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A Life-Cycle Model with Unemployment Traps*

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Abstract

This paper extends the life-cycle model allowing for a small risk of long-term unemployment with permanent effects on labour income. Such nonlinear income risk dampens both early consumption and early investment in equities. Their optimal portfolio share becomes flat in age due to the resolution of uncertainty, as the worker ages, concerning this personal disaster. Adopting such flatter investment profile, instead of an age rule, delivers mean welfare gains that are three times larger than in models with linear income shocks. The uninsured share of long-term unemployment risk therefore matters for the design of optimal portfolios during working years.

Keywords: disaster risk, life-cycle portfolio choice, unemployment risk, human capital depreciation, age rule.

JEL classification: D15, E21, G11

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1 Introduction

Unemployment leads to large and persistent earnings losses that increase in its duration due to skill deterioration. The magnitude of this effect varies over time and across industries and demographic groups (Rhum, 1991; Jacobson, Lalond and Sullivan, 1993a; Davis and vonWacther 2011) as well as countries (Machin and Manning, 1999). Recently, the average duration of unemployment spells in developed economies has remarkably increased. For example, in the US the share of unemployed workers who are jobless for more than one year doubled over the Great Recession episode, reaching 24% of total unemployment in 2014. Krueger, Cramer and Cho (2014) and Kroft, Lange, Notowidigdo and Katz (2016) show that the re-employability of the long-term unemployed progressively declines over time, so that they are more likely to exit the labour force. More job openings do not lead to more employment among those who are jobless for more than six months, a pattern holding across all ages, industries and education levels (Ghayad and Dickens 2012). On the whole, these findings indicate that long-term unemployment may become a trap, often not supported by supplementary income provisions, given that unemployment benefits usually decline rapidly with unemployment duration.

In this paper, we embed the possibility of entering long-term unemployment with permanent consequences on human capital in a life-cycle model of consumption and portfolio choice. We model working life careers as a three-state Markov chain driving the transitions between employment, short-term and long-term unemployment states, as in Bremus and Kuzin (2014), calibrated to broadly match observed US labour market features. Importantly, we allow for (a small probability of) human capital erosion during unemployment. When unemployed, individuals receive benefits but simultaneously experience a cut in the permanent component of labour income which captures diminished future income prospects. This represents the observed permanent earning losses (Arulampalam, Booth and Taylor, 2000; Arulampalam, 2001; Schmieder, von

Wachter and Bender, 2016) due to skill loss during long-term unemployment (Neal, 1995; Keane and Wolpin, 1997; Edin and Gustavsson, 2008).

Such potential loss in human capital considerably lowers the optimal portfolio share invested in stocks with respect to the case of no unemployment risk. Importantly, optimal stock investment is no longer decreasing with age but remains remarkably flat over the whole working life, in line with evidence on U.S. portfolios (Amkeriks and Zeldes, 2004). On the contrary, traditional life-cycle models imply that households should reduce exposure to risky stocks as they approach retirement (Bodie, Merton and Samuelson, 1992; Viceira 2001; Cocco, Gomes and Maenhout 2005). The reason is that human capital provides a hedge against shocks to stock returns, making financial risk bearing more attractive. Investment in stocks should therefore be relatively high at the beginning of working careers, when human capital is large relative to accumulated financial wealth, and then gradually falling until retirement as human capital decreases relative to financial wealth. This model implication is embodied in the popular financial advice of a stock exposure gradually decreasing with age, the so-called "age rule". In our model with unemployment traps, such effect is instead moderated by the resolution of uncertainty concerning labour and pension income, as the worker safely approaches retirement age. Since the risk of long term unemployment falls together with human capital as retirement approaches, the resolution of uncertainty compensates the hedge effect and the optimal investment in stocks is relatively flat over the life-cycle.

The optimal risky portfolios are highly heterogeneous in models without long-term unemployment traps. On the contrary, a small probability of such personal disaster shrinks the heterogeneity of optimal portfolio choices across agents characterized by different employment histories. In the face of possible, albeit rare, human capital depreciation, individuals accumulate substantially more financial wealth during working life to buffer possible adverse labour market outcomes. Optimal early consumption consequently falls, becoming higher during both late working years and retirement years. The working-year responses to unemployment risk, including the flat age profile in stock investment, are remarkably robust to changes in preferences on the intertemporal correlation of shocks. Allowing for Epstein-Zin preferences only causes slower wealth decumulation and less risk taking during retirement years. Similarly, an increase in the correlation between stock returns and labour income shocks leaves the flat shape of optimal equity investment during working age unaltered, only increasing the portfolio share allocated to the riskfree asset. Thus, it is unemployment risk, due to uninsured long term unemployment, the first order determinant of the optimal financial risk taking at different ages.

This is not the first paper that explicitly connects life cycle precautionary savings to social insurance in general (Hubbard, Skinner and Zeldes, 1995) and to the partial one against employment risk in particular (Low, Meghir and Pistaferri, 2010). Our analysis uncovers the link between the share of long-term unemployment risk that is left uninsured and the path of optimal equity risk taking during working years. In this respect, acknowledging the presence of unemployment traps delivers relevant consequences for the design of pension fund default investment rules. Absent traps, welfare losses due to the use of simple age rules are below 1% of annual consumption in standard calibrations (Cocco, Gomes and Maenhout, 2005; Love, 2013), while they are above 3% when allowing for traps that are left uninsured by unemployment benefits. Such losses easily reach 10% of consumption for investment rules mimicking the ones embedded in Target Date Funds. They are due to both excessive financial risk taking when the young worker confronts higher uncertainty about his future labour and pension income, and by insufficient financial risk taking when this uncertainty is resolved. These suboptimal rules lead to lower consumption during retirement.

The above results obtain in calibrations to US data. The implied unconditional probabilities of being short-run unemployed (3.84%) and long-run unemployed (0.16%)

are rather conservative, and the unemployment benefit replacement rate is set at the average level observed for the US. As for human capital erosion, it is respectively equal to 0% and 60% in case of short-term and long-term unemployment spells in a calibration that captures the relatively slow re-employment process experienced by US workers. We select the human capital erosion during long-term unemployment considering both the total loss of human capital for the fraction of workers abandoning the labor force, and the partial loss for those who are able to find a job. Results go through when the erosion parameter is reduced to 50%, and when the probability of moving into long-term unemployment from an initial unemployment state is reduced by a third (from 0.15 to 0.10). We also experiment with a stochastic human capital loss conditional on long-term unemployment, to represent the possibility of incurring into large losses only in deep crisis situations rather than in normal business cycle downturns. In this case, our results go through even when the expected erosion is as low as 10%-20% of the permanent labour income component after the second year of unemployment.

Previous life-cycle models with unemployment and self-insurance leave the observed age pattern of stock holding during working life largely unexplained. Some versions of the life-cycle model account for the risk of being unemployed by introducing a (small) positive probability of zero labour income: in these models unemployment risk affects income only during the unemployment spell with no consequences on subsequent earnings ability (Cocco, Gomes and Maenhout, 2005) even when unemployment is persistent (Bremus and Kuzin, 2014). With no permanent consequence on subsequent earnings ability, the stock holding is still counterfactually decreasing in age till retirement although, on average, lower than what obtained without unemployment risk. Thus, it is the possibility of unemployment traps - rather than unemployment per se - that restrains risk taking by the young and middle-aged workers. Therefore, our model draws attention to a scenario opposite to the one depicted by Bodie, Merton and Samuelson (1992) and Gomes, Kotlikoff and Viceira (2008), where the worker is able - if employed - to modify labour supply to buffer income shocks. In fact, we know that flexible labour supply may enhance risk-taking thereby compressing precautionary saving and reducing consumption after retirement. However, this option is available ex-post only to those long-term unemployed who do find a new job, while it is the ex ante risk of permanently losing human capital that drives our results.

Several papers already investigate alternative hypotheses that may deliver the relatively flat stock profile observed in the data, departing from the pattern implied by traditional life-cycle models. Some of this prior research already relate the resolution of uncertainty over working life to the flattening of the age profile of stock investment. Hubener, Maurer and Mitchell (2016) point to the risk of changing family status during working age (i.e. marriage, fertility, divorce..), which affects consumption both directly and through labour supply. In Bagliano, Fugazza and Nicodano (2014), such flattening depends on the presence of both another risky asset, besides equities, and a positive correlation between stock returns and permanent labor income shocks. Moreover, it only appears when risk aversion or the variance of labor income shocks are higher than in the baseline calibration of Cocco Gomes and Maenhout (2005). In our model, the flat profile is robust to an in-depth sensitivity analysis and derives from the possibility of a rare personal labour market disaster, in an otherwise standard baseline setting.

This disaster differs from both the individual stock market disaster modeled in Fagereng, Guiso and Gottlieb (2017) and the aggregate economic collapse explaining asset pricing puzzles in Barro (2006), both of which concern financial wealth and may occur during retirement, as well. It instead reminds of the rare idiosyncratic disaster in Schmidt (2016), that appears to capture both the magnitude and the dynamics of the equity risk premium. Such rare personal disaster makes returns to human capital negatively skewed, a feature recently uncovered in the data by Guvenen, Karahan, Ozkan and Song (2015). In this light, our paper documents the large effects of non-normal shocks to labour income on life-cycle savings and investment, following the suggestion in Blundell (2014) of introducing higher moments and nonlinearities in shocks to labour income. Thus, our paper extends the literature on portfolio choice that has so far focused only on non-Gaussian returns to financial assets (see e.g. Guidolin and Timmerman, 2008).

The rest of the paper is organized as follows. Section 2 presents the benchmark life-cycle model and briefly outlines the numerical solution procedure adopted. We detail the model calibration in Section 3 and discuss our main results in Section 4. Section 5 provides a quantitative assessment of the welfare loss entailed by departing from optimal asset allocation profiles and adopting conventional age-related investment rules. Several robustness checks are presented in Section 6. Section 7 concludes the paper.

2 The life-cycle model

We model an investor who maximizes the expected discounted utility of consumption over her entire life and wishes to leave a bequest as well. The investor starts working at age t_0 and retires with certainty at age t_0+K. The effective length of her life, which lasts at most T periods, is governed by age-dependent life expectancy. At each date t, the survival probability of being alive at date $t+1$ is p_t, the conditional survival probability at t (with $p_{t_0-1}=1$). Investor's i preferences at date t are described by a time-separable power utility function:

$$\frac{C_{it_0}^{1-\gamma}}{1-\gamma}+E_{t_0}\left[\sum_{j=1}^{T}\beta^j\left(\prod_{k=0}^{j-2}p_{t_0+k}\right)\left(p_{t_0+j-1}\frac{C_{it_0+j}^{1-\gamma}}{1-\gamma}+(1-p_{t_0+j-1})\,b\frac{(X_{it_0+j}/b)^{1-\gamma}}{1-\gamma}\right)\right] \quad (1)$$

where C_{it} is the level of consumption at time t, X_{it} is the amount of wealth the investor leaves as a bequest to her heirs in case of death, $b\geq 0$ is a parameter capturing the strength of the bequest motive, $\beta < 1$ is a utility discount factor, and γ is the constant relative risk aversion parameter.

