the situation when the greedy heuristic produces very similar solutions at every run. The outcome is a very low population diversity during the first iterations of the algorithm, that makes the cross-over not efficient. Hence, to boost up the diversity, very high mutation rate is used to start with. Assuming that the diversity increases as the algorithm runs, the mutation rate decreases in favour of the crossover rate with every iteration. Finally, the rates converge to the settings that are consistent with the usual MOGA parameter selection recommendations (Purshouse, 2004). It should be noted that in the case study for three products (that will be presented in the following section), the greedy heuristic produces a number of diverse solutions, and the described above "rates tuning" procedure is not necessary.

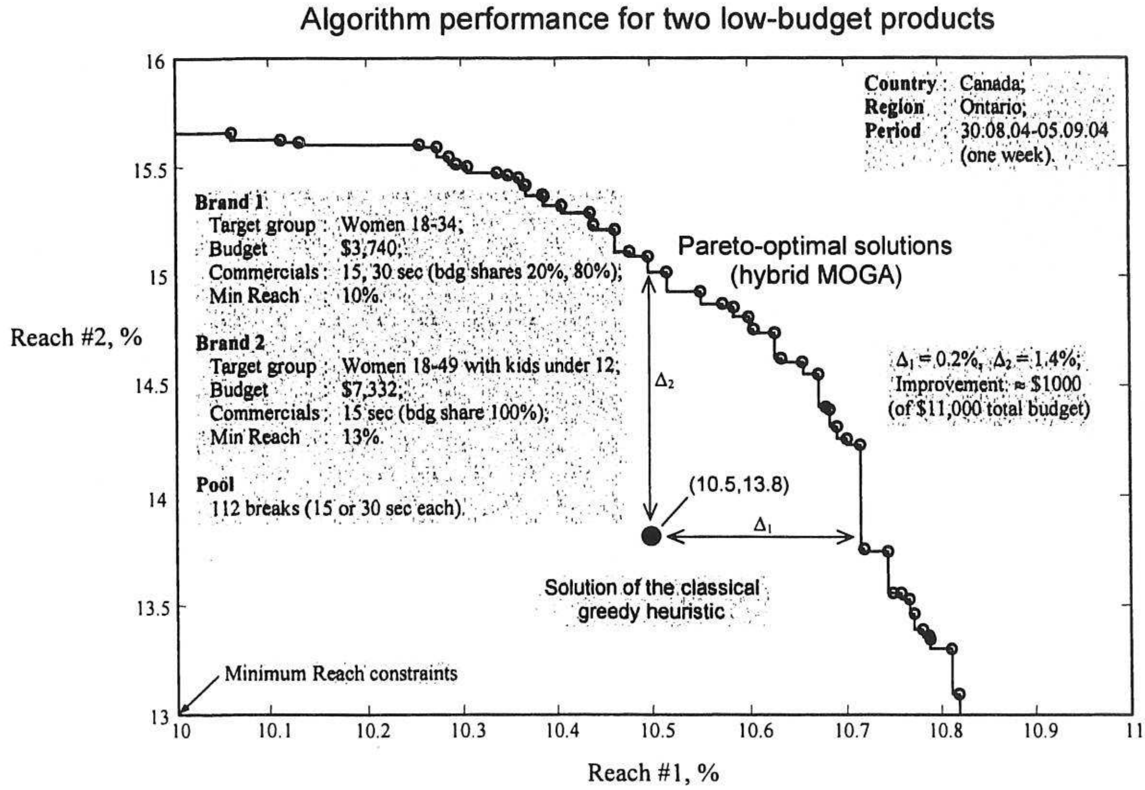

Figure 7. Classical greedy heuristic vs. the developed modified multi-objective genetic algorithm

To develop intuition about the encoding technique proposed for the algorithm, Table 1 enumerates the states of a 30 seconds long break, where both of the products p_1 and p_2 can be aired. As follows from the table, the state 0 corresponds to the empty break, the states 1-3 stand for airing a single commercial, while the state 4 refers to advertising both products in the break, using 15 seconds long commercials.

