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Abstract

The scattered and pervasive variability of material objects, being a conspicuous part of the very experience of Early-Modern and Modern science, challenges its purely theoretic character in many ways. Problems of this kind turn out in such different scientific contexts as Galilean physics, chemistry, and physiology. Practical answers are offered on the basis of different approaches, among which, in particular, two can be singled out. One is made out by what is often called an 'art' (thus not a science, rather an informed practice) of experiments. From the Renaissance until J. H. Lambert's writings of the 1750–1760s, we can follow a train of reflections on the art of making experiments that deal precisely with the persistence of contingency in the matterly objects of pure science. The other is the analysis of contingency in probabilistic terms. They develop subsequently and eventually meet, as it can be seen precisely in Lambert's work: among the first to pursue this path are Jakob Bernoulli and Leibniz.

# Chapter 14
# *Ars experimentandi et conjectandi*. Laws of Nature, Material Objects and Contingent Circumstances

**Enrico Pasini**



The scattered and pervasive variability of material objects, being a conspicuous part of the very experience of Early-Modern and Modern science, challenges its purely theoretic character in many ways. Problems of this kind turn out in such different scientific contexts as Galilean physics, chemistry, and physiology. Practical answers are offered on the basis of different approaches, among which, in particular, two can be singled out. One is made out by what is often called an 'art' (thus not a science, rather an informed practice) of experiments. From the Renaissance until J. H. Lambert's writings of the 1750–1760s, we can follow a train of reflections on the art of making experiments that deal precisely with the persistence of contingency in the material ~~matterly~~ objects of pure science. The other is the analysis of contingency in probabilistic terms. They develop subsequently and eventually meet, as it can be seen precisely in Lambert's work: among the first to pursue this path are Jakob Bernoulli and Leibniz.

## 14.1 ~~Introduction~~ Obsessed

Early modern natural philosophers were, in a way, obsessed with the lofty pessimism of Hippocrates' '*ars longa, vita brevis*'.[1] It was urgent, in their view, to make the learning of art quicker through method, while life would be made longer thanks to a new science of medicine. But they could have adopted the last part of this aphorism as a motto: '*experimentum fallax, iudicium difficile*', as it sounds in the famous

[1] See for instance Yeo 2014, 91–95.

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1494 Latin rendition.[2] Experiment is deceptive and judgement difficult—nature is elusive.

Natural objects like birds and stones, salts and acids, seeds and plants, present to the early-modern and modern scientific eye a stubborn quality of erraticism: they are citizens of an overly populated realm of unpredictability, and, although in some respects they obey general laws of nature, in some others they are, in Wallace Stevens' words, "inconstant objects of inconstant cause / In a universe of inconstancy" (Stevens 2011, 389).

This unpredictability poses theoretical and practical problems to both natural scientists and practitioners that go way beyond the traditional oppositions of stability and change, identity and multiplicity, (epistemic) necessity and contingency, and other basic metaphysical dichotomies. It is usually perceived as a problem peculiar to disciplines based on classification; as a difficulty to be inevitably tackled by the founding fathers of certain fields which oscillated in their beginnings between 'science' and natural history. But it is also, when considered in connection to the issue of natural laws ~~laws of nature~~ and of the uniformity of properties pertaining to the nature of things, a major problem for experimenters, both in the medical and life sciences as well as in more matterly pursuits more concerned with matter, like alchemy, and later chemistry. It even grows into an early disturbance in mathematical physics and mechanics.[3] AU1

In this last domain, Galileo Galilei was among the first to thematize this matter and to sketch some practical answers. When Galileo's methodological contributions are considered, much weight is normally put on his *Il saggiatore*: a beautiful book indeed, and of a kind historians of science relish because of charming quotes on the mathematization of *Weltanschauung*, but also with an excess of general flourishes and a strong dose of 'anything-goes-if-I-am-to-put-Jesuits-down'. One can easily sympathize, but might wonder whether *Il saggiatore* was really so relevant for developing scientific *Selbstbewusstsein* and methodology. Would not scientists read Galileo's *Dialogo* and *Discorsi*[4] instead? Well, as for our present purpose, it is also in these other works that we really find Galileo both enforcing the rule of mathematics *and* taking in the contingency of nature.

The first example of such convergence that I have in mind can be found in the third day of the *Dialogo*, where the theme of observational errors in astronomy is



[2] "Vita brevis: ars longa: occasio praeceps: experimentum fallax: iudicium difficile est" (Hippocrates 1494, a3*v*).

[3] On contingency and laws of nature see chapers. 1, 2, 7 of Daston and Stolleis (2008). On the historical evolution of observation as a canonical form of learned experience in late medieval and early modern Europe see Pomata and Siraisi (2005), Gaukroger (2006), and chapters 1–3 in Daston and Lunbeck (~~2011~~ 2010). On the context and development of a culture of experimental facts in ~~16th~~ sixteenth to eighteenth century England, see Shapiro (1979, 1983, 2000). We are considering observation of the 'normal' variability of things; of course 'monsters' and 'wonders' pose even more complex problems (see Daston and Park 1998), yet these are outside the scope of this chapter.

[4] *Dialogo sopra i due massimi sistemi del mondo, tolemaico e copernicano* (1632, nine years after *Il saggiatore*); *Discorsi e dimostrazioni matematiche intorno a due nuove scienze attenenti alla mecanica e i movimenti locali* (1638).

raised. Galileo wrote in a letter that he wanted to dispel "Chiaramonti's blinders" (OG 17, 194)[5] on novae (in particular that of 1604), which Chiaramonti maintained to be sublunar optical phaenomena, thus preserving Aristotelian incorruptibility of the skies, and to show the shortcomings in his opponent's "method in confuting the Astronomers, who affirm the new Stars to be superiour to the Orbs of the Planets" (Galilei 1661, 253; OG 7, 303).[6] It is plain, according to Salviati, that since "the new star could not possibly be in many places", diverging calculations imply "an errour in the observations", and only converging observations might be right, or better "may happily be the non-erroneous, but the others are all absolutely false" (Galilei 1661, 261; OG 7, 313).

Regrettably, observations of the nova are not repeatable, since it disappeared. A discussion of Chiaramonti's method for the choice of the 'rightful' group of observations ensues, introduced by an explanation of the calculation of parallaxes. Salviati admits that the combination of a multiplicity of measurements, and of contingencies due to the variety of circumstances, observers and instruments, makes errors unavoidable:

> for the observations being four in number that serve for one working, that is, two different altitudes of the Pole, and two different elevations of the star, made by different observers, in different places, with different instruments, who ever hath any small knowledge of this art, will say, that amongst all the four, it is impossible but there will be some error (Galilei 1661, 262; OG 7, 314).

Yet one must not believe, as a disingenuous Chiaramonti appears or pretends to, that the magnitudes of relative errors are proportional to the differences in the results of measurements, "and that by conversion, from the greatness of the exorbitancies, may be argued the greatnesse of the error" (Galilei 1661, 265; OG 7, 317). This overlooks the fact that errors in the instrument have a certain proportion to the calculated results.[7] The correct procedure to assess the probable accuracy of groups of observations is, in the discussed case, rather the opposite:

> the greatnesses of the error (to so speak) instrumental, are not to be valued by the event of the calculation, but by the quantity itself of degrees and minutes numbred upon the instrument, and these observations are to be called more just or less erroneous, which with the addition or substraction of fewer minutes, restore the star to a possible situation; and amongst the possible places, the true one may be believed to have been that, about which a greater number of distances concurre upon calculating the more exact observations (Galilei 1661, 265; OG 7, 318).



[5] I shall use the following abbreviations: OG = Galilei 1964; A = Leibniz 1923; AG = Leibniz 1989; GM = Leibniz 1849; GP = Leibniz 1875; NE = *Nouveaux essais sur l'entendement humain*, according to A VI 6; RB = Leibniz 1996, with the same pagination as A VI 6.

[6] For the sake at least of testifying to the influence of these works, I shall be using coeval English translations of Galileo.

