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Quantifying bounded rationality: Managerial behaviour and the Smith Predictor

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QUANTIFYING BOUNDED RATIONALITY: MANAGERIAL BEHAVIOUR AND THE SMITH PREDICTOR

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ABSTRACT: The concept of bounded rationality in decision making and research on its relation to aggregate system dynamics is examined. By recasting one such example of a dynamic system, the Beer Game, as a Smith predictor control system a natural measure of the level of bounded rationality in the system is derived. A stability analysis is then employed to support and qualify the assertion that the level of bounded rationality can adversely affect the aggregate dynamic behaviour of such supply chains. The analytical basis of these calculations enables the quantification of the potential cost improvements resulting from more desirable supply chain dynamics. This approach is designed to inform the strategic investment decision to purchase computational aids in order to overcome the level of bounded rationality in the system.

1) Introduction

The disciplines of Management Science and behavioural psychology have produced a lot of evidence supporting the assertion that there are severe limitations on the thinking and reasoning power of the human mind [Simon, 1979; Hogarth and Reder, 1987]. Since many companies today still rely substantially on human decision making, both with and without the use of computational aids, a method for moving from the micro-level of the individual actor to the resultant behaviour of the macro-system should enlighten understanding of undesirable system behaviour. A seminal piece of work [Sterman, 1989] accomplished this task by generating the macro-dynamics of a multi-echelon production-distribution system experimentally in a role playing exercise nicknamed 'The Beer Game'. An anchoring and adjustment heuristic for stock management was found, statistically, to reproduce the actual dynamics well. The parameter values thus calculated were taken as evidence of the cognitive characteristics of human decision making in this scenario. A useful insight into the causes of the dysfunctional performance of the overall system was achieved and supply chain thinking was subsequently influenced by this work (see [Riddalls and Bennett, 2000] and the references therein).
The complexity of Sterman's discrete time model militated against the quantitative analysis of the dynamic properties of the system and so the influence of model parameters on system performance was gauged only through observations based on the empirical results. Although these proved to be insightful in the context of management science, they neglected some important factors. For instance, the effect of different lead time delays was not examined. By considering a linear continuous time delay differential equation model of the beer game we aim to redress this analytical deficit and investigate the impact on the system dynamics of all the pertinent model parameters. This work enables a more detailed quantification of the influence of the various components of the decision making heuristic and their interaction with external parameters like lead time delays.
In the next section we introduce the model postulated by Sterman and enumerate his conclusions. These relate to the theory of bounded rationality [Simon, 1979] which

we set forth, as related to this area. In section three we develop our delay differential equation model of the beer game and recreate the characteristic dynamics observed by Sterman, which thus serve to validate this approach. We then undertake a detailed analysis of the dynamic properties of the system and relate these back to Sterman's conclusions. Consideration of the model as a control system yields particular insight since a little manipulation recreates the system as a Smith predictor, a well known mechanism for controlling time delay systems [Marshall, 1979]. The new interpretation of a particular model parameter as the degree of model mismatch in the Smith predictor explains the poor system performance and supports the theory of bounded rationality. Similarly, knowledge of general properties of Smith predictors prompts simple remedies for improving the system's performance.

In section four we use the ideas thus quantified in a method to reconcile cost based trade-offs designed to improve the performance of the system by overcoming bounded rationality.

2) The Beer Game and Bounded Rationality

Before the advent of the notion of bounded rationality classical economic theory was underpinned by the assumption that human behaviour is rational and optimising [Sterman, 1987]. The invariable axioms of omniscience and optimality attributed to humans in effect obviated their consideration in aggregate system models which, incidentally, were rendered much simpler as a result. One of the first researchers to question this approach was Herbert Simon, a recipient of the Nobel prize for his work in this area, among others [Simon, 1979]. Simon defines bounded rationality in the following way [Simon, 1957]:

> 'The capacity of the human mind for formulating and solving complex problems is very small compared with the size of the problems whose solution is required for objectively rational behaviour in the real world or even for a reasonable approximation to such objective rationality.'

Behavioural Decision Theory (BDT) was developed to identify the cognitive limitations in the perception and processing of information [Sterman, 1987]. It also seeks to illuminate how decisions are made and highlight systematic deviations from objective rational behaviour. What has become known as the Carnegie School of Thought [Morecroft, 1983] contends that the behaviour of complex organisations can only be understood by taking into account the psychological and cognitive limitations of its members. However, Sterman points out that 'Discovering and representing the decision rules of actors is subtle and difficult' [Sterman, 1987]. Direct experiment offers a means of elucidating these rules and simultaneously creating a link with the consequent aggregate performance of a system.

One such experiment, a stock management problem, is documented in [Sterman, 1989]. The stock management problem is an archetypal dynamic decision making task in which the stock level (system state) is regulated about a desired target in response to an external demand. Typically, there is a lag between the initiation of the control action and its effect and some mechanism to account for this delay is desirable. In the beer game a four echelon production-distribution system is simulated and each echelon can be described in two parts, by (i) the equations governing the physical stock and flow structure of the system and (ii) the decision rule used by the manager to regulate the stock level. Using Sterman's variables, the stock and flow structure equations are

$$S(t)=\int_{t_0}^{t}[A(\tau)-L(\tau)]d\tau+S(t_0) \quad (1)$$

$$SL(t)=\int_{t_0}^{t}[O(\tau)-A(\tau)]d\tau+SL(t_0) \quad (2)$$

where $S(t)$ is the stock level, which is the accumulation of the acquisition rate, $A(t)$, over the loss rate, $L(t)$, all at time t. Losses arise from demands being placed upon the inventory stock and acquisitions constitute arrivals of stocks from the suppliers. In a manufacturing echelon acquisitions would comprise production of the finished good. $S(t_0)$ and $SL(t_0)$ are the values of the inventory stock and supply lines at the initial time, t_0. The control variable is $O(t)$, the orders placed on the supplier. One of the consequences of bounded rationality is that humans recognise their computational limitations and rely on rules of thumb to make decisions [Cyert and March, 1963; Kleinmuntz, 1985]. Sterman postulates the following rule of thumb for determining orders

$$O(t)=\hat{L}(t)+\alpha_S\{S^*(t)-S(t)\}+\alpha_{SL}\{SL^*(t)-SL(t)\} \quad (3)$$

