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Efficient Downlink Channel Estimation Scheme Based on Block-Structured Compressive Sensing for TDD Massive MU-MIMO Systems

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Abstract—**In this letter, an efficient channel estimation approach based on the emerging block-structured compressive sensing is proposed for the downlink massive multiuser (MU) MIMO system. By exploiting the channel properties of block sparsity and channel reciprocity in TDD mode, the auxiliary information based block subspace pursuit (ABSP) algorithm is proposed to recover the downlink channels, where the path delays acquired from uplink training is utilized as the auxiliary information. Unlike traditional approaches where the channel estimation overhead is proportional to the number of BS antennas, the proposed approach could provide an accurate channel estimation approaching the performance bound while reduce the pilot overhead by nearly one-third.**

Index Terms—**Massive MU-MIMO, channel estimaion, block compressive sensing.**

I. Introduction

As a promising technology for future communication systems, massive multiuser (MU) multiple-input multiple-output (MIMO) has received more and more attentions [1]. Similar with classical MIMO system, one massive MU-MIMO base station (BS) with lots of antennas serves many single antenna user terminals (UTs) simultaneously. Such that, the spectrum efficiency and data rates could be substantially increased. However, to implement this technique in practice, there are still many issues that need to be properly addressed. For example, the exact channel state information (CSI) is crucial to the massive MU-MIMO system, since it has significant impact on the accuracy of signal detection. As the number of BS antennas increases, the acquisition of CSI becomes challenging due to the large channel matrix that has to be estimated. The downlink channel estimation is even more difficult since the time required to transmit downlink pilot symbols is proportional to the number of antennas at the BS side, which is unaffordable in a massive MIMO system. For this reason, most of the researches avoid the downlink channel estimation and prefer the time division duplexing (TDD) in massive MIMO systems, owing to the channel reciprocity whereby the UTs could use the CSIs estimated from uplink training directly. However, the uplink CSIs could be inaccurate or even outdated for the downlink in fast time-varying channel conditions, which will consequentially lead to performance deterioration.

Recently, some researches become aware of this issue. In [2], the authors propose a channel reciprocity based beamforming training scheme which utilizes the precoded sequence to estimate the effective channel. However, this scheme is not reliable enough since it overly depends on the uplink training. In order to efficiently estimate the downlink channel in massive MIMO systems, the authors in [3] propose a compressive sensing (CS) based channel estimation scheme. However, this scheme only considers systems in frequency division duplexing (FDD) mode. To the authors' best knowledge, the pilot based downlink channel estimation has not been considered in TDD massive MIMO systems.

In this letter, we propose an efficient channel estimation approach under the framework of block-structured CS. This work is inspired by the recently proposed idea of common support in sparse channels [5], whereby the channel impulse response (CIR) between different transmit and receive antenna pairs exhibits sparse block structure. In addition, channel reciprocity in TDD mode enables us to use the path delays estimated from uplink training as an auxiliary information to improve the channel estimation performance in downlink. Therefore, we propose the Auxiliary information based Block Subspace Pursuit (ABSP) method which could acquire the channel parameters with only few pilots[1]. Compared with conventional CS based channel estimation methods, the proposed method could reduce the computational complexity and pilot overhead significantly while providing superior mean square error (MSE) performance.

The rest of the paper is organized as follows. We first describe the massive MU-MIMO system model in Section II. Then the ABSP algorithm is introduced in Section III. Section IV presents the performance analyses of the proposed method. Numerical experiments are presented in Section V. Finally, section VI concludes the paper.

Notations: Throughout this paper, boldface lower and upper case symbols represent vectors and matrices, respectively. Operators ^{T}, ^{H} and † represent transpose, Hermite and Moore-Penrose matrix inversion, respectively. $\|x\|_p$ and $supp_K(x)$ denote the ℓ_p-norm and the largest K elements in the support

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[1]It is worth noting that the proposed algorithm is different from the conventional channel estimation approache called Auxiliary information based Subspace Pursuit (A-SP) in [7] which acquires the path delays by time domain pilot sequences and is only applicable to single-input single-output (SISO) systems.

of x, respectively.

