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# QUANTIZED FUSION RULES FOR ENERGY-BASED DISTRIBUTED DETECTION IN WIRELESS SENSOR NETWORKS

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***Abstract*—We consider the problem of soft decision fusion in a bandwidth-constrained wireless sensor network (WSN). The WSN is tasked with the detection of an intruder transmitting an unknown signal over a fading channel. A binary hypothesis testing is performed using the soft decision of the sensor nodes (SNs). Using the likelihood ratio test, the optimal soft fusion rule at the fusion center (FC) has been shown to be the weighted distance from the soft decision mean under the null hypothesis. But as the optimal rule requires a-priori knowledge that is difficult to attain in practice, suboptimal fusion rules are proposed that are realizable in practice. We show how the effect of quantizing the test statistic can be mitigated by increasing the number of SN samples, i.e., bandwidth can be traded off against increased latency. The optimal power and bit allocation for the WSN is also derived. Simulation results show that SNs with good channels are allocated more bits, while SNs with poor channels are censored.**

## I. INTRODUCTION

Distributed detection has been attracting significant interest in the context of WSNs [1] and [3]. This is due to the flexibility of WSNs, which can be seamlessly deployed over a wide geographic area for military monitoring and surveillance purpose [2]. However, WSNs suffer from constrained bandwidth and limited on-board power. This poses challenges in the design of distributed detection algorithms, especially when the intruder's signature is unknown to the WSN. The main issue is to improve the detection by fusing the measurements provided by various SNs in a manner that efficiently utilizes the scarce bandwidth and overcomes the limitations of a fading wireless channel.

The problem of decentralized detection in bandwidth constrained sensor networks has been addressed in [4], where the authors investigated the design of sensor messages sent to the FC that minimize the error probability. The problem of detecting a known deterministic parameter is investigated in [5] under restricted channel capacity. The channel fading effect on distributed detection was tackled in [6]. In [7], the authors addressed both issues of limited bandwidth and channel imperfections. They optimized the transmission power, which consequently dictated the number of allocated bits, for the detection of a known signal.

In this paper, we consider the detection of an unknown signal, which is the case in many WSN applications. We find the optimal fusion rule for energy-based soft decision, through the use of the likelihood ratio test. However, it turns out that this rule is difficult to implement in practice. So we will suggest realizable suboptimal fusion rules that weight the soft decisions of the SNs based on the measurement's quality. Furthermore, a linear optimal fusion rule is derived that can serve when the probability distributions of the soft decisions are not known a-priori. Similarly, a simpler fusion rule is proposed based on the linear rule. Then, the previous algorithms are revisited under noisy, flat fading channels with limited bandwidth. Finally, the SN's transmitted powers are optimized to achieve the best probability of detection.

Section II presents the system model. Soft decision fusion rules are proposed in Section III. The quantization effect is discussed in Section IV and the optimal power allocation is derived in Section V. Simulation results are given in Section VI and conclusions are presented in Section VII.

## II. SYSTEM MODEL

Consider a WSN with $M$ sensor nodes reporting to a FC tasked with the detection of any intruders. The intruder leaves a signature signal that is unknown to the WSN but it is assumed to be deterministic. The $i^{th}$ SN collects $N$ samples that are corrupted by additive white Gaussian noise (AWGN) with zero mean and known variance $\sigma_i^2$. So, the measured signal takes one of the following forms, depending on the underlying hypothesis:

$$\mathcal{H}_0 : x_i(n) = w_i(n) \quad (1)$$

$$\mathcal{H}_1 : x_i(n) = s_i(n) + w_i(n) \quad (2)$$

where $n = 1, 2, \ldots, N$, $i = 1, 2 \ldots, M$, $x_i(n)$ is the $n^{th}$ sample of the measured signal at the $i^{th}$ sensor, $s_i(n)$ is the intruder's signature signal and $w_i(n) \sim \mathcal{N}(0, {\sigma_i}^2)$ is the AWGN. Furthermore, the noise samples are assumed to be identically and independently distributed (iid) across time and space.