Table 1. Encoding example for the case study #1

Break state z	0	1	2	3	4
Product #1 commercials	–	15 sec	30 sec	–	15 sec
Product #2 commercials	–	–	–	15 sec	15sec

The results of the study are presented in Figure 7. As follows from the figure, the solution of the classical greedy heuristic can be essentially improved and is dominated by the Pareto-optimal solutions generated by the developed modified MOGA. An aggregated improvement of the MOGA vs. the classical approach can be roughly estimated as \$1,000 vs. about \$11,000 total budget of the product, that is a very significant increase. This figure was calculated based on the cost per gained Reach point of the greedy solution, and can be interpreted as how much money the agency would have to spend to achieve the results of the developed technique while using the greedy heuristic to solve the problem.

Table 2 compares the greedy heuristic solutions (only one is shown due to their extreme similarity) to the solutions from the Pareto-optimal set that dominate it. As follows from the table, the solutions generated by the developed in the paper algorithm ensure not only the superior Reach index values, but also better GRP gain, as well as lower cost per Reach point and cost per GRP. Besides, the mediaplanner can choose the most attractive solution from the Pareto-optimal set and it is not limited to a single point, as for the classical approach.

Figure 8 shows the percent gain of the Pareto-optimal individuals generated by the developed modified multi-objective genetic algorithm, over the greedy solution. In addition to Reach and GRP, the following performance indices are shown for each brand: cost per Reach (CpR) and cost per GRP (CpG). As follows from the parallel axis visualization, all but one Pareto-optimal individuals outperform the greedy heuristic for all efficiency indices, and the excepted solution has slightly lower GRP for the product p_1, while possessing extremely high Reach value for the product p_2. Besides, the developed Pareto-optimal solutions are clearly better then the original mediaplan, that did not satisfy the minimum Reach constraint for the product p_2. These results also confirm the superiority of the developed optimization technique over the classical heuristic.

Table 2. The greedy solution (GS) vs. the developed Pareto-optimal solutions (PO) for the case study #1, compared to the original mediaplan

	Brand #1					Brand #2				
Solution	Reach	GRP	Cost	Cost per Reach	Cost per GRP	Reach	GRP	Cost	Cost per Reach	Cost per GRP
PO # 1	10.51	15.65	3,612	343.79	230.76	15.04	41.36	7,310	486.08	176.74
PO # 2	10.51	15.04	3,508	333.68	233.24	15.02	40.43	7,324	487.73	181.16
PO # 3	10.52	15.68	3,606	342.86	230.01	15.02	41.36	7,317	487.27	176.89
PO # 4	10.55	15.71	3,629	343.93	230.95	14.93	42.25	7,327	490.85	173.41
PO # 5	10.57	15.73	3,607	341.14	229.21	14.87	41.18	7,315	491.86	177.64
PO # 6	10.58	15.70	3,621	342.13	230.62	14.86	41.16	7,306	491.66	177.47
PO # 7	10.60	15.77	3,629	342.34	230.19	14.81	42.10	7,327	494.60	174.03
PO # 8	10.61	15.77	3,638	343.07	230.71	14.75	42.03	7,318	495.94	174.09
PO # 9	10.63	15.79	3,629	341.51	229.79	14.74	42.00	7,321	496.80	174.30
PO #10	10.63	15.76	3,643	342.62	231.24	14.62	41.59	7,317	500.44	175.94
PO #11	10.66	15.83	3,652	342.75	230.76	14.60	41.55	7,313	500.79	175.99
PO #12	10.67	15.84	3,681	344.92	232.35	14.55	42.47	7,323	503.35	172.43
PO #13	10.68	15.85	3,692	345.72	232.94	14.40	42.26	7,311	507.67	172.99
PO #14	10.69	15.86	3,714	347.41	234.14	14.31	42.16	7,285	509.11	172.80
GS	10.49	15.24	3,721	354.72	244.16	13.76	40.04	7,329	532.63	183.04
Original	10.23	15.51	3,740	365.59	241.13	12.81	39.01	7,332	572.37	187.95