[7] "Fix it well in your mind, that in the highest distances, that is v.g. the height of *Saturn*, or that of the fixed Stars, very small errors made by the Observator, with the instrument, render the situation determinate and possible, infinite and impossible. This doth not so evene in the sublunary distances, and near the earth" (Galilei 1661, 265; OG 7, 317).

This might well be hailed as the inception of the modern theory of errors. An appreciation of the genesis of both gross and random errors, as we would call them, in observations involving multiple measures, is adroitly connected to a precise analysis of the proportion between contingent imprecisions due to the measurement condition and the magnitude of the true error.

Yet it is in the *Discorsi* that Galileo focuses on the origin of the uncertainty of measurements in the observation of natural events, or of their experimental counterparts, and identifies this origin in the numberless multiplicity of contingent circumstances, or 'accidents'. In the first theorem of the fourth day, when discussing resistance (*impedimento del mezo*; OG 8, 275) it is said:

> Again, as to the Irregularity proceeding from the Impediment of the Medium, this, we grant, is more considerable, and by Reason of its so manifold Varieties can't possibly be reduced to certain Laws: For if we should only consider the Impediment which the Air causes in such Motions as we have th[o]roughly examin'd into, we should plainly find it to disorder them all, and that after infinite Ways, according as the Figures, Gravities and Velocities of the Moveables are infinitely varied (Galilei 1730, 383–384).

A greater velocity will meet with a greater impeding effect of the medium, a fact which is more visible in the case of lighter mobiles. Even if, according to the known laws of the fall of bodies, a continual acceleration is expected, "yet such will be the Impediment of the Air, that the Body will be deprived of any further Increase of its Velocity" (Galilei 1730, 384). This circumstance is perceived by Galileo not as an object of possible measurement and calculation, but as a hindrance to the correct measurement and calculation of the principal object of enquiry:

> And inasmuch as these Accidents of Gravities and Figures are subject to infinite Mutations, we can come at no certain Knowledge[8] concerning them: Wherefore to treat of this Matter scientifically, 'tis requisite to abstract from them (Galilei 1730, 384).

It is easily seen that the mathematical analysis of errors and the abstraction from the infinity of accidental circumstances are but two faces of the same coin–forbearance, so to speak, and nonacceptance. That is, the controlled execution of experiments is the only possible countermeasure to the scattered and pervasive variability of material objects.

## 14.2 ~~2.~~ An urge is born AU2

An urge is born, we may say, that will become open in the eighteenth century, for an 'art of experiments', or *ars experimentandi*. This expression does not appear immediately in the writings of the natural philosophers but after some time, and it is rare and remarkable. Consequently, as I often happen to do with such clauses, it can be treated as a symptom: maybe just the symptom of some *malaise*, and, as hinted by its wording, of the desire for an artful cure.

[8] In the Italian text: "ferma scienza" (OG, 8, 276).

For instance, the German astronomer and cartographer Johann Heinrich Müller wrote in 1721 a *Collegium experimentale, in quo ars experimentandi […] explanatur ac illustratur*. He plainly defines the art, in traditional guise, as a habit of conducting experiments[9]: "*Ars experimentandi* est habitus instituendi experimenta." It is interesting that he shows an awareness that 'experimentandi' might sound like an awkward neologism and ("ne quis […] vocis novitate offendatur") he quotes Bacon's *De augmentis scientiarum* as a previous and authoritative use (Müller 1721, 1).[10] According to Müller, the natural variety of circumstances is mirrored in the innumerable ways in which an experimental 'artificial state' can be produced: "There are innumerably innumerable variations in which the artificial state of every natural body subjected to the control of the artificer can be modified".[11] Correspondingly, in the section devoted to *Prolegomena*, a unique *Axiom* concerning experimentation is introduced: "Only under precise conditions and circumstances a natural body exhibits and demonstrates what and how much it can act or be acted upon".[12]

Again, the control of experiments, although it is based here more on methodology than on mathematization of procedures, is central in tackling the variety of material contingency. But an art of experiments cannot be an affair of pure theory. It is true that this expression is connected to a strong methodological commitment; nevertheless, it is not difficult to detect in many scientific writers of the time a clear awareness also of an intrinsic ambivalence.

Very different, for instance, is the attitude of Thomas Sprat, a clergyman who contributed to founding the Royal Society in 1660 and eventually became an Anglican bishop. As early as 1667, Thomas Sprat published a *History of the Royal Society*, in which he intended to vindicate the scientific progress they had already made,[13] and in which a long section was devoted to a "defence and recommendation of *Experimental Knowledge* in general" (Sprat 1667, B4*v*). Nevertheless, when Sprat discusses the "Art of Experiments", and opposes it to cavilling on arguments, he not only maintains that it "consists not in Topicks of reas'ning, but of working", he adds as well that it "indeed is full of doubting and inquiry, and will scarce be

[9] *Ars* is considered an *habitus intellectualis* in the whole Latin Aristotelian tradition on the basis of *Eth. Nic*. VI. It is remarkable that, in contrast to Bacon and certain strands of Aristotelianism, the pertinent instrument is not a 'logic'.

[10] Although using a not dissimilar definition, the most important contemporary German philosopher, Christian Wolff, is surprisingly deaf to any practical aspect of these issues: for him, very simply, "*Ars experimentandi* est, qua experimentis veritates eruuntur"—*i.e.* the art in which we find truths by means of experiments, as a subdivision of the *ars inveniendi a posteriori*, in a sort of belated Ramist treatment of the *ars inveniendi* in general (*Psychologia empirica*, §459; Wolff 1732, 357–58).

[11] "Innumerabiliter […] innumerae erunt variationes, quibus corporum naturalium omnium, potestati artificis subjectorum, status artificialis mutari potest" (Müller 1721, 7).

[12] "Corpus naturale non nisi sub certis conditionibus et circumstantiis, quid et quantum agere vel pati possit, prodit ac demonstrat" (Müller 1721, 7).

[13] Not without a conspicuous, maybe inevitable share of idealizations; see Wood (2009).

brought to settle its assent"—although, as expectable, this is "such a doubting as proceeds on Trials, and not on Arguments" (Sprat 1667, 332).

To remain on Baconian English land, Robert Boyle, the standard-bearer of experimental philosophy, has an even clearer insight into the pitfalls of experimentation, which is exemplarily testified by his composing, some years later, an essay *Of the unsuccessfulness of experiments*:

> Several observations or experiments […] though communicated for true by candid authors or undistrustful eye witnesses, or perhaps recommended to you by your own experience, may upon further trial, disappoint your expectation, either not at all succeeding constantly, or at least varying much from what you expected (Boyle 1772, I. 318–319).

Such disruptions can be looked upon as "the effects of an unfriendliness in nature or fortune to your particular attempts, as proceed but from a secret contingency incident to some experiments" (Boyle 1772, I. 319).[14] To the diversity of these 'secret' contingencies Boyle is very attentive. In the *Experimental history of colours*, for instance, it is correspondingly remarked that "the Fineness or Coarseness of the Papers, their being carefully or slightly Colour'd, and divers other Circumstances, may so vary the Events of such Experiments as these" (Boyle 1772, I. 726), such that only very careful repetition can secure the results. There is a variety of contingent material circumstances affecting the bodies we have experience of, which themselves can be made object and source of experimentation: "several bodies, which experience assures us imbibe or retain something from the air, as calcined minerals, marcasites, salts, factitious and natural, and may be often expos'd to it, and then weigh'd again, and farther diligently examined", as we read in the *Suspicions about some hidden qualities in the air* (Boyle 1772, IV, 97). Experiments themselves can be varied accordingly to the variety of circumstances:

> experiments may be varied with a good magnet, by exposing it long to the air, in regions differing much in climate, foil, or both; by exposing it by day only, or by night, at several seasons of the year, in several temperatures of the air, at several considerable aspects of the stars and planets, by making it more or less frequently part with what it has gained from the air; and in short, by having regard to that variety of circumstances which human sagacity will suggest (Boyle 1772, IV, 97).

And "by thus diversifying the experiment many ways", it becomes possible to "make some unexpected, and yet important discovery" (*ib.*). Mirroring the contingent variety of material conditions in experiments becomes thus both a problem and an opportunity, provided that it is absorbed into an art of experiments. Again the relation is double-faced, but, in contrast with Galileo, it is not a mathematical theory or meta-theory of experiments that is proposed, but a general technique of experimental praxis, in short: again an *art* of experiments.