For optimal detection, the SNs should send their measurements to the FC, where the ultimate detection decision about the intruder's presence will be made. However, this approach is not always feasible in the context of WSNs due to the limited bandwidth available. Thus, the WSN adopts a distributed detection algorithm in which the SNs send their quantized soft decisions (i.e., the quantized local test statistics) to the FC, which combines them to arrive at the global decision.

Since the intruder's signal is unknown at the SNs, the optimal detector in this case would be the energy detector, which is implemented at the $i^{th}$ SN as follows:

$$T_i = \sum_{n=1}^{N} |x_i(n)|^2 . \tag{3}$$

The local soft test statistic $T_i$ is then quantized with $L_i$ bits and transmitted to the FC with power $p_i$ over a wireless channel. The channel suffers from zero mean AWGN with a variance of $\zeta_i$. Moreover, the wireless channel between the $i^{th}$ SN and the FC experiences flat fading with a channel gain $h_i$ (also assumed to be iid). The number of quantization bits at the $i^{th}$ SN must satisfiy the channel capacity constraint:

$$L_i \leq \frac{1}{2}\log_2\left(1+\frac{p_i h_i^2}{\zeta_i}\right) \text{bits},\ i=1,2,\ldots,M. \tag{4}$$

We will assume that the maximum channel capacity is utilized by the SNs. So our objective is to find the best soft fusion rule first, and then optimize the allocated power to maximize the detection probability.

## III. Soft Decision Fusion Rules

In this section, the optimal soft decision fusion rule is investigated given infinite bandwidth for each WSN, i.e., no quantization is required. However, it turns out that the optimal rule requires prior information about the signal's energy, which cannot be known in practice. Hence, suboptimal rules are proposed as an implementable alternative.

### A. Optimal Fusion Rule

Given the local soft test statistic defined in (3), the optimal fusion rule follows from the likelihood ratio test:

$$\text{LRT}(\boldsymbol{T}) = \frac{p\{T_1,T_2,...,T_M|\mathcal{H}_1\}}{p\{T_1,T_2,...,T_M|\mathcal{H}_0\}} \geq \gamma \tag{5}$$

where $p\{T_1,T_2,...,T_M|\mathcal{H}_j\}$ is the joint probability distribution of local soft decisions under the $j^{th}$ hypothesis. However, $T_i$ has a $\chi^2$ distribution under $\mathcal{H}_0$ and a non-central $\chi^2$ under $\mathcal{H}_1$, which means evaluation of the LRT in (5) is complicated. Consequently, we evoke the central limit theorem to simplify the distribution of $T_i$ when $N$ is sufficiently large. So the distribution of any $T_i$ can be adequately approximated by a Gaussian distribution with the following mean and variance:

$$\text{E}\{T_i|\mathcal{H}_0\} = N\sigma_i^2, \qquad \text{Var}\{T_i|\mathcal{H}_0\} = 2N\sigma_i^4 \tag{6}$$

$$\text{E}\{T_i|\mathcal{H}_1\} = N\sigma_i^2(1+\xi_i), \text{Var}\{T_i|\mathcal{H}_1\} = 2N\sigma_i^4(1+2\xi_i) \tag{7}$$

where $\xi_i = \sum_{n=1}^{N} s_i^2(n)/N\sigma_i^2$ is the SNR at the $i^{th}$ SN.

Since the noise at different SNs is independent, it can easily be shown [11] that the log-likelihood ratio test (LLR) takes the form

$$T_f = \sum_{i=1}^{M}\left(\frac{\left(T_i - N\sigma_i^2\right)^2}{2N\sigma_i^4} - \frac{\left(T_i - N\sigma_i^2(1+\xi_i)\right)^2}{2N\sigma_i^4(1+2\xi_i)}\right) \geq \gamma' \tag{8}$$

where $\gamma' = 2\ln\left(\prod_{i=1}^{M}\gamma\left(\frac{\sqrt{2N\sigma_i^4}}{\sqrt{2N\sigma_i^4(1+2\xi_i)}}\right)\right)$.