II. MASSIVE MU-MIMO SYSTEM MODEL

Consider a massive MU-MIMO system where the BS is equipped with M antennas and serving a large number N_u autonomous single-antenna UTs ($M > N_u$). The downlink transmission is organized in OFDM symbols where N_p pilots t_i are randomly allocated in N subcarriers. To reduce the pilot overhead, pilots in different transmit antennas share the same locations, but each pilot sequence $\boldsymbol{t}_m = [t_1^m, t_2^m, \cdots, t_{N_p}^m],\ m = 1, 2, \cdots, M$ is unique, in order to distinguish the CIRs corresponding to different transmit antennas. In this letter, we simply generate the pilot sequence by setting $t_i^m = 1$ or $t_i^m = -1$ following the identically and independently distributed (i.i.d) random Bernoulli distribution [3].

The CIR vector between the ith transmitting antenna and a certain UT can be denoted as $\boldsymbol{h}_i = [h_i(0), \cdots, h_i(L-1)]^T$, where L is the maximum delay spread of the CIR. Due to the sparse nature of MIMO channels [3], there are only K nonzero or significant elements in $\boldsymbol{h}_i$, and $K \ll L$.

Next, let $\boldsymbol{y}$ be the received pilot sequence after cyclic prefix removal and DFT , then we have

$$\begin{aligned} \boldsymbol{y} &= \sum_{m=1}^{M} diag\{\boldsymbol{t}_m\}\boldsymbol{F}_{N_p,L}\boldsymbol{h}_m + \boldsymbol{w} \\ &= \sum_{m=1}^{M} \boldsymbol{T}_m\boldsymbol{F}_{N_p,L}\boldsymbol{h}_m + \boldsymbol{w} \end{aligned} \tag{1}$$

where $\boldsymbol{T}_m = diag\{\boldsymbol{t}_m\}$ is the diagonal matrix with $\boldsymbol{t}_m$ on its main diagonal, $\boldsymbol{F}_{N_p,L}$ is the sub-matrix of the $N \times N$ DFT matrix $\boldsymbol{F}$ collecting the N_p rows according to $\boldsymbol{t}_m$ and first L columns of $\boldsymbol{F}$. $\boldsymbol{w}$ represents the additive white Gaussian noise (AWGN) with zero mean and variance σ^2. Let $\boldsymbol{P}$ denote the $N_p \times LM$ matrix as

$$\boldsymbol{P} = [\boldsymbol{T}_1\boldsymbol{F}_{N_p,L}\ \boldsymbol{T}_2\boldsymbol{F}_{N_p,L}\ \cdots\ \boldsymbol{T}_M\boldsymbol{F}_{N_p,L}], \tag{2}$$

and $\boldsymbol{h} = [\boldsymbol{h}_1^T, \cdots, \boldsymbol{h}_M^T]^T$, then we can rewrite (1) as

$$\boldsymbol{y} = \boldsymbol{P}\boldsymbol{h} + \boldsymbol{w}. \tag{3}$$

A traditional approach to recover the channel $\boldsymbol{h}$ from (3) is the least square (LS) method [2], whereby the solution will be obtained as $\hat{\boldsymbol{h}} = (\boldsymbol{P}^H\boldsymbol{P})^{-1}\boldsymbol{P}^H\boldsymbol{y}$. Note that $N_p \ll ML$ is always satisfied in the massive MU-MIMO system, due to the large number of BS antennas M and limited number of pilots N_p. Thus, equation (3) is underdetermined, where infinite choices of $\boldsymbol{h}$ exist for a given $\boldsymbol{y}$. However, since the channel is sparse in nature, the CS based channel estimation methods could be used to recovery the high-dimensional CIR $\boldsymbol{h}$ from the low-dimensional pilot vector $\boldsymbol{y}$.

TABLE I
PARAMETERS OF COMMUNICATION SYSTEMS

Standard	Bandwidth(B)	$d_{max} = \frac{C}{10B}$	$d = \frac{\lambda}{2}$
CDMA2000	1.25 MHz	24 m	0.15m
3GPP LTE	20 MHz	1.5 m	0.058m

