Fostering critical thinking in primary school within dynamic geometry environments

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Abstract

The paper presents the design of a geometric inquiring-game activity in primary school and discusses the first results of its experimentation, through a case-study methodology. The activity, developed within a dynamic geometry environment, makes students investigate comparisons between the rhombus and the rectangle area. The goal of the study is to analyse the evolution of the arguments produced by students in order to connect and make sense of the results observed within the different registers of semiotic representation involved in the game.

Introduction and theoretical framework

The development of critical attitudes is necessary to act and to evaluate information in everyday life. This is emphasized in the CIEAEM Manifesto (p. 6), as well as in the CIEAEM 70 Discussion document, in which questions about "how to empower people to think critically and to adopt critical attitudes", and "how mathematics education could emphasise more the development of judgement and wisdom rather than of particular skills" are posed. Also the Italian educational guidelines underline the importance of nurturing students' argumentative competences since an early age, so that they may become citizens that fully participate to society (MIUR, 2012).

Previous research has shown that Dynamic Geometric Environments (DGEs) are particularly apt for triggering an inquiring approach in geometry (Yerushalmy, Chazan & Gordon 1990, Arzarello et al. 2002, Olivero & Robutti 2007, Baccaglini & Mariotti 2010, Sinclair & Robutti 2013). The dynamism allows the exploration of different examples of geometric configurations and the discovery of their invariant properties. Students' discoveries at first are stated in the form of conjectures and later are mathematically evaluated and checked. As J. Dewey (1938) wrote "[…] all logical forms arise within the operation of inquiry and are concerned with control of inquiry" (p.3). In other words, the operations of inquiry and investigation trigger students' critical thinking, promoting the passage from an empirical to a more detached and theoretical approach to geometry.

In this contribution we describe and analyse a mathematical inquiring activity that has been developed and experimented in two 5th grade Italian classrooms. Specifically, the activity is based on an *inquiring-game* consisting in a game to be played on a GeoGebra diagram and a worksheet task containing questions related to the geometric properties on which the game is based. The design of the games is inspired by the Logic of Inquiry (1999) elaborated in the '70s by the logician Jaakko Hintikka. Within this logic, in order to establish the truth of statements, Hintikka made use of semantical games (1998), i.e. games of verification/falsification. For example, to verify a statement expressed in the form $\forall x\ \exists y\ S(x, y)$, imagine a game between a verifier V who controls the variable y and a falsifier F who controls the variable x. F starts the game by choosing a value x_0 for the variable x and then the turn moves to V who should find a value y_0 for y such that $S(x_0, y_0)$ is true. According to Hintikka, the discovery of the y_0 by V is a reliable test of truth if the choice of the x_0 by F has been made in order to create the "worst-case scenario" to the verifier.

Within DGEs, different kinds of mathematical representations are explored during the inquiring-game. Duval (2006) highlights the central role of semiotic representations, which are "essential condition[s] for the development of mathematical thought" (p.107) since they allow "not only to designate mathematical objects or to communicate but also to work on mathematical objects and with them" (p.107). In particular, Duval stresses the dialectics between verbal language and visualization:

> "in geometry it is necessary to combine the use of at least two representation systems, one for verbal expression of properties or for numerical expression of magnitude and the other for visualization. What is called a "geometrical figure" always associates both discursive and visual representations [...]" (p.108)

Duval points out two different operations with registers, which he calls treatment and conversion. Treatments are transformations of representations within the same register of representation, while conversions are transformations that involve the passage from a register to another. The last ones are more complex since necessitate the recognition of the same represented object between different representations. “[T]he ability to change from one representation system to another is very often the critical threshold for progress in learning and for problem solving” (pp. 10).

Methodology

On the base of the presented theoretical framework and according the design-based research paradigm (Cobb et al., 2003), we designed inquiring-game activities focused on area equivalence and isoperimetric rectangles. The design method, previously experimented with secondary school students (Soldano & Arzarello 2016), has been adapted for primary level. Three game-activities were designed and experimented in two Italian classrooms of 5th grade students. The games are played by pairs of students on tablet or computers using GeoGebra. Beside playing, students have to answer to some questions contained in a task worksheet. Finally, students’ observations are shared through a class discussion in which the mathematical properties involved in the game are deeply analysed.

The authors participated as participant observers in the classroom and helped the teacher to manage technology as well as class discussion. The data collected from each experiment consist of the captured screen and dialogue of two pairs of students, the completed worksheets from all the students and the videotaped class discussion.