The LLR can be further simplified by completing the square in (8) to yield

$$T_f = \sum_{i=1}^{M} a_i (T_i - b_i)^2 \tag{9}$$

$$a_i = \frac{\xi_i}{N\sigma_i^4(1+2\xi_i)} \tag{10}$$

$$b_i = \frac{N\sigma_i^2}{2}. \tag{11}$$

The fusion rule in (9) has an interesting interpretation. It is, in fact, the *weighted distance* in the $M$-dimensional space between the local soft test statistic and half of its mean under the null hypothesis (see (6)). It is also clear that SNs with lower noise get more weight in the fusion process. Another interesting note here is that at high SNR ($\xi_i$) the weight $a_i$ depends only on the noise power at the SN and not on the measured signal energy.

### B. Suboptimal Fusion Rules

Now since the optimal fusion rule in (9) requires the exact knowledge of the SNR ($\xi_i$), it cannot be realized in practice. However, its structure can be used to formulate implementable suboptimal rules. So now we propose three suboptimal rules: weighted fusion, equal fusion and optimum linear fusion.

*1) Weighted and Equal Fusion Rules:* The weighted fusion rule takes the same structure as (9). However (for large $\xi_i$) $a_i$ in (10) is replaced by $a_i^w = 1/2N\sigma_i^4$ and we let $b_i^w = b_i$. This rule approaches the optimal one when the SNR is large, as discussed earlier.

As for the equal fusion rule, equal weight is given for all the SNs, i.e., $a_i^e = 1$ for all $i = 1, 2, \cdots, M$. Also, $b_i^e = b_i$.

*2) Optimum Linear Fusion Rule:* Now we examine the (sub-optimal) linear fusion rule:

$$T_f^l = \sum_{i=1}^{M} \alpha_i T_i. \tag{12}$$

and the optimal weights to maximize the probability of detection are

$$\alpha_i = \frac{\xi_i}{N\sigma_i^2(1+2\xi_i)}. \tag{13}$$

Due to the lack of space however, the proof is omitted but a detailed discussion can be founded in [11]. However, the above weights are not realizable due to requiring a-priori knowledge of $\xi_i$.

## IV. Quantized Soft Decision Fusion

The previous fusion rules assume the availability of an infinite bandwidth to send the exact $T_i$. So let the quantized test statistic ($\hat{T}_i$) at the $i^{th}$ sensor be modeled (with $L_i$ bits) as

$$\hat{T}_i = T_i + v_i \tag{14}$$

where $v_i$[1] is the quantization noise with uniform distribution in the interval $[-B, B]$ and variance

$$\sigma_{v_i}^2 = \frac{B^2}{3\times 2^{2L_i}}. \tag{15}$$

[1] $v_i$ in (14) is the quantization noise independent of $w_i(n)$ (in (1) and (2)) for all $n$ and $i$.

However, the distribution of $\hat{T}_i$ can be approximated [11] by a Gaussian distribution with mean and variance:

$$\begin{aligned}\mathrm{E}\left\{\hat{T}_i|\mathcal{H}_0\right\} &= N\sigma_i^2,\ \mathrm{Var}\left\{\hat{T}_i|\mathcal{H}_1\right\} = 2N\sigma_i^4\left(1+2\xi_i\right)+\sigma_{v_i}^2\\ \mathrm{E}\left\{\hat{T}_i|\mathcal{H}_1\right\} &= N\sigma_i^2\left(1+\xi_i\right), \mathrm{Var}\left\{\hat{T}_i|\mathcal{H}_0\right\} = 2N\sigma_i^4+\sigma_{v_i}^2.\end{aligned} \tag{16}$$

### A. Quantized Optimal/Suboptimal Fusion Rule

Since the $\hat{T}_i$'s are now Gaussian, then in a similar manner to Section III, the log-likelihood ratio test with quantization can be shown to be

$$T_f^q = \sum_{i=1}^{M}\left(\frac{\left(\hat{T}_i - N\sigma_i^2\right)^2}{2N\sigma_i^4+\sigma_{v_i}^2} - \frac{\left(\hat{T}_i - N\sigma_i^2\left(1+\xi_i\right)\right)^2}{2N\sigma_i^4\left(1+2\xi_i\right)+\sigma_{v_i}^2}\right) \gtrless \gamma'' \tag{17}$$

where $\gamma'' = 2\ln\left(\prod_{i=1}^{M}\gamma\left(\frac{\sqrt{2N\sigma_i^4+\sigma_{v_i}^2}}{\sqrt{2N\sigma_i^4(1+2\xi_i)+\sigma_{v_i}^2}}\right)\right)$. As before, (17) can be now written in the following form