III. ABSP ALGORITHM FOR MASSIVE MU-MIMO SYSTEMS

A. Analyses of Block Sparsity and Channel Reciprocity

It is suggested in [4] that two channel taps are resolvable if the time interval of arrival is larger than $\frac{1}{10B}$ where B is the bandwidth of signal. Therefore, it is obvious that the CIRs measured at different antennas share a common support [5] if $\frac{d_{max}}{C} \leq \frac{1}{10B}$, where d_{max} is the maximum distance between two BS antennas and C is the speed of light. In other words, the path delays of nonzero elements in CIRs between different transmit-receive pair are identical while the path gains are distinct, e.g.,

$$supp(\boldsymbol{h}_i) = supp(\boldsymbol{h}_j),\ i \neq j, \tag{4}$$

where $supp(\boldsymbol{h}_i)$ denotes the support of $\boldsymbol{h}_i$ defined as

$$supp(\boldsymbol{h}_i) = \begin{cases} 1 & h_i(l) \neq 0 \\ 0 & h_i(l) = 0 \end{cases},\ 0 \leq l \leq L-1. \tag{5}$$

In Table 1 we summarize the system parameters of two classical communication standards in terms of the bandwidth B, maximum distance d_{max} and distance between two adjacent antennas $d = \frac{\lambda}{2}$ where λ is the signal wavelength [6]. For example, in the 3GPP LTE standard, the maximum distance between two BS antennas of a 16×16 array is $15d < d_{max}$, which proves the reliability of this assumption[2].

Thus, it natureally leads us to exploit the inherent block structure of massive MU-MIMO channels. By rearranging the elements of $\boldsymbol{h}$ as $\boldsymbol{c} = [\boldsymbol{c}_0^T, \cdots, \boldsymbol{c}_l^T, \cdots, \boldsymbol{c}_{L-1}^T]^T$ with $\boldsymbol{c}_l = [h_1(l), \cdots, h_M(l)]^T$, we have

$$\boldsymbol{y} = \sum_{l=0}^{l=L-1} \Psi_l\boldsymbol{c}_l + \boldsymbol{w} \quad = \Psi\boldsymbol{c} + \boldsymbol{w}, \tag{6}$$

where $\Psi = [\Psi_0, \cdots, \Psi_l, \cdots, \Psi_{L-1}]$, $\Psi_l = [\boldsymbol{p}_l, \cdots, \boldsymbol{p}_{(M-1)L+l}]$ is a $N_p \times M$ matrix, where $\boldsymbol{p}_l$ is the lth column vector of $\boldsymbol{P}$. Therefore, based on the assumption of sparse common supports within different CIRs, the channel vector $\boldsymbol{c}$ shows block sparsity, which could be an additional constrain to solve the underdetermined problem in (3) [10].

So far, most researches solve the channel estimation problems by exploiting the channel reciprocity in TDD massive MIMO systems, assuming the CSIs do not change during an uplink-downlink duration so that the CSIs estimated at the BS in uplink could be directly feedback to the UTs. However, this assumption is sometimes unrealistic in time varying channels where the uplink CSIs could be inaccurate or even outdated for the downlink, resulting in significant performance deterioration [2][3]. In this letter, we adopt a more reliable assumption requiring only the path delays remain unchanged during an uplink-downlink process, while the path gains could be quite different. This assumption is reasonable since the coherence time of path gains is inversely proportional

[2]For the larger antenna array, e.g. a 40×40 array with the maximum distance $39d > d_{max}$, we can still utilize the property of common support based on the information exchange strategy between neighboring antennas. For details, readers are referred to [6].

to the frequency of system carrier, while the coherence time for path delays is inversely proportional to the signal bandwidth [8]. For example, the variation rate of path delay is about 100 times lower than that of the path gains in the digital terrestrial multimedia/television broadcasting (DTMB) system with carrier frequency of 770MHz and signal bandwidth of 7.56 MHz [9].

B. Auxiliary Information based Block-Structured Subspace Pursuit Algorithm

To take practical advantage of block sparsity and channel reciprocity of the massive MU-MIMO channel, we exploit the block-structured CS framework and then propose the ABSP algorithm to improve the accuracy of downlink channel estimation for massive MIMO. Note that the ABSP is similar with the classical subspace pursuit (SP) algorithm [11] but with three main differences:

1) Initial Configuration. In the SP algorithm, the initial approximation of the support set Γ is set to the $\tilde{K}$ indices of the largest magnitude entries in the matching vector $\boldsymbol{x} = \Psi^H \boldsymbol{y}$, where $\tilde{K}$ is the approximated channel sparsity, if no prior knowledge of the target signal is available. In the proposed ABSP scheme, by exploiting the channel reciprocity, we can set $\Gamma = \tilde{\Upsilon}$ directly, where $\tilde{\Upsilon}$ is the approximated path delays estimated from the uplink training containing the accurate path delays with overwhelming probabilities. By this way, we use the path delays estimation as auxiliary information to improve the channel estimation performance of SP without any additional overhead.