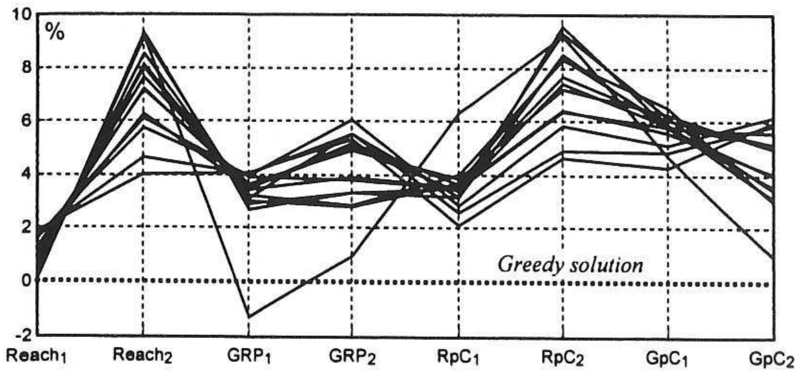

Figure 8. Parallel axis representation for the percent gain of the Pareto-optimal solutions over the greedy solution, case study #1 (for two primary objectives and six additional performance indices)

7.2. Case Study #2: Three Products with Averages Budget Sizes and Competing Constraints

In the case study #2, there were selected three products p_1, p_2 and p_3 with average budget sizes, and the products p_1 and p_2 were competing with each other. As in the previous example, the pool of the commercial breaks was generated as all the advertising time that was assigned to these three products by the agency in the original mediaplan. Subsequently, this pool was distributed between the products using two optimization approaches:

(i) the greedy heuristic;

(ii) the developed modified multi-objective genetic algorithm.

Finally, the Pareto-optimal solutions generated by the developed modified MOGA were compared to the "greedy" solutions.

The selected products p_1, p_2 and p_3 had the budgets \$34,384, \$15,312 and \$42,351 respectively, and were aimed at the very similar target groups "Women 18-34", "Women 25-54" and "Women 25-54". The first product p_1 had the commercials of length 15 and 30 sec with the corresponding budget shares 40% and 60%, the second product p_2 had only the commercials of 30 seconds long, and the third product p_3 had the commercials of length 15 and 30 sec with the shares 60% and 40%. The minimum Reach constraints were set at 40%, 30%, 30% respectively, while the GRP goals were defined as 90%, 50% and 100%. The pool consisted of 1,364 commercial breaks with about 8% of 15 seconds and 92% of 30 seconds long.

The following parameter values were selected for the developed modified MOGA: the population of 50 individuals, 250 iteration count. The initial population was generated by running the greedy heuristic 50 times. Also the obtained solutions were quite diverse in the decision variable space (similarity from 50% to 77%), they all were very close in the objectives space. Standard crossover and mutation rates of 0.8 and 0.1 were used, while still relying on the high mutation parameter value 0.2 to stimulate the exploration of new regions of the search space. Table 3 presents an encoding example for a 30 seconds break that was admissible for all of the considered products.

Table 3. Encoding example for the case study #2

Break state z	0	1	2	3	4	5	6
Product #1 commercials	–	15 sec	30 sec	–	–		15 sec
Product #2 commercials	–	–	–	30 sec	–		
Product #3 commercials	–	–	–	–	15 sec	30 sec	15 sec

Table 4. The greedy solutions (GS) vs. the developed Pareto-optimal solutions (PO) for the case study #2, compared to the original mediaplan