In addition, a cut-off comes in to effect when orders reach zero so they never go negative. This rule of thumb is known as an anchoring and adjustment strategy [Tversky and Kahneman, 1974]. It estimates an unknown quantity by first recalling a known reference point, (the anchor, $\hat{L}(t)$) and then adjusting for the effects of less salient factors whose calculation requires some 'mental simulation'. In this case the anchor is a forecast for demand based on recent levels. $S^*(t)$ and $SL^*(t)$ are the desired stock level and supply line, respectively. The parameters α_S and α_{SL} are the proportions of the discrepancies between the actual an desired stock level, and the actual and desired supply lines, respectively, that are fed back into oeders at any one time. It is these equations which create a negative feedback structure in the model and prompted the Authors' consideration of it as a control system. Indeed, Morecroft [1983] asserts that bounded rationality is embodied in the feedback structure of such systems. Expounding this idea, Cyert and March [1963] observe that bounded rationality causes decisions to be based on relatively few sources of information of low uncertainty and which are the focus of symptoms. This creates feedback loops in the sense that decision makers favour using information based on the current conditions of the local environment.

In Sterman's experiments groups of players constituting a four echelon supply chain make ordering decisions in isolation based on local information. The simulation is split into weeks and the experiment is thus regarded as discrete time system. The parameters α_S and α_{SL}, amongst others, are estimated for each echelon from the role playing, which is carried out many times with different participants. Certain notional costs are attached to holding stocks and to stockouts and players are told to minimise these following a step increase in demand at the retailer echelon.

The trials are characterised by instability, oscillation, phase lag and amplification. Inventory levels oscillate wildly and with increasing amplitude and lag away from the retailer. This behaviour would be undesirable in a real system because it implies there are alternating periods when inventory stocks are surplus to requirements, which increases the storage and handling costs and risks of obsolescence. Similarly it increases the risk of stockouts when inventory is depleted, thus damaging service levels. For the factory echelon wildly varying production rates inhibit the most economical scheduling of jobs on machines. Similarly, uncertain demand may increase the frequency of ordering, which in turn may increase costs if each order incurs a fixed charge or quantity discounts are not exploited. Wildly oscillatory inventory levels and order rates also make the implementation of the rules of thumb decision policy more difficult since volatility reduces the timeframe during which data is accurate and thus hastens the calculation of policy decisions, which may therefore be impaired.

Observations of the dynamics highlight a strong correlation between poor system response (as outlined above) and the ratio of the two feedback parameters. Sterman thus defines a new variable, $\beta = \alpha_{SL}/\alpha_S$ which, through experimentation, is found to be less than one. This is a plausible result since managers may be expected to pay more attention to their inventory stock, which has a direct and immediate impact on profitability, than on the supply line, which is often hard to perceive and quantify. In an attempt to explain the influence β on the system dynamics Sterman interprets β as the fraction of the supply line taken into account by each decision maker. If $\beta = 1$ he asserts that subjects fully recognise the supply line and do not over order. If $\beta = 0$ the amount of goods on order is ignored. This is an appealing interpretation since it is intuitively reasonable that an imperfect accounting of the supply line should lead to transient periods of over-ordering when demand increases and an eventual surfeit of stocks which triggers cuts in ordering rates which are similarly disproportionate. This would lead to oscillation and amplification (along the supply chain) of both ordering rates and inventory levels. However, it is clear that α_{SL}, not β, represents the proportion of the supply line taken into account, by definition. If $\alpha_S < 1$ but $\beta = 1$, this does not mean the supply line is fully accounted for, merely that it is accounted for in the same proportion as the inventory discrepancy. Hence when α_{SL} is small managers are still prone to over-ordering, following the sequence of events enumerated above, and yet, through observations of the beer game paper, if $\alpha_S \approx \alpha_{SL}$, this does not lead to oscillations. An explanation is not intuitively accessible. Also, Sterman offers no opinion on the influence of α_S or the delay magnitude on the dynamics. In the next section, by recasting the system as a Smith predictor we show that β has a natural interpretation as a direct measure of the level of bounded rationality evinced by decision makers. In this light, an explanation for its influence on the system dynamics accrues from the well known theory of Smith predictors and is consistent with the thesis that bounded rationality impairs the aggregate performance of complex systems [Morecroft, 1983]. We shall also investigate analytically the influence of α_S on the dynamics and interactions with the magnitude of delivery lead time delays.

Sterman's statistical analysis supports the conclusion that in the absence of a calculus to determine optimal inventory targets, subjects aim to anchor their stocks at their initial levels. The resulting estimates (using (5)) of SL^* are observed to be smaller than that needed to sustain an adequate supply of arriving goods during the order lead time. The desired supply line should be

$$SL^* = \hat{\lambda}\Phi^*(t) \tag{4}$$

where $\hat{\lambda}$ is the estimated order leadtime and $\Phi^*(t)$ the desired throughput. Sterman attributes low values of SL^* to a misperception of time lags but, in fact, it is not clear from the analysis whether this is instead due to inadequate estimates of $\Phi^*(t)$, or a combination of both. We shall see in the next section that β itself also has an effect on this calculation.

