THINKING IN MOVEMENT AND MATHEMATICS: A CASE STUDY

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This paper discusses recent theories in mathematics education that, while studying the role of the body in mathematics, reveal growing interest in the dynamic nature, or flow, of mathematical activity, rather than in what the activity allows to achieve and how. Pursuing the lines of flight offered by these studies on the visceral role of movement, we draw on the idea of "thinking in movement" by Sheets-Johnstone (2009, 2011) to elucidate the interconnection between moving and thinking. We use this perspective to analyse an interview in which a grade 4 student is engaged with spatio-temporal relationships, as a way to better study his encounter with graphical configurations.

INTRODUCTION

Since the corporeal turn prompt by theories of embodied mathematics in the 2000s, studies that take into account the role of the body in mathematics education research often discuss bodily movement in the classroom as a crucial resource for teaching and learning (e.g. Radford, Edwards & Arzarello, 2009; Radford, 2013; Edwards et al., 2014). These studies often strive to code multimodal engagement—hand gesture, eye gaze, prosody in speech (high-low pitch), bodily posture, and so on—as a way to infer correspondences with a particular cognitive stage, level of understanding or step in a learning trajectory, within constructivist or acquisitionist perspectives. But the risk of such associations is to see bodily engagement as a placeholder of cognitive schemas already existing in mind and, therefore, to fall into old body/mind splits, instead of thinking of the body and bodily activity as "genuinely constitutive of knowing" (Nemirovsky *et al*., 2013).

In the last decade, researchers have offered ways to rethink the notion of embodiment through new perspectives that insist in dissolving any conceptual-perceptual cut or dualism. For example, Nemirovsky and colleagues (2013) take a non-dualistic stance on learning in informal settings to describe how perceptuomotor integration partakes in mathematical thinking about graphing motion, pursuing a phenomenology of lived experience. De Freitas and Sinclair (2014) propose the idea of assemblage within a new materialist perspective to address the issue of the body in learning in a wider sense, which also comprises the body of mathematics. They want to extend thinking beyond the single individual effort and to show how it occurs as distributed through material encounters of human and non-human bodies. Enactivist researchers (e.g. Maheux & Proulx, 2015) posit a shift in the way they consider enacted mathematical activity as knowing itself, in a dynamic process that involves the learner acting and immersed in the environment. Others explore the image of a growing-making mathematics (Roth, 2016), use theories on material phenomenology (Hwang & Roth, 2011) or extend the idea of sensuous cognition (Radford, 2013), in an attempt to investigate classroom situations arguing for monistic views of cognition.

This brief review on recent theories of embodiment in mathematics education makes the following point clear: there is growing interest in the dynamic nature, movement or flow, of the mathematical activity rather than in what the activity allows learners to achieve and the way it does so. To say it differently, focus is more and more shifting to the proper encounters of learners with mathematical concepts and to the relational entanglement of movement and thinking in these encounters. We pursue here this line of flight, drawing attention to the way in which movement and thinking are contiguous and push each other forward in mathematics. In particular, focus is on an informal conversation about graphical representations of motion in the context of an interview between a student and a researcher.