[14] And it is even more important to "consider the contingencies, to which experiments are obnoxious, upon the account of circumstances, which are either constantly unobvious, or at least are scarce discernible till the trial be past" (Boyle 1772, I. 334). On the role of contingencies and experimental miscarriages in Boyle's scientific programme see Sargent (1994).

## 14.3 ~~3.~~ Complication

The common element to all these cogitations from astronomy, physics and chemistry, is, as we have seen at length, the contingency of the world of material objects as represented by the unruly innumerability of particular elements or circumstances. The situation was not any different in the realms of anatomy, zoology, botany that is, in the sciences of the living. Here, though, the issues at stake were amplified by the revolution brought about by microscopical observations, with a new infinitely small complication of contingent particulars proposing itself as a new object of the life sciences. The answer, at least of some prominent scientists, was again to be given in the framework of the art of experiments.

With "our Modern Engine (the Microscope)", enthusiast Henry Power[15] would write in the 1660s, "you may see what a subtil divider of matter Nature is" (Power 1664, c2*r*). Even Adam in his prelapsarian state could not see the satellites of Jove, and "so doubtless the Minute Atoms and Particles of matter, were as unknown to him, as they are yet unseen by us" (a4*v*). But the "Experimental and Mechanical Philosopher" (194) would be able "to attempt even Impossibilities" (191), and indeed,

> if the Dioptricks further prevail, and that darling Art could but perform what the Theorists in Conical sections demonstrate, we might hope, ere long, to see the Magnetical Effluviums of the Loadstone, the Solary Atoms of light (or *globuli aetherei* of the renowned Des-Cartes) the springy particles of Air, the constant and tumultuary motion of the Atoms of all fluid Bodies, and those infinite, insensible Corpuscles which daily produce those prodigious (though common) effects amongst us (Power 1664, c3*v*–c4*r*).

The work comprised three books and was titled *Experimental philosophy*. One of the books was entirely devoted to microscopical experiments, or experimental observations made with microscopes. It is curious, and provides us with a nice bridge to the authors we shall consider hereafter, that in the same book Power also proposed (albeit very naively) the experimental study of "insensible Transpiration in plants" (Power 1664, 29).

It is well-known that 'Galilean' approaches were soon applied to the understanding of living bodies. Among the best-known attempts is the study of muscular motion attempted by Borelli with the use of geometrical schemes. But if we are looking for experimental strategies that tackle, in the domain of observability, the multiplicity of minute contingent entities and events, we should bring our attention instead to Santorio Santorio, the Istrian physician who taught in Padua and befriended Paolo Sarpi, Sagredo, Acquapendente, and maybe Galileo himself. Although Santorio wrote his *Statica medicina* in aphorisms to imitate Hippocrates (Santorio 1614, A3*r*-*v*), experiments were his forte. The quantification of unobservable particularities has had an important role in medicine since the time of Santorio's

[15] Henry Power's experimental philosophy joined "claims for experimental liberty and devotion" (Shapin and Schaffer 2011, 304). Power put great weight indeed on experimentation: "the true Lovers of Free, and Experimental Philosophy […] are the enlarged and Elastical Souls of the world" (Power 1664, 191). See also the still useful Cowles 1934.

studies of insensible perspiration, which were presented by the author himself as a first attempt to submit this invisible process to an experimental enquiry, since all he wrote had been corroborated by the use of his famous weighing chair. In the Epistle to the Reader he proudly states: “Ego vero primus periculum feci” (Santorio 1614, a7*v*). *Periculum* means here ‘experiment’, as it is correctly translated in the coeval English version:

> It is a thing new, and not before heard of, in Medicine, that any one should be able to find out the exact weight of insensible perspiration, nor has any one of the Philosophers or Physicians attempted the doing of any thing in that part of the medical faculty. I am the first who made the experiment, and (if I am not mistaken) brought the Art to perfection, by reason, and the experience of 30 years (Santorio 1676, A3r).

Measurement of and dealing with the minute phenomena of living bodies are methodically related to experiments in another, equally seminal work that we may introduce symmetrically with Santorius’ *Statica medicina*, that is Stephen Hales’ *Vegetable Staticks* (1727). In the Dedicatory Epistle, Hales declares that his studies will show that plants get food not only from the earth, but also from the air, “that wonderful fluid […] which by infinite combinations with natural bodies, produces innumerable surprizing effects” (Hales 1727, A3*r*-*v*). And ‘innumerable’ is the buzz-word of this book, where, inside Malpighian-inspired descriptions of the finest structures of vegetable organisms, we meet “innumerable fine capillary vessels” (149), “innumerable little pores of leaves” (155), “innumerable little vesicles” (345, 359), “innumerable minute” (244) and “innumerable narrow meanders” (376); while “innumerable air-spheres”, and “innumerable bubbles of air” turn out *passim*. Hales also returns a couple of times to the “infinite varieties of combinations of the common principles of vegetables” (360) and the “infinite combinations, action, and re-action of those principles” (319).

Faced with this explosion of contingencies, Hales explains in the Introduction how the natural philosopher should proceed:

> Since we are assured that the all wise Creator has observed the most exact proportions, of *number, weight and measure*, in the make of all things; the most likely way, therefore, to get any insight into the nature of those parts of the creation, which come within our observation, must in all reason be to number, weight, and measure. And we have much encouragement to pursue this method of searching into the nature of things, from the great success that has attended any attempts of this kind (Hales 1727, 1–2).

Hales points, more precisely, to the success of astronomy and to the most recent results in the study of “animal œconomy” (2). The allusion gives a strong experimental tinge to the request for precision and measurement. And in fact, in the Preface Hales proposes yet another instance of the conjunction of complicated natural contingency and artful experimentation that is becoming, I suppose, familiar to us:

> the wonderful and secret operations of Nature are so involved and intricate, so far out of the reach of our senses, as they present themselves to us in their natural order, that it is impossible for the most sagacious and penetrating genius to pry into them, unless he will be at the pains of analysing Nature, by a numerous and regular series of Experiments; which are the only solid foundation whence we may reasonably expect to make any advance in the real knowledge of the nature of things. (Hales 1727, ix–x).

## 14.4 ~~4.~~ Paths of unification

It is fairly evident that the two strands I have presented here—one based on the precondition of abstraction, associated with mathematical procedures of control, and the other more attentive to the repetition of experiments and the large scale of experimentation on multiple aspects of the natural world—are not only, as per the subject of this chapter, the most qualified answers in early modern science to the contingency of material objects, but also correspond to well-known alternative views on experimental science and its development. It is also apparent that the two views might be complementary but, in the sources we have used, seem to be independent. In the eighteenth century we begin to see—still in conjunction with the appreciation of the contingent variability of nature—paths of progressive unification.

An outstanding example in this field in the eighteenth century is the work of Johann Heinrich Lambert, in particular in the domain of the measurement of light and heat. "*Rien de plus difficile, que la mesure de la lumière*": nothing is more difficult than measuring light, we read in the Foreword to Lambert's *Remarkable properties of the light path*, "when one wants to follow all its modifications and all the phenomena it offers us" (Lambert 1758, 6). Two years later he published his *Photometria*, in which he founded a new specialized field of study devoted to the measurement of light and illumination.

On the one hand, in both works Lambert's approach has a strong deductive component: starting from certain basic properties axiomatically expressed, he demonstrates other, non-trivial properties and laws. On the other hand, especially in the *Photometria*, he establishes a complex and innovative experimental apparatus to complement, verify and extend his findings. He insists, in particular, on the recursive role of experiments in the training and adjustment of the 'instrument', that is, the eye. So he writes in the Preface:

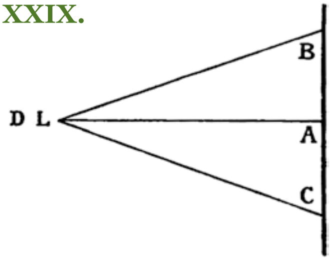
> I have noted throughout that all photometric experiments still depend on the judgment of the eye. So if anyone should repeat that which I have described, and sees it in a different way, I am confident that I am not going to be faulted. The acuteness and sensitivity of my eyes was explored in experiments I furnished […] and I added the precautions which I used, to the extent I was able [*quantum in me fuit experimentis exploratam dedi atque insuper cautelas adiunxi, quibus usus sum*] (Lambert 1760, 5*v*; 2001, vii).