Solution	Brand #1					Brand #2					Brand #3				
	Reach	GRP	Cost	Cost per Reach	Cost per GRP	Reach	GRP	Cost	Cost per Reach	Cost per GRP	Reach	GRP	Cost	Cost per Reach	Cost per GRP
GS # 1	44.37	104.63	34,379	774.77	328.56	33.27	56.29	15,310	460.05	272.00	34.17	57.31	17,502	512.28	305.41
GS # 2	44.27	104.26	34,379	776.59	329.73	33.20	55.94	15,307	461.04	273.62	34.46	58.19	17,501	507.88	300.72
GS # 3	44.32	104.45	34,381	775.67	329.16	33.43	56.10	15,311	458.04	272.95	34.14	57.81	17,500	512.69	302.73
GS # 4	44.28	103.88	34,381	776.41	330.97	33.09	55.64	15,311	462.66	275.19	34.52	58.47	17,500	507.02	299.31
GS # 5	44.33	104.45	34,380	775.59	329.13	33.21	56.06	15,309	461.01	273.11	34.35	57.84	17,503	509.52	302.62
GS # 6	44.32	104.01	34,382	775.70	330.56	33.07	55.53	15,311	463.02	275.73	34.51	58.46	17,499	507.09	299.35
GS # 7	44.36	104.09	34,378	774.93	330.29	34.22	57.81	15,310	447.42	264.86	33.28	55.99	17,503	525.81	312.60
GS # 8	44.36	104.33	34,378	774.97	329.50	33.27	56.25	15,311	460.15	272.18	34.25	57.55	17,503	511.05	304.10
GS # 9	44.36	104.12	34,382	775.02	330.21	34.29	57.94	15,311	446.49	264.24	33.21	55.82	17,500	526.89	313.49
GS #10	44.36	104.56	34,379	775.05	328.83	34.09	57.28	15,308	449.06	267.25	33.40	56.49	17,500	523.93	309.81
GS #11	44.32	103.98	34,379	775.68	330.63	33.37	56.28	15,310	458.82	272.03	34.22	57.69	17,503	511.50	303.38
GS #12	44.32	104.45	34,378	775.59	329.13	33.42	56.08	15,310	458.08	273.00	34.14	57.82	17,499	512.51	302.65
GS #13	44.37	104.51	34,379	774.88	328.96	34.16	57.63	15,311	448.22	265.70	33.33	56.11	17,501	525.15	311.90
GS #14	44.28	103.85	34,377	776.40	331.04	34.12	57.65	15,307	448.67	265.52	33.53	56.44	17,502	521.95	310.10
GS #15	44.37	104.50	34,378	774.86	328.98	34.17	57.64	15,311	448.15	265.66	33.32	56.10	17,503	525.29	311.98
GS #16	44.32	104.45	34,378	775.59	329.13	34.24	57.58	15,311	447.14	265.90	33.32	56.32	17,503	525.27	310.78
GS #17	44.33	104.43	34,382	775.66	329.23	34.03	57.45	15,311	449.93	266.50	33.54	56.48	17,498	521.79	309.80
GS #18	44.36	104.54	34,378	775.06	328.84	34.29	57.95	15,311	446.58	264.21	33.21	55.82	17,502	526.99	313.53
GS #19	44.35	104.07	34,379	775.18	330.35	33.11	55.47	15,307	462.28	275.96	34.42	58.41	17,500	508.36	299.60
GS #20	44.26	103.75	34,380	776.78	331.37	34.26	57.73	15,307	446.73	265.15	33.42	56.56	17,500	523.57	309.40
Mean	44.33	104.27	34,379	775.51	329.73	33.72	56.20	15,309	454.55	269.54	33.84	57.08	17,501	517.33	306.66
PO # 1	44.05	98.43	29,361	666.40	298.29	33.47	56.91	15,146	452.47	266.14	53.74	129.54	34,461	641.31	266.03
PO # 2	44.06	98.81	29,501	669.63	298.56	33.48	56.89	15,198	453.93	267.14	53.63	129.63	33,975	633.50	262.09
PO # 3	44.19	102.87	32,589	737.43	316.80	33.24	54.10	15,106	454.40	279.22	53.46	128.65	36,027	673.86	280.04
PO # 4	44.18	103.57	29,345	664.23	283.34	33.06	55.83	15,304	462.94	274.11	53.52	131.96	33,357	623.23	252.77
PO # 5	44.48	103.72	33,450	752.00	322.51	33.89	58.08	15,311	451.79	263.61	52.36	125.05	37,488	716.01	299.77
PO # 6	44.48	103.74	33,453	752.04	322.45	33.88	58.00	15,268	450.633	263.24	52.35	124.33	37,489	716.01	301.52
PO # 7	44.48	104.18	33,455	752.00	321.10	33.89	58.02	15,296	451.32	263.63	52.34	126.14	37,482	716.10	297.13
PO # 8	44.24	103.17	29,347	663.37	284.46	33.02	55.67	15,297	463.26	274.79	53.46	132.48	33,241	621.76	250.92
PO # 9	44.48	105.33	33,490	752.96	317.94	33.90	58.07	15,295	451.17	263.40	52.34	124.64	37,446	715.44	300.42
PO #10	44.18	103.42	29,470	667.06	284.95	33.05	55.90	15,297	462.73	273.65	53.48	132.72	32,600	609.60	245.63
PO #11	44.48	103.88	33,332	749.35	320.88	33.90	58.06	15,288	451.03	263.32	52.34	125.65	37,575	717.93	299.05
PO #12	44.20	103.00	29,334	663.66	284.81	33.02	55.69	15,311	463.64	274.92	53.47	132.26	32,436	606.52	245.22
PO #13	44.18	102.58	29,314	663.50	285.76	33.03	55.88	15,291	462.98	273.65	53.48	131.74	32,462	606.94	246.42
PO #14	44.06	102.18	34,362	779.92	336.29	35.52	60.26	15,303	430.79	253.94	50.68	120.74	35,245	695.48	291.92
PO #15	44.01	101.08	34,332	780.06	339.66	35.55	60.52	15,307	430.54	252.93	50.45	120.46	34,278	679.45	284.55
PO #16	44.02	101.29	34,316	779.51	338.78	35.60	60.92	15,307	430.00	251.27	50.29	118.71	34,257	681.23	288.57
PO #17	44.77	103.50	32,548	727.08	314.48	33.29	56.63	14,858	446.25	262.37	50.08	116.81	37,158	741.96	318.11
PO #18	44.81	103.82	32,559	726.60	313.60	33.04	56.00	14,798	447.88	264.23	50.23	117.73	34,023	677.29	288.99
PO #19	44.79	104.85	34,377	767.48	327.87	33.17	54.09	15,088	454.87	278.95	49.24	117.44	33,879	687.95	288.47
PO #20	44.48	105.33	33,474	752.61	317.80	33.89	58.08	15,294	451.29	263.32	52.35	125.58	37,507	716.51	298.66
Original	35.28	101.52	34,384	974.33	338.67	24.79	38.15	15.312	617.69	401.33	56.54	163.30	42,351	749.02	259.34

