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Motion and observation in a single-particle universe

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Abstract. We outline an argument that a single-particle universe (a universe containing precisely one pointlike particle) can be described mathematically, in which observation can be considered meaningful despite the a priori impossibility of distinguishing between an observer and the observed. Moreover, we argue, such a universe can be observationally similar to the world we see around us. It is arguably impossible, therefore, to determine by experimental observation of the physical world whether the universe we inhabit contains one particle or many – modern scientific theories cannot, therefore, be regarded as descriptions of 'reality', but are at best human artefacts. Our argument uses a formal model of spacetime that can be considered either relational or substantivalist depending on one's preferred level of abstraction, and therefore suggests that this long-held distinction is also to some extent illusory.

Keywords: First-order relativity theory, formal philosophy, philosophy of spacetime, observational indistinguishability, underdetermination, formal physical models.

Abbreviations: CST – causal set theory; FOL – first order logic; FORT – first order relativity theory; GR – general relativity; SPU – single-particle universe.

Dedicated to István Németi on the occasion of his 70th birthday.

1. Introduction

It has long been understood that the concepts of space, time and motion are intimately linked, so that questions relating to the nature of spacetime depend inherently on one's understanding of motion itself (Huggett and Hoefer, 2009). Since motion is commonly taken to involve changes in the relative positions or velocities of distinguishable entities, the question arises whether motion can ever be considered empirically meaningful in the context of a universe containing precisely one pointlike particle (hereafter, a *single-particle universe*). Such a universe contains no distinguishable particles whose separation in space or time might be used as an empirical measuring rod, and there is no a priori distinction between an observer and what it observes.

In this paper we outline a mathematical description of a single-particle universe, and explain how a non-standard conception of motion can be defined within it. We then show how this form of motion can be used to define what it means for an observation to be made, in such a way that the single-particle universe is observationally similar to the universe we see around us. It is, therefore, arguably impossible to determine by scientific experimentation whether the universe we inhabit contains one particle or many. This in turn has implications for the debate concerning absolute vs relational motion, the question of substantivalism, and the status of scientific theory.

The structure of the paper is as follows. In section 2 we describe the underlying mathematical constructs use to express our model: Andréka et al.'s first-order relativity theories (section 2.2), and Stannett's finitary reformulation of quantum theory (section 2.3). In section 3 we describe our example of a single-particle universe and explain why (subject to certain caveats) one would expect it to be observationally similar to the world we see around us. We conclude in section 4 by considering some implications of our model.

2. Theoretical Development

If motion means anything in the context of a single-particle universe, it cannot mean relative motion, since there are no bodies relative to which changes in position can be defined. Moreover, because there are no bodies between which non-trivial relations can be defined, the underlying spacetime cannot be relational. In short, if we are to define what it means for the particle to 'move' within its universe, we must necessarily start by providing an account based on absolute motion in a substantivalist spacetime. Consider, therefore, a container spacetime $\mathcal{M} \equiv (M, g)$, where M is a continuously differentiable manifold[1] with associated metric tensor g. Let P be a single pointlike particle, and assume that P's motion is defined in absolute terms as a function $p : T \to \mathcal{M}$ where T is a partially ordered set (poset) of *absolute proper times* and $p(\tau)$ indicates the absolute spacetime position of P at absolute proper time $\tau \in T$.

The (relative) motions we see around us on a daily basis suggest that the paths followed by particles moving in $\mathcal{M}$ should be continuous

[1] Following Earman and Norton (1987), we consider the accompanying stress-energy tensor T to be contained within, rather than a constituent part of, spacetime; but we reject their identification of spacetime with the manifold M, adopting instead Hoefer's view that spacetime is more properly represented by the metric tensor g: "To give the metric field without specifying the global topology—always possible for at least small patches of space-time—is to describe at least part of space-time. By contrast, to give the manifold without the metric is not to give a space-time, or part of a space-time, at all." (Hoefer, 1996, pp. 24–25)