3) Derivation of the Beer Game Model

In this section we develop a continuous time model of a single echelon in the discrete time beer game system. Continuous systems are analytically more tractable than discrete systems and under certain circumstances can approximate the behaviour of the latter. We have run a beer game simulation model in its original form and a continuous approximation and found the dynamics to be similar for systems with significant delays. The nonlinearity of the beer game model constitutes an impediment to progress in its analysis and understanding. Recognising this we shall adopt a few tricks to circumvent the introduction of nonlinearities into our model and thus facilitate analysis based on linear system theory. The first nonlinearity arises in (3) when the order rate falls below zero, which is physically unrealisable. By simply restricting our attention to high volume systems and defining the time unit in the 'per unit time' variables (e.g. orders per unit time) to be sufficiently large we eliminate the nonlinearity. Indeed, in all but one of the typical responses plotted by Sterman, order rates never reach zero, even without using this device. The second nonlinearity emanates from stockouts and a subsequent inability to supply all that is demanded of a particular echelon. Negative inventory itself is acceptable if we define $S(t)$ to be the excess of stock on hand over backorders (orders not yet satisfied). Yet interruptions to the supply line caused by stockouts constitute a nonlinearity which may instead be mimicked by transient increase in the delay magnitude, thus preserving the linearity of the system at a cost of introducing non-stationarity. We shall see in the next section that stockouts can push the system into regions of instability.

To close the loop in the system we must define the time delay for which orders are satisfied. In many supply chains this is contractually fixed between partners, but may vary in the event of stockouts. This means that

$$A(t) = O(t-\lambda) \tag{5}$$

where λ is the delay between placing an order on an upstream echelon and receiving it. Now the supply line can be measured thus

$$SL(t) = \int_{t-\lambda}^{t} O(s)ds \tag{6}$$

Using (4), (5) and (6), (3) becomes

$$O(t) = \hat{L}(t) - \alpha_S S(t) + \alpha_{SL}\left[\hat{\lambda}\hat{\Phi}(t) - \int_{t-\lambda}^{t} O(s)ds\right] \tag{7}$$

where we can assume, without loss of generality since the system is linear, that $S^* = 0$. Now differentiate (2) and (7) then combine and use (5), to get

$$\frac{dO(t)}{dt} = -\alpha_{SL}O(t) - (\alpha_S - \alpha_{SL})O(t-\lambda) + \frac{d\hat{L}(t)}{dt} + \alpha_{SL}\hat{\lambda}\frac{d\hat{\Phi}(t)}{dt} \tag{8}$$

Or, equivalently,

$$\frac{dO(t)}{dt} = -\alpha_S \beta O(t) - \alpha_S (1-\beta)O(t-\lambda) + \frac{d\hat{L}(t)}{dt} + \alpha_S \beta\hat{\lambda}\frac{d\hat{\Phi}(t)}{dt} \tag{9}$$

This delay differential equation forms the basis of the subsequent analysis of a single echelon system. Figure two shows a representation of the system as a control system in the Laplace domain.

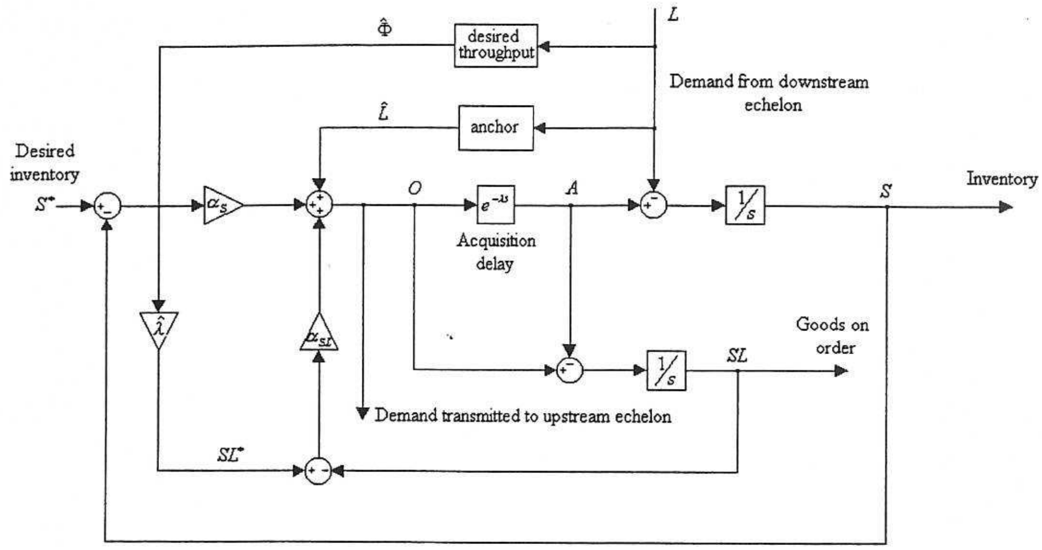

Figure 1. Representation of a single echelon in the beer game.

One of the most realistic ways to calculate both $\hat{L}$ and $\hat{\Phi}$ is thus:

$$\hat{L}(t) = \frac{1}{T}\int_{t-T}^{t} L(s)ds \tag{10}$$

i.e. an average of the past T weeks of demand. Using the same method for both calculations yields

$$\begin{aligned}\frac{dO(t)}{dt} &= -\alpha_{SL}O(t) - (\alpha_S - \alpha_{SL})O(t-\lambda) \\ &+ \frac{1}{T}(\alpha_{SL}\hat{\lambda} + 1 + \alpha_S T)L(t) - \frac{1}{T}(\alpha_{SL}\hat{\lambda} + 1)L(t-T)\end{aligned} \tag{11}$$

Figure 2 shows the response of the system with the following parameters and various combinations of α_S and α_{SL}.

$\lambda = 6$ weeks, $\hat{\lambda} = 6$ weeks, $T = 10$ weeks, $\bar{i} = 200$ units.