MOVEMENT AND THINKING

In a very recent work, Roth and Maheux (2015) propose a "dynamic approach to mathematical thinking", addressing the issue of how we might exhibit mathematical thinking in movement in a way that learning and movement are not reduced to schemas. De Freitas and Ferrara (2015) take a similar, more philosophical, stance as they show that mathematical concepts themselves are mobile, but the most freedom of movement belongs to thought. The dynamic, mobile nature that is to be characterised in these studies belongs not just to the process of knowing or to the body but to thought, and to mathematics itself, in resonance with Châtelet's (1993/2000) view of the virtual dimension of mathematics. In this paper, we share with these authors the same concerns around the non-representational character of gesture and bodily movement and a visceral interest in the way in which movement might be better characterized and studied in the context of classroom situation, in order to embrace the mathematical activity of students and teachers (and researchers) in its whole complexity and profundity. In particular, we aim to contribute to this area of research by showing how movement and thinking sustain and build up each other in mathematical situations. To do so, we draw on Sheets-Johnstone (2009; 2011), whose work is mainly dedicated to elucidating the nature of movement. Grounding her studies in Husserl's phenomenology and Merleau-Ponty's theory of perception, she expands their vision and elucidates the primacy of movement in the life of animate beings. For Sheets-Johnstone, movement is not equivalent to a mere local change in position, but is our primary way of making sense of the world at both human and evolutionary scale (that is to say, in terms of human development and with respect to the evolution of animate forms). In particular, she proposes a ground-breaking interconnection among moving, feeling and thinking. Even though we recognise the immense power of her theory of affect grounded in movement (affective/tactile-kinesthetic body), in light of the purposes of our paper, we are more interested in deepening co-constitutional relationships between moving and thinking. Sheets-Johnstone (2011) examines the experience of "thinking in movement" and its foundational character to the creation of a *kinetic bodily logos*. By proposing a first-person experience of an improvisational dance, she describes a paradigmatic example of thinking in movement. She shows how in the dance "[q]ualities and presence are enfolded into [her] own ongoing kinetic presence and quality" (*ibid*., 2011), engaging her directly with the here and now, without any gap between the "global dynamic world" which is perceived and "the kinetic world" in which she is moving. The world that she is *exploring* in movement cannot be separated from the world she is *creating* in movement: "the idea that thinking is separate from its expression—a thought in one's head, so to speak, existing always prior to its corporeal expression—is a denial of thinking in movement". By the same token, saying that thinking in movement is a way of being in the world and a natural mode of being a body, the author is also challenging representational visions of the body, that is, of "a body that mediates its way about the world by means of language". This standpoint has at least two important consequences: Sheets-Johnstone first proposes that we must rethink what it means 'to have meaning', and, secondly, argues that movement might be meaningful in itself. Therefore, in our understanding of it, the expression "thinking in movement" can be 'read' not only left-to-right but also in the opposite direction. In both ways, it implies not just a temporal overlapping but the mutual constitution and implication of the two processes: movement is thinking and thinking is moving. Moreover, there is a manifold of possibilities "contained" in any movement, which can be disclosed through "certain felt tensional quality, linear quality, amplitudinal quality, and projectional quality" (Sheets-Johnstone, 2011). These four primary qualitative structures of movement relate to force or effort, to space and to time. They are "separable only reflectively, that is, analytically, after the fact; experientially, they are all of a piece in the global qualitatively felt dynamic phenomenon of self-movement." (*ibid*., 2011)

We draw here on these theoretical aspects to take a perspective on moving-thinking that helps us examine the dynamic nature of mathematical activity of students, who deal with graphical representations of spatio-temporal relationships.