2.1 Labour and retirement income

During *working life* individuals receive exogenous stochastic earnings as compensation for labour supplied inelastically. Working life careers are modelled as a three-state Markov chain considering employment (e), short-term (u_1) and long-term (u_2) unemployment. Individual labour market dynamics are driven by the following transition matrix:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} \pi_{ee} & \pi_{eu_1} & \pi_{eu_2} \\ \pi_{u_1e} & \pi_{u_1u_1} & \pi_{u_1u_2} \\ \pi_{u_2e} & \pi_{u_2u_1} & \pi_{u_2u_2} \end{pmatrix} = \begin{pmatrix} \pi_{ee} & 1-\pi_{ee} & 0 \\ \pi_{u_1e} & 0 & 1-\pi_{u_1e} \\ \pi_{u_2e} & 0 & 1-\pi_{u_2e} \end{pmatrix} \tag{2}$$

where $\pi_{ij} = \text{Prob}\,(s_{t+1} = j|s_t = i)$ with $i, j = e, u_1, u_2$. If the worker is employed at t ($s_t = e$), she continues the employment spell at $t+1$ ($s_{t+1} = e$) with probability π_{ee}, otherwise she enters short-term unemployment ($s_{t+1} = u_1$) with probability $\pi_{eu_1} = 1 - \pi_{ee}$. Since to become long-term unemployed she must first experience short-term unemployment, we set the probability for the employed to directly enter long-term unemployment at zero, $\pi_{eu_2} = 0$. The short-term unemployed at t ($s_t = u_1$) exits unemployment ($s_{t+1} = e$) with probability π_{u_1e} or becomes long-term unemployed ($s_{t+1} = u_2$) with probability $\pi_{u_1u_2} = 1 - \pi_{u_1e}$; consequently we set $\pi_{u_1u_1} = 0$. Finally, if she is long-term unemployed at t ($s_t = u_2$), she is re-employed in the following period ($s_{t+1} = e$) with probability π_{u_2e} and remains unemployed with probability $\pi_{u_2u_2} = 1 - \pi_{u_2e}$.

As in Cocco, Gomes and Maenhout (2005), the employed individual receives a stochastic labour income driven by permanent and transitory shocks. In each working period, labour income Y_{it} is generated by the following process:

$$Y_{it} = H_{it} N_{it} \qquad\qquad t_0 \leq t \leq t_0 + K \tag{3}$$

where $H_{it} = F(t, \mathbf{Z}_{it}) P_{it}$ represents the permanent income component. In particular, $F(t, \mathbf{Z}_{it}) \equiv F_{it}$ denotes the deterministic trend component that depends on age (t) and a vector of individual characteristics ($\mathbf{Z}_{it}$) such as gender, marital status, household composition and education. Consistent with the available empirical evidence, the logarithm of the stochastic permanent component is assumed to follow a random walk process:

$$\log P_{it} = \log P_{it-1} + \omega_{it} \tag{4}$$

where ω_{it} is distributed as $N(0, \sigma_\omega^2)$. N_{it} denotes the transitory stochastic component and $\log(N_{it})$ is distributed as $N(0, \sigma_\varepsilon^2)$ and uncorrelated with ω_{it}.

In our set-up, differently from Bremus and Kuzin (2014), labour income received by the employed individual at time t depends on her past working history. In particular, we allow unemployment and its duration to affect the permanent component of labour income, H_{it}. Since the empirical evidence suggests that the longer the unemployment spell the larger is the worker's human capital depreciation (Schmieder, von Wachter and Bender, 2016), we let human capital erosion increase with unemployment duration. Thus, after one-year unemployment the permanent component H_{it} is equal to H_{it-1} eroded by a fraction Ψ_1, and after a two-year unemployment spell the permanent component, H_{it-1}, is eroded by a fraction Ψ_2, with $\Psi_2 > \Psi_1$. This introduces a non-linearity in expected permanent labour income. In compact form, the permanent component of labour income H_{it} evolves according to:

$$H_{it} = \begin{cases} F\left(t, \mathbf{Z}_{it}\right) P_{it} & \text{if } s_t = e \text{ and } s_{t-1} = e \\ (1-\Psi_1)H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_1 \\ (1-\Psi_2)H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_2 \end{cases} \qquad t = t_0, ..., t_0 + K \tag{5}$$

In the short-term unemployment state ($s_t = u_1$) individuals receive an unemployment benefit as a fixed proportion ξ_1 of the previous year permanent income $H_{it-1} = F_{it-1}P_{it-1}$, whereas in the long-term unemployment state ($s_t = u_2$) no benefits are available: $\xi_2 = 0$. Thus, the income received during unemployment is:

$$Y_{it} = \begin{cases} \xi_1 H_{it-1} & \text{if } s_t = u_1 \text{ and } s_{t-1} = e \\ 0 & \text{if } s_t = u_2 \text{ and } s_{t-1} = u_1 \text{ and } s_{t-2} = e \end{cases} \qquad t = t_0, ..., t_0 + K \tag{6}$$

making the unconditional distribution of labour income no longer log-normal.

Finally, during *retirement*, income is certain and equal to a fixed proportion λ of the permanent component of labour income in the last working year:

$$Y_{it} = \lambda F\left(t, \mathbf{Z}_{it_{0+l}}\right) P_{it_{0+l}} \qquad t_0 + K < t \leq T \tag{7}$$

where retirement age is $t_0 + K$, $t_0 + l$ is the last working period and λ is level of the replacement rate.

2.2 Investment opportunities

We allow savings to be invested in a short-term riskless asset, yielding each period a constant gross real return R^f, and one risky asset, characterized as “stocks” yielding stochastic gross real returns R_t^s. The excess returns of stocks over the riskless asset follows

$$R_t^s - R^f = \mu^s + \nu_t^s \tag{8}$$

where μ^s is the expected stock premium and ν_t^s is a normally distributed innovation, with mean zero and variance σ_s^2. We do not allow for excess return predictability and other forms of changing investment opportunities over time, as in Michaelides and Zhang (2017).

At the beginning of each period, financial resources available to the individual for consumption and saving are given by the sum of accumulated financial wealth W_{it} and current labour income Y_{it}, that we call *cash on hand* $X_{it} = W_{it} + Y_{it}$. Given the chosen level of current consumption, C_{it}, next period cash on hand is given by:

$$X_{it+1} = (X_{it} - C_{it})R_{it}^P + Y_{it+1} \tag{9}$$

where R_{it}^P is the investor's portfolio return:

$$R_{it}^P = \alpha_{it}^s R_t^s + (1 - \alpha_{it}^s)\, R^f \tag{10}$$

with α_{it}^s and $(1 - \alpha_{it}^s)$ denoting the shares of the investor's portfolio invested in stocks and in the riskless asset respectively. We do not allow for short sales and assume that the investor is liquidity constrained, so that the nominal amounts invested in stocks and in the riskless asset are non negative in all periods. All simulation results presented below are derived under the assumption that the investor's asset menu is the same during working life and retirement.

2.3 Solving the life-cycle problem

In this intertemporal optimization framework, the investor maximizes the expected discounted utility over life time, by choosing the consumption and the portfolio rules given uncertain labour income and asset returns. Formally, the optimization problem is written as:

$$\max_{\{C_{it}\}_{t_0}^{T-1},\{\alpha_{it}^s\}_{t_0}^{T-1}} \left(\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0} \left[\sum_{j=1}^{T} \beta^j \left(\prod_{k=0}^{j-2} p_{t_0+k} \right) \left(p_{t_0+j-1} \frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} + \right. \right. \right.$$
$$\left. \left. \left. + \left(1 - p_{t_0+j-1}\right) b \frac{\left(X_{it_0+j}/b\right)^{1-\gamma}}{1-\gamma} \right) \right] \right) \tag{11}$$

$$s.t. \quad X_{it+1} = \left(X_{it} - C_{it}\right) \left(\alpha_{it}^s R_t^s + \left(1 - \alpha_{it}^s\right) R^f \right) + Y_{it+1} \tag{12}$$

with the labour income and retirement processes specified above and the no-short-sales and borrowing constraints imposed. Given its intertemporal nature, the problem can be restated in a recursive form, rewriting the value of the optimization problem at the beginning of period t as a function of the maximized current utility and of the value of the problem at $t+1$ (Bellman equation):

$$V_{it}\left(X_{it,}P_{it}, s_{it}\right) = \max_{\{C_{it}\}_{t_0}^{T-1},\{\alpha_{it}^s\}_{t_0}^{T-1}} \left(\frac{C_{it}^{1-\gamma}}{1-\gamma} + \beta E_t \left[p_t V_{it+1}\left(X_{it+1,}P_{it+1}, s_{it+1}\right) \right. \right.$$
$$\left. \left. + \left(1 - p_t\right) b \frac{\left(X_{it+1}/b\right)^{1-\gamma}}{1-\gamma} \right] \right) \tag{13}$$

At each time t the value function V_{it} describes the maximized value of the problem as a function of three state variables: cash on hand at the beginning of time t (X_{it}), the stochastic permanent component of income at beginning of t (P_{it}), and the labour market state $s_{it}(= e,\ u_1,\ u_2)$. The Bellman equation can be written by making the expectation over the employment state at $t+1$ explicit, as:

$$
\begin{aligned}
V_{it}\left(X_{it,}P_{it},s_{it}\right) = & \max_{\{C_{it}\}_{t_0}^{T-1},\{\alpha_{it}^s\}_{t_0}^{T-1}} \left(\frac{C_{it}^{1-\gamma}}{1-\gamma}\right. \\
& + \beta\left[p_t \sum_{s_{it+1}=e,u_1,u_2} \pi\left(s_{it+1}|s_{it}\right)\widetilde{E_t V}_{it+1}\left(X_{it+1,}P_{it+1},s_{it+1}\right)\right. \\
& \left.\left. + (1-p_t)\, b \sum_{s_{it+1}=e,u_1,u_2} \pi\left(s_{it+1}|s_{it}\right)\frac{\left(X_{it+1}/b\right)^{1-\gamma}}{1-\gamma}\right]\right)
\end{aligned} \tag{14}
$$

where $\widetilde{E_t V}_{it+1}$ denotes the expectation operator taken with respect to the stochastic variables ω_{it+1}, ε_{it+1}, and ν_{it+1}^s. The history dependence that we introduce in our set-up, by making unemployment affect subsequent labour income prospects, prevents us to rely on the standard normalization of the problem with respect to the level of P_t. To highlight how the evolution of the permanent component of labour income depends on previous individual labour market dynamics we write the value function at t, in each possible state, as (dropping the term involving the bequest motive):

$$
V_{it}(X_{it},P_{it},e) = u(C_{it}) + \beta p_t \left\{
\begin{array}{l}
\left\{\begin{array}{ll}
V_{it+1}(X_{it+1},P_{it+1},e) & \text{with prob. } \pi_{e,e} \\
\text{with } \ P_{it+1} = P_{it}e^{\omega_{it+1}} \quad \text{and} & \\
X_{it+1} = (X_{it}-C_{it})R_{it}^p + F_{it+1}P_{it+1}e^{\varepsilon_{it+1}} &
\end{array}\right. \\
\\
\left\{\begin{array}{ll}
V_{it+1}(X_{it+1},P_{it+1},u_1) & \text{with prob. } 1-\pi_{e,e} \\
\text{with } \ P_{it+1} = (1-\Psi_1)P_{it} \quad \text{and} & \\
X_{it+1} = (X_{it}-C_{it})R_{it}^p + \xi_1 F_{it}P_{it} &
\end{array}\right.
\end{array}\right.
$$