Lambert repeatedly states, in not-so-different words, that "the precautions must be related by which the errors of the eye are to be remedied [*cautelae, quibus occurrendum est oculi fallaciis*]" (1760, §255; 2001, 87). Moreover, he devises specific experiments to train and test the eye:

> For instance, after having positioned a candle at L and placed a wooden board at D, casting a shadow on the entire room by the candle, I stepped back from the wall 10 or 12 feet, and then looking at the wall with not only the naked eye but also armed with a concave lens, I sought the interval or extent BC, in which the eye could discern no difference in brightness which was sensible [*in quo nullam deprehendere valuit oculus claritatis differentiam, quae sensibilis esset*] (Lambert 1760, §265; 2001, 92). [Fig. 14.1].

Lambert also takes into account the familiar dispersion of contingent circumstances: "Given a roughness of infinite degree [*infiniti asperitatis gradus*], reflected

**Fig. 14.1** ~~Caption~~ **Lambert, Photometria, Fig. XXIX.** AU3




and dispersed light are mixed in an infinity of ways" (Lambert 1760, §627; 2001, 221). But he treats infinite variations of different kinds in different ways. For Lambert, to whom infinitesimal calculus and infinite series are available, infinite variations and infinite small elements can be treated mathematically. Conversely, the infinite variability of experiments cannot be treated with any *a priori* technique:

> If the same experiment is repeated several times, with varying circumstances, and the error turns out greater or less by reason of the circumstances, either the universal truth of the law is to be doubted, or some particular law is to be suspected depending on these circumstances (Lambert 1760, §273; 2001, 95).

In accordance with the principle of abstraction from the infinite contingent circumstances, an ideal manifesto of which we have found in Galileo's *Discorsi*, Lambert says that we should not calculate in all details every case corresponding to a certain problem, but he looks for a different kind of remedy:

> Since these cases could be endless, and many of them would have to be pursued by very laborious calculation—if in fact you wish to consider all the details and have a reckoning of them—it has been proposed to present a certain average one [*medium quoddam in his tenere*] from among these. The details [*minutias*] themselves we will ignore to the extent it will be possible without noticeable error, and we will survey chiefly those cases which are more frequent, and which will need to be used for establishing and describing many subsequent experiments, and finally we will take care so that the formulas derived are well-ordered, so they can more easily be applied in practice (Lambert 1760, §486; 2001, 170–171, modif.).

The mention of 'frequency' patently situates this 'method' inside the newly founded domain of probabilistic anticipation, that is, inside the process of the formalisation of the treatment of *conjectures*. A conjecture is made – for example, that experiments concerning certain cases will be more fruitful – on the basis of frequency records. Briefly stated, then, Lambert appears to be supplementing the *ars experimentandi* with an *ars conjectandi*, an 'art of conjecturing'.

In point of fact, so runs the title of one of the founding works of probability theory, composed at the beginning of the eighteenth century by Jakob Bernoulli and published after his death: a book with which "the story of the emergence of probability comes to an end" (Hacking 1975, 166). The appropriation of this title for our reasoning is somewhat abusive—Nicolas Bernoulli, who edited its posthumous publication, wrote that the author's intent had been to show the importance of this part of mathematics in civil life.[16] Nothing particular concerning natural observations and experiments was featured in the book, although the collection of experimental data concerning, for example, life expectancy, one of the great themes of the early debates on probability, was considered.[17]

Most of the book dealt with games of chance. The fourth part was expressly devoted to the *ars conjectandi* and had been devised as a continuation of the last chapters of Arnauld's and Nicole's *Art de penser* (the famous *Logique de Port Royal*, titled *Ars cogitandi* in the Latin translations that had appeared since 1674). These chapters dealt with certainty and probability in human affairs ("évenemens humains"), where no necessary truth is involved, "ces évenemens estant *contingens* de leur nature"[18] (Arnauld and Nicole 1981, 339).

Bernoulli has a peculiar and innovative approach: on the one hand, everything in the created world is determined and certain in itself (*objective et in se*)—as it is mandatory, one may say, according to mechanistic science as well as rational theology.[19] On the other hand, *subjective et in ordine ad nos*, our knowledge of truths concerning the world is 'certain' up to a measure. But if this is the case, knowledge can be more or less precisely measured, and 'certainty' is a *mensura cognitionis nostrae circa hanc veritatem*:

> The certainty of anything is considered either objectively and in itself or subjectively and in relation to us. Objectively, certainty means nothing else than the truth of the present or future existence of the thing. Subjectively, certainty is the measure of our knowledge concerning this truth.[20] In themselves and objectively, all things under the sun, which are, were, or will be always have the highest certainty (Bernoulli 1713, 210; 2006, 315).

Conjectures based on 'experiments' are therefore the backbone in the conclusive and fundamental section of Bernoulli's work. And such conjectures are ruled not by weighing the authority of testimonies, but by mathematical rules that concern, first of all, the role of frequency. Even some *stupidissimus*, Bernoulli writes, would recognize, at least by some natural instinct,

> that to judge in this way concerning some future event it would not suffice to take one or another experiment, but a great abundance of experiments would be required [*magna experimentorum requiratur copia*], given that […] the more observations of this sort are made, the less danger there will be of error (Bernoulli 1713, 225; 2006, 328).



[16] "Propositum fuit Auctori monstrare eximium usum quem in vita civili habet ea. Matheseos pars, a paucis hactenus tractata, quæ de probabilitatibus dimetiendis agit" (Bernoulli 1713, n.n.).

[17] On the connection of primeval probability theory and such questions of civil and political import see Poovey (1998).

[18] My italics. This passage ("since such events are contingent by their own nature") was the only appearance of the term 'contingent' in the text of the *Logique*. On the pre-Port Royal and pre-Pascalian history of rational methods of dealing with uncertainty see Franklin 2001.

[19] Plainly for God, and in general for any omniscient being, 'chance' does not exist. Incidentally, Bernoulli's first studies had been in theology.

[20] This is a first approximation to distinctions that will be crucial in probability theory, to which, yet, Bernoulli's concepts are not perforce to be mapped: "There is no need to foist a single probability idea on to Bernoulli" (Hacking 1975, 149). On the peculiar way in which both Bernoulli and Lambert kept in view non-additive notions of probability see Shafer (1978).

Bernoulli wants, firstly, to properly demonstrate this natural thing that everybody knows. Moreover, he wants to provide a means of calculating the expectation of certain outcomes, positive, for example, from the ratio of favorable outcomes in series of tests or experimentations – i.e. from what will become the classical measure of probability. Now this requires him to ask:

> Whether, as the number of observations increases, so the probability increases of obtaining the true ratio between the number of cases in which some event can happen and not happen, such that this probability may eventually exceed any given degree of certainty. Or whether, instead, the problem has an asymptote, so to speak […] (Bernoulli 1713, 225; 2006, 328).

Bernoulli was indeed the first to demonstrate a limit theorem in probability theory. He calls favorable cases ‘fertile’, and works with natural numbers and rational fractions: if the number of ‘fertile’ and ‘sterile’ cases have, exactly or approximately, the ratio $r/s$, fertile cases are to all the cases in the ratio $r/(r + s)$, which Bernoulli shortens to $r/t$,

> Which ratio is bounded by the limits $(r + 1)/t$ and $(r - 1)/t$. It is to be shown that so many experiments can be taken that it becomes any given number of times (say $c$ times) more likely that the number of fertile observations will fall between these bounds than outside them, that is, that the ratio of the number of fertile to the number of all the observations will have a ratio that is neither more than $(r + 1)/t$ nor less than $(r - 1)/t$ (Bernoulli 1713, 236; 2006, 337).