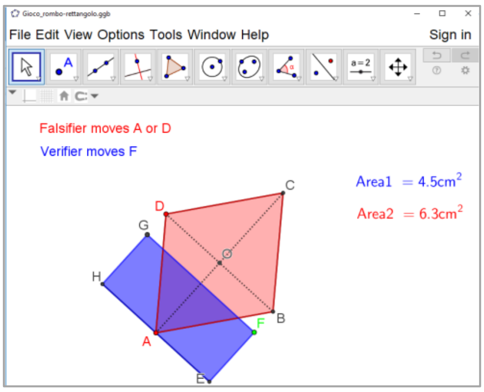


Figure 2: Dynamic diagram on which the game is played

In this paper, we focus on the inquiring-game activity designed to investigate the relationship between the area of a rectangle and a rhombus whose diagonals are of equal length as the sides of the rectangle. As shown in Figure 2, sides *EH* and *EF* of the rectangle have been robustly constructed on lines parallel to the rhombus diagonals and the vertex A of the rhombus lays in the midpoint of segment *EH*. Using the drag tool on point *F* it is possible to vary the length of the side *EF* of the rectangle and consequently the value of its area (Area 1). Using the drag tool on point *A* it is possible to vary the length of the diagonal *AC* of the rhombus and consequently the value of its area (Area 2). By moving the point *D*, it is possible to rotate the two figures, maintaining constant ratio of area. In the didactical design we do not tell the students the geometric nature of the objects involved in the game, we just give to them the GeoGebra file and the rules of the game, whose English translation is reported in Table 1.

Within your pair, choose a verifier and a falsifier.

- *The <u>falsifier</u> can move point A or D*
- *The <u>verifier</u> moves point F.*

Each match is made of two moves and the first one is always made by the falsifier. During the moves it is possible to interrupt the dragging for making zoom, moving the screen etc., and then ending the move.

<u>GOALS:</u>

The goal of the verifier is to make Area 1 twice the size of Area 2, while the goal of the falsifier is to prevent the verifier from reaching the goal.

The player who reaches the goal at the end of the verifier's move wins the match.

Table 1: Rules of the inquiring-game.

The verifier can always win the game transforming any configuration produced by the falsifier into a winning configuration. However, it may also happen that the falsifier wins due to tool affordances or to manual abilities.

Using Duval's frame, we can interpret players' moves as treatments in the figural register which may have an effect also within the numeric ones. The verifier's moves transform the rectangle so that his size is the double of the size of the rhombus. The equivalence is observable both within the figural register (imagining to reconfigure and overlap the triangles outside of the rhombus) and within the numeric register (the values of the areas shown in GeoGebra are equal, see Figure 3).

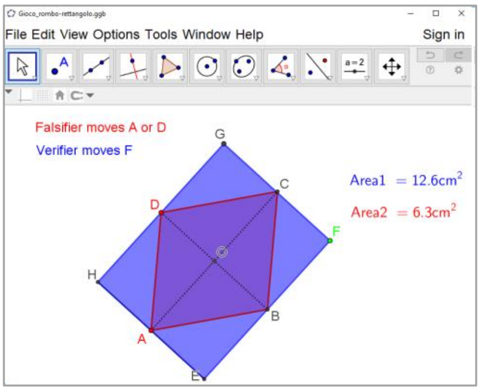


Figure 3: A possible winning configuration for the verifier

The guiding questions contained in the worksheet task are meant to shift students' attention from the game to the *geometric properties* of the game (Table 2). The first two questions trigger geometric exploration. The third and the fourth ask to justify *why* the rhombus is twice/equal the size of the rectangle. Finally, the fifth focuses the attention on a degenerate case.

1) *Which geometric figures are the 'red' and the 'blue' ones?*
2) *Consider the case of verifier's winning. Write here your observations.*
3) *Explain why each time the verifier reaches the goal, Area 1 is twice the size of Area 2.*
4) *Why if point B coincide with point F the two figures have equal area?*
5) *Have you ever find the situation in which the two areas are zero while playing? If not, play a new match to get this result. Describe what happened to the figures. If you want, make a drawing.*

Table 2: Worksheet task of the rhombus-rectangle game

We analyse the case study of two video-recorded girls Rose and Lily; According to their teacher, they are medium level students, Rose is more reflective then Lily, who is more skilled in practical duties.