$$
V_{it}(X_{it},P_{it},u_1) = u(C_{it}) + \beta p_t \left\{
\begin{array}{l}
\left\{\begin{array}{ll}
V_{it+1}(X_{it+1},P_{it+1},e) & \text{with prob. } \pi_{u_1,e} \\
\text{with } \ P_{it+1} = (1-\Psi_1)P_{it-1}\, e^{\omega_{it+1}} = P_{it}\, e^{\omega_{it+1}} \quad \text{and} & \\
X_{it+1} = (X_{it}-C_{it})R_{it}^p + F_{it-1}P_{it+1}e^{\varepsilon_{it+1}} &
\end{array}\right. \\
\\
\left\{\begin{array}{ll}
V_{it+1}(X_{it+1},P_{it+1},u_2) & \text{with prob. } 1-\pi_{u_1,e} \\
\text{with } \ P_{it+1} = (1-\Psi_2)(1-\Psi_1)P_{it-1} = (1-\Psi_2)P_{it} \ \text{and} & \\
X_{it+1} = (X_{it}-C_{it})R_{it}^p &
\end{array}\right.
\end{array}\right.
$$

$$
V(X_t, P_t, u_2) = u(C_t) + \beta p_t \begin{cases} \begin{cases} V_{it+1}(X_{it+1}, P_{it+1}, e) & \text{with prob. } \pi_{u_2,e} \\ \text{with } \; P_{it+1} = P_{it} e^{\omega_{it+1}} & \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} + F_{it-2} P_{it+1} e^{\varepsilon_{it+1}} \end{cases} \\ \\ \begin{cases} V_{it+1}(X_{it+1}, P_{it+1}, u_2) & \text{with prob. } 1 - \pi_{u_2,e} \\ \text{with } \; P_{it+1} = (1 - \Psi_2) P_{it} \text{ and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} \end{cases} \end{cases} \tag{15}
$$

This problem has no closed form solution: hence we obtain the optimal values for consumption and portfolio shares, depending on the values of each state variable at each point in time, by means of numerical techniques. To this aim, we apply a backward induction procedure starting form the last possible period of life T and computing optimal consumption and portfolio share policy rules for each possible value of the continuous state variables (X_{it} and P_{it}) by means of the standard grid search method.[1] Going backwards, for every period $t = T-1, T-2, ..., t_0$, we use the Bellman equation (14) to obtain optimal rules for consumption and portfolio shares.

[1] The problem is solved over a grid of values covering the space of both the state variables and the controls in order to ensure that the obtained solution is a global optimum.

3 Calibration

Parameter calibration concerns investor's preferences, the features of the labour income process during working life and retirement, and the moments of the risky asset returns. For reference, we initially solve the model abstracting from the unemployment risk as in Cocco, Gomes and Maenhout (2005). Then, we introduce unemployment risk and consider two scenarios: (i) unemployment spells cause only temporary income losses as in Bremus and Kuzin (2014), and (ii) unemployment has permanent consequences on the worker's earnings ability.

Across all scenarios, the agent begins her working life at the age of 20 and works for (a maximum of) 45 periods (K) before retiring at the age of 65. After retirement, she can live for a maximum of 35 periods until the age of 100. In each period, we take the conditional probability of being alive in the next period p_t from the life expectancy tables of the US *National Center for Health Statistics*. As regards to preferences, we set the utility discount factor $\beta = 0.96$, and the parameter capturing the strength of the bequest motive $b = 2.5$ (which bears the interpretation of the number of years of her descendants' consumption that the investor intends to save for). Finally, the benchmark value for the coefficient of relative risk aversion is $\gamma = 5$. The latter choice is relatively standard in the literature (Gomes and Michaelides 2005; Gomes, Kotlikoff and Viceira 2008), capturing an intermediate degree of risk aversion, though Cocco, Gomes and Maenhout (2005) and Bremus and Kuzin (2014) choose a value as high as 10 in their benchmark setting. The riskless (constant) interest rate is set at 0.02, with an expected equity premium μ^s fixed at 0.04. The standard deviation of the return innovations is set at $\sigma_s = 0.157$. Finally, we impose a zero correlation between stock return innovations and aggregate permanent labour income disturbances ($\rho_{sY} = 0$). Table 1 summarizes the benchmark values of relevant parameters as well as their changes considered in our subsequent analysis and robustness checks.

3.1 Labour income and unemployment risk

The labour income process is calibrated using the estimated parameters for US households with high-school education (but not a college degree) in Cocco, Gomes and Maenhout (2005).

The share of long term unemployment in total unemployment is fairly similar across all education groups (Katz et al., 2016). For instance, Mayer (2015) observes that the percentage of unemployed workers who have been out of work for two years or more with a high school degree (12.8%) and with a bachelor's degree (13.5%) do not statistically differ. For the high-school group we analyze, the variances of the permanent and transitory shocks (ω_{it} and ε_{it} respectively) are equal to $\sigma_\omega^2 = 0.0106$ and $\sigma_\varepsilon^2 = 0.0738$, and, after retirement, income is a constant proportion λ of the final (permanent) labour income, with $\lambda = 0.68$[2]. The parameter values assumed above are maintained across all scenarios.

The resulting labour income process does not capture the evidence in Krueger, Cramer and Cho (2014) that the long-term unemployed experience a progressive declining re-employability over time and are more likely to exit the labour force. We use data from the Current Population Survey (CPS) to calibrate the transition probabilities from employment to unemployment to reflect the risk of entering unemployment beside the observed average unemployment rates at different durations. According to the evidence based on CPS reported in Kroft, Lange, Notowidigdo and Katz (2016), the annual transition probability from employment to unemployment is 4%. Given the duration dependence and the steady decline in the annual outflow rate from unemployment to employment during the first year of unemployment (Kroft, Lange, Notowidigdo and Katz, 2016), we set the probability to leave unemployment after the first year at 85%. Our calibration appears quite conservative, since the chance of being employed 15 months later for those who had been unemployed 27 weeks or more is

[2] For a more realistic Social Security System design and its implications on retirement, consumption and investment decisions see Hubener, Maurer and Mitchell (2016).

only 36% (see the evidence on CPS data in Krueger, Cramer and Cho, 2014).

The annual transition probabilities between labor market states are chosen to match the average annual unemployment rate in the US:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} 0.96 & 0.04 & 0 \\ 0.85 & 0 & 0.15 \\ 0.85 & 0 & 0.15 \end{pmatrix} \tag{16}$$

The assumed transition matrix (16) yields quite conservative unconditional probabilities of being short-run (4.7%) and long-run unemployed (0.8%), compared to the 2015 overall (5,3%) anhd long-term (1.7%) unemployment rates.

In our baseline calibration with "*unemployment traps*" we assume a non-negligible human capital depreciation following a two-year unemployment spell. While Ψ_1 is kept at 0, Ψ_2 is increased up to 0.6, implying a 60% erosion of the individual permanent labour income component after the second year of unemployment, capturing the long lasting effects of protracted inactivity on job careers. Well established empirical evidence on job displacement shows that job losses affect earnings far beyond the unemployment spell, though the range of the estimated effects varies considerably. For example, the estimates for immediate losses following displacement may range from 30% (Couch and Placzek, 2010) to 40% of earnings (Jacobson, Lalond and Sullivan,1993b). Earnings losses are shown to be persistent in a range from 15% (Couch and Placzek, 2010) to about 25% (Jacobson, LaLonde and Sullivan, 1993a) of their pre-displacement levels. These estimates abstract from the effect of unemployment duration, while Cooper (2013) finds that earnings losses are larger the longer unemployment lasts. Also, based on administrative data, Jacobson, LaLonde and Sullivan (2005) estimate that earnings losses for displaced workers amount, on average, to 43-66% of their pre-displacement wage. This body of evidence, combined with a probability of finding a job after being unemployed for 24 months as low as 40% (Kroft, Lange, Notowidigdo and Katz, 2016), leads us to calibrate a substantial expected drop in human capital following a long term unemployment spell. More precisely,we derive the baseline value for the parameter Ψ_2 by considering a probability of leaving the labour force, losing all human capital, equal to 0.3 and a 0.7 probability of finding a new job with a 40% cut in wage.

Unemployment benefits are calibrated on the US unemployment insurance system. In particular, considering that the replacement rate with respect to last labour income is on average low and state benefits are paid for a maximum of 26 weeks, we set $\xi_1 = 0.3$ in case of short-term unemployment spells and set a value of $\xi_2 = 0$ for the long-term unemployed. No additional weeks of federal benefits are available in any state: the temporary Emergency Unemployment Compensation (EUC) program expired at the end of 2013, and no state currently qualifies to offer more weeks under the permanent Extended Benefits (EB) program.[3]

For comparison, we also consider a calibration of the model without unemployment risk. This "no unemployment risk" scenario corresponds to the standard life-cycle set up with $\pi_{ee} = 1$ and all other entries equal to zero in the transition probability matrix (2). In addition, to highlight the effects of permanent consequences of unemployment on future earnings prospects, we consider a third calibration by adding the unemployment risk embedded in the transition probability matrix (16) with no human capital erosion. In this "*unemployment with no traps*" scenario, unemployment has no permanent consequences on future earnings (i.e. $\Psi_1 = \Psi_2 = 0$) but entails only a cut in current income. This case closely correspond to the set-up studied by Bremus and Kuzin (2014) who focus only on temporary effects of long-term unemployment.

[3] Low, Meghir and Pistaferri (2010) acknowledge that layoffs are partially insured by the unemployment insurance system, while individual productivity shocks, other than major observable health shocks, are rarely insured in any formal way. As for other welfare programs, we do not model basic consumption needs and therefore overlook basic consumption insurance.

4 Results

4.1 Optimal policies

Figure 1 compares investors' optimal stock shares in the standard case of "*no unemployment risk*" (panel (a)) and in our preferred scenario with "*unemployment traps*" (panel (b)). In particular, the figure plots the optimal stock share as a function of cash on hand for an average level of the permanent labour income component of investors of three different ages (20, 40, and 70). In the case with no unemployment risk, standard life-cycle results obtain. Labour income acts as an implicit risk-free asset and affects the optimal portfolio composition depending on investor's age and wealth. For example, at age 20 the sizable implicit holding of the risk-free asset (through human capital) makes it optimal for the less wealthy investors to tilt their portfolio towards the risky financial asset. Indeed, for a wide range of wealth levels, agents optimally choose to be fully invested in stocks. The optimal stock holding decreases in financial wealth because of the relatively lower implicit investment in (risk-free) human capital.

When the model is extended to allow for permanent effects of unemployment spells on labour income prospects at re-employment ("*unemployment traps*"), setting the parameters governing the proportional erosion of permanent labour income at $\Psi_1 = 0$ after one year of unemployment and at $\Psi_2 = 0.6$ after two years, the resulting policy functions are shifted abruptly leftward. The optimal stock share is still decreasing in financial wealth but a 100% share of investment in stocks is optimal only at very low levels of wealth. In this case, long-term unemployment implies the loss of a substantial portion of future labour income which severely reduces the level of human capital and increases its risk at any age. Thus, for almost all levels of financial wealth, stock investment is considerably lower than in the case of no unemployment risk.