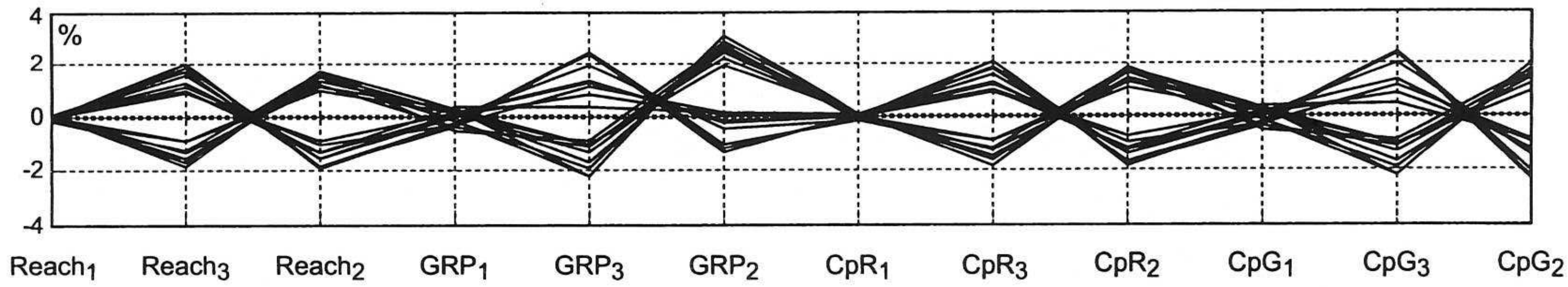

Figure 9. Parallel axis representation for the difference of the greedy solutions from the mean, case study #2