In this way, epistemic probability would be connected in a mathematically demonstrative way to the quantitative evaluation of past knowledge (in the form of series of tests converging on some value of ‘probability’), although it is manifest that the measure of the probability of conjectures would be sourced from an *a posteriori* evaluation of contingent circumstances.

Let us now get back to Lambert. He considers frequency as the starting point and, as a concept (as became customary in the eighteenth century) originally integrated into chance:

> Since positive and negative aberrations are equally possible […], it is a consequence that they will also be equally frequent [*aeque quoque eas fore frequentes*] if the experiment is repeated many times (Lambert 1760, §277; 2001, 96).

On this basis, some general methods of probabilistic correction and assessment of experimental measurements can be developed. For example, the difference between the arithmetic mean of all observations, and the arithmetic mean of all but the observation most diverging from the first mean is the measure of the uncertainty of the series of observations considered, or, in Lambert’s words, how ‘dubious’ it is: “quousque dubium est medium ex omnibus invento” (Lambert 1760, §294; 2001, 101). Plainly, Lambert considers the “quantitas vera determinanda”, the true quantity to be determined (1760, §296; 2001, 102), as something that exists in reality, as it was with Bernoulli’s *summa certitudo*. Yet in principle it is inaccessible, since it is impossible to know whether any single experiment gives the ‘true quantity’ as result. Consequently, a method is devised that allows the true quantity to be approximated, if not with absolute certainty, with the greatest probability:

> Since in individual experiments in Photometry, as in countless [*infinitis*] others, aberrations are not equally frequent, another method is provided for determining the mean quantity from a finite number of them, so that the probability is greatest that it will differ least from the true quantity of them all [*ut maxime probabile sit, eam a vera omnium minime discrepare*]. [Thus] everything should be brought to the highest degree of probability [*ad probabilitatem maximam esse perducendam*], when absolute certainty in every detail cannot be attained (Lambert 1760, §277; 2001, 102 modif.).

## 14.5 5. Experiment connects to probability

So there must be some point, be it in time or in the sequence of events, at which experiments begin to connect to probability. The connecting link might be searched for, from the point of view of historiography, in the form of some train of quotations, or of the formulation of a specific doctrine. But the connecting link, in truth, is the real-life necessity that had developed in the meantime, and textual manifestations do but record or mirror—in some cases sooner, in others later—precisely that necessity. This having been said, we must add that among the first to express and conceptualize these needs with an eye on the development of mathematical tools and doctrines, we find, as it is often the case in the 17th and 18th centuries, Leibniz.

In his youth, as a reader of Bacon, of Ramists, and of all kinds of writings on method, Leibniz had composed among others a short memoire on an 'art of finding theorems'—that is, propositions of mathematics and of mathematical physics. There, an 'art of experiments' was also brought to the fore:

> Then there is the method of investigation by induction. But, since we are unable to look into all the cases, it pertains to the art to choose which shall be considered before the others, and this comes down precisely to analogy; therein consists the entire art of experiments.[21]

This is an art of devising experiments, not of performing or comparing them, as becomes clear in the following lines, where we read: "If we are simply looking for the experiments that are to be done on a given subject, these will be found by means of previous experiments, through analogy".[22] Such self-sufficiency of experiments is strictly Baconian. In Bacon's view, the scattering of natural particularities that we have dwelt upon is an opportunity rather than a hindrance, in relation to the knowledge of general laws of nature: "you may see great Axioms of Nature, through small and contemptible Instances and Experiments" (*Sylva sylv.* I, §91; Bacon 1857–1874, II, 377). Bacon, moreover, radically pitted probable reasoning against experience: erstwhile philosophers, he wrote in one of his customary unfavorable assessments of past doctrines in the Preface to *Instauratio magna*, have "followed only probable



[21] "Superest methodus investigandi per inductionem, sed cum omnia percurrere nequeamus, artis est. eligere præ cæteris examinanda, et hoc jam reducitur ad Analogiam; et in eo consistit tota ars experimentorum" (A VI, 3, 425).

[22] "Sed simpliciter experimenta quaerere dato subjecto, hoc faciendum est, ope jam cognitorum experimentorum per analogiam" (A VI, 3, 425).

reasoning", while nobody "has spent an adequate amount of time on things themselves and on experience" (Bacon 2000, 9). In the *Novum organum*, §36 ends so: "We have deliberately taken quite a long time to deal with [crucial instances], so that men may gradually learn the habit of forming judgements of a nature by crucial instances and illuminating experiments, and not by probable reasoning" (Bacon 2000, 168).

Interestingly, a little further in the same text we are quoting from, Leibniz introduces another art, concerned with hypotheses instead of experiments, under a name that is again familiar to us:

> The art of making hypotheses, or the art of conjectures [*ars conjectandi*], has different kinds: the art of explaining cryptograms pertains to it and is the highest specimen of the pure art of conjectures, abstract from matter, from which such rules can be derived that will eventually be applied to matter.[23]

Leibniz seems somewhat to be cognizant of the very same sensitive questions that we have singled out in the writing of many relevant early-modern scientists and natural philosophers, and, moreover, to have at least a feeling of a connection, rather than an opposition, between them. Nonetheless, his way of treating such issues seems to bear but a feeble similarity to the path we have been following, and his notions, at least in the beginning, stem from a very different approach. This concerns, first of all, his idea of probability.

In Leibniz's youthful writings we mostly find a "protoconceptualisation juridique" (Parmentier 1995, 8) of probability. For a long time, 'probable' is taken by him in the original sense of likely, verisimilar, plausible, or credible.[24] Yet, in his view, it is worthy of belief in a measure that can be tested or probed in some way,[25] on the model of the evaluation of witnesses in a trail. It is consilient with this starting point that in 1677 Leibniz equated probable arguments with 'semi-proofs'.[26] Probability is a property of propositions that, in the framework of a more general reformation of logic, should be subject to some sort of dependable and procedural assessment, and be metaphorically weighed.[27] To move from this to a probabilistic appreciation of contingent natural phenomena, however, an important change of perspective is needed.

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[23] "Ars faciendi Hypotheses, sive Ars conjectandi diversi generis est., huc pertinet ars explicandi Cryptographemata, quae pro maximo haberi debet specimine artis conjectandi purae et a materia abstractae, unde regulae duci possunt quas postea etiam materiae applicare liceat" (A VI, 3, 426).

[24] In the terms of Aristotelian tradition, upon which Leibniz so often relies, it is a matter of dialectical reasoning, instead of demonstrative. Conclusions are not necessary, in this case, but probable, basing on, so to say, inconclusive inferences. The distinction is also presented as one "between examination of matters of necessity and examination of contingencies" (Jardine 1991, 118).

[25] "A gifted mind ignorant of the doctrine of chances but able to apprehend the fact that evidence and causation are in different categories could perfectly well start measuring epistemic probability. The proof of this is that Leibniz did" (Hacking 1975, 85).

[26] "Reasons are either proofs, or presumptions, or semi-proofs (*semiprobationes*) or probabilities" (A VI, 4, 2167; Dascal 2007, 53).

[27] Marcelo Dascal has pointed to Leibniz's frequent use of the word "balance" and its synonyms (Dascal 2008, 68–69).

The knowledge of contingent small things is just as much an object of interest for Leibniz as it is for anyone else, if not more,[28] and his concept of contingency, together with the complex metaphysical framework that underpins it, has been the subject of ample literature. For sure this matter is exceedingly intriguing, but has not very much to do with our subject here. For Leibniz events and entities are contingent, if they are not 'of logical necessity' – that is, if their existence depends principally on God's free choice to create a certain possible world (the best of them all, to be sure). Contingent events have infinite conditions, since, according to Leibniz, the apparent material world is infinitely divided into actual living immaterial beings, each one representing the whole universe, past, present, and future, from its point of view, and at the same time acting as a center of force. This inexhaustible complexity implies that no demonstrative knowledge is possible of a contingent truth, since a demonstration is such, according to Leibniz, only if it can be carried through in a finite number of steps.