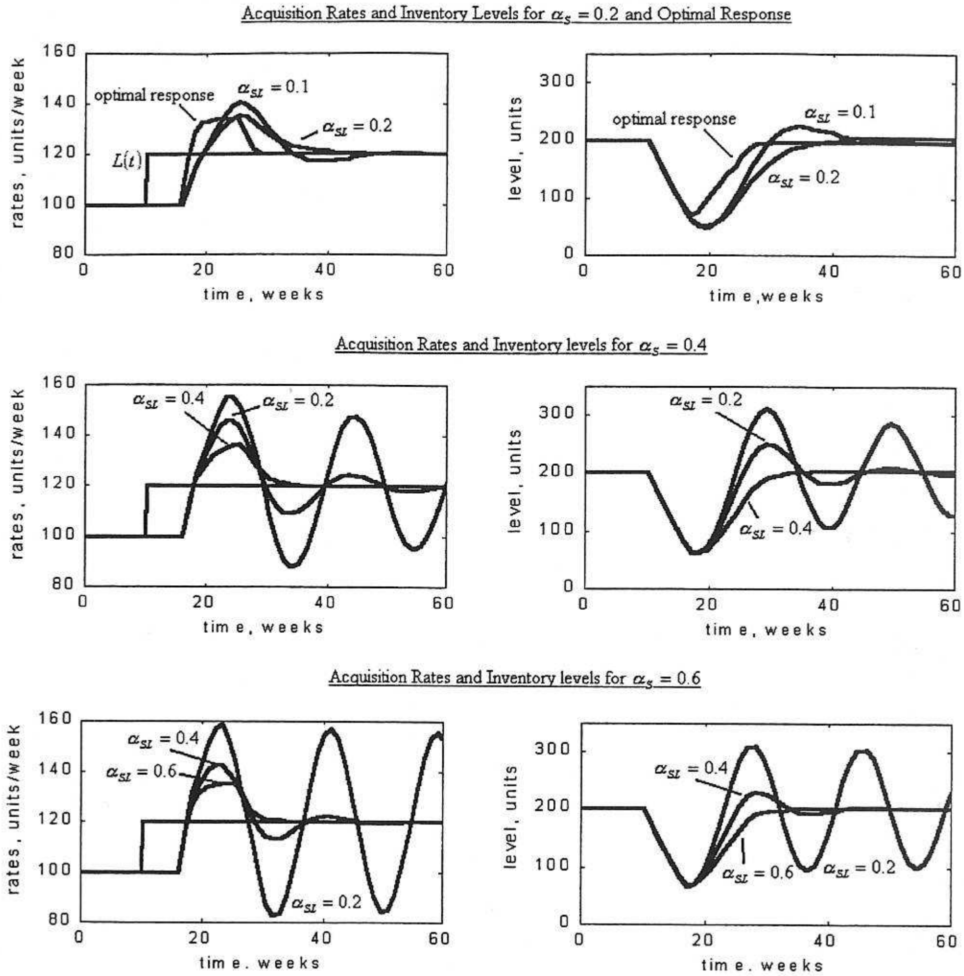

Figure 2. Dynamics of a single echelon in the beer game.

As one can see, the demand is a step function (as used by Sterman) of 20% of the initial rate. Since the system is linear, the superposition principle ensures that step functions can be used to approximate any integrable signal and so its choice is entirely general and not calculated to achieve any particular kind of pathological behaviour. The top graphs show the 'optimal' response, when $\alpha_S = \alpha_{SL} = 1$, i.e. each discrepancy is entirely accounted for. Notice this simulation exhibits the swiftest response yet the smallest peak and inventory depletion and no oscillatory behaviour. Looking at all the responses one concludes that it is not the specific values of α_S and α_{SL} in isolation that cause oscillatory behaviour, rather their ratio, β. Perhaps this is why it is specifically defined by Sterman in terms of the other parameters. In each case when $\alpha_S = \alpha_{SL}$ or $\beta = 1$, the response is swift and non-oscillatory. For smaller values of α_S the response is slower and the inventory discrepancy thus greater, yet these effects are small compared to the behaviour engendered by even a small difference between α_S and α_{SL}. In the top two graphs a discrepancy of $\beta = 0.5$ results in slight oscillation, but as this ratio decreases in the lower graphs to 0.25 and 0.33, so too does the amplitude of the oscillation. Sterman concludes that small values of β reflect an incomplete accounting for the 'supply line' leading to transient periods of overordering when demand increases and an eventual surfeit of stock which triggers cuts in ordering rates which themselves are similarly disproportionate. This leads to oscillation both in ordering rates and inventory levels. However, given our assertion that α_{SL} itself, not β directly measures the level of accounting of the supply line (by definition), it is not intuitively clear why this disruptive behaviour occurs irrespective of the particular values of α_S and α_{SL} and is only sensitive to their ratio. Given our interpretation, when α_{SL} is small the manager is still prone to over-ordering and yet if $\alpha_S \approx \alpha_{SL}$ then this does not lead to oscillation. In the next section we shall quantify the influence of β through a stability analysis and offer a consistent and intuitively attractive explanation for this that relates to the bounded rationality of the decision makers.

4) Analysis of the Beer Game Model

The preservation of linearity in the beer game system enables the consideration of the dynamics of each echelon in isolation with the understanding that the global dynamics are composed additively of those of each echelon. The asymptotic stability of a single isolated echelon in the beer game is determined by the unforced part of (9):

$$\frac{dO(t)}{dt} = -\alpha_S \beta O(t) - \alpha_S (1-\beta) O(t-\lambda) \tag{12}$$

The stability properties of such a system were derived in [Bellman and Cooke, 1963] for constant delays and [Niculescu et al., 1997] for varying delays, and result in the following:

The system (12) is asymptotically stable independent of the delay magnitude (IoD) if and only if $\beta \geq \frac{1}{2}$.

If, on the other hand, $\beta < \frac{1}{2}$, *then the maximum delay for which (14) is asymptotically stable is*

$$\lambda^* = \frac{\arccos\left(\frac{\beta}{\beta - 1}\right)}{\alpha_S \sqrt{1-2\beta^2}}, \quad or \quad \lambda^* = \frac{1}{\alpha_S (1-\beta)} \tag{13}$$

for constant and varying time delays, respectively.

