

# Financial Constraints in Search Equilibrium: Mortensen Pissarides meet Holmstrom and Tirole

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## Abstract

A key lesson from the Great Recession is that firms' leverage and access to finance are important for hiring and firing decisions. It is now empirically established that bank lending is correlated with employment losses when credit conditions deteriorate. We provide further evidence of this and make causal inferences on the effect of leverage on job losses drawing on a new firm-level dataset that we assembled on employment and financial positions of European firms. Yet, in the Diamond Mortensen Pissarides (DMP) model there is no role for finance. All projects that display positive net present values are realized and financial markets are assumed to be perfect. What if financial markets are not perfect? Does a different access to finance influence the firm's hiring and firing decisions? The paper uses the concept of limited pledgeability proposed by Holmstrom and Tirole to integrate financial imperfections and labor market imperfections. A negative shock wipes out the firm's physical capital and leads to job destruction unless internal cash was accumulated by firms. If firms hold liquid assets they may thus protect their *search capital*, defined as the cost of attracting and hiring workers. The paper explores the trade-off between size and precautionary cash holdings in both partial and general equilibrium. We find that if labor market frictions disappear, so does the motive for firms to hold liquidity. This suggests a fundamental complementarity between labor market frictions and holding of liquid assets by firms.

**Keywords:** Search frictions, financial frictions, leverage, liquidity, pledgeability.

**JEL codes:** G01, J64.

# 1 Introduction

The 2008 financial crisis and the associated increase in unemployment on both sides of the Atlantic sparked a new interest in the relationship between financial imperfections and labor market dynamics. In the aftermath of the crisis, a growing empirical literature studied the links between financial conditions and employment adjustment. A key lesson that we have learned from this flourishing literature is that firms' leverage and access to finance are strongly and significantly correlated to hiring and firing decisions. More specifically, it is now empirically accepted that frictions in bank lending are