functions of type $\mathbb{R} \to M$, so that $T = \mathbb{R}$. At quantum scales we know that the structure of spacetime may be considerably more complicated than this suggests (Doplicher et al., 1995; Bojowald, 2012), but in fact the assumption that *absolute* proper time should be representable by $\mathbb{R}$ is not even justifiable at macroscopic scales, because it is important to distinguish between time *as experienced* (by a particle), and time *as it appears* (to an observer of that particle). The process of observation, by which absolute motions are re-interpreted in relational terms, need be neither passive nor straightforward, and there is no a priori reason why time-as-experienced and time-as-observed should carry the same order structure. (It is also necessary, therefore, to distinguish between the underlying manifold M in which an observer considers itself to move, and the manifold M' in which it is perceived to move by an observer. Observations are the basis of appearances, and appearances, we know, can be deceiving.) All we can safely suggest at this stage is that observation is a process by which observers perceive absolute trajectories (paths of type $T \to M$) as relative ones (paths of type $\mathbb{R} \to M'$, say).

As we argue below, it is entirely possible to obtain a coherent theory of observation even when absolute motion is assumed to be discrete ($T = \mathbb{N}$), and our model has the surprising consequence that 'observed reality' in a single-particle universe can appear essentially the same as the world we see around us. We are not claiming, of course, that a single-particle universe *must* be represented as delineated below, only that it is *possible* to define a 'reasonable' single-particle model in which the observable structures of everyday life emerge naturally.

The approach we adopt below requires us to consider the empirical (relational) behaviour of physical systems, so our first task is to explain how a relational description of spacetime can arise as a result of discrete observations by a point particle in an absolute single-particle universe. Since P is the only inhabitant of its universe we first need to ask, what do we mean by 'observation' in this context? This question cannot be answered definitively, so we adopt the strategy of defining 'observation' as loosely as possible.

2.1. Manifolds generated by P's observations

Let us assume that P performs an 'observation' at each absolute proper time $t \in T$. We have no way of establishing what P sees when it performs an observation, but for convenience we will refer to the observation outcomes as *locations*. For our purposes it is enough to assume that observations follow one another, so we shall take $T = \mathbb{N}$. We assume no notion of 'duration', since we want the concepts of (relational) space and time to be generated intrinsically from P's observations. We will nonetheless need to impose some structure on the set of locations, and to the extent that this structure will be ordered, algebraic, etc, we can say that its representation involves the use of "numbers" or "coordinates" – but it should be remembered that this is just a linguistic convenience. We are not suggesting that the particle itself is able to 'perceive' numeric or geometric structure. Formally, then, we assume

- a set $\mathcal{L}$ of locations;
- an internal clock, $T = \langle 0, 1, 2, \ldots \rangle$;
- a function $\mathbf{at} : T \to \mathcal{L}$.

We read "$x = \mathbf{at}(t)$" as "P coordinatizes itself to be at location x on its t'th internal clock tick". Note, we are not saying that P can identify its location in M – the set $\mathcal{L}$ is entirely abstract, and has no intersection with M'.

Our goal is to endow $\mathcal{L}$ with enough topological structure that it sensibly reflects the properties of a suitable general relativistic (GR) manifold $M' \equiv (M', g')$ – recall that we cannot assume that M and M' are identical – and we do so by choosing an M' in which $\mathcal{L}$ can be embedded as a dense subset. Given this paper's goals, we also require M' to be a model of the first-order theory GenRel^+ of general relativity (see section 2.2 for an overview of first-order relativity theories), but fortunately this constraint is automatically satisfied:

> "... our notion of timelike geodesic coincides with its standard notion in the literature on general relativity. All the other key notions of general relativity, such as curvature or Riemannian tensor field, are definable from timelike geodesics. Therefore we can treat all these notions (including the notion of metric tensor field) in our theory GenRel^+ in a natural way. ..." (Székely, 2009, p. 98)

The approach we adopt uses ideas from the *causal set theory* (CST) approach to quantum gravity (Dowker et al., 2004; Dowker, 2005). CST starts by assuming that the universe is inherently discrete, but differs from the development presented here in that it assumes an underlying notion of causality from which the structure of observed spacetime is induced. In contrast, we do not assume that P's observations carry an up-front notion of causality ($\mathcal{L}$ is simply a set); this is important, because only a small proportion of causal sets can be embedded faithfully in relativistic spacetimes (Smolin, 2006; Wüthrich, 2012).