CASE STUDY AND METHODOLOGY

The data we present in this paper come from a pilot experiment, which was conducted in 2016 for a wider study on the role of movement in mathematics. The experiment was designed as a classroom-based intervention (Stylianides & Stylianides, 2013) and carried out with the twofold aim of (1) improving classroom practices proposing a graphical approach to functional relationships at primary school and (2) deepening the ways in which a specific technology that required bodily movements might be fuelling understanding in this context. The pilot study involved a class of grade 4 students (aged 8-9 years old) in graphing motion activities with the software WiiGraph. WiiGraph leverages two remote controllers of the game console Wii to graphically capture the positions of two students as they move in an interaction space, while pointing the remotes to a sensor bar. When the students move back and forth, farther and closer with respect to the bar, two real-time graphs originate on a Cartesian plane. Each graph captures one user's distance in time, for a duration of usually 30 seconds. In our setting, we used a 4-meter masking tape strip to create two parallel corridors (orthogonal to the sensor) so that each user could move freely in one of them. During the three 2-hour meetings, the students were mainly involved in using the software in collective discussions led by one of the researchers (the first author). The graphs were projected on an interactive whiteboard. At first, pairs of students moved in the interaction space and the entire class began to explore the encountered graphical representations in terms of movements. Later, the students were asked to move in order to obtain specific configurations on the screen (e.g. a couple of horizontal straight lines, or of parallel slanted lines), and to imagine and draw the graphs eventually produced by two people moving without the real-time feedback of the software. These activities aimed at creating space for mathematical explorations on spatio-temporal relationships to take place in the classroom, through bodily interactions and narratives around movements and experiments with the technology. The students also worked in groups solving written tasks. We filmed the classroom discussions with two cameras and collected the students' written protocols. After a 6-month period from the intervention, five students participated in 10-minute individual semi-structured interview with one researcher (the second author). The interviews were informal conversations regarding memories of the students about what they experienced and liked during the pilot experiment. The focus of this paper is on one of these interviews, which were all filmed. The data were transcribed and analysed following a microethnographic methodology (Streeck & Mehus, 2005), in the attempt of taking into account micro-movements that emerge in the activity through the body (talk, gesture, posture, diagramming, gaze, rhythm, and so on). This choice is in line with our theoretical commitment on movement, and with the crucial role of movement in the students' activity with the technology.

CROSSING LINES

In the following we present and then analyse a 1-minute segment of the interview in which a student, Luca, discusses with the researcher the event of crossing lines. We chose this episode for various reasons. One is that Luca was involved—months before, during class discussion—in the following experiment. He moved together with a classmate, Giulia (Fig. 1a), to fulfil the request of the researcher, that of producing two parallel (slanted) lines with WiiGraph. At the beginning of the experiment, the children seemed to be coordinating with each other to move at the same pace while going farther from the sensor. But, as Giulia decelerated approaching the far end of the corridor, Luca reached her, provoking the software to display a pair of intersecting lines (Fig. 1b) and creating amazement in all the students.

Making sense of the intersection and, more generally, dealing with two distances plotted in Cartesian coordinates is not trivial at all. Studies in the literature highlight that it is difficult for young children "to work out relation of different positions plotted in this way" (Bryant, 2009). There are important, intertwined aspects to be taken into account. For each user, WiiGraph captures spatio-temporal relationships that involve one variable (distance) which depends on another (time). In an experiment, it captures two distances at the same time: two variables that change together without depending on one another. Similarly, two movements occur simultaneously but independently. Therefore, relations between movements can be established but need to be directly explored and maintained by the students (e.g. the same pace) to produce a specific pair of graphs that preserve specific relationships.

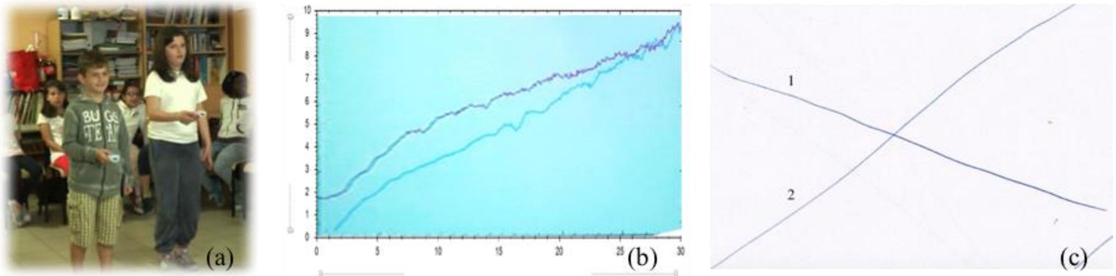

Figure 1. (a) Luca and Giulia moving; and (b) their graphs; (c) two crossing lines.

Going back to Luca and Giulia's experiment, a *choreography* in which two people start close to the sensor, 1 metre away from each other, and, moving at a constant speed, both walk away from the sensor, is one that allows for the creation of parallel straight lines with positive slope. It is not the only one: there is an entire bundle of possible pairs of movements that gives rise to a similar diagram (two parallel lines). There is a manifold of interactions between movers and nuances or little variations in movement virtually contained in that *configuration*. A similar point may be stressed if we speak of crossing lines: Figures 1b and 1c show two of the possible configurations.