$$T_f^q = \sum_{i=1}^{M} a_i^q\left(\hat{T}_i - b_i^q\right)^2 \tag{18}$$

$$a_i^q = \frac{\xi_i}{N\sigma_i^4\left(1+2\xi_i+\frac{\sigma_{v_i}^2}{2N\sigma_i^4}\right)\left(1+\frac{\sigma_{v_i}^2}{2N\sigma_i^4}\right)} \tag{19}$$

$$b_i^q = \frac{N\sigma_i^2}{2} - \frac{\sigma_{v_i}^2}{4\sigma_i^2}. \tag{20}$$

Note that $T_f^q \to T_f$ as $\sigma_{v_i}^2 \to 0$ for all $i$. Consequently, $a_i^q \to a_i$ and $b_i^q \to b_i$ under the previous condition as well. More interestingly however, is that $T_f^q \to T_f$ as $N \to \infty$, regardless of $\sigma_{v_i}^2$. This implies that bandwidth can be saved but at the expense of increasing both the number of collected measurements and also the detection delay.

As for the suboptimal (quantized) fusion rule, it can be easily shown that

$$a_i^{wq} = \frac{1}{N\sigma_i^4\left(1+\frac{\sigma_{v_i}^2}{2N\sigma_i^4}\right)^2} \tag{21}$$

$a^{eq} = 1$ and $b^{eq} = b^{wq} = b_i^q$.

### B. Quantized Optimal Linear Fusion Rules

The quantized version of the linear fusion weights in (13) can be shown to be [11]

$$\alpha_i^q = \frac{\xi_i}{2\sigma_i^2\left[1+2\xi_i+\frac{\sigma_{v_i}^2}{N\sigma_i^2}\right]}. \tag{22}$$

If the SNR is large, i.e., when either $\sigma_{v_i}^2 \to 0$ or $N \to \infty$ then it follows that $\alpha_i^q \to \alpha_i$.

## V. Optimum sensor transmit power allocation

The performance of the proposed quantized fusion rules approach the performance of their unquantized counterparts if the number of (test statistic) bits is sufficiently large. However, this entails a large transmission power as predicted by (4). So, we desire to strike a trade-off between the fusion rule's performance and transmit power. To this end, we first need to adopt an optimization criterion. A natural one is the probability of detection, which depends on the distribution of the fusion rule. So letting $U_i = \left(\hat{T}_i - b_i\right)^2$ then the optimum fusion rule can be written as

$$T_f^q = \sum_{i=1}^{M} a_i^q U_i. \tag{23}$$

The mean and variance of $U_i$ under $\mathcal{H}_0$ and $\mathcal{H}_1$ are now given in (24). Using the central limit theorem, $T_f^q$ can be approximated by a Gaussian distribution

$$T_f^q \sim \begin{cases}\mathcal{N}\left(\mathrm{E}\left\{T_f^q|\mathcal{H}_0\right\}, \mathrm{Var}\left\{T_f^q|\mathcal{H}_0\right\}\right) \text{ under } \mathcal{H}_0\\ \mathcal{N}\left(\mathrm{E}\left\{T_f^q|\mathcal{H}_1\right\}, \mathrm{Var}\left\{T_f^q|\mathcal{H}_1\right\}\right) \text{ under } \mathcal{H}_1\end{cases} \tag{25}$$

where

$$\begin{aligned}\mathrm{E}\left\{T_f^q|\mathcal{H}_0\right\} &= \sum_{i=1}^{M} a_i^q \mathrm{E}\left\{U_i|\mathcal{H}_0\right\}\\ \mathrm{E}\left\{T_f^q|\mathcal{H}_1\right\} &= \sum_{i=1}^{M} a_i^q \mathrm{E}\left\{U_i|\mathcal{H}_1\right\}\\ \mathrm{Var}\left\{T_f^q|\mathcal{H}_0\right\} &= \sum_{i=1}^{M} \left(a_i^q\right)^2 \mathrm{Var}\left\{U_i|\mathcal{H}_0\right\}\\ \mathrm{Var}\left\{T_f^q|\mathcal{H}_1\right\} &= \sum_{i=1}^{M} \left(a_i^q\right)^2 \mathrm{Var}\left\{U_i|\mathcal{H}_1\right\}.\end{aligned} \tag{26}$$