4.2 Life-Cycle Profiles

On the basis of the optimal policy functions, we simulate the whole life-cycle consumption and investment decisions for 10,000 agents. Figure 2, panel (a), shows the average optimal stock shares plotted against age obtained when unemployment risk is ignored and when it is accounted for. In the case of no unemployment risk (dotted line), the well known result on the age profile of optimal stock portfolio shares obtains. Over the life cycle the proportion of overall wealth implicitly invested in the riskless asset through human capital declines with age. Consequently, at early stages of the life cycle, optimal stock investment is about 100% and decreases with age to reach around 80% at retirement. When unemployment risk without human capital erosion is considered (dashed line), the optimal portfolio share of stocks still declines with age, though being slightly lower at all ages, with a 100% optimal stock share only for very young investors.

However, when long-term unemployment implies a rare but large skill erosion (solid line), the optimal stock investment is sizably reduced at any age and almost flat around 55-60%. The risk of permanently losing a substantial portion of future labour income prospects reduces the level of human capital and increases its riskiness, the more so the younger is the worker, inducing a lower optimal stock investment conditional on financial wealth especially when young. Consequently, the age profile remains remarkably flat over the whole working life.[4] These results highlight that possible long-run consequences of unemployment significantly dampen the incentive to invest in stocks, under standard calibrations, whereas unemployment persistence, with only temporary income losses as in Bremus and Kuzin (2014), has almost no effect on the age profile of optimal portfolio composition.

The reduction in the optimal portfolio share allocated to stocks is due to higher

[4] The relatively low investment in stocks during retirement is due to the presence of a positive bequest motive, common to all parametrizations considered in the paper.

wealth accumulation, in turn induced by larger precautionary savings.[5] Panel (b) of Figure 2 displays the average financial wealth accumulated over the life cycle for the three scenarios considered. In the face of possible, albeit rare, human capital depreciation, individuals accumulate substantially more financial wealth during working life to buffer possible disastrous labor market outcomes. Optimal consumption when young consequently falls, but it is much higher during both late working years and retirement years. Figure 3 displays the life-cycle profile of the ratio between savings and total (financial plus labour) income, comparing the case without unemployment risk with the one with unemployment traps. When the worker is 20 years old, the average propensity to save is especially high in the latter case, reaching 0.8 compared with less than 0.2 when unemployment risk is absent. Such propensity monotonically decreases in age, converging to the known pattern when the worker is in her forties. The figure clearly depicts the impact on savings of the resolution of uncertainty as individuals age.

4.2.1 Heterogeneity

The above results imply that the optimal stock investment is flat in age, even for a moderately risk averse worker. In the face of a very rare but large human capital depreciation, workers on average invest in stocks about 55% of their financial wealth. This average pattern may hide considerable differences across agents. The present section investigates the distribution across agents of both conditional optimal stock share and accumulated wealth.

The case of no unemployment risk is displayed in panel (a) of Figure 4, which shows the 25^{th}, 50^{th} and 75^{th} percentiles of the distributions. Both the optimal stock share and the stock of accumulated financial wealth are highly heterogeneous across

[5] Love (2006) shows that higher unemployment insurance benefits reduce calibrated contributions to pension funds by the young, suggesting that precautionary savings when young is due to unemployment risk.

workers as well as retirees, with the exception of young workers who tilt their entire portfolio towards stocks given the relatively riskless nature of their human capital. Heterogeneity of portfolio shares depends on the shape and movements through age of the policy functions displayed in Figure 1, relating optimal stock shares to the amount of available cash on hand, and on the level of cash on hand itself. Relatively steep policy functions imply that even small differences in the level of accumulated wealth result in remarkably different asset allocation choices. At the early stage of the life cycle, when accumulated financial wealth is modest, it is optimal for everybody to be fully invested in stocks. As investors grow older, different realizations of background risk induce large differences in savings and wealth accumulation pushing investors on the steeper portion of their policy functions and determining a gradual increase in the heterogeneity of optimal risky portfolio shares during their working life. After retirement, investors decumulate their financial wealth relatively slowly, due to the bequest motive, and still move along the steeper portion of their relevant policy functions; as a consequence, the dispersion of optimal shares tends to persist.

Panel (b) of Figure 4 displays the life-cycle distribution of stock share and financial wealth for the case with unemployment risk and human capital erosion. Compared with the case of no unemployment risk, the distribution of optimal stock shares is much less heterogeneous over the whole life-cycle. In particular, heterogeneity shrinks during working life even for young workers given the high human capital risk they bear at the beginning of their careers. In case of unemployment risk, policy functions are relatively flat (see panel (b) of Figure 1) implying that even large differences in the level of accumulated wealth result in homogenous asset allocation choices. Then, as in the previous case, the shape of heterogeneity of stock shares and accumulated financial wealth over the life-cycle is due to different realizations of background risk.

5 Welfare analysis of suboptimal choices

Workers usually delegate to their pension funds the task of managing long term saving on their behalf. The strategies proposed by portfolio managers often embed the feature of a decreasing age profile of investment in the riskier assets, with a portfolio share in stocks often in excess of 80% when young. These strategies resemble the ones that are optimal in the absence of unemployment risk. In fact, Cocco, Gomes and Maenhout (2005) measure that the representative worker should enjoy only a slightly higher (0.64%) consumption level to be compensated for the adoption of a suboptimal "age rule" by her pension fund. Bagliano, Fugazza and Nicodano (2014) find that a compensation of a similar amount is needed when the investor's asset menu includes also bonds, unless there is positive correlation between stock returns and permanent income shocks.[6] Love (2013) finds even lower welfare losses when optimizing over the parameters of the rule of thumb.

This section provides a quantitative assessment of the welfare loss associated with the adoption of simple portfolio allocation rules of thumb related to age when there are rare unemployment traps. We also explore an alternative suboptimal situation, where the worker adopts the utility-maximizing consumption and portfolio allocation but ignores the possibility of unemployment traps. This case is inspired by the scant discussion of long-term unemployment in the US prior to the recent crisis, which points to underestimation of the problem before 2007. The crisis may have generated a structural break in the economy, aggravating the long-term unemployment problem; or it may have enhanced the awareness of the rare occurrence of unemployment traps. The analysis of this case also delivers an upper bound to the welfare gains achievable when workers switch to the optimal asset allocation taking into account the potential occurrence of long-term unemployment spells with permanent consequences on their

[6] Only in the case of positive correlation, the compensating consumption is higher and may reach 3.9% for the benchmark risk aversion parameter ($\gamma = 5$).

earnings prospects.

In particular, we consider two suboptimal asset allocation patterns related to the investor's age. The first is the typical "age rule" analyzed by Cocco, Gomes and Maenhout (2005), with a risky portfolio share set at 100 less the investor's age.[7] The second rule of thumb, denoted as "target-date fund (TDF) rule", comes closer to actual strategic asset allocation patterns adopted by Target-Date Funds. As shown in panel (a) of Figure 5, the stock portfolio share is set at 90% until the age of 40, is gradually decreased over the remaining working life down to 50% at the retirement age (65), and is further reduced in the early retirement period to reach a bottom of 30% at the age of 72. This TDF rule reminds of the one investigated by Bagliano, Fugazza and Nicodano (2014) with also bonds in the investor's asset menu. The metric used to perform welfare comparisons is the standard consumption-equivalent variation employed by Cocco, Gomes and Maenhout (2005). We compute the percentage increase in annual consumption required by the investor to obtain the same level of expected utility warranted by the optimal life-cycle strategy for each suboptimal asset allocation rule. The consumption-equivalent variation is obtained by simulating consumption and wealth accumulation choices of 10,000 agents following the optimal asset allocation strategy and each of the alternative (suboptimal) investment rules, and deriving the associated expected discounted life-time utility levels. From the average expected discounted utility across individuals, the constant consumption stream needed to compensate investors (in each period and state) for the adoption of suboptimal strategies is computed using the CRRA utility function. Throughout our comparisons, we adopt the benchmark calibration parameters reported in Table 1.

The left-hand side of Table 2 shows the welfare gains associated with switching from the "age rule" to the optimal portfolio choice. Both the mean and the median

[7] In a variant of this "age rule", the worker starts saving for retirement 40 years before the target retirement date, setting the initial share of stocks at 80% and letting it fall to 40% at retirement (Bodie, Treussard and Willen, 2007).

increase in welfare-equivalent consumption are equal to 3.3%. Welfare gains are three times larger than prior estimates in the literature. Such gains derive from the fact that consumption and savings are distorted by both the higher risk taking when the worker faces a large amount of uncertainty about future labour and pension income, and by lower risk taking when uncertainty is resolved. Average consumption (panel (b) of Figure 5) is close to the optimal one during early working years under the "age rule", but it is much lower during retirement. Moreover, as shown in panel (c), while wealth accumulation until age 55 is close to the optimal one, average financial wealth at retirement and thereafter turns out to be lower under the "age rule". This pattern is due to agents who, having incurred in a trap, save less and ultimately - given the quick reallocation towards the riskless asset - are also able to consume and bequeath less. Those workers who do not experience personal disasters are able to set aside more wealth, but gradual conversion into the riskless asset reduces the return on the financial wealth relative to investors adopting optimal portfolio shares. The pattern of welfare gains across income brackets is however surprising, as revealed by panel (b) of Table 2. Mean welfare gains when income, at age 64, is below the 5th percentile of income distribution is lower than for agents with incomes above the 95th percentile (1.6% versus 2.4%). We tentatively ascribe such result to the fact that a distorted portfolio rule delivers lower utility losses at the bottom of the income distribution, because of lower financial wealth.

The middle column of Table 2 displays welfare gains when the investor adopts the optimal asset allocation pattern instead of the "TDF rule", with the stock portfolio share exceeding the one dictated by the "age rule" until age 55 and later falling below it. Given that there is higher exposure to financial risk early in life and lower exposure during retirement, mean and median welfare gains from adopting the optimal portfolio rule are much higher (above 10% of yearly consumption). This pattern emerges despite two seeming improvements, highlighted in Figure 5. The first is that mean consumption under a TDF portfolio allocation is higher, from age 30 until age 80, than consumption under the optimal portfolio rule. The second is that mean financial wealth exceeds the optimal one until past age 80. The increased consumption during working life and early retirement is thus more than offset by the marked reduction during later retirement years, followed by lower bequest. Once again, the mean welfare gains are lower for those with income (at age 64), below the 5th percentile of income distribution than for those with incomes above the 95th percentile (9.5% versus 12.3%). Both experiments suggest that distortions in asset allocation produce larger welfare losses for richer households. Figure 6 confirms the above interpretation displaying mean income profiles for individuals conditional on welfare gains. Income for workers with welfare gains below the 5th percentile exceeds mean income (across the whole population) at all ages. In turn, the latter is well above the income accruing to those enjoying the largest welfare gains.