Our choice of M' is essentially unconstrained: it is sufficient for us to consider any spacetime $M' \equiv (M', g')$ and any dense sequence

$\langle x_0, x_1, \dots \rangle$ of points within it, provided the set $\{x_n \mid n \in \mathbb{N}\}$ represents a random sampling of the points in M'. If we embed $\mathcal{L}$ in M' via the mapping $\mathbf{at}(n) \mapsto x_n$ and endow $\mathcal{L}$ with the topology inherited from M':

- randomness ensures that the symmetries of $\mathcal{L}$ are well matched[2] by those of M';
- density ensures that every path in M' can be approximated arbitrarily closely by finite subpaths of P's trajectory.

We will write $\mathcal{L}'$ for the set $\mathcal{L}$ equipped with the topological and metric structure inherited from M'.

This approach has a further consequence that will prove useful below, when we come to discuss dynamical properties like action (section 2.3). The Einstein field equations for M' can be written $T_{\alpha\beta} = (G_{\alpha\beta} + \Lambda g'_{\alpha\beta})/(8\pi)$, where the Einstein tensor G can be determined from $g' \equiv g'_{\alpha\beta}$ (Misner et al., 1973, 410). Since all of the terms on the right hand side of this equation can be determined from g' once a value for Λ (the cosmological constant) has been specified, this allows us to determine the associated energy-momentum tensor, $T_{\alpha\beta}$. In other words, even though we have not assumed that P carries mass, we can nonetheless consider M' (and hence its dense subspace $\mathcal{L}'$) to carry a mass-energy distribution which correctly generates the curvature inherited via the metric tensor g'.

2.2. First-Order Relativity Theory (FORT)

Andréka, Németi and their colleagues have argued extensively that both the elegance and the subtlety of relativity theories can best be appreciated when formulated within first order logic (FOL) (Andréka et al., 2004, 2008, 2012; Madarász et al., 2007; Németi and Andréka, 2006).

In their first-order relativity theories (FORT), $(1+n)$-dimensional spacetime is represented as a vector space $\mathcal{Q}^{n+1}$ over an ordered field $\mathcal{Q}$ of coordinates (or *quantities*). In general, $\mathcal{Q}$ is assumed to be Euclidean (positive quantities have square roots in $\mathcal{Q}$) or real-complete ($\mathcal{Q}$ satisfies exactly the same first-order statements as $\mathbb{R}$). *Bodies*, which are represented as objects of a second sort $\mathcal{B}$, include both photons and inertial observers, as well as more-general observers and particles depending

[2] "The way to protect the embedding against a loss of Lorentz invariance is by sprinkling the points randomly. Causal set theory uses a ... Poisson sprinkling [which] exhibits exact Lorentz invariance for Minkowski spacetime." (Dowker, 2005, p. 451)

on the particular version of relativity theory under discussion. We will generally write x for points in $\mathcal{Q}^{n+1}$, m and k for inertial observers, and b for general bodies.

The central FORT construct is the *worldview relation* $\mathbf{W}$ of type $\mathcal{B} \times \mathcal{B} \times \mathcal{Q}^{n+1}$, where we interpret the statement $\mathbf{W}(m, b, x)$ to mean "*observer m considers body b to be present at spacetime location x*". The worldline $w_m(b)$ followed by b according to m is then definable as $w_m(b) = \{x \mid \mathbf{W}(m, b, x)\}$, and we can similarly use $\mathbf{W}$ to define a *worldview transformation* $\mathbf{wvt}$, which expresses the relationship between two observers' views of events taking place around them: if m sees body b at spacetime location x, then k sees it at $\mathbf{wvt}(k, m, b)$.