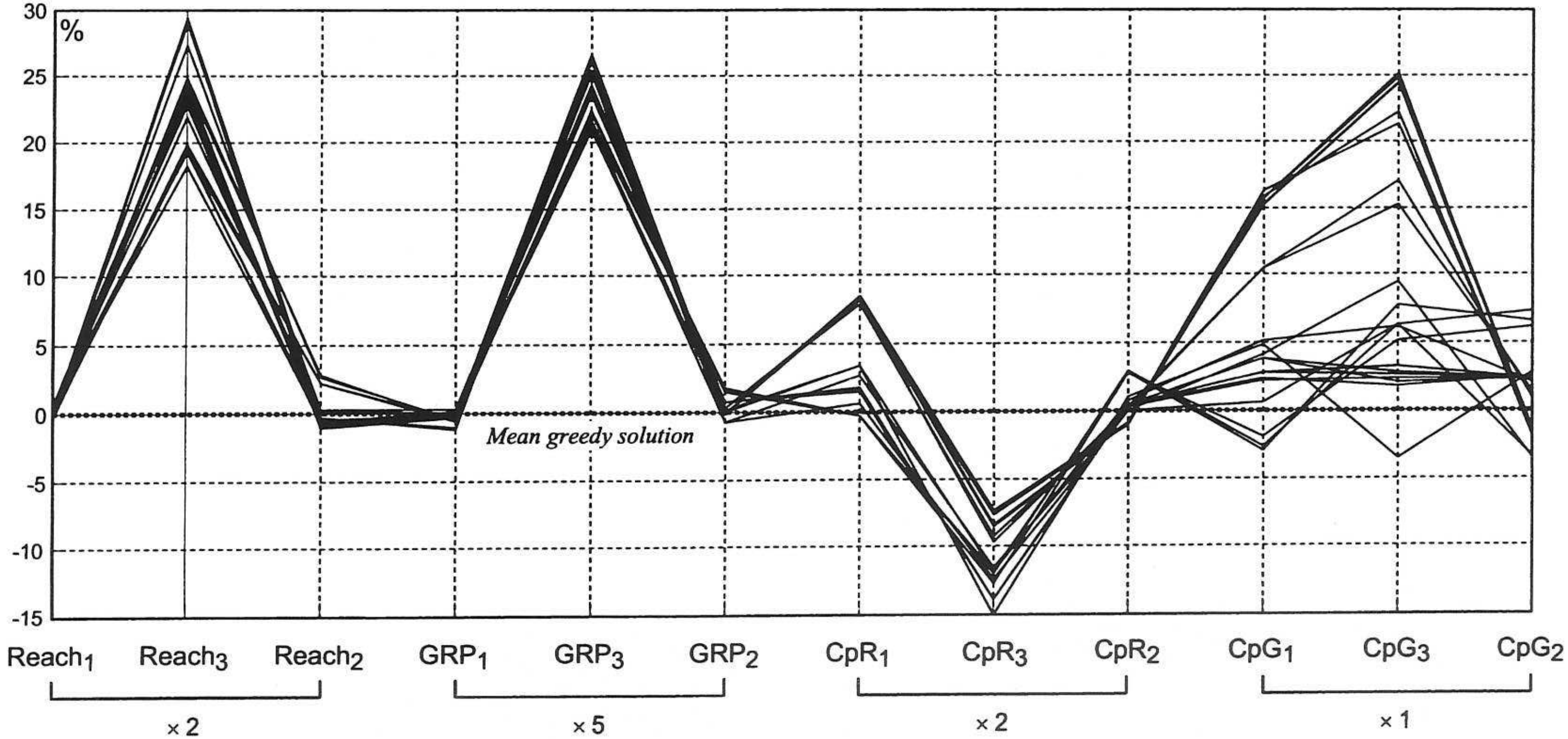

Figure 10. Parallel axis representation for the percent gain of the Pareto-optimal solutions over the mean greedy solution, case study #2 (for two primary objectives and nine additional performance indices)

Figure 9 shows the parallel axis representation for the relative difference of the greedy solutions from their mean. As follows from the figure, the greedy solutions are very close in the objective space, and have similar values of the additional performance indices: GRP, Reach per Cost (RpC), GRP per Cost (GpC). For this reason, the mean of the greedy solutions was used when assessing the performance of the developed modified MOGA.