2) Matching Vector. The matching vector $\boldsymbol{x} = \Psi^H \boldsymbol{y}$ is used to determine the support set Γ in each iteration. Different from the SP algorithm which calculates the matching vector for only one channel vector, the proposed ABSP exploits the block sparsity and computes matching vector corresponding to all the channel vectors simultaneously and then obtain the joint matching vector as

$$\boldsymbol{m}_{\boldsymbol{x}}^B = [m_{\boldsymbol{x}}^B(0), \cdots, m_{\boldsymbol{x}}^B(j), \cdots, m_{\boldsymbol{x}}^B(L-1)], \tag{7}$$

where $m_{\boldsymbol{x}}^B(j)$ $(0 \leq j \leq L-1)$ is the $B-order\ sum$ of matching vector $\boldsymbol{x}$ defined as [10]

$$m_{\boldsymbol{x}}^B(j) = \sum_{i=1}^{M} \sum_{r=1}^{B} |\boldsymbol{x}(i+jM)|^r \tag{8}$$

where $B \geq 1$ is an integer.

3) Iteration Number. Compared with the conventional SP which needs at least $M \times K$ iterations, the required number of iterations in ABSP is sharply reduced since the path delay is already known. For example, we consider system model with $M = 16$, $K = 6$ and assume $\|\Upsilon - \tilde{\Upsilon}\|_0 = 1$, where Υ is the actual path delays in the downlink channel. Then we can use only one iteration to reconstruct the channel by ABSP algorithm while 96 iterations are required by SP. The main steps of ABSP algorithm is summarized in Algorithm 1.

Algorithm 1

Input: Received pilot sequence $\boldsymbol{y}$, sensing matrix Ψ, approximated path delays $\tilde{\Upsilon}$, approximated channel sparsity $K = \|\tilde{\Upsilon}\|_0$
Initialization:
The initial residual $\boldsymbol{v}_0 = \boldsymbol{y}$, the estimated channel matrix $\tilde{\boldsymbol{c}} = \boldsymbol{0}$, $\Gamma = \tilde{\Upsilon}$ and $k = 1$
while $\|\boldsymbol{v}_k\|_2 < \|\boldsymbol{v}_{k-1}\|_2$ **do**
$\boldsymbol{x} \leftarrow \Psi^H \boldsymbol{v}_k$
$\Gamma \leftarrow \Gamma \cup supp_K(\boldsymbol{m}_{\boldsymbol{x}}^2)$
$\boldsymbol{x} \leftarrow \Psi_\Gamma^\dagger \boldsymbol{y}$
$\Gamma \leftarrow supp_K(\boldsymbol{m}_{\boldsymbol{x}}^1)$
$\tilde{\boldsymbol{c}} \leftarrow \Psi_\Gamma^\dagger \boldsymbol{y}$
$\boldsymbol{v}_k \leftarrow \boldsymbol{y} - \Psi^H \tilde{\boldsymbol{c}}$
$k \leftarrow k+1$
end while
Output: The estimatied CIR matrix $\tilde{\boldsymbol{c}}$

IV. PERFORMANCE ANALYSIS OF ABSP BASED ON MASSIVE MU-MIMO SYSTEM

In this section, we analyze the performance of the proposed ABSP algorithm in terms of the spectral efficiency and computational complexity.

A. Spectral Efficiency

From [10], we know that the scales of the required pilots for estimating the CIRs by the block-structured CS method is $N_p = \mathcal{O}(MK + Klog(L/K))$, which is a substantial improvement over $N_p = \mathcal{O}(MKlog(L/K))$ required by the conventional CS methods. Considering a massive MU-MIMO system with $N = 4096$, $M = 16$, $K = 6$ and $L = 256$. The conventional CS method requires at least $N_t = 16 \times 6 \times log(256/6) \approx 160$ pilots while the proposed ABSP needs only 106 pilots, reducing the pilots by nearly one-third. Moreover, let $N_p = 128$ with some margin, the required pilots by ABSP only occupy 3.1% subcarriers of the total $N = 4096$ subcarriers, with the average pilot occupancy of 0.19% on each antenna. For comparison, the average pilot occupancy of structured CoSaMP is 0.3% [3], where channel reciprocity is not taken into account.