This profound connection between actual infinity and contingency cannot blend well, as we shall see, with the study of distributions in series of events governed by the law of (finite) great numbers. Most of Leibniz's texts concerning contingency express his satisfaction at discovering the opposition of moral necessity and metaphysical necessity; the principle of the best on which the choice is based and on which moral necessity depends, and the discovery that the 'root' of contingency is to be found in infinity. Much rarer, yet extant in some quantity, are texts in which Leibniz directly confronts this infinity of circumstances in relation to actual knowledge of natural events, processes, and laws:

> For instance, whether such-and-such a fixed star is larger or smaller than the sun, or whether Vesuvius will erupt in such-and-such a year—knowledge of these facts is beyond us, not because they are above reason but because they are above the senses. After all, we could judge very soundly about these matters if we had more perfect organs and more information as to the facts [*plus d'information des circonstances*] (NE IV, 17, §23; RB, 493).

When there is not enough information as to circumstances, we might be left with mere contingent connections of contingent facts, contingently similar. But these connections are the stuff of purely empirical knowledge, which Leibniz regards as typical of beasts, of incognizant humans, and of adepts of certain epistemologies, for example obstinately 'empirical' physicians:

> Beasts are purely empirical […] whereas man is capable of demonstrative knowledge [*sciences demonstratives*]. […] The consequences beasts draw are just like those of simple empirics, who claim that what has happened will happen again in a case where what strikes them is similar, without being able to determine whether the same reasons are at work. This is what makes it so easy for men to capture beasts, and so easy for simple empirics to make mistakes (NE, *Préf*; A VI, 6, 50; AG, 293).

This especially affects "people in civil and military affairs", when they "rely too much on their past experiences" (*ib.*). Empirics and beasts enjoy but 'a shadow of reasoning', whereas only reason is capable of establishing sure rules. And since in

[28] May I refer on this to Pasini (2016).

many instances such rules cannot be established with enough certainty and precision, what is needed is a new kind of logic dealing with the probable.[29]

But a proper definition of chance seems impossible, if not in purely epistemic (ignorance-based) terms, in Leibniz's world, where an all-embracing principle of reason is unrestrictedly valid—that is, "the great principle, little used, commonly, that nothing takes place without sufficient reason, that is, that nothing happens without it being possible for someone who knows enough things to give a reason sufficient to determine why it is so and not otherwise" (GP 6, 602; AG, 209–10). Leibniz makes it clear, in §36 of the *Monadology*, that the infinity of contingent circumstances on which we have ruminated since the beginning of this chapter does not escape that law:

> There must also be a sufficient reason in contingent truths, or truths of fact, that is, in the series of things distributed throughout the universe of creatures, where the resolution into particular reasons could proceed into unlimited detail because of the immense variety of things in nature and because of the division of bodies to infinity (GP 6, 612–13; AG, 217).

In fact, the 'probability' that Leibniz most often discusses is still a property of statements in terms of credibility, rather than a property of events in terms of expectations.[30] His main aim is to provide new instruments, formal and operational, for properly assessing testimonies and historical sources. In the *Nouveaux essais* Leibniz insists that an opinion based on likelihood (*l'opinion, fondée dans le vraisemblable*) is a legitimate kind of knowledge, and such is the case, in particular, for historical knowledge. Precisely in this context he introduces the theme of probability:

> I maintain that the *study of the degrees of probability* would be very valuable and is still lacking, and that this is a serious shortcoming in our treatises on logic. For when one cannot absolutely settle a question one could still establish the degree of likelihood on the evidence [*ex datis*], and so one can judge rationally which side is the most plausible (NE IV, 2, 14; RB, 372).

Leibniz, it can be remarked, keeps here his notion of probability away from the more traditional meanings related to persuasiveness. Moralists, he writes, have had "an inadequate and over-narrow notion of probability, which they have confused with Aristotle's *endoxon* or *acceptability*". Probability or likelihood is broader than plausibility, he adds. Unfortunately, in this passage he just explains the foundation of likelihood in the sense that "it must be drawn from the nature of things" (*ib.*), but



[29] One must agree at least with the second part of this comment: "By 'a kind of logic', Leibniz means a calculus of probabilities, whose development he foresaw with his usual prescience. […] Yet once again, Leibniz overstates the virtues of formalization and its role in practical deliberation" (~~Rolfe~~ Grosholz 2008, 176).

[30] To circumvent this problem, some interpreters have invested in what Hacking (1975, 138) called "the probability-possibility-facility-creatability nexus", which is a rather weak solution, at least because it seems to impose either epistemic or practical limitations on the Creator, or to misread important Leibnizian texts (as *e.g.* in Krüger 1981).

it is clear, at least, that the basis on which probability should be assessed has to do with 'things' and 'data', that is, it must be tested against knowledge of the world.[31]

Leibniz knew the work of Pascal, Huygens, and De Witt, who had initiated the mathematical study of chance, starting on the one hand from an interest in games and the probabilities of different results in the case of unfinished parties, on the other hand from the calculus of life expectations based on statistical data, for the use of insurances and political economy, of medicine and public health, and of politics.[32] These were indeed analyses, up to some point, of the nature of things, in particular of such things that apparently happen by chance. AU4

What of 'chance', then, in Leibniz's view? There are many phenomena of interest, and that can be the object of important knowledge, that Leibniz seems to consider as submitted to chance and fortuitousness. A conspicuous example is provided by the peculiarities of native tongues, which do not arise only from a change of customs: "le hazard y a aussi sa part", there is a role for chance as well (NE II, 22, §6; A VI, 6, 214). In historically developed languages, there is a combination of 'nature' and 'chance', *hazard*: this is because root words are formed not only by chance, but "sur des raisons physiques", on physical grounds as well (NE III, 2, §1; A VI, 6, 278 and 281). Moreover, when striking similarities and common roots are found between different natural languages, it is not chance that will be searched for, but reasonable historical explanations, such as the effects of commerce and migrations: "on ne le sauroit attribuer au seul hazard, ny même au seul commerce, mais plustost aux migrations des peuples" (*ib.*).

Sometimes Leibniz even deals with 'coincidences', also considered as the effect of 'chance'.[33] And he is confronted with the concept of 'chance' in probability theory, when reading Jan de Witt's *Waerdye van Lyf-Renten* (*Valuation of Life Annuities*, 1671) in the autumn of 1683. In the margin of his notes, Leibniz remarks: "I suppose that *Kansse*, or *expectative* in Dutch, comes from the French *chance*. In German I would say Schanße, like when they say 'to lose or forfeit a chance'".[34] Thus he does not immediately connect 'chance' with casual distributions of equi-



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[31] This allows him to negate any special role of the "opinion of weighty authorities", which can contribute to the likelihood of an opinion, but "does not produce the entire likelihood by itself" (NE IV, 2, 14; RB, 373) as regards the nature of things. For instance, Copernicanism was decidedly likely even when Copernicus was isolated in his opinion.

[32] On Leibniz's attentiveness to these matters see Leibniz (1995, 2000), Cussens (2004), ~~Mora Charles and Sol de~~ De Mora Charles (1992), Parmentier (1999), Rohrbasser and Véron (2001, 2002), Schulenburg and Thomann (2010).

[33] "I have been assured that a lady at a well-known court saw in a dream the man she later married and the room where the betrothal took place, and she described these to her friends, all before she had seen or known either the man or the place. This was attributed to some secret presentiment or other; but chance could produce such a result since it happens rather rarely; and in any case the images in dreams are a little hazy, which gives one more freedom in subsequently relating them with other images" (NE IV, 11, §11; RB, 445).

[34] "*Kansse* oder *expectative* hollandisch, puto esse ex Gallico chance germanice exprimerem Schanße, wie man sagt die schanße verlieren oder verscherzen" (A IV, 6, 705).

probable events,[35] but in the idea of the opportunity for an event to take place there is an implicit hint in that direction.

Huygens was well aware of this ambiguity of probability and so, in his analysis of games of chance, he preferred to discuss 'expectations' (in the sense of legitimate objective expectations of gamblers). The word Leibniz prefers, in order to express this aspect, is *apparence*, appearance:

> Now, since all these reasonings are based on reasonable appearances [*des apparences raisonnables*], it is timely to explain, first of all, what is an appearance and how it should be estimated. I say that the appearance is nothing else but the degree of probability: e.g. a dice like those used to gamble has six equal sides, and so the appearance is equal for each side. (A IV, 3, 457).