It can be readily shown that the detection probability as a function of the false alarm probability has the form

$$P_d = Q\left(\frac{Q^{-1}\left(P_{fa}\right)\sqrt{\mathrm{Var}\left\{T_f^q|\mathcal{H}_0\right\}} - \Psi}{\sqrt{\mathrm{Var}\left\{T_f^q|\mathcal{H}_1\right\}}}\right) \tag{27}$$

where $Q(\cdot)$ is the $Q$-function and $\Psi = \mathrm{E}\left\{T_f^q|\mathcal{H}_1\right\} - \mathrm{E}\left\{T_f^q|\mathcal{H}_0\right\}$. The probability of detection implicitly depends on the transmission power through the relationships (24) and (26). Based on this, we can optimize the transmission powers $(p_i)$ to maximize $P_d$ under the constraint of a maximum aggregate transmit power budget $(P_t)$:

$$\begin{aligned}\boldsymbol{p}_{opt} = &\ \arg\ \max_{\boldsymbol{p}} P_d\left(\boldsymbol{p}\right)\\ \text{subject to} &\sum_{i=1}^{M} p_i \le P_t \text{ for } \ p_i \ge 0,\ i = 1,\ldots,M\end{aligned} \tag{28}$$

where $\boldsymbol{p} = [p_1, p_2, \ldots, p_M]$. Now (28) is difficult to solve and there is no closed form solution. Hence, we propose a numerical solution by adopting the spatial branch-and-bound

$$\mathrm{E}\{U_i|\mathcal{H}_0\} = 2N\sigma_i^4 + N^2\sigma_i^4 + \sigma_{v_i}^2 - 2b_i N\sigma_i^2 + {b_i}^2, \quad \mathrm{Var}\{U_i|\mathcal{H}_0\} = \mathrm{Var}\left\{\hat{T}_i|\mathcal{H}_0\right\}\left[4N^2\sigma_i^4 + 2\mathrm{Var}\left\{\hat{T}_i|\mathcal{H}_0\right\} + 4b_i^2 - 8Nb_i\sigma_i^2\right]$$
$$\mathrm{E}\{U_i|\mathcal{H}_1\} = \mathrm{E}\left\{\hat{T}_i|\mathcal{H}_1\right\}^2 + \mathrm{Var}\left\{\hat{T}_i|\mathcal{H}_1\right\} - 2b_i\left(N\sigma_i^2 + N\sigma_i^2\xi_i\right) + b_i^2$$
$$\mathrm{Var}\{U_i|\mathcal{H}_1\} = 4\mathrm{E}\left\{\hat{T}_i|\mathcal{H}_1\right\}^2 \mathrm{Var}\left\{\hat{T}_i|\mathcal{H}_1\right\} + 2\mathrm{Var}\left\{\hat{T}_i|\mathcal{H}_1\right\}^2 + 4b_i^2\mathrm{Var}\left\{\hat{T}_i|\mathcal{H}_1\right\} - 8b_i\mathrm{E}\left\{\hat{T}_i|\mathcal{H}_1\right\}\mathrm{Var}\left\{\hat{T}_i|\mathcal{H}_1\right\} \quad (24)$$

strategy [8] using the YALMIP optimization tools [9]. In the first step of the algorithm, we start by applying a standard nonlinear solver to obtain a locally optimal solution and then set it as an upper bound on the achievable objective. Secondly, in each node, a convex relaxation of the model is derived, and the resulting convex optimization problem is solved. We then assign this as a lower bound. Bound tightening using [9] is applied iteratively to detect and eliminate redundant constraints and variables, and tighten the bounds where possible. The algorithm outline is summarized in $Algorithm1$ [11].