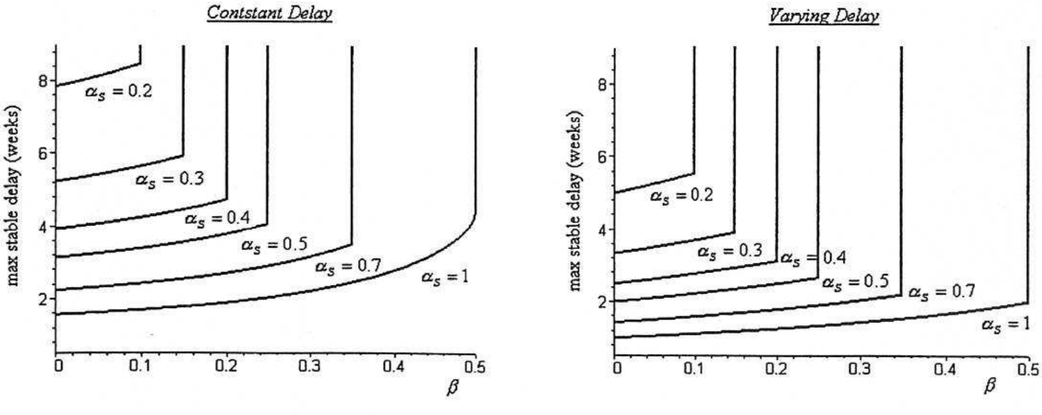

Figure 3. Stability regions for (12) and various parameter combinations

The stability regions for various α_S and varying β are shown in figure 3, for both fixed and varying delay systems, as the area to the right and below each curve. The curves become straight lines when the system is stable IoD. It is apparent that β plays the most important role in determining stability, especially stability IoD. For small values of β the range of delays resulting in a stable system is severely restricted given a particular α_S. When delays vary this range is reduced further as shown in the right hand plot. We can conclude that for robust stable systems (i.e. those which cope well with interruptions to the expected supply line), managers should attach at least half as much importance to the supply line as the inventory level. For good dynamic behaviour (i.e. swift response, no overshoot, small inventory discrepancy) we have seen that systems with $\beta = 1$ perform best and are 'most' stable (i.e. furthest away, in the parameter space, from unstable systems). Having quantified the significance, already qualitatively known, of β, we now offer an intuitive interpretation of its role based on well known principles from control theory. Some block diagram manipulation renders figure one as a Smith predictor control system as shown in figure four [Marshall, 1979].

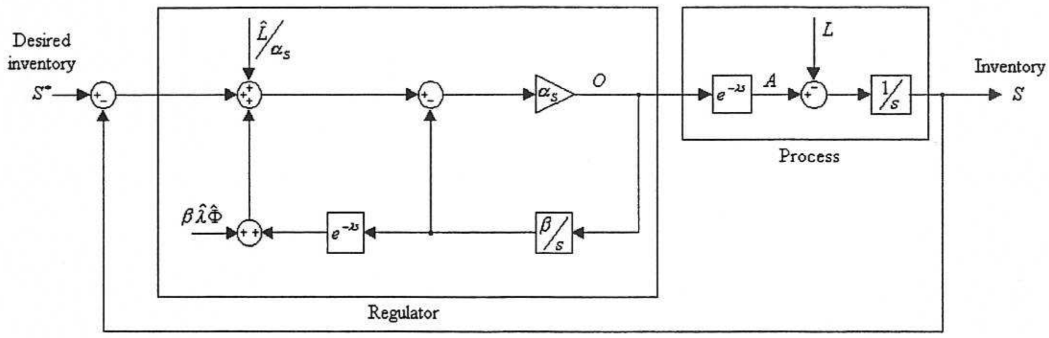

Figure 4. Single echelon as a Smith predictor system.

Smith developed a delay compensation technique in which the regulator (shown in the left hand box in figure 4) uses a mathematical model of the actual process (shown in the right hand box). What this means in our application is that the production manager subconsciously uses his own cognitive model of the system to guide decision making. When his cognitive model and the actual process are identical the time delay is removed from the characteristic equation and control can be based on the delay free part of the process (in this case a pure integrator, $1/s$). With this interpretation α_S fulfils the role of the controller gain, which accords with its influence over the speed of response, as demonstrated in figure 2. More importantly, β now has a natural interpretation as the measure of the mismatch between the process model and the model use by the regulator (i.e. subconsciously by the production manager). When $\beta = 1$ the two models are equal, but lower values of β signify a degree of mismatch. It is well known [Marshall, 1979] that significant mismatch causes instability via the introduction of a time delay component in the characteristic equation. Thus, contrary to Sterman's interpretation of β, we now see that it is a direct measure of the level of bounded rationality inherent in decision making since it measures the difference

between the model of the process used for decision making and the actual process. Furthermore, for this application, we have quantified and calibrated how bounded rationality can lead to poor performance in a dynamical system, as outlined in [Morecroft, 1982]. Regarding the system in control theoretic terms, it is now apparent why bounded rationality can be overcome by a regulating rule with $\alpha_S \approx \alpha_{SL}$: If $\alpha_{SL} < 1$ reflects uncertainty over the supply line, then one way to compensate for this 'model uncertainty' is to reduce the gain, α_S so that $\alpha_S \approx \alpha_{SL}$ or $\beta \approx 1$. This is a standard procedure in control system design.
Returning to figure 4, the load disturbance, $L(t)$ is compensated for in the usual way with an additional input to the controller, $L(t)/\alpha_S$. However, unlike most Smith predictor regulation systems, ours demands a nonzero steady state error to ensure a pipeline of orders which are not yet in receipt. Therefore an addition input, $\beta\hat{\lambda}\hat{\Phi}$ is required in the controller. Since we actually require a supply line of $\hat{\lambda}\hat{\Phi}$, we see that low values of β affect desired levels for the supply line in exactly the same way as it does their variations.