In both cases considered above, the worker is aware of unemployment traps and optimally chooses saving and consumption given the rule-of-thumb portfolio allocation. Our welfare analysis concludes with the suboptimal case when the worker maximizes expected utility oblivious of rare personal disasters. The right-hand column of Table 2 delivers an upper bound on median welfare gains from becoming aware of unemployment traps (more than 200%). Mean welfare gains are even higher (642%). This pattern derives from a much higher average consumption until age 40, leading to lower buffer savings, for the unaware worker who chooses consumption and portfolios composition as in Cocco, Gomes and Maenhout (2005) (see panel (b) in Figure 5). If a long-term unemployment spell occurs, consumption and bequest drop dramatically, in some cases almost to zero. This implies a large utility loss for a tiny share of workers, causing a sizable increase in mean welfare loss relative to the median value. Since close-to-zero consumption is likelier at the bottom of the income distribution, mean welfare gains from becoming aware of traps are higher for those with low rather than high income, at age 64 (1024% against 218% as shown in panel (b) of Table 2). Clearly, the earlier the traps occur, the lower is buffer wealth, the worse are the consumption and welfare consequences. The latter observation hints at a welfare improving scheme designed to support the fraction of workers hit by long-term unemployment when young, along the lines suggested by Michelacci and Ruffo (2015). More generally, the size of the welfare losses suggests to move towards schemes that protect against longer term unemployment (see, for instance, Setty 2017).

6 Robustness

This section sheds additional light on the strength of our results, that radically depart from the known wisdom concerning optimal life cycle behavior during working years and hold across the entire distribution of investors.

A first robustness check concerns the sensitivity of our results to a lower probability of personal disaster. In performing such analysis, we also allow for an asymmetric reduction in the probability of long-term unemployment with respect to workers' age. Recent data from US labour market statistics indeed show that the composition of long-term unemployment is shifting towards the elderly. In 2015 the overall and the long-term unemployment rates were about 5.7% and 1.7% respectively, with the share of long-term unemployment in the overall unemployment rate differing widely among age groups: from 20% among young workers (16-24 years old), to 35% among prime age workers (25-55), and up to 41% among older workers (over 55).

A second check regards the modelling of the link between unemployment risk during working life and retirement income, so as to make sure that our results do not originate exclusively from long-term unemployment occurring during the very last working years, which heavily reduces retirement income.

Further, since previous literature highlights that the power utility function implies that the worker is indifferent to intertemporal correlation of consumption shocks, we adopt Epstein-Zin preferences to investigate whether positive correlation aversion boosts the impact of unemployment traps. A similar motivation pushes us to analyze the sensitivity of the equity-investment profile to positive correlation between stock returns and labour income shocks.

Finally, we experiment with more conservative calibrations and alternative modelling of human capital erosion, so as to represent the possibility of incurring in large losses only in deep crisis situations rather than in normal business cycle downturns.

6.1 Age-dependent unemployment risk

In this section, we calibrate our model with unemployment traps, allowing for both a smaller and age-dependent long-term unemployment risk. We change the transition probability from (short-term) to long-term unemployment, denoted as $\pi_{u_1 u_2}$ in the following transition probabilities matrix:

$$\Pi_{s_t, s_{t+1}} = \begin{pmatrix} 0.96 & 0.04 & 0 \\ 1 - \pi_{u_1 u_2} & 0 & \pi_{u_1 u_2} \\ 0.85 & 0 & 0.15 \end{pmatrix}$$

with respect to the baseline calibration in (16) where $\pi_{u_1 u_2} = 0.15$ irrespective of the worker's age. In "*case 1*", the probability of entering long-term unemployment is reduced by one third (from 0.15 to 0.10) only for workers younger than 50 years old. In "*case 2*", we further reduce the probability of entering long-term unemployment for very young workers, setting $\pi_{u_1 u_2} = 0.075$ for workers less than 30 years old. In all scenarios, transition probabilities are rather conservative implying steady state long-term unemployment rates lower than the actual one. For reference, in the baseline case, the steady state long-term unemployment rate is 0.8%, while it is 0.5% and 0.4%, in case 1 and 2, respectively.

Figure 7 reports the life-cycle profiles for the optimal conditional stock holding and financial wealth accumulation when the long-term unemployment risk is age-dependent. Compared with the baseline case, the age profile of stock investment is only slightly modified. A lower long-term unemployment risk at young ages implies a moderately higher stock share during prime age but does not significantly alter investors' behavior later over the working life and during retirement, and has virtually no effect on wealth accumulation.

6.2 Unemployment risk and retirement income

In our model, pension benefits are a fixed proportion of the last labour income earned prior to retirement age. Such income is especially sensitive to human capital loss due to the occurrence of long-term unemployment in years just before retirement. Thus, we analyze whether our results are robust to changes in modelling the link between long-term unemployment at old ages and subsequent pension provisions.

To begin with, we assume no human capital loss in the event that unemployment occurs in the years immediately before retirement. Our simulation results show that the flattening of the optimal stock share profile carries over to this setting, suggesting that it is not an artifact of how we model pension income. In a second check, we take the solution of our original model (calibrated in the case of unemployment risk with human capital erosion) and focus on simulated life-cycle profiles for two selected groups of agents. The first group includes workers who have experienced just one long-term unemployment spell of 5 years over the entire working life at the beginning of their job career (i.e. before the age of 35), whereas the other group is contains workers who have experienced just one long-term unemployment spell of 5 years over the entire working life but at end of their job career (i.e. after the age of 60). We find that in both cases, average life-cycle stock share profiles exhibit the flattening property. This experiment confirms that the flattening is due to the riskier nature of human capital, together with the resolution of uncertainty during working age, and is not affected by specific assumptions on the determinants of pension income.

6.3 Boosting the effect of unemployment traps

In this section, we consider two possible avenues that might reinforce our results. The first one is to allow for a positive correlation between stock return innovations and the innovations in permanent labor income ($\rho_{sY} > 0$), on top of human capital erosion. Results in Bagliano, Fugazza and Nicodano (2014) show that a realistically small correlation has large effects on life-cycle choices when it interacts with a higher variance of the permanent component of labor income shocks. One may therefore expect a similar effect in the presence of unemployment traps. Empirical estimates of the stock return-labor income correlation differ widely even when we restrict attention to the US economy. Cocco, Gomes and Maenhout (2005) report estimated values not significantly different from zero across various education groups, in line with Heaton and Lucas (2000), whose estimates range from -0.07 to 0.14. However, Campbell, Cocco, Gomes and Maenhout (2001) find higher values, ranging from 0.33 for households with no high-school education to 0.52 for college graduates. In the simulations below we adopt an intermediate positive value of $\rho_{sY} = 0.2$.

Figure 8 shows optimal portfolio shares of stocks and the pattern of financial wealth accumulation with no correlation and with a positive correlation between labor income shocks and stock returns. While the shape of life-cycle profiles is relatively unaffected, the average stock share is lower at all ages. In case of positive correlation, labour income is closer to an implicit holding of stocks, reducing the incentive to invest in stocks at all ages. More specifically, investors are relatively more exposed to stock market risk and will find it optimal to offset such risk by holding a lower fraction of their financial portfolio in stocks if compared with the case of no correlation. The stock share remains substantially flat over the whole working life, displaying limited variability around a level of about 50%. At the retirement age of 65, human capital becomes riskless, since pension income is certain and therefore uncorrelated with stock return innovations. Thus investors rebalance their portfolio towards stocks: during retirement, the level and time profile of the stock share are very close to the case of no correlation. Further, the relative increase in human capital risk due to a positive correlation does not substantially alter the pattern of financial wealth accumulation.

The second experiment implements a change in preferences that allows for intertemporal correlation aversion (Bommier, 2007). With a power utility function, the worker is indifferent to positive or negative intertemporal correlation of consumption lotteries (shocks). With Epstein-Zin preferences, the worker is averse to positive correlation when the coefficient of relative risk aversion is greater than the inverse of the elasticity of intertemporal substitution (EIS). Adopting a recursive (Epstein-Zin) formulation for preferences and keeping the risk aversion parameter constant ($\gamma = 5$), we simulate the model with positive ($EIS = 0.5$) and negative ($EIS = 0.1$) correlation aversion, comparing the results with our baseline case of indifference (i.e. power utility, $EIS = 0.2$). Figure 9 shows that aversion to positive correlation has a negligible effect during working years, while it causes a slower wealth decumulation and less risk taking during the retirement period, especially as death approaches. This is consistent with the known property that higher mortality risk magnifies the effects of intertemporal correlation aversion (Bommier, 2013).

Overall, the experiments above confirm the robustness of the flattening of the life-cycle profile to changes in both hedging opportunities in the stock market and to the intertemporal elasticity of substitution, pointing to the dominance of the personal disaster effect.

6.4 Sensitivity to human capital erosion

Finally, we check the sensitivity of life-cycle profiles with respect to the magnitude of the human capital erosion effect due to long-term unemployment, captured by the parameter Ψ_2 (set equal to 0.6 in our baseline calibration). Since, as discussed in Section 3, available estimates of earnings losses due to long-term unemployment as as low as 15% of the level of pre-displacement earnings, Figure 10 shows the results of an experiment with the human capital erosion parameter Ψ_2 in the range $0.2 - 0.6$. Once this parameter falls to 0.4, the hump shape in the portfolio share of stocks appears again over working life. The life-cycle profiles of both the optimal risky portfolio share and financial wealth accumulation track closely the case of no unemployment when the potential human capital loss is equal to 20%.

So far, the human capital erosion parameters Ψ_1 and Ψ_2 are set equal to constants. However, it could be more appropriate to model them as stochastic variables. After all, the tragic human capital consequences of the unemployment trap emerged in the US during the Great Recession, but were less evident in the past. This may correspond to high (low) realizations of Ψ_2 for a large number of workers in the Great Recession (business cycle expansions), and an intermediate value in ordinary business cycle contractions.

The Beta distribution can be usefully employed to model the behavior of random variables that take values limited to finite intervals. In our case, random variables Ψ_i ($i = 1, 2$) are fractions limited to the interval $[0, 1]$. We therefore assume that Ψ_i follow Beta distributions with support $[0, 1]$ and shape parameters (a_i, b_i): thus, $\Psi_i \sim Beta(a_i, b_i)$. The flexibility of the distribution enables us to concentrate nearly all the probability mass toward the most probable values of proportional human capital erosion, leaving the possibility of extremely unlikely realizations open. The most revealing case is displayed in Figure 11. While Ψ_1 is kept at 0, Ψ_2 follows the Beta distribution with expected value ranging between 0.10 and 0.20 and standard deviation between 0.21 and 0.32, implying an expected $10\% - 20\%$ erosion of the individual permanent labor income component after the second year of unemployment. More precisely, the calibrated distribution for Ψ_2 implies a median value for the proportional human capital erosion lower than 1%, while the 75^{th} percentile ranges between 6% and 25%. It turns out that under all distributional assumptions, life cycle profiles are very similar to the case of a non-random loss of human capital $\Psi_2 = 0.6$.

This provides a final suggestion that extremely rare but potentially disastrous labour income shocks may be relevant to understand cautiousness by young workers and their limited risk taking in the stock market.

7 Conclusions

As the recent Great Recession episode has highlighted, long-term unemployment spells may persistently damage workers' human capital. With this background motivation, this paper investigates the effects of unemployment traps on life-cycle savings and portfolio choice, introducing higher moments and non-linearities in the labour income process. This methodological innovation delivers new insights. Even a small probability of experiencing human capital erosion due to long-term unemployment is able to generate optimal conditional stock shares more in line with those observed in the data. Due to the possibility of human capital loss, young workers face higher uncertainty concerning future income and social security pension levels than older ones. At the same time, young workers with continuous careers have larger human capital than older ones. When a highly unlikely unemployment spell may potentially lead to considerable human capital erosion, the first effect offsets the second and the optimal investment in stocks is relatively flat over the life-cycle. This result departs from the implications of previous models with linear income shocks highlighting the importance of unemployment traps for the life-cycle portfolios.