The aim of the algorithm is to obtain the global minimum of the function $\beta(\boldsymbol{p}) = \frac{Q^{-1}(P_{fa})\sqrt{\mathrm{Var}\{T_f^q|H_0\}} - \Psi}{\sqrt{\mathrm{Var}\{T_f^q|H_1\}}}$ over the solution space $\wp_{start}$ where $\boldsymbol{p} \in \wp_{start}$. For any $\wp \subseteq \wp_{start}$ we define $F_{lb}$ (the upper bound) and $F_{ub}$ (the lower bound) as functions that satisfy: $F_{lb}(\wp) \leq F_{min}(\wp) \leq F_{ub}(\wp)$. Then, the global optimum solution $\beta^* = F_{min}(\wp_{start}) = \inf_{\boldsymbol{p}\in\wp_{start}}\beta(\boldsymbol{p})$.

## VI. Simulation Results

We simulate a WSN of $M$ SNs detecting an intruder with $s_i(n) = A$, where $A = 0.1$. The communication noise variances are arbitrarily set to $\zeta_i = 0.1$ for all $i = 1, 2, \cdots, M$ (for simplicity). The measurement noise variances are generated randomly and used throughout all the simulations. The average measurement SNR for the network is defined as $\xi_a = 10\log_{10}\left(\frac{1}{M}\sum_{i=1}^{M}\xi_i\right)$. In all simulations we assume perfect knowledge of $\xi_i$. Fig. 1 shows the receiver operating characteristic for six different fusion rules. It is clear that the optimal fusion rule attains the best performance for ($\xi_a = -8.5$ dB) whereas the worst performance is that of the equal weight linear combining rule. However, all the rules converge when the $P_{fa}$ increases. In Fig. 2 the effect of the number of measurement samples ($N$) on $P_d$ is shown at a fixed $P_{fa}$. Obviously, as $N$ increases $P_d$ improves for all algorithms. Interestingly, the optimal linear fusion rule outperforms the suboptimal LRT-based one. This is explained by the structure of (21) where for large (but finite) $N$ the effect of $\sigma_{v_i}^2$ (quantization noise variance) is still noticeable. A similar trend is noticed in Fig.3, in which $P_d$ is plotted against the number of SNs, ($M$), for a fixed $N$. The $P_d$ performance of both LRT-based and linear combining schemes as a function of the average SNR ($\xi_a$) is shown in Fig. 4. Fig. 5 on the other hand, exhibits the effect of the transmission power $p_i$ on $P_d$. Increasing $p_i$ leads to a larger number of allocated bits, through (4), and consequently less quantization variance, which ultimately improves the detection performance. Interestingly, the dependence of $P_d$ on $p_i$ is alleviated when

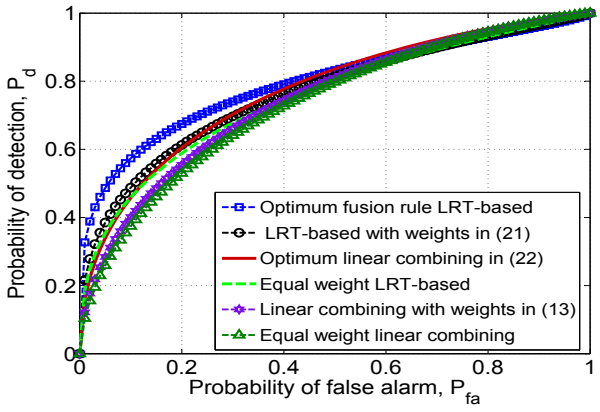


Fig. 1. Receiver operating characteristics of six different fusion rules for $N = 10$, $M = 10$, $\xi_a = -8.5$ dB and $B = 0.5$.

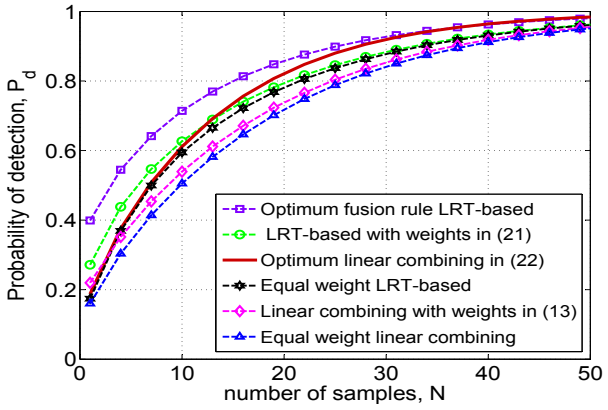


Fig. 2. Probability of detection ($P_d$) versus the number of samples ($N$) with $M = 20$, $P_{fa} = 0.1$, $B = 0.5$ and $\xi_a = -8.5$ dB.