What follows specifically refers to Luca's interview, during which the issue of crossing lines comes again to the fore. In the first part of the interview, Luca is asked what he liked and remembers about the classroom intervention (the software is not in use). On the table, two remotes and the sensor bar are at disposal, together with some pens and a sheet of paper. He begins telling that "two children held the remotes, and they had to do lines on that graphical area, pointing the remotes to the sensor". Then he speaks of the case of parallel slanted lines as that in which "two students had to go forward keeping the distance fixed". Holding the remotes, Luca and the researcher simulate this experiment following the indication given by Luca.

After few minutes, the interviewer askes Luca about the crossing lines (in the transcript we use R = researcher; L = Luca; L/RH = left/right hand):

1 R: What if I wanted to create two-o lines that cross each other, at some point?

2 L: It is needed that a child goes forward (*RH, holding a pen, moves rapidly towards his torso, then comes back to the starting position, in front of him*), th-, the other goes faster (*LH goes shortly back and then with impulse reaches RH*) and then they have to meet (*slowing down speed*, *LH reaches RH*. *Looks at R*) (pause) in a point (*still gazes at R*)

3 R: Can we try out? What would you do? (*takes one remote in RH and keeps it pointed to the sensor in front of her*)

4 L: (*takes the other remote with RH, gazes at R's remote*) I start ahead, then you go faster (*moves LH index finger from the R's remote position towards*

the sensor), I go slowly and then they meet (*LH reaches his remote*, *fingers extended and kept in the same position for few seconds: Fig. 2a*)

5 R: And do we both move forward? (*LH rapidly points to the sensor*)

6 L: No, then they meet (*LH goes back*, *then slowly goes forth again and overtakes his RH*), then you go forward and I stay behind (*RH zigzags moving a little closer to his LH*)

7 R: Ok

8 L: So, you do, (*LH points to the sensor*) you go ahead

9 R: Tell me when to go (*keeps the remote still*)

10 L: Go (*gazes at R's remote. R and L move the remotes towards the sensor*). You do like this (*moves his remote a little back*), you overtake me and I stay behind (*looks at R, moves again his remote towards the sensor*)

11 R: Ok (*interrupts her movement*). So, how are the lines showing up?

12 L: (*puts the remote on the table*) Criss-crossed (*cross arms: Fig. 2b*)

13 R: How?

14 L: Hm (*cross arms again, turned to a different slope*, *takes a pen*), do I draw them…? (*softly speaking*)

15 R: Yes, yes, as you want (*puts her remote on the table*)

16 L: One like this (*draws line 1 in Fig. 1c*), the other one like this (*draws line 2 in Fig. 1c*)

17 R: (pause) (*gazes at the drawing*) So-o (*points to the drawing*)

18 L: Hm, no (*with closed fists, RH ahead and LH back are swapped in position*), I start ahead and then (pause) I start ahead (*points ahead with RH*), you start from behind (*points back with LH*) and then (*suddenly, swaps hands' positions again*) they cross each other

19 R: So, is this drawing (*points to it again*) of another movement, for you?

20 L: Yes (*takes the remote*)

21 R: So (*takes the remote too*)

22 L: I start ahead, you [start] from behind, they cross, then (*Fig. 2c captures the experiment performed by Luca and the researcher*)