Our calibrations also suggest an alternative, and more balanced, design for target-date investment funds. Such modified design implies an average 3% -10% increase in welfare-equivalent annual consumption, depending on the benchmark age rule. This more balanced design fits different kinds of workers, given the limited heterogeneity in life cycle investments induced by the threat of personal disasters. More generally, our analysis indicates that the pattern of risk taking at different ages in Target Date Funds should be related to the share of uninsured long-term employment risk.

Our analysis implies that we should observe changes in the life-cycle profile of household portfolios, both across cohorts and across countries, in response to the coverage of long term unemployment. Clearly there are other sources of human capital erosion, as well as other partial insurance vehicles. The optimal flat asset allocation will extend to such scenarios to the extent that such additional shocks remain partially uninsured by additional hedges, and that they have worse consequences the earlier they hit the worker.

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Table 1. Calibration parameters

This table reports benchmark values of relevant parameters.

Working life (max)	20 -65
Retirement (max)	65 -100
Discount factor (β)	0.96
Risk aversion (γ)	5
Replacement ratio (λ)	0.68
Variance of permanent shocks to labour income (σ^2_ε)	0.0106
Variance of transitory shocks to labour income (σ^2_n)	0.0738
Riskless rate	2%
Excess returns on stocks (μ^s)	4%
Variance of stock returns innovations (σ^2_s)	0.025
Stock ret./permanent lab. income shock correlation (ρ_{sY})	0

	No unemployment risk	**Unemployment risk with no traps**	**Unemployment traps**
Unemployment benefits			
Short term unemployed (ξ_1)	-	0.3	0.3
Long term unemployed (ξ_2)	-	0	0
Human capital erosion			
Short term unemployed (Ψ_1)	-	-	0
Long term unemployed (Ψ_2)	-	-	0.6

Table 2. Welfare gains

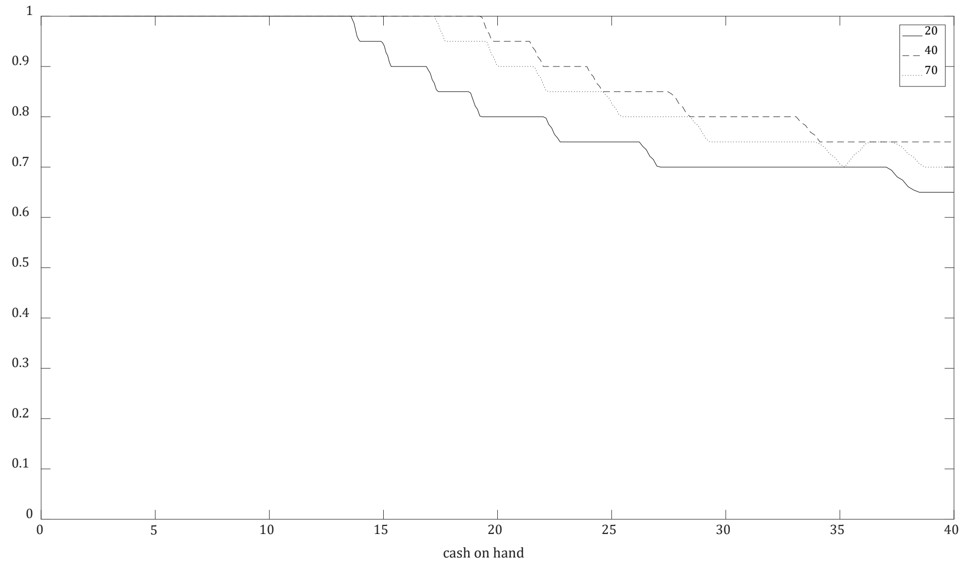
This table reports welfare gains (in percentage points) due to the adoption of optimal choices instead of suboptimal ones. The first and second columns display gains from respectively abandoning suboptimal “Age” and “Target Date Fund” rules for asset allocation. The third column refers to gains deriving from taking account of unemployment traps in the optimization process. Panel (a) displays the distribution of welfare gains. Panel (b) compares mean gains for workers with income at age 64 below the 5th percentile and above the 95th percentile of income distribution.

	Age Rule	TDF rule	Unaware of Traps
a. Distribution of welfare gains (% points)			
Mean	3.3	12.0	642.5
Median	3.3	11.8	215.8
5^{th}	1.5	8.0	-40.5
95th	5.4	17.0	573.6
b. Welfare gains conditional on income at age 64 (% points)			
Below 5^{th} percentile	1.6	9.5	1024.0
Above 95^{th} percentile	2.4	12.3	218.2

Figure 1. Policy functions

This figure shows the portfolio rules for stocks as a function of cash on hand for an average level of the stochastic permanent labor income component. The policies refer to selected ages: 20, 40, and 70. Panel (a) and (b) refer to the case without unemployment risk and with unemployment traps, respectively. In the latter, the parameters governing the human capital erosion during short-term and long-term unemployment spells are Ψ_1=0 and Ψ_2=0.6.

a. No unemployment risk



b. Unemployment traps

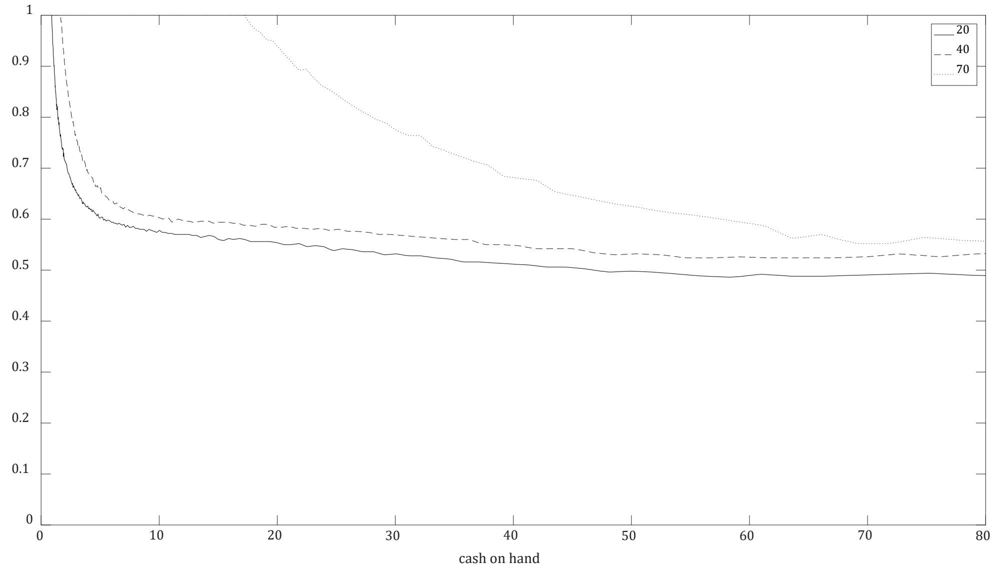
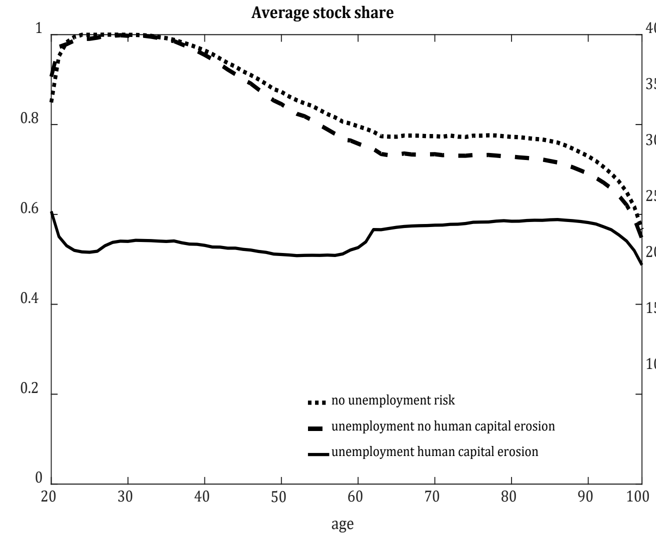


Figure 2. Life-cycle average profiles

This figure displays the mean simulated stock investment and financial wealth accumulation life-cycle profiles. Age ranges from 20 to 100. The three cases correspond to no unemployment risk (dotted line); unemployment risk with no traps (dashed line); unemployment risk with traps, with human capital erosion Ψ_1=0 and Ψ_2=0.6 (solid line).

(a)



(b)

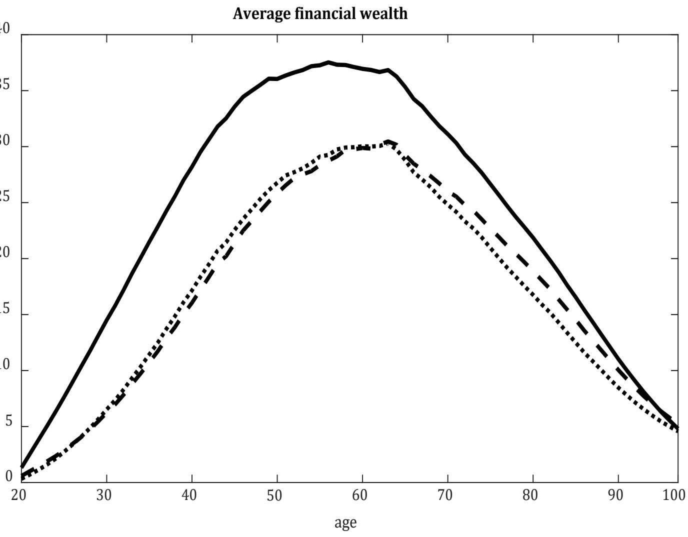


Figure 3. Life-cycle profiles of the savings rate

This figure displays the savings dynamics for individuals of age 20 to 100, relative to total income (i.e. labor income plus financial income). The two cases correspond to no unemployment risk (dotted line) and unemployment trap with human capital erosion: Ψ_1=0 and Ψ_2=0.6 (solid line).