5) Overcoming Bounded Rationality

We have seen how bounded rationality impairs the dynamic performance of each echelon in the beer game and have justified the choice of β as a direct measure of its extent. Relating the stability characteristics of each echelon to β and the other system parameters (figure 3) encourages the investigation of steps to overcome the level of bounded rationality and so improve system performance. However, these steps may be costly, perhaps involving training or the purchase of computational aids. So managers may be forgiven for their reluctance to embark on such steps without an idea of the potential rewards. As we shall see in one example, it may be that managers' existing rules-of-thumb are already suited to the cost structures in the system and the other dynamic factors (like the delay length). This section aims to indicate when change may be necessary and quantify the potential rewards and so furnish managers with a dynamic supply chain trade-off methodology. It might seem obvious from the preceding results that poor system performance can be mitigated through steps which lead to an ordering policy with $\beta = 1$. Since then bounded rationality is eliminated and oscillations are suppressed. As we shall see, however, although low values of β do reflect a mismatch between the manager's mental model of the supply chain and the actual supply chain, this may reflect genuine variability. Furthermore, the particular cost structure of an echelon may favour slight oscillation (in order to achieve a swift response) at the expense of more inventory costs.
The methodology proceeds as follows:

i) Estimate α_S and α_{SL}.
ii) Locate parameter values on a stability chart.
iii) Simulate step response for alternative α_S and α_{SL} and use stability chart to rationalise best choice. Calculate cost improvement of this step response.
iv) Estimate cost of effecting improved parameter values.
v) Carry out cost trade-off.

Step (i)

α_S and α_{SL} (and thus β) may be calculated from historical data using least squares estimation in much the same way as the beer game experiment.

Step (ii)

Figure 5 shows the stability regions in the α_S-α_{SL} plane, as calculated from (13), for various delay values. The utility of such a diagram lies in its qualitative description of the current stability characteristics (point P, say) and possible improvement options (indicated by the arrows) and therefore the potential for dynamic improvements. For each system the degree of oscillation is most strongly influenced by the delay magnitude and the proximity of P to the unstable parameter region (the grey area), as measured by the 'distance' d. The smaller the distance, the more oscillatory the response, other things remaining equal. We illustrate this idea by simulating the step responses in figure 6 of two systems with the same α_S and β (as shown in figure 5) but different delay values $(\lambda = 4 \text{ weeks and } \lambda = 10 \text{ weeks})$. Since d is much smaller for the larger delay there is much more oscillation in inventory levels and orders made and thus arrivals.

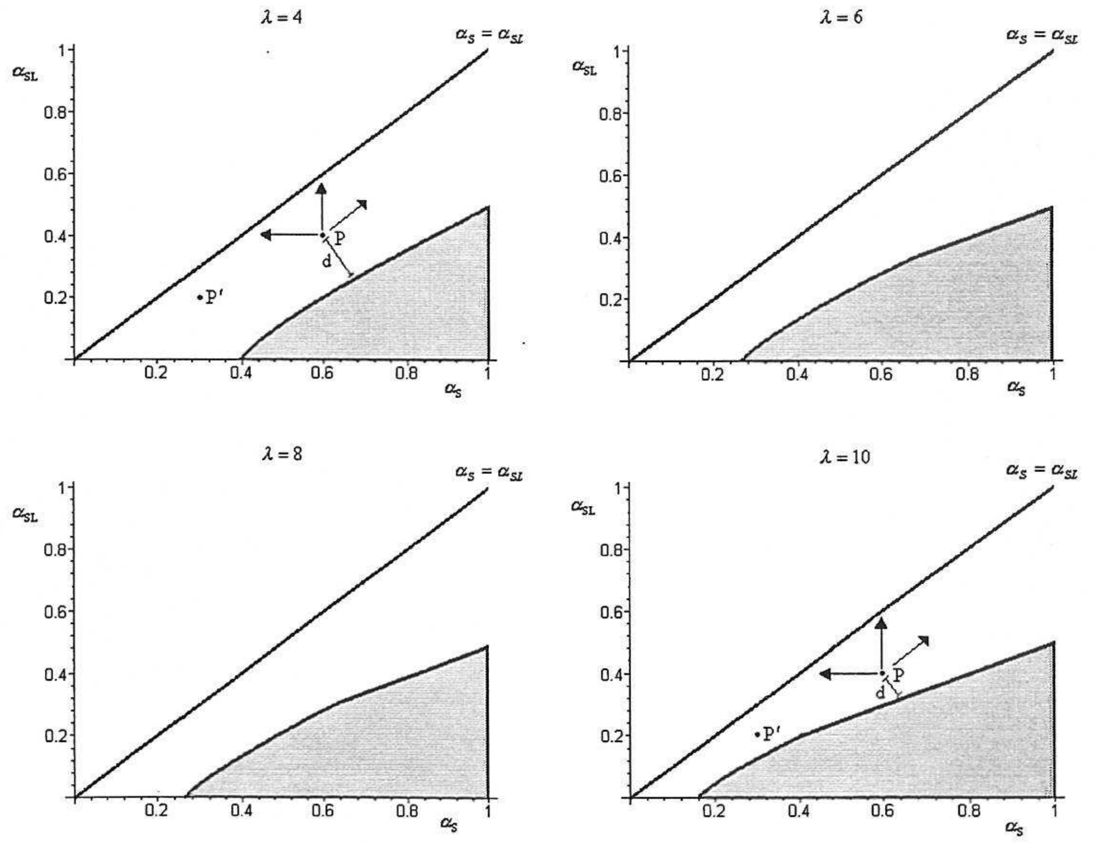

Figure 5. Stability regions for various delay values.