This definition is followed up by a rule for finding 'mean appearances' ('Regle pour trouver les moyennes apparences, aux quelles il se faut arrester dans l'incertitude'). Do just like the peasants do, Leibniz intimates: have three groups of estimators of the value of a good, and take the mean value of their estimates. This proceeding, he adds, "est fondé en raison demonstrative" (A IV, 3, 458), which means, among other things, that it can be submitted to logic, as we have seen that Leibniz, in his maturity, consistently maintains. In particular it can be submitted to an especially innovative branch of his already innovative universal logic, or *characteristica universalis*. The new branch of logic devoted to the degrees of probabilities, then, would be of the kind he always had in mind: a formal logic, an 'art of characters'. This he had written, for instance, to Princess Elizabeth in 1678:

> In order to reason with evidence in all subjects, we must hold some consistent formalism [*formalité constante*]. There would be less eloquence, but more certainty. But in order to determine the formalism that would do no less in metaphysics, physics, and morals, than calculation does in mathematics, that would even give us degrees of probability when we can only reason probabilistically, I would have to relate here the thoughts I have on a new symbolic analysis [*characteristique*], something that would take too long. (A II, 1, 666; AG, 239, modif.)

Of course the 'calculation' of a degree can be performed in many different ways and Leibniz seems to be thinking of some kind of direct measure. Possibly he is thinking of a set of criteria, and of rules for the composition of values corresponding to the composition of sentences.[36] In a way, nevertheless, it also had to be a proper calculus, a set of mathematical devices, since "the degrees of probability or likeli-



[35] A pioneer of the study of lotteries, Gataker, admitted that the word 'chance' could by some be "utterly condemned, and held foolish and heathenish", yet it was a term "according to the iust analogie and proportion of Tongues and Languages, used by the Holy Ghost himselfe in Gods booke both in the Old and New Testament" (Gataker 1627, 9–10). On him see Daston (1988, 155).

[36] As it is hinted at by, for example, this passage: "The question of how inevitable a result is, is heterogeneous from—*i.e.* cannot be compared with—the question of how good or bad it is. […] The fact is that in this as in other assessments which are disparate, heterogeneous, having more than one dimension (so to speak), the magnitude of the thing in question is made up proportionately out [*en raison composée*] of two estimates […] As for the inevitability [*grandeur*] of the result, and degrees of probability, we do not yet possess that branch of logic which would let them be estimated" (NE II, 21, § 67; RB, 206–7).

hood in conjectures or proofs" are capable of an estimation "as sure as that of numbers [*aussi asseurée que les nombres*]" (A VI, 4, 689).

For Leibniz, 'logic', 'calculus', and 'formal argument', are very broad notions and comprise very disparate ways of proceeding:

> The entire form of judicial procedures is, in fact, nothing but a kind of logic, applied to legal questions. Physicians, too, can be observed to recognize many differences of degree among their signs and symptoms. Mathematicians have begun, in our own day, to calculate the chances [*les hazards*] in games (NE IV, 16, §9; RB, 465).

Here hazard and its laws are, on the one hand, sheer constructs, since there is no real 'chance' in Leibniz's universe, as we have seen. On the other hand, in epistemic terms they can be the essential theoretic components of the pure analytic modelling of a possible situation, like that of the dice with the equal 'appearance' for each side. How far, then, can modelling answer the problem of evaluating contingencies, natural and practical?

It is telling that, when confronted in 1703 with Jakob Bernoulli's first attempts at mathematizing the art of conjectures, Leibniz immediately sees there "a difficulty", that is, "that contingencies, *i.e.* those things which depend upon infinite circumstances, cannot be determined through a finite number of experiments". Things change and, day in, day out, new illnesses appear: "who can state whether the next experiment will not depart from the law that all the former ones have followed?"[37]

It is Leibniz's usual mistrust of induction,[38] together with his usual enthusiasm for the infinite inexhaustibility of every parcel of creation, that makes him recoil at the thought of his friend Bernoulli's great-numbers laws. To this general attitude, a formal demonstration of the inevitable and inexhaustible variability of reality, and even of its analytical representations, is added:

> Given any number of points, it is possible to find infinite geometric lines that pass through them. I demonstrate it so. I postulate (and it can be demonstrated) that given any number of points, a regular line can be found that passes through them.[39] Suppose that it is found, and let us call it *A*. Now take another point in the same region, but such that does not belong to that line: let a line pass through both the formerly given points and the new point, which is made possible by the same postulate. By necessity this will be *a priori* a different line, and yet it will pass through the same points through which the former one passed. Since points



[37] Leibniz to Jacob Bernoulli, Dec 3, 1703: "Difficultas in eo mihi inesse 84 videtur, quod *contingentia* seu quae ab infinitis pendent circumstantiis, per finita experimenta determinari non possunt […] quis dicet, an sequens experimentum non discessurum sit nonihil a lege omnium praecedentium? ob ipsas rerum mutabilitates. Novi morbi inundant subinde humanum genus" (GM 3, 83–84).

[38] "That is why geometers have always held that what is proved by induction or by example in geometry or in arithmetic is never perfectly proved"; even "if one tried a hundred thousand times, […] one can never be absolutely certain of this unless one learned the demonstrative reason for it, something mathematicians discovered long ago. […] In fact, there are experiments that succeed countless times, and ordinarily succeed, yet in some extraordinary cases we find that there are instances where the experiment does not succeed" (GP 6, 504–5; AG, 190).

[39] This idea (that depends much on the definition of a 'regular' line) returns often in Leibniz's writings, together with the optimistic notion that all curves can have an analytical expression: see for example §6 of the *Discours de métaphysique* (A VI, 4, 1537–1538).

> can infinitely vary, more and more lines will be possible, up to infinity. Now these points can be compared to the observed instances, and the regular line can be compared to the rule, or estimation, that is drawn from those instances.[40]

In fact, Leibniz's idea was that, with his desired logic of probability we would not be entitled to certainty—that would be the same as pretending to demonstrate some truth by induction—but to a reasonable preference between opposing appearances, as he had written ~~1~~ one year before to Queen Sophie Charlotte:

> But the force of the demonstrations depends upon intelligible notions and truths, which alone are capable of allowing us to judge what is necessary. In the conjectural sciences they are even capable of demonstratively determining the degree of probability, given certain assumptions, so that we may reasonably choose, among opposing appearances, the one which is most probable. But this part of the art of reasoning has not yet been developed as much as it should be (GP 6, 504; AG, 189).

Now that a tentative foundation of this part of the art of reasoning has been presented to him, he writes to Bernoulli explicitly: "The estimation of probabilities is of the utmost utility, yet in juridical and political matters what is needed is not some subtle calculus, but rather an accurate enumeration of all circumstances."[41] Jacob, after a short-lived display of patience, answers in his customary uncompromising style:

> That the theory of the estimation of probabilities in juridical matters comprises not only the enumeration of circumstances, but this very reasoning and calculus as well, that we usually employ in comparing the lots of players, this is something that I have been taught by the various enquiries on insurances, on life rents, on dotal pacts, on presumptions, and still others.[42]

He proposes then to Leibniz an example that is intended to show him the correspondence between abstract reasoning and contingent natural reality. Just as, by numerous repeated extractions, there is an increasing probability of determining the unknown ratio of white and black pebbles in an urn, so by numerous repeated observations it is possible to determine with the necessary precision the frequency of illnesses in human individuals:



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[40] "Datis quotcunque punctis inveniri possunt lineae infinitae per ipsa transientes. Quod sic demonstro: Postulo (quod demonstrari potest) datis quotcunque punctis inveniri posse lineam aliquam regularem, per ipsa transeuntem. Inventa illa esse ponatur et sit A. Sumatur jam aliud punctum inter data, sed extra hanc lineam; et per puncta initio data et punctum novum transeat linea, quod fieri potest per idem postulatum: hanc necesse est esse diversam a priori, at tamen per eadem transire puncta data, per quae prior. Et cum punctum infinities variari possit, etiam aliae atque aliae in infinitum lineae erunt possibiles. His autem punctis comparari possunt casus observati et lineae regulari regulae seu aestimationes ex casibus ducendae" (GM 3, 84).