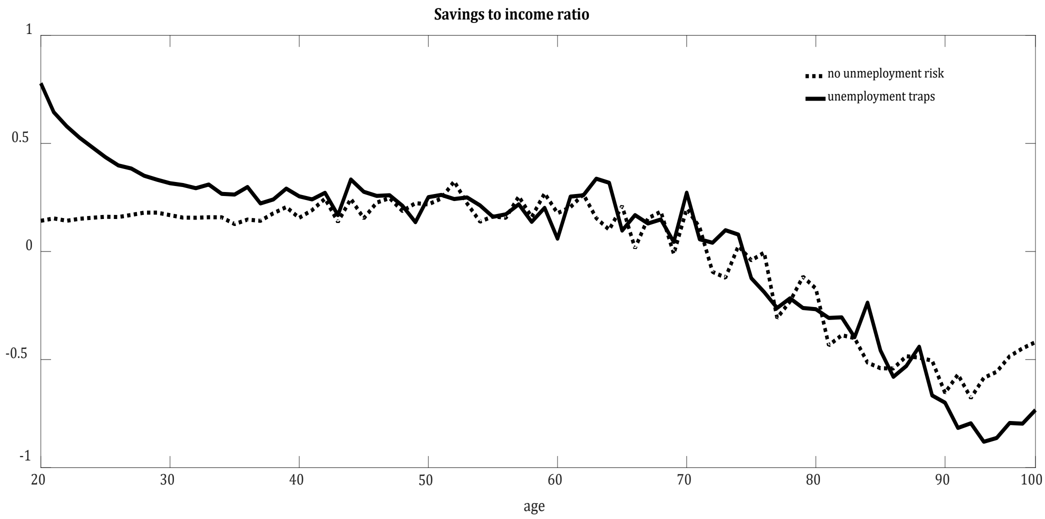
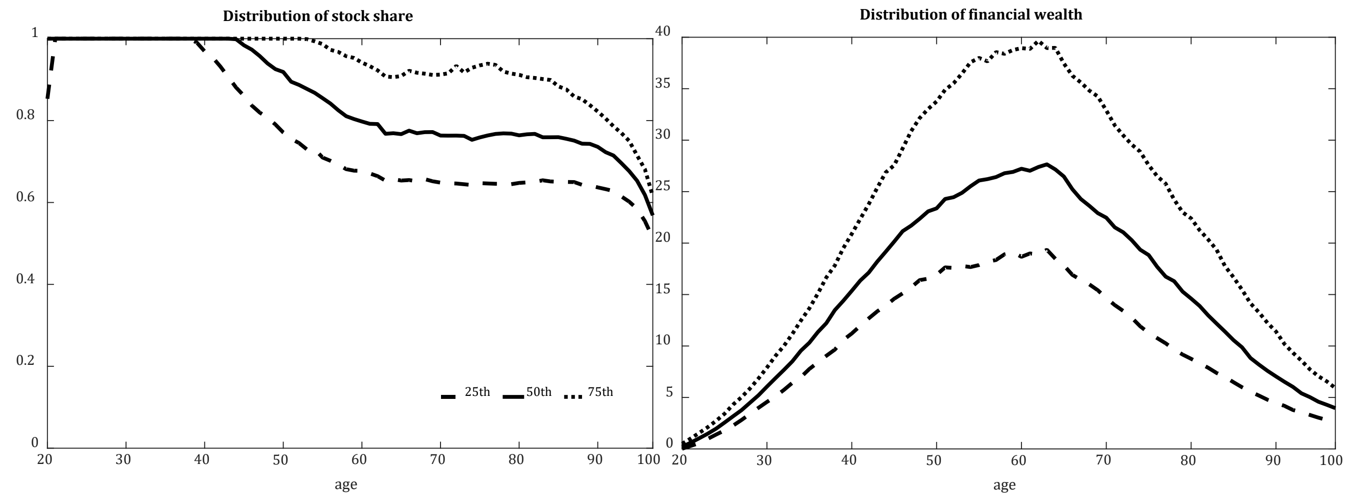


Figure 4. Life-cycle percentile profiles

This figure displays the distribution of simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100 in the case of unemployment risk (panel (a)) and unemployment traps (panel (b)). The parameters governing the human capital erosion during short-term and long-term unemployment spells are Ψ_1=0 and Ψ_2=0.6.

a. No unemployment risk



b. Unemployment traps

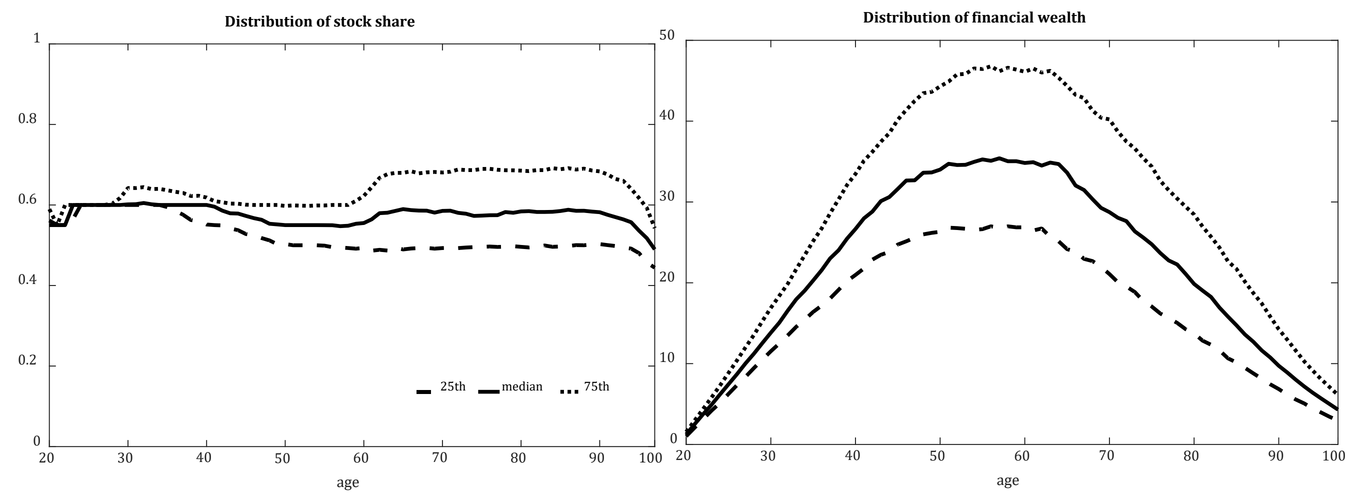
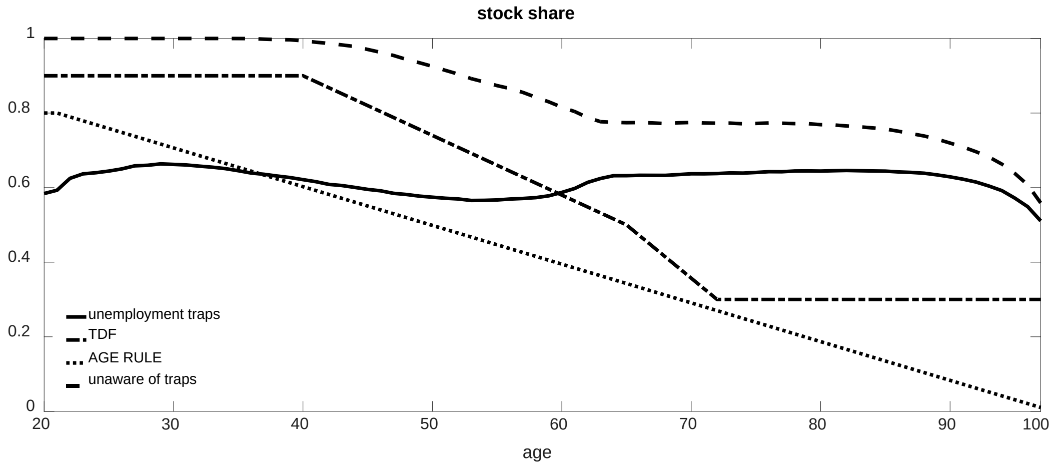


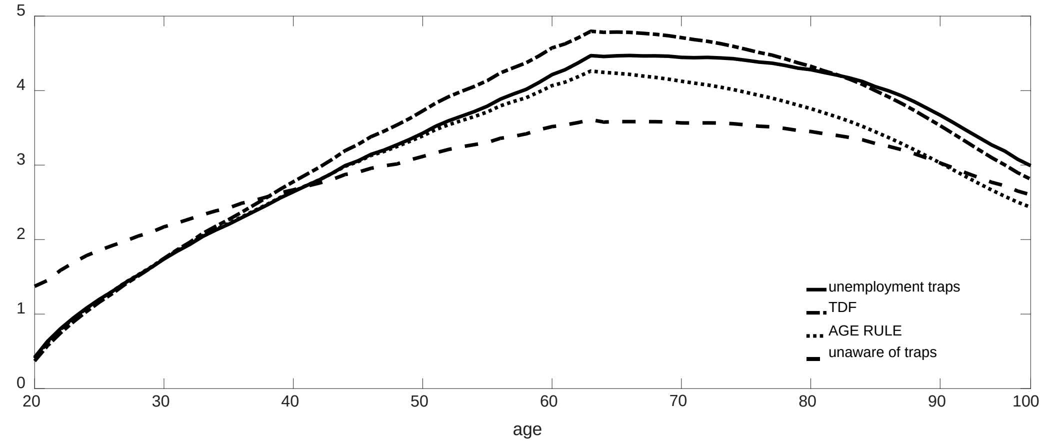
Figure 5. Optimal and suboptimal life-cycle profiles

This figure contrasts the optimal (solid line) and suboptimal life cycle profiles (dotted line: "Age Rule"; dashed-dotted line: "Target Date Fund rule"; dashed line: "unaware of traps", i.e. optimization without taking into account the existence of unemployment traps).

a. Equity portfolio share



b. Consumption



c. Financial Wealth

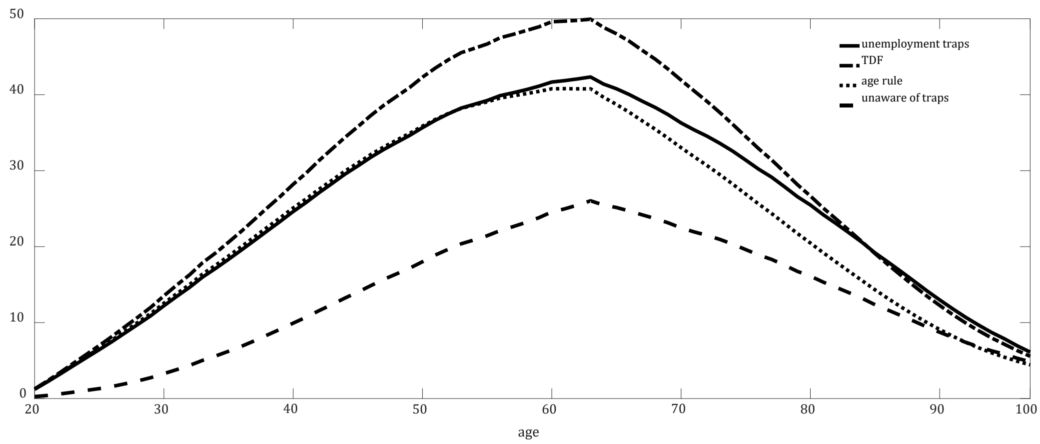


Figure 6. Income profiles conditional on welfare gains.

This figure displays mean income profiles for individuals conditional on welfare gains from becoming aware of traps. The dotted line represents the mean income profile for individuals with welfare gain below the 5-th percentile. The solid and dash-dotted lines refer to individuals with welfare gains respectively at the average and above the 95th percentile.