Data analysis

At first, while playing, Rose and Lily do not consider the geometrical properties involved therein. On the contrary, they explore configurations with very big and small sizes of area, with the goal to prevail on the opponent. Table 2 shows the configurations produced during the 5th and the 7th matches:

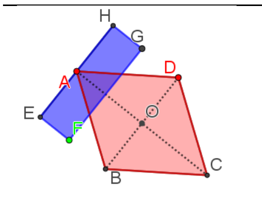
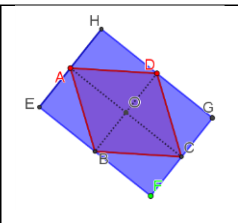
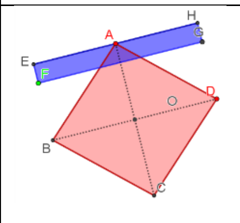
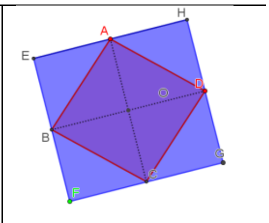
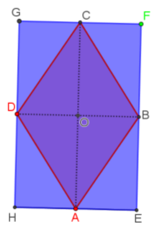
5th match		7th match	
			
Area 1 = 0,2 cm^2 Area 2 = 0,5 cm^2	Area 1 = 1,0 cm^2 Area 2 = 0,5 cm^2	Area 1 = 18,9 cm^2 Area 2 = 74,4 cm^2	Area 1 = 148,8 cm^2 Area 2 = 74,4 cm^2

Table 3: 5th and 7th matches in Rose and Lily's game

Since the dragging is not very accurate when the size of the area is very big/small, the moves become longer in time and the verifier has to put a great effort into the move. In such configurations, it could happen that an always-true geometric property can be falsified by the instrument inaccuracy. The activation of students' critical thinking is therefore fundamental to detach from the empirical situation and to establish the player who can always win from a theoretical point of view. The misalignment between what is possible to do empirically and what is true according to the mathematical theory can provide insights on students' approach to geometric figure.

1	**Rose**	We observe that the two figures are overlapped and their colour turn into purple		Area 1 = 6,4 cm^2 Area 2 = 3,2 cm^2
2	**Lily**	What does it matter that the colour turns into purple, Rose?		

3 **Rose** So tell me what do you want to say!

4 **Lily** We can say that the figures are overlapped and we can understand that from the remaining parts of the rectangle it is possible to make the same figure again, namely the rhombus. Because this one plus this one, plus this one plus this one make the rhombus *(pointing to the triangles AHD, DGC, CFB, BEA namely the parts of the rectangle not overlapped by the rhombus).*

[…] because if this one *(pointing to the rectangle EFGH)* is the double of this one *(pointing to the rhombus ABCD)*….

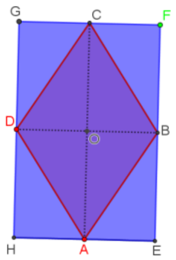
Not only the explored values of areas, but also students' dialogue reveals that, in a first phase, they are not paying attention to geometrical properties (lines 1-2). While Rose focuses on the colours, Lily observes that it is possible to make another rhombus equivalent to ABCD by decomposing and rearranging the parts of the rectangle which are not overlapped by the rhombus (lines 4). After some minutes the researcher (R) asks the students to better explain their reasoning:

5 **R** Can you explain me better why if we don't divide AC times BC by two we obtain the area of this rectangle?

6 **Lily** Because the rhombus is made by two triangles and in triangles we have to divide by two.

7 **R** And you Rose, did you observe the same?

8 **Rose** Because if we don't divide by two, we make 'base', that we can imagine here *(pointing to the side EF of the rectangle)* and which is also here *(pointing to the minor diagonal AC of the rhombus),* 'times height', which is the rhombus one *(pointing to the major diagonal BD of the rhombus)* but which is also the rectangle one *(pointing the side FG of the rectangle)*



The students want to justify the property they have observed, namely the double size of the rectangle, by using the formulas for the area of a rectangle and a rhombus. Only Rose provides a mathematically correct argument, in which the formula of the rhombus area is interpreted in the figural register (line 8). In this way Rose accomplishes a *conversion* from the symbolic to the figural register and vice-versa.

Results

The game prompts students to make treatments within the figural register: each match provides a new configuration in which the area of the rectangle is twice the area of the rhombus (verifier's move) and a new configuration in which it is not (falsifier's moves). The analysis of the recorded video shows an evolution in students' observations and argumentations. Initially, students mix properties that are geometrically relevant with those which are not (line 1). After some attempts, they observe that the area of the rectangle is twice the area of the rhombus; first they observe this property in the numerical register, since the numeric value of the rectangle area is the double of the rhombus area when the verifier reaches the goal. Then Lily argues the same property within the figural register (by decomposing and rearranging parts of the rectangle EFGH it is possible to make another rhombus equivalent to ABCD, line 4). Successively, by investigating in more depth the situation, students' arguments acquire a more theoretical reference (e.g. Rose, line 8). The simultaneous presence of two different and dynamically linked registers of representation (figural and numeric) pushed students to make sense of the result in the conversion between the two registers. This evolution is also fostered by the specific didactical design of the activity, which besides the DGE game includes the written questions that students have to answer after playing the game.

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