B. Computational Complexity

Since the computational complexity of most recovery algorithms are proportional to the number of measurements, any reduction in the number of required pilots N_p would also reduce the total complexity. In addition, owing to the introduction of auxiliary information, the proposed ABSP algorithm convergents in much faster speed as discussed in Section III-B, which could further reduce the computational complexity. In details, the main computational load on the proposed algorithm is computing the joint matching vector, corresponding a complexity of $\mathcal{O}(LM + LlogL)$ in each iteration [10]. Therefore, the overall complexity of ABSP comprising S iterations is $\mathcal{O}(S(LM + LlogL))$, where $S = \|\Upsilon - \tilde{\Upsilon}\|_0$. On the contrary, the total complexity of SP is

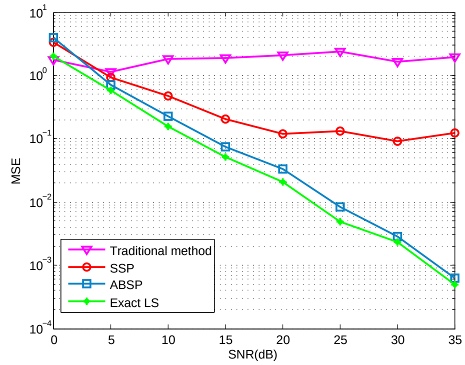

Fig. 1. MSE performance versus SNR

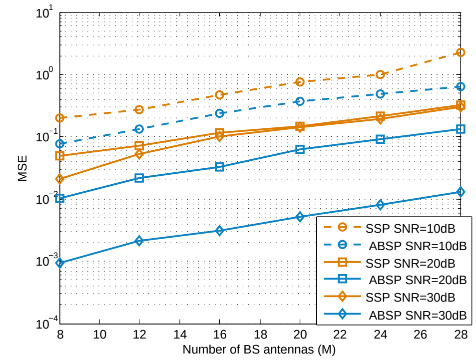

Fig. 2. MSE performance versus different number of BS antennas and SNRs

$\mathcal{O}(MKN_p(L+K^2))$, which is much higher than the proposed method.

V. SIMULATION RESULTS

In this section, we conduct some simulation studies to investigate the performance of the proposed ABSP algorithm. Consider a massive MU-MIMO system where the number of BS antennas $M = 16$. The number of total subcarriers in a OFDM symbol is $N = 4096$, among which $N_p = 128$ pilot subcarriers are randomly placed in the frequency domain. The Rayleigh fading channel with 6-tap multipath and maximum delay spread $L = 100$ is considered. In addition, we assume the channel reciprocity is imperfect where the CIRs of uplink and downlink are not precisely identical.

Firstly, we compare the proposed ABSP with the structured subspace pursuit (SSP) [3] that has already shown to outperform SP in Fig. 2. Meanwhile, the traditional way that utilizes the uplink CIRs directly in the downlink channel recovery (namely traditional method) and the exact least square (LS) channel estimation which perfectly knows the common support Υ are also included for comparison. It can be seen that the traditional method cannot work since the channel reciprocity is imperfect. Moreover, the MSE of the conventional SSP algorithm drops slowly with the increase of SNR and then goes flat when $SNR > 20$. On the contrary, the proposed ABSP algorithm achieves good performance close to the exact LS estimation, thanks to the use of auxiliary information.

Next, we illustrate the MSE comparison of ABSP and SSP with different number of BS antennas M and SNRs in Fig. 3. From the figure we can see that both of the channel estimation methods suffer from performance degradations when M becomes larger, due to the insufficient number of pilots. However, the proposed ASBP is superior to SSP for all the SNRs, especially when SNR increases from 20 to 30 where ABSP shows a substantial improvement while SSP only reaps little benefit.

VI. CONCLUDING REMARKS

This letter considers the downlink channel estimation for TDD massive MU-MIMO system. By exploiting the inherent block sparsity and channel reciprocity, an auxiliary information based block-structured SP algorithm is proposed to efficiently solve the pilot overhead problem in downlink training within massive MIMO systems. Theoretical analysis has demonstrated the effectiveness of the proposed method, in terms of spectral efficiency and computational complexity, while simulation results showed its good channel estimation performance in terms of MSE.

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