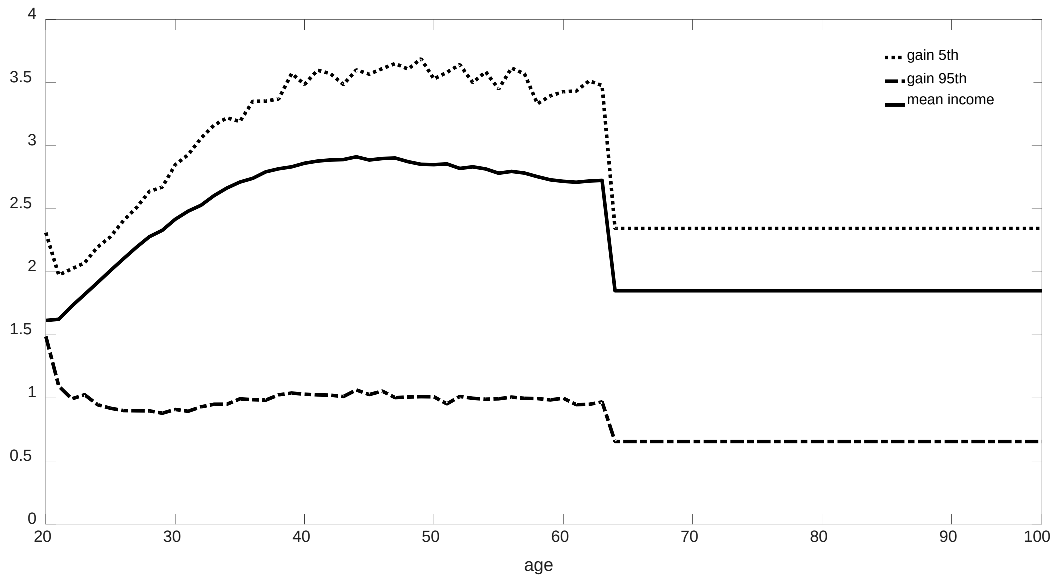


Figure 7. Life-cycle profiles with unemployment traps: age-dependent long-term unemployment risk

This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. The probability of entering long-term unemployment for an unemployed worker is set to 0.15 in the baseline case, to 0.10 only for workers younger than 50 in Case 1, to 0.10 for all workers in Case 2. Human capital erosion: Ψ_1=0 and Ψ_2=0.6.

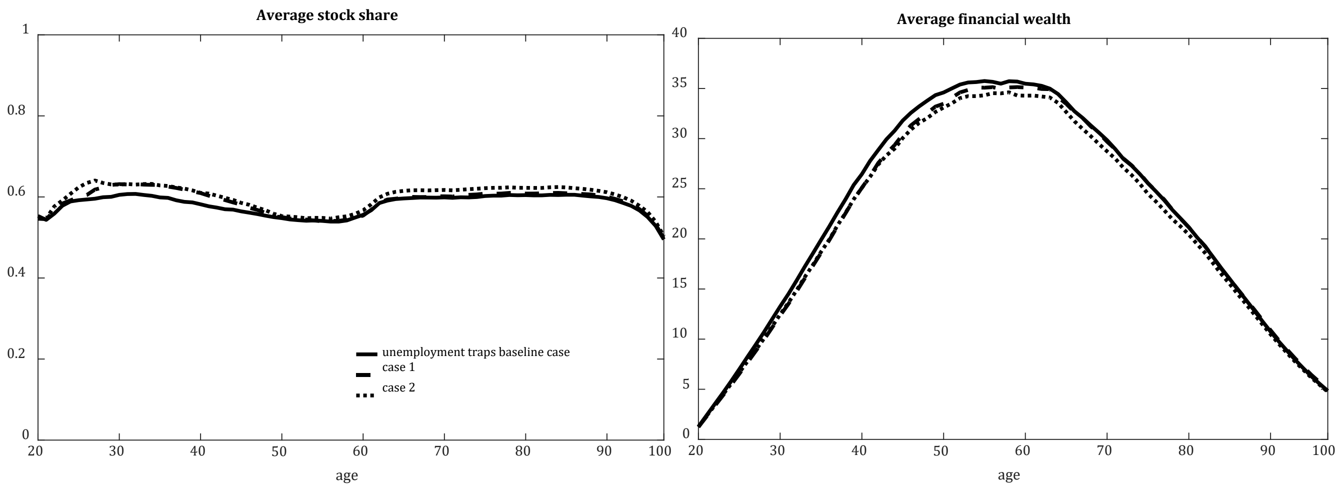


Figure 8. Life-cycle profiles with unemployment traps: positive correlation between labor income and stock returns

This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. Human capital erosion: Ψ_1=0 and Ψ_2=0.6. Positive correlation between labor income shocks and innovation to stock returns: ρ_{sY} =0.2.

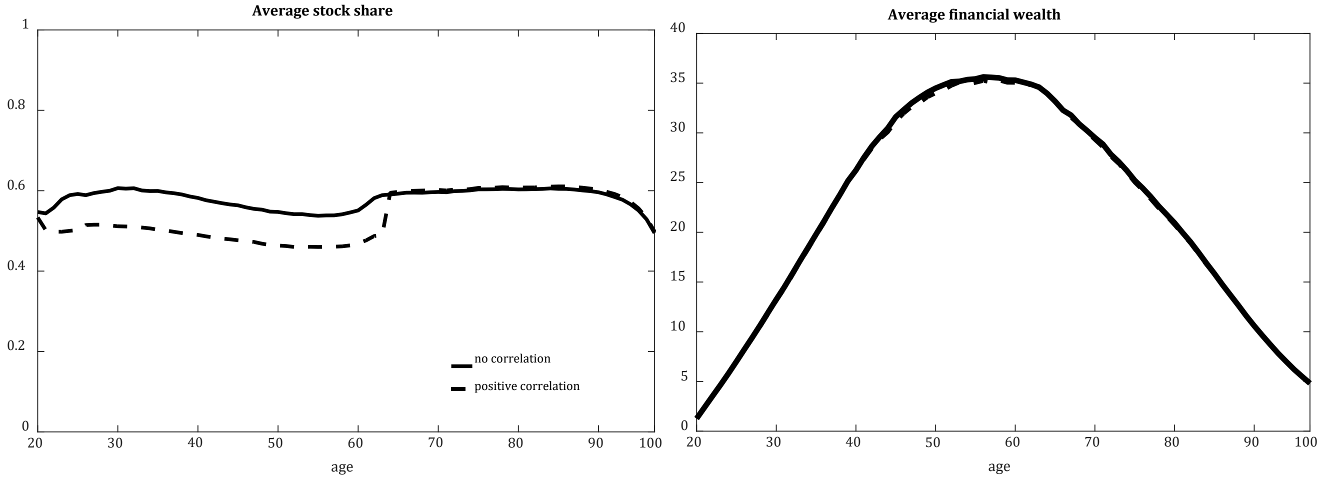


Figure 9. Life-cycle profiles with unemployment traps: recursive preferences

This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. Preferences over consumption are recursive, represented by an Epstein Zin utility function. Elasticity of intertemporal substitution varies from 0.1 to 0.5. Human capital erosion: Ψ_1=0 and Ψ_2=0.6.

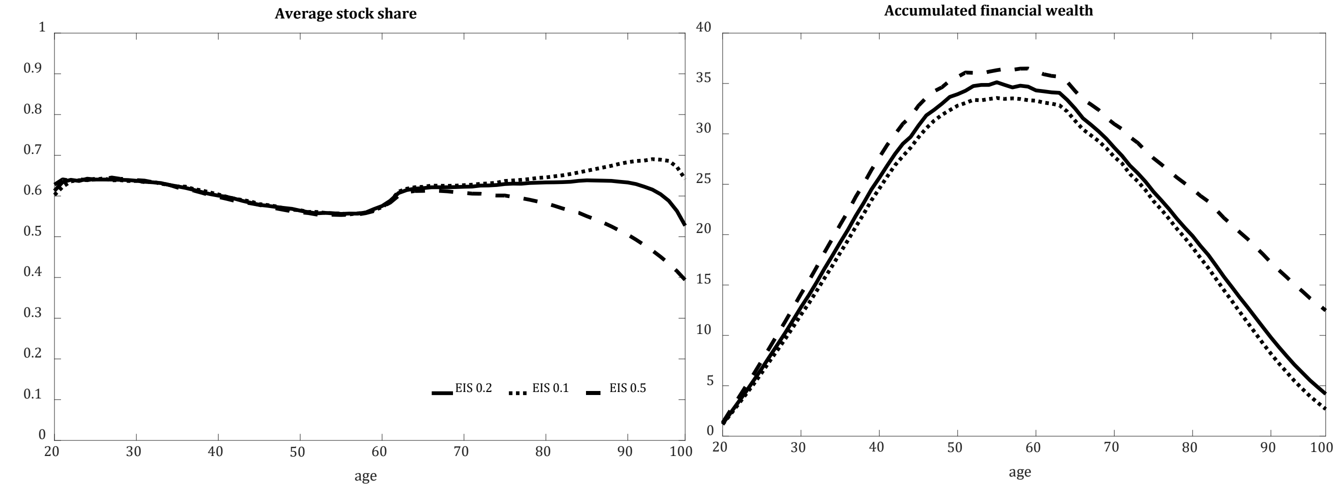


Figure 10. Life-cycle profiles with unemployment traps: sensitivity to human capital erosion

This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. Various cases are considered*:* no unemployment risk; unemployment traps with alternative values of human capital erosion, with Ψ_2 decreasing from 0.6 to 0.2 (Ψ_1=0 in all cases).

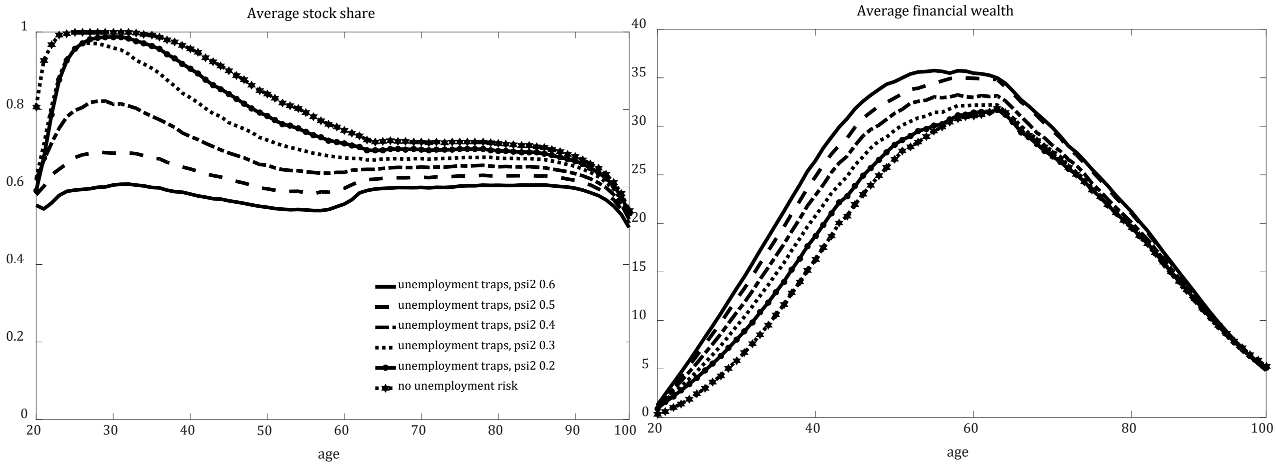


Figure 11. Life-cycle profiles with unemployment traps: stochastic human capital erosion

The figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. Various cases are considered*:* no unemployment risk; unemployment traps with deterministic human capital erosion (Ψ_1=0 an d Ψ_2 = 0.6), unemployment traps and stochastic human capital erosion. In the latter case, Ψ_1=0 and Ψ_2 follows a beta distribution with shape parameters *a* and *b*. In particular, *a*=0.1 and *b*=0.4, 0.7 and 0.9, implying an expected value for Ψ_2 equal to 0.20, 0.12 and 0.10, respectively.