An important principle of the FORT approach is that axioms should be as simple as possible. From a pedagogic standpoint, this makes the proofs relatively believable for non-experts, since they assume only things which can be explained easily in natural language; from a logical standpoint, the approach ensures that proofs can be analysed to determine whether any of the axioms used are redundant or can be weakened still further. For example, typical axioms[3] of the theory SpecRel (used to capture special relativity) include, where x_j denotes the j^{th} component of x and x_0 is its 'time component':

AxSelf : $\mathbf{W}(m, m, x) \Rightarrow (\forall j > 0)(x_j = 0))$
"every observer considers itself to be at rest spatially"

and

AxEv : $\mathbf{W}(m, b, x) \Rightarrow (\exists y)\mathbf{W}(k, b, y)$
"all observers live in the same universe (see the same events)".

In keeping with Einstein's (1920) conception of spacetime as arising from the relationships between the objects within it, together with the observation that the interactions underpinning such relations can only be verified empirically by observing induced motions, the FORT approach generally assumes that b is fully specified from m's point of view (and hence from any other observer k's, via the worldview transformation) once its worldline $w_m(b)$ is given. In other words, as far as FORT is concerned, a particle *is* its worldline. While it is a general theorem of FORT that worldlines are (the images of) continuous paths drawn on spacetime, Stannett (2009a) notes that the worldview relation $\mathbf{W}$ captures only static information about the worldline as a whole, and cannot identify the manner in which it is 'populated.'

For example, AxSelf tells us that any observer m will consider its worldline to be the time axis. Consider the interval $[0, 3]$ along that

[3] Universal quantification over free variables is assumed implicitly in these axioms.

axis. While it is natural to think of the particle moving continuously 'up' the axis from 0 to 3, the same interval would also be swept out by a particle performing the three consecutive continuous motions $0 \to 1$, $2 \to 3$ and $1 \to 2$. Such higher-level dynamics are invisible within Andréka et al.'s theories, and we cannot argue within those theories what form the underlying motion actually takes.

Far from being a shortcoming of the FORT approach, this inability to reflect higher-level dynamics may be regarded as highly fortuitous, since it provides a way to link first order relativity theories to a theory of observation inspired by quantum theory, thereby allowing us to examine the single-particle universe in more detail. To this end, we briefly review Stannett's argument (2009a; 2009b; 2012) that continuous motion in FORT can be regarded as a form of "quantum illusion" within an inherently discrete finitary quantum theory. We then show that a single-particle universe can be described within an enhanced variant of Stannett's finitary model, and that its worldlines are again precisely those of FORT.

2.3. Finitary quantum theory

The theory described in this section was introduced in (Stannett, 2009b) as a finitary analogue of the 'path integral formulation', one of the standard formulations of modern quantum mechanics (Feynman and Hibbs, 1965). In order to find the probability that a particle moves from one location to another by a path lying entirely in some spacetime region R, the path integral formulation starts by assigning each possible path a probability amplitude (a complex number). We then integrate over all possible paths (taking care to include suitable normalisation factors if appropriate) to find the amplitude that the motion occurs via a path lying entirely in R: the magnitude of the result is the probability required. The finitary approach is identical, except that 'path' is redefined to mean a finite random sequence of discontinuous 'hops'.

Suppose $M' \equiv (M', g')$ is some spacetime for which there is a global time function $T' : M' \to \mathbb{R}$ such that T' increases as one moves in the future direction along any timelike curve, and consider a particle (for example, the particle P in the manifold in which $\mathcal{L}'$ is embedded) which moves through M' in a sequence of random 'hops'. In other words, the particle is equipped with an internal clock, $t = 0, 1, 2, \ldots$, and its relative positions $\langle x_t \mid t \in \mathbb{N} \rangle$ in M' form a stochastic sequence, where all positions are measured by some inertial observer $\mathcal{O}$ whose identity doesn't concern us.

By the principle of stationary action, the worldline followed by a particle moving from location x to location y will the one for which action is stationary (to first order). We will call the action associated with this path the *standard action* for the particle to move freely from x to y, written $S(x, y)$.