Table 4 and Figure 10 compare the Pareto-optimal solutions developed by the modified multi-objective genetic algorithm with the greedy solutions, as well as with the original mediaplan. Due to the similarity of the greedy solutions in the objective space, twenty typical solutions are given in the table. The Pareto-optimal set of solutions generated by the developed algorithm consisted of 1,304 individuals. Twenty solutions, that seem the most promising from the practical view point, are presented in the table. Eleven of these solutions were selected based on the maximum sum of Reaches for all the brands, and for each product, three solutions, that ensure the maximum value of its Reach, were picked.

From Table 4, it is clearly seen that the greedy heuristic fails to produce good Reach for the product p_3 (also it has large budget surpluses), while outperforming the original mediaplan for the products p_1 and p_2. In comparison, the developed multi-objective optimization technique produces solutions that provide slightly lower Reach for p_1 or p_2, when compared to the greedy heuristic, but leads to very good Reach for the product p_3. For example, the solution PO #16 leads to the Reach of 44.02% for the product p_1 (when the mean of greedy solutions is 44.33%), the value 35.60% for the product p_2 (33.72% for the greedy heuristic), and the value 50.29 for the product p_3 (vs. 33.84% for the classical technique). Thus, by allowing a negligible decrease of the first objective, the modified MOGA ensures the better value of the second objective and significantly increase the third objective. Besides, it outperforms the original mediaplan for absolutely all performance indices, and is not limited to a single point in the objective space.

Figure 10 shows the percent gain of the Pareto-optimal individuals generated by the developed modified multi-objective genetic algorithm, over the greedy solution. As follows from the parallel axis visualization, the Pareto-Optimal solutions are significantly better then the mean of the greedy solutions for all criteria, except of the Reach per Cost index for the product p_3. This inconsistency is explained by the saturation property of the Reach index, and the heavily underspent budget of the product p_3 in the greedy solutions.

Hence, as in the case study #1, the obtained computation results confirm the superiority of the developed multi-objective genetic algorithm over the classical optimization technique.

8. Conclusions

A multi-objective algorithm approach has been proposed for the problem of optimal TV advertising campaign generation for multiple products. To solve this *NP*-hard combinatorial optimization problem with numerous constraints, the greedy heuristic is used by the advertising agencies. While this traditional approach is limited to the solutions that are closely related in the objective space, the developed modified multi-objective genetic algorithm produces a Pareto-optimal set of solutions that

(i) outperforms the greedy heuristic;

(ii) allows the decision maker choosing from a variety of optimal trade-off alternatives.

To achieve the high performance, the problem-specific solution encoding, the genetic operators, and the original local optimization routine were developed for the algorithm. These techniques allow the algorithm manipulating with only feasible individuals, thus significantly improving its convergence that is complicated by the problem constraints. The efficiency of the developed modified multi-objective genetic algorithm is verified using the case studies for the real data sets from the Canadian advertising market.

Future work will deal with improving the algorithm operators to let them take into account additional constraints that may arise in the considered problem. Besides, an additional study needs to be performed to allow choosing the best algorithm parameters in different application settings. Another challenge will be including to the problem additional optimization objectives for each of the brands.

After this work is accomplished, an evolutionary multi-objective reasoning will be employed in a mediaplanning optimization system, thus providing the advertising agency with significantly more advanced technique for developing the optimal mediaplans.

Acknowledgments

This research was supported by the INTAS grant YSF 03-55-869. The authors are also grateful to the Omega Software GmbH for the data used in the case studies.

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