[41] "Utilissima est aestimatio probabilitatum, quanquam in exemplis juridicis politicisque plerumque non tam subtili calculo opus est, quam accurata omnium circumstantiarum enumeratione" (GM 3, 83).

[42] "Quod Doctrina de probabilitatibus aestimandis in materiis juridicis non sola circumstantiarum enumeratione, sed eodem illo ratiocinio et calculo indigeat, quo alias in sortibus aleatorum comparandis uti solemus, docent me variae quaestiones de Assicurationibus, de Reditibus ad vitam, de Pactis dotalibus, de Praesumtionibus, aliaeque" (GM 3, 87).

> Now, if you replace the urn with a human body, that be the body either of an old or of a young man, this body contains in itself the source of illnesses like the urn contains the pebbles, and you can determine in the same way, by observations, how much closer to death the former man is than the latter.[43]

If new illnesses should appear, then it would simply be required to institute new observations and experiments. This is, then, Bernoulli's conclusion:

> The ratio between the numbers, albeit infinite, of illnesses, can be determined by a finite amount of experiments, indeed without complete accuracy, yet so much as it is needed in practice, approximating nearer and nearer, until the error becomes negligible.[44]

Did Jacob Bernoulli manage to convince Leibniz? In any event, in the *Nouveaux essays*, which Leibniz composed between 1704 and 1705, we encounter this decisive overture:

> I have said more than once that we need a new kind of logic, concerned with degrees of probability, since Aristotle in his *Topics* could not have been further from it […] Anyone wanting to deal with this question would do well to pursue the investigation of *games of chance* (NE IV, 16, §9; RB, 466).

## 14.6 ~~5.~~ Conjectures

It may be remarked that conjectures on natural phenomena assessed with cutting-edge mathematical tools were also a way to go beyond, or leave behind the debates on Cartesian 'hypotheses'.[45] Emilie Du Châtelet, the socialite turned scientist who delivered the best translation ever of Netwon's *Principia Mathematica*, who proposed relevant insights on the estimation of forces and kinetic energy, and wrote some of the best pages on scientific methodology after Bacon himself, was among the first to rehabilitate hypotheses in natural science after Newton's anti-Cartesian offensive. The enabling instrument, so to speak, for the recovery of hypotheses was the evaluation of degrees of probability:

> Since hypotheses are only made in order to discover the truth, they must not be passed off as the truth itself, before one is able to give irrefutable proofs. So it is very important for the progress of the sciences not to delude oneself and others with the hypotheses one has invented, but one should estimate the degree of probability in a hypothesis [*estimer le degré de probabilité qui s'y trouve*]. (Du Châtelet 1740, ~~1,~~ 1: 89; 2009, 152).



[43] "Quod si nunc loco urnae substituas corpus humanum senis aut juvenis, quod fomitem morborum in se velut urna calculos continet, poteris eodem modo determinare per observationes, quanto ille quam iste morti sit vicinior" (GM 3, 88).

[44] "Rationem inter numeros morborum etsi infinitos determinare possumus finitis experimentis non praecise, sed quantum ad praxin sufficit accedendo subinde propius donec error insensibilis fiat" (GM 3, 91).

[45] See nonetheless McClaughlin (1996) for an investigation of French Cartesian empiricism.

Hypotheses, then, "are only probable propositions that have a greater or lesser degree of certainty,[46] depending on whether they satisfy a more or less great number of circumstances attendant upon the phenomenon that one wants to explain by their means." Hypotheses would finally become truths, "when their probability increases to such a point that one can morally present them as a certainty" (Du Châtelet 2009, 154). This is a noteworthy point, on which we shall conclude this chapter.

Jacob Bernoulli's *Ars conjectandi* ends with the unexpected surfacing, from the deep pools of frequentist probability and the law of great numbers, of a vindication of universal determinism, associated by Bernoulli with 'apocatastasis'—the final 'restitution' or 're-establishement' of all things—the two doctrines being drawn together under the possibly inappropriate banner of Platonist philosophy.[47] From what has been demonstrated, in fact,

> at last this remarkable result seems to follow, that if the observation of all events were continued for the whole of eternity (with the probability finally transformed into perfect certainty) then everything in the world would be observed to happen according to precise proportions and under constant laws of alteration *certis rationibus et constanti vicissitudinis lege*: so we would have to admit a sort of necessity and, so to speak, fatality, even in the most casual and fortuitous things. I do not know whether or not Plato already wished to assert this result in his dogma of the universal return of things to their former positions, in which he predicted that after the rolling of innumerable centuries everything would return to its original state (Bernoulli 1713, 239; 2006, 339, modif.).

It is not devoid of interest that the same notion of a universal return of all things to the initial conditions, as an ἀποκατὰστασις πὰντον, had been since the late 1690s the object of several short writings of Leibniz's.[48] The bases for Leibniz' treatment were not probabilistic, but rather combinatorial, allowing him to conclude *a priori* what follows:

> If the human kind continued for long enough, the time would necessarily come when nothing would be said that had not been already said before. But it is not certain that a time will come when nothing can be said that has not been said before. For it could happen that certain things were never said, even through all eternity. […] But suppose that 1 day nothing could be said that had not already been said before, then there must also be a time when the same events reoccur and when nothing happens which did not happen before, since events provide the matter for words (Leibniz 1991, 59; Coudert 1995, 113, modif.).

A similar result, in the last writing dedicated to such speculations that Leibniz composed in 1715, is produced by a simple demonstration of the finiteness of all possible written descriptions of any interval of time in the history of the universe, so



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[46] In the first edition, instead of *certitude*, Du Châtelet (1740, 1, 86; 1742, 1, 91) wrote *probabilité*.

[47] On the historical development of the doctrine of *apocatastasis* from its Stoic origins to Christian adaptations see Ramelli 2013. Bernoulli (1713, 53; 2006, 177) had also been using the term in a weaker sense, for the return of the lots to their original states in a game continued for long enough (*apocatastasis sortium*).

[48] There is some Origenism in the background of these discussions, that saw Leibniz debate with people like Petersen and Overbeck on the theme of universal renovation and salvation; see Costa 2014.

that Leibniz, in the draft of this last take of his on “*apocatastasis*”, wrote that, if the demonstration were true,

> if the human kind continues in its present state for long enough, a time will come when even single lives would repeat themselves in minute detail, and in the same circumstances: I myself, for instance, would return to live once again in the city called Hanover, on the banks of the river Leine, once again busy studying the history of the Brunswick house, and writing to the same friend letters with the same content (Leibniz 1991, 64).

But the demonstration is not applicable to the real vicissitudes of creation. As we have seen, contingent events, according to Leibniz, are governed by infinity: actually infinite circumstances enter in the constitution of the smallest parcel of the created universe. This richness corresponds to divine wisdom: the creator has produced nothing uniform, nothing monotone, and diversity marries plenitude in the whole of nature. That infinity likewise ensures the well-known fact that there exist no distinct indiscernible objects in Leibniz’s world, and thus also the repetition of identical events is made impossible by the same element—divine wisdom and choices of necessity only moral—by which the contingency of the world is granted.

Lambert’s answer, not many years later, to what might be called Bernoulli’s *paradoxum apocatastaticum*, would be much simpler: *cum vero nullum experimentum infinities repetatur,* “since no experiment can be repeated infinitely,[49] what can be deduced from a finite number of experiments must be considered instead” (Lambert 1760, §280; 2001, 96, modif.). Thus, we might say, the appreciation of contingency that was becoming the privileged object of the art of conjectures would be saved, once again, by a most basic principle of the *ars experimentandi*.



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[49] It is in reality a self-reply to a concern he expressed in the preceding paragraph: “So if the same experiment is taken to have been repeated infinitely, it is clearly right (*omni iure licet*) to consider the mean among all to not differ from the truth. This is apart from defects of instruments” (Lambert 1760, §279; 2001, 96).

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