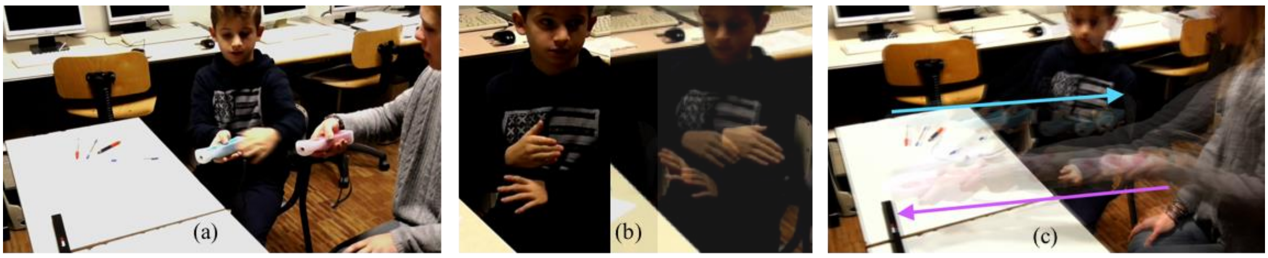

Figure 2. (a) 1st choreography; (b) gesture for intersecting lines; (c) 2nd experiment.

We see how, in the interview, two different mathematical events mainly resonate with the question and the experience of crossing lines (cf. diagrams in Fig. 1b and 1c). These events emerge as intertwined out of movement and fuel Luca's thinking in movement. Bodily movements actualize specific choreographies, perform simulations of experiments, establish shapes for diagrams and arrangements of lines (configurations), or even mix the three aspects together. We can capture a sequence from the episode. A first choreography sees hands, people, remotes going in the same direction, then meeting and eventually one overtaking the other (3 times: [2], [4], [6]; Fig. 2a). Then, a first experiment also engages the researcher in performing such choreography: [10]. The following configuration (2 gestures, 1 diagram: [12], [14], [16]; Fig. 2b and 1c) with the emerging diagram is a turning point as it reconfigures previous movements and engenders a second choreography. The new choreography (2 times: [18]) is still evoking the crossing relational movement, but now involves two hands, people, remotes swapping positions. Finally, a second experiment, rhythmically dictated by Luca's narrative, closes the episode ([22]; Fig. 2c). Each moment fluently evolves into the next in the experience of thinking in movement, which we characterize as follows. On the one side, the diagram reconfigures boundaries between the two choreographies by unfolding a new point of view that is also able to capture a crossing event. Hesitation and suddenness destabilize homogeneous continuity in the temporal overlapping of the two possibilities, as well as of the processes of thinking and moving. On the other side, repetitions of a choreography entail little variations within movements, as it is in the case of the first choreography, where a zigzagging of the remote is added in a way that stresses relative positions between hand and remote, and therefore in the two movements. This sheds light on the complexity within the process of movement in thinking and the potential dimension of both moving and thinking. Ambiguity between the choreographies is generative of new meanings that are still open to mobility within the mathematical event, which is at the core of the episode. Such mobility and openness resonates with the creative power of explosion attributed from Leibniz to points, when thought of as generated by the intersection of two lines or curves, in his account of the virtuality of mathematical concepts (as shown in Châtelet, 1993/2000).

CONCLUSIVE REMARKS

Further research may elucidate how explorations of crossing lines could be considered pivotal in thinking of couples of graphs with WiiGraph. Discussion around this point in the classroom created new meanings for the intersection as "swapping places" or "overtaking the other", which are the configurations captured by the choreographies in the segment of Luca's interview. As researchers, this point made clear for us the importance for students of experiencing and making sense of the intersection of lines in order to relate not simply each of the graphs to an individual movement, but the graphs themselves, as well as the movements, to each other. Examining movement in thinking in Luca's interview, we offer a way of drawing attention to how the flow of movements implicates dynamic thinking about pairs of graphs and their relations, being generative of mathematical meanings beyond its own meaningfulness. We use superposition of subsequent video frames with increasing transparency filter (see Fig. 2b and 2c) to induce a sense of movement which cannot be otherwise grasped by still images. In fact, there arises a delicate methodological issue that needs to be further examined: to develop ways that allow for better addressing and capturing the complexity of movement without reducing it. This also points out the richness and hidden beauty that emerge from the challenging matter of movement in/of mathematical concepts, which may be infinite source of delight or, as Châtelet would say, "enchant(e)ment".

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