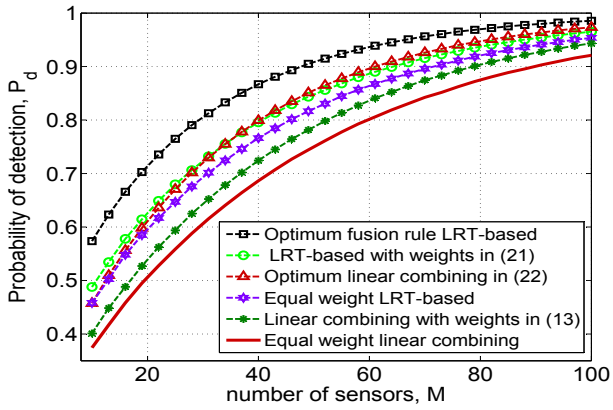


Fig. 3. Probability of detection ($P_d$) versus number of sensors ($M$) for $N = 10$, $P_{fa} = 0.1$, $\xi_a = -8.5$ dB and $B = 0.5$.

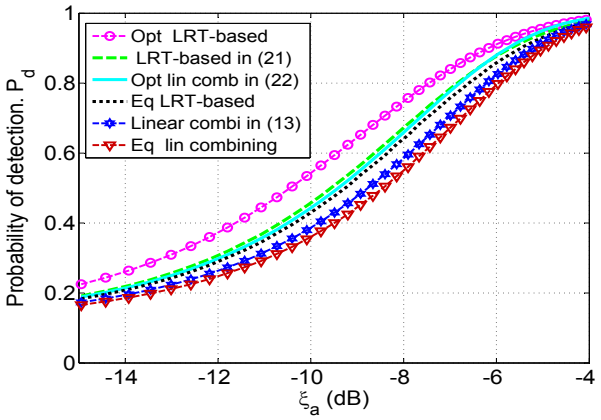


Fig. 4. Probability of detection ($P_d$) versus the signal to noise ratio ($\xi_a$) for $M = 20$, $N = 10$, $P_{fa} = 0.1$ and $B = 0.5$.

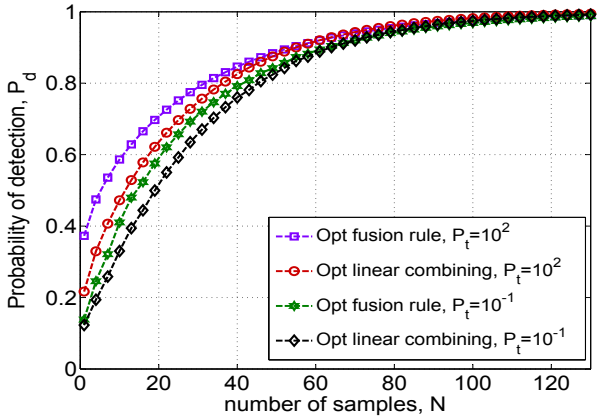


Fig. 5. Probability of detection ($P_d$) versus the number of samples ($N$) for $M = 10$ sensors, $P_{fa} = 0.1$, $\xi_a = -8.5$ dB and $B = 1$.

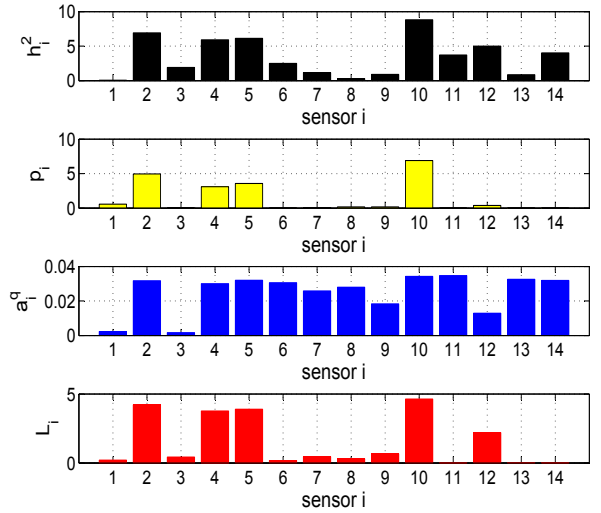


Fig. 6. Optimum sensor transmit power and channel quantization bits allocation for $N = 10$, $P_{fa} = 0.1$, $\xi_a = -8.5$ dB and $P_t = 20$.