We have called the action S 'standard' because it corresponds to the classical conception of motion as continuous displacement along an $\mathbb{R}$-parametrized future-pointing timelike path. Since random hop-based motion need not respect T' in this way, we define a separate *hop action*, $s : M' \times M' \to \mathbb{R}$, by

$$s(x, y) = \begin{cases} S(x, y) & \text{if } T'(y) > T'(x) \\ \bar{S}(y, x) & \text{if } T'(x) \geq T'(y) \end{cases}$$

where $\bar{S}(y, x)$ is the standard action for the particle's anti-particle to move from y to x. The status of this definition is slightly delicate. While we can, if we wish, limit our attention to particles following only timelike trajectories, it is also possible within FORT to study the dynamics of particles travelling at superluminal speeds along spacelike paths (Madarász et al., 2014). This draws attention to the significance of the observer, since it is possible for one observer to see a faster-than-light particle moving forward in time while another sees it moving backwards in time. For negative-energy particles the hop action can be seen as partially implementing the Stückelberg-Feynman-Sudarshan-Recami "switching principle" (Stückelberg, 1941; Feynman, 1949; Bilaniuk et al., 1962), that "any negative-energy particle P travelling backwards in time can and must be described as its antiparticle $\overline{P}$, endowed with positive energy and motion forward in time" (Sudarshan, 1970). However, first-order relativity theories can accommodate motions of all kinds, depending on the choice of underlying axiom set and so we make no such assumption.

As in the path integral formulation, we now associate each (hop-based) path with a probability amplitude[4]: the *amplitude* of a hop $x \rightsquigarrow y$ is defined to be the complex value $\langle y \mid x \rangle \equiv \exp\{\imath s(x, y)/\hbar\}$, where $\hbar$ is Plank's constant. A *finitary path* f from a to b in M' is a finite sequence $(a =)x_0, x_1, \ldots, x_n, x_{n+1}(= b)$ of (possibly repeated) points in M', and its amplitude is defined to be

$$\langle f \rangle = \prod_{j=0}^{n} \langle x_{j+1} \mid x_j \rangle$$

[4] We are grateful to an anonymous referee for noting that a related construction was given by Adolphe Bühl in 1934. This construction, which potentially provides a physical meaning to the sums of certain 'sawtooth' hop trajectories, is described in (Bachelard, 1968).

To find the 'hop-based' amplitude that a particle travels from a to b via a path lying entirely in a region $R \subseteq M'$, we again apply certain normalisation factors and then integrate $\langle f \rangle$ over paths $f = \langle a, x_1, \ldots, x_n, b \rangle$ for which all of the points $a, x_1, \ldots, x_n, b$ lie in R.

The key result we need in what follows is:

THEOREM 2.1 (Stannett, 2009b, §4.5). *It is possible to define appropriate normalisation factors so that the hop-based amplitude that P travels from a to b via a path lying entirely in R is precisely equal to the amplitude given by the path-integral formulation of quantum mechanics.* □

Notice that it is not necessary for hops to have access to *all* points in R. We can, if we wish, restrict attention to hops moving between points in a dense subset D of R: given any finitary path f in R we can approximate it as the limit of a net (f_λ) of finitary paths lying entirely in D, so we can replace the amplitude $\langle f \rangle$ in all relevant formulae with the limit of the amplitudes $\langle f_\lambda \rangle$ (Stannett, 2012).

3. A Single-Particle Universe Model

We are now ready to complete the description of our single-particle universe model.

Let $M' \equiv (M', g')$ be any GR manifold for which a global time function function T' can be defined, and let $\mathcal{L}'$ be a densely embedded subspace generated by random sampling in M'. Although P is the only particle in the absolute single-particle universe M, the freedom we have in selecting (M', g'), coupled with the formal relationship between g' and the matter distribution in M', means that M' can generally be considered to contain a wide range of material particles (see section 2.1), some of which may be located at points in $\mathcal{L}'$. We shall refer to these as *apparent* particles.

Suppose, then, that we wish to determine the probability that an apparent particle P' will move between locations x and y in $\mathcal{L}'$ via a finitary path lying entirely in some region $R \subseteq \mathcal{L}'$. Since each such path is also a finitary path in M', we can use the procedure described in section 2.3 to assign it an associated hop-based action, and this action will be consistent with the metric structure of $\mathcal{L}'$ by density of $\mathcal{L}'$ in M'. Integrating over all paths then gives us the overall amplitude for the motion, and hence the required probability.