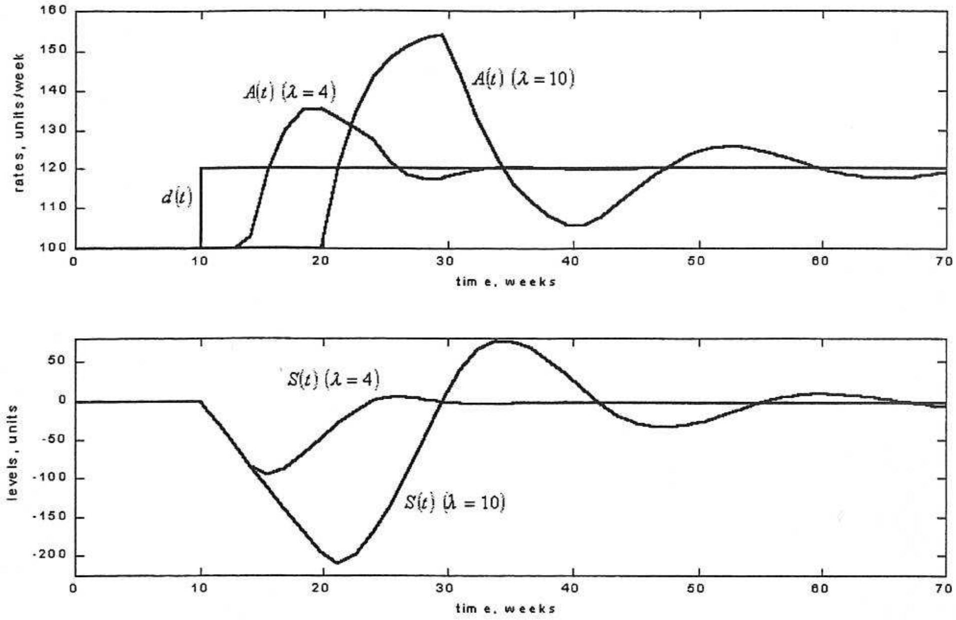

Figure 6. Step responses for different delay values.

Step (iii)

The stability charts in figure 5 can be used to suggest the best ways to improve the dynamic response of the system. On the basis of these indicators, this step quantifies the cost improvements resulting from these suggestions. The system response depends on the current parameter values and the delay magnitude. We have seen that one of these parameters, β, is a direct measure of the level of bounded rationality implicit in decision making. In terms of system stability, high values of β are desirable since then oscillation is eliminated. The cost implications are clear: Periods of excessive and depleted inventory levels are reduced. Order variability is also reduced cutting fixed order costs and transportation costs. Therefore we shall concentrate on steps to increase β. On the stability diagram well behaved systems with high values of β (i.e. near 1) can be found near the 45° line. There are various ways to increase β since it is the ratio of two constituent variables. We now pick three of them and, in order to evaluate the gains achievable, simulate the step response of each, attaching costs to excessive and depleted inventory holdings as follows:

$$\text{Total cost} = C = \int_{t_0}^{t_\infty} \left[c_1 u(S(t)) + c_2 u(-S(t))\right] dt, \tag{14}$$

where u is the unit step function, $[t_0, t_\infty]$ is the time interval of the transient dynamics, c_1 is the cost penalty per unit per week of holding excess inventory (over zero, without loss of generality) and c_2 is the cost penalty per unit per week of inventory depletions. More complicated cost structures may be used, for instance by increasing c_2 as a quadratic in S to penalise actual stockouts more than depletions below a safety stock level. A term accounting order variability may also be explicitly included since this affects fixed ordering and deliver costs. However (14) tends to account for this implicitly since variations in inventory levels usually imply variations in orders (since demand is a step response). Figure 7 shows the percentage cost change, over the standard system response as shown in figure 6, resulting from 3 methods used to increase β. The top plots use the cost structure $c_1 = 1, c_2 = 2$ and, for comparison, the bottom plots use $c_1 = c_2 = 1$. Note that the qualitative results are relatively insensitive to the cost assumptions made and thus are primarily influenced by the changing values of β. This supports the applicability of this methodology even when detailed costings are unavailable.

Decreasing α_S slows the responsiveness of the system to demand changes. This may be desirable when damping down oscillations but increases the risk of stockouts through greater inventory depletions. The leftmost plots in figure 7 show that such a policy marginally reduces costs for the system with the longer delay. Figure 5 demonstrates why: the distance, d, from the instability region increases, damping volatility. The concomitant loss of responsiveness is partially offset because the oscillatory nature of the original system (as shown in figure 6) contributes to the responsiveness. This highlights the complex interactions between stability, oscillations and responsiveness. Were the system at the point P', reducing α_S would prove even less effective since the large instability region (figure 5) implies that d can only be increased marginally. Decreasing α_S for the smaller delay system increases costs because there was very little oscillation to start with (d is relatively large and the delay magnitude small) and so all the additional costs emanate from a greater inventory depletion. Reducing α_S can be regarded as a negative step since it reduces the agility of the system. Since it delivers only small cost savings in some instances, we do not recommend it.