$N$ is increased, since the effect of the quantization noise is mitigated as predicted by (19) and (22). In Fig. 6, we report the optimized sensor transmit power and the corresponding number of bits allocated to quantize $T_i$ by applying the branch and bound algorithm [8]. Clearly we allocate more power and bits to the best channels. However, note that the power and bit allocation is also affected by the weights $a_i^q$ in (19) which are a function of the signal to noise ratio $\xi_i$. For instance, consider sensor 12 which has a relatively good channel gain, but the corresponding local $\xi_i$ is bad. Hence, it will allocate a relatively small amount of the transmit power. Those SNs with bad channels are allocated *zero* bits, i.e., they will be censored or prevented from transmission.

## VII. CONCLUSION

We have shown that the optimal fusion (see (9)) for energy-based soft decisions is actually the weighted distance of the decisions from their mean under the null hypothesis. Realizable suboptimal fusion rules derived from the optimal one are proposed as well, in which more weight in the actual fusion are given to decisions with better sensing quality. We show that the effect of quantization on the detection performance can be mitigated by increasing the number of measurements ($N$), or equivalently incurring more delay in the system. Finally, the SN's transmission power has been optimally allocated. Intuitively, more power is given to SNs having better channel gains and consequently increased number of bits.

## REFERENCES


[1] J. F. Chamberland and V. V. Veeravalli, "Wireless sensors in distributed detection applications," *IEEE Signal Processing Magazine*, vol.24, no.3, pp.16-25, May 2007.

[2] P. Chen, et al. "Instrumenting wireless sensor networks for real-time surveillance," *Robotics and Automation, 2006. ICRA 2006. Proceedings 2006 IEEE International Conference on*, vol.15, no.19, pp.3128-3133, May 2006.

[3] S. Barbarossa, S. Sardellitti, and P. Di Lorenzo, "*Distributed Detection and Estimation in Wireless Sensor Networks*," In Rama Chellappa and Sergios Theodoridis eds., Academic Press Library in Signal Processing, Vol. 2, Communications and Radar Signal Processing, pp. 329-408, 2014.

[4] J. F. Chamberland and V. V. Veeravalli, "Decentralized detection in sensor networks," *Signal Processing, IEEE Transactions on*, vol.51, no.2, pp.407,416, Feb 2003.

[5] J. J, Xiao and Z. Q. Luo, "Universal decentralized detection in a bandwidth-constrained sensor network," *Signal Processing, IEEE Transactions on*, vol.53, no.8, pp.2617-2624, Aug 2005.

[6] B. Chen, L. Tong, and P. K. Varshney. "Channel aware distributed detection in wireless sensor networks," *IEEE Signal Processing Mag*, vol.24, no.4, pp.16-26, July 2006.

[7] S. Barbarossa and S. Sardellitti, "Optimal bit and power allocation for rate-constrained decentralized detection and estimation," *Proc. EUSIPCO*, Marrakech, Morocco, 9-13 Sept. 2013.

[8] E. L. Lawler and D. E. WoodSource, "Branch-And-Bound Methods: A Survey," *Operations Research*, vol. 14, pp. 699-719, no. 4 (Jul. - Aug., 1966).

[9] J. Lofber, "YALMIP : A toolbox for modeling and optimization in MATLAB," *CACSD, IEEE International Symposium on*, vol.4, pp.284-289, Sept 2004.

[10] E. Nurellari, D. McLernon, M. Ghogho and S. Aldalahmeh, "Optimal quantization and power allocation for energy-based distributed sensor detection," *Proc. EUSIPCO*, Lisbon, Portugal, 1-5 Sept. 2014.

[11] E. Nurellari "Optimum fusion rules with optimized sensor transmit power for distributed sensor detection," *University of Leeds, technical report*. Available: http://bit.ly/Twsa6o.