In other words, the unstructured absolute 'observations' performed by the solitary particle P can be re-interpreted in a relational setting as observations of apparent particles within a dense countable subspace $\mathcal{L}'$ of any manifold M' of the appropriate form – arguably including the universe in which we find ourselves on a daily basis.

3.1. Caveats

The construction we have presented above is necessarily informal, and should therefore be considered indicative rather than definitive. It should also be noted that formal results concerning the use of finitary paths to compute the probabilities of particle motions being observed have only been published in the context of non-relativistic (Stannett, 2009b) and special relativistic (Stannett, 2012) spacetimes. The extension to general relativistic spacetimes is implied (Stannett, 2012, p. 57), but in the absence of detailed proofs the justification presented there should also be regarded as informal.

4. Implications and Further Questions

Our model has a number of implications. We consider two of these here, though in the light of the caveats outlined in section 3.1 these require further investigation.

Substantivalist or relational?
Our model is based on the supposition that a particle P exists in an absolute spacetime $M = (M, g)$, and that it has a clock capable of showing absolute proper time. The set $\mathcal{L}$ of observations is entirely abstract, and $\mathcal{L}'$ inherits its structure from an absolute container spacetime $M' \equiv (M', g')$. The universe-as-assumed is, therefore, defined entirely in absolute substantivalist terms. Nonetheless, the universe-as-observed appears, depending on one's choice of M', to contain a wide range of material objects, and if we choose M' to be locally identical to the observable 'real world', the equivalence of the finitary and path integral formulations implies that $\mathcal{L}'$ will appear under observation to be essentially identical to the relational world of everyday empirical experience. It follows that the world we see around us can be regarded as either relational or substantivalist depending on ones preferred level of abstraction. In essence, therefore, our model provides a purely formal demonstration in support of Rynasiewicz's view (1996, p. 279) that the question whether spacetime is substantivalist or relational is "no longer a meaningful one." Rather, substantivalism and relationalism can be seen as two sides of the same coin. As Glymour (1972, p. 215) puts it, "Some conventions in physics arise just because more than one theory

is in fact true, and in such cases any appearance of contradiction is illusory."

Science as artefact
Observations of relative motion lie at the heart of all modern scientific theories of physical interaction – indeed, one may reasonably argue that physical theories are *nothing but* theories of observed motion, since they necessarily seek to explain and predict effects that must ultimately be checked by human observers if their replicability is to be verified. Our findings therefore suggest that scientific theories can at best be regarded as human artefacts, since we can equally well explain all of these underlying motions using a finitary model in which all observed motion is illusory, or even a single-particle universe model in which relative motion has no inherent meaning.

This view of science as an artefact that serves its purpose without necessarily providing a 'true' picture of the world around us is, of course, not new. It is implicit, for example, in Kuhn's theory of paradigm shifts (Kuhn, 1962). Modern scientific theories have replaced earlier theories, and it is reasonable to assume that they will eventually be replaced in their turn. Just as we can understand the rationale for ancient beliefs and cosmologies while at the same time considering them to be scientifically untenable, one can only wonder what the thinkers of the next millennium will think of our current scientific beliefs.

4.1. Further Questions

Given the caveats outlined in section 3.1, it remains unclear to what extent the finitary hop description of observations extends fully to general relativistic manifolds. Stannett (2012, p. 57) argues that the method can be extended "to any model of spacetime in which it is possible to specify which regions of spacetime are accessible from each current location", but does not provide detailed proofs, whereas our treatment here focusses on spacetimes in which a global time function can be defined. It would be useful to have a clearly formulated mathematical proof as to the spacetimes in which observations can necessarily be expressed using finitary path calculations, since it is precisely these spacetimes from which M' should be chosen. It would also be interesting to see how easily purely formal proofs can be generated within different FORT variants, describing (for arbitrary finite or infinite m and n) the conditions under which an m-particle universe can be simulated by an n-particle universe.[5]

[5] We are grateful to an anonymous referee for this suggestion.

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