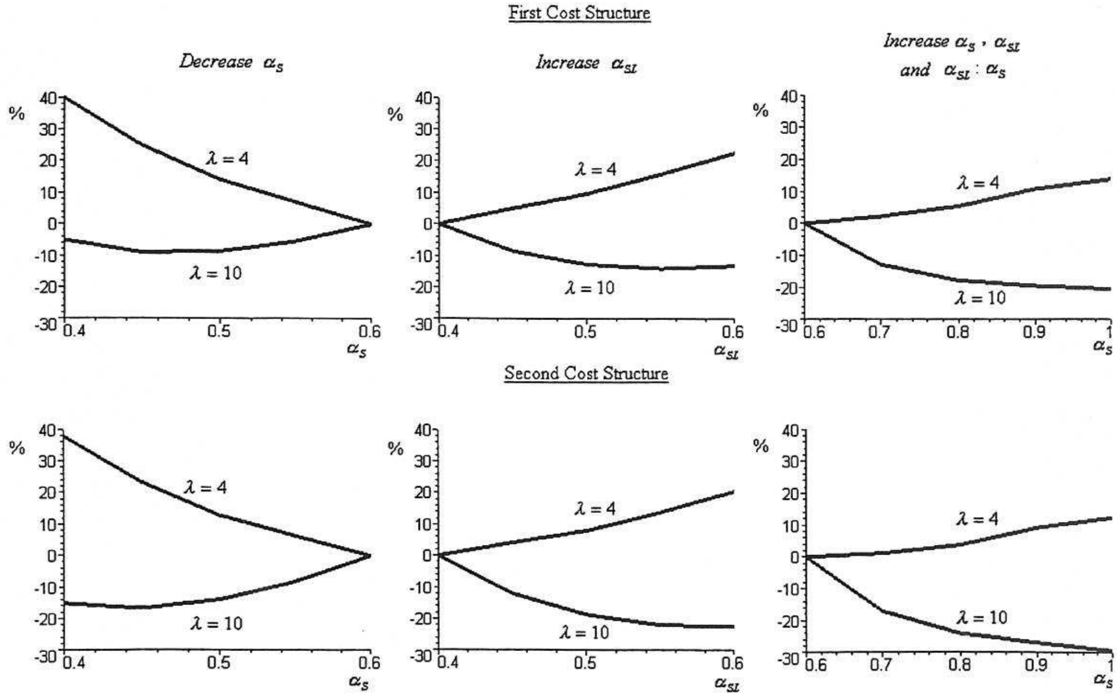

Figure 7. Percentage cost changes with different parameter combinations.

A more obvious way to increase β is by increasing α_{SL}, the results of which are shown in the central plots of figure 7. Again, for the longer delay, costs are reduced since d is increased from a small value to a relatively large one and oscillations are therefore diminished. However, for the smaller delay value, increasing α_{SL} adds costs to the step response of our standard system. Simulations show that again the small amount of oscillation in the original response is eliminated but again this was quite minor and its reduction has slowed the response speed thus incurring greater inventory discrepancy costs. Put simply, the system with $\lambda = 4$ is behaving pretty well to start with.

This conclusion is supported by the rightmost plots in figure 7, which show the effects of increasing both α_{SL} and α_S whilst increasing their ratio, β, until it is 1. For the larger delay the original small value of d is increased substantially and hence oscillation is eliminated, thus reducing costs substantially. However, for the smaller costs actually increase slightly, for the reasons just noted. In his paper, Sterman asserted that the optimum system parameters were both unity, since then all discrepancies are fully accounted for in decision making. However, we have seen that even for a relatively simple cost structure, there is a subtle trade-off between volatility and the speed of response which implies that other parameter choices may be more suitable. For real inventory systems fixed ordering and transportation costs may imply that lower values of α_S are more desirable. In the light of our interpretation of α_S as the controller gain in a control system, it is directly related to the responsiveness of the system to varying demand. When $\alpha_S = 1$ the system corrects for inventory discrepancies as fast as possible given the delay and other parameter values. However, it may not be cost effective to follow a varying demand signal with a similar order pattern if this implies placing more frequent orders (each perhaps incurring a fixed charge) and increased transportation costs through having to deliver more one week (perhaps requiring 2 lorries) and less the next (the lorry being half empty). Hence it may be more cost effective to smooth orders and hold more safety stocks.

Step (iv)

Steps (ii) and (iii) indicate, for a particular system, whether overcoming bounded rationality has the potential to secure cost savings. We saw that for systems with relatively small delays discounting both supply line and inventory discrepancies was more efficient to an extent since it smoothed orders and inventory fluctuations whilst retaining a degree of responsiveness. In a real system we could not tell whether these parameter choices resulted from bounded rationality or the intuitive reasoning by managers as the correct choice. It may be in this case that bounded rationality has fortuitously resulted in acceptable behaviour. In any case, no action need be taken.

It is, however, intuitively reasonable that longer delays create more uncertainty and that therefore more mental effort is required to generate desirable dynamics. In these cases figure 7 shows that $\alpha_S = \alpha_{SL} = 1$ is the best parameter choice. Thus the improvement option is to train staff to make decisions via the heuristic (3) and (4) with these parameter values. The purchase of a computational aid to record unreceived orders and perhaps carry out this calculation itself may be required especially if there are many product lines to track.

Step (v)

The cost trade-off is relatively straightforward. To link the current cost scenario with the potential benefits shown in figure 7, we need an estimate of the total current yearly inventory costs of the company. Reducing these by the various percentages indicated in figure 7, for different combinations of α_S and α_{SL} produces a yearly saving which we can compare to the cost of effecting such a change as outlined in step (iv). In this way one can calculate the yearly saving (if any) accruable by overcoming bounded rationality. If there is no saving in year one it would be easy to calculate over what period an investment might pay off.
The use of percentage cost improvements based on the step response is justified by the observation that any demand signal can be approximated by a sum of step functions and, since the system is linear, the superpostion principle ensures that the system response is composed of the individual responses. When stockouts in the upstream echelon significantly affect the supply line this can be incorporated in the calculation by weighting the results with a step response using a longer delay. In the absence of long term demand predictions, this is the best way to gauge the potential benefits.

Example

Returning to the system with $\lambda = 10$ and observing that the lowest costs result from $\alpha_S = \alpha_{SL} = 1$, we make the following trade-off: Suppose the current annual inventory cost is £10,000. It is postulated that bounded rationality can be overcome by the purchase of a PC to record goods on order and calculate orders to ensure a steady supply of goods. Figure 7 show that a step response cost saving of 20% can be secured in this manner (using cost structure 1) therefore an estimate of the potential inventory cost savings per year is £2000. If the cost of purchasing a PC and training staff to use it is less than this amount then the investment may be expected to pay off within a year.

6) Conclusions

By recasting the beer game as a Smith predictor control system we have found a natural measure for the level of bounded rationality in the system. We then used a stability analysis to support the assertion that the level of bounded rationality can adversely affect the aggregate dynamic behaviour of the system. The analytical basis of these calculations enabled the reconciliation of the trade-off calculation concerned with overcoming the level of bounded rationality in the system and thus improving supply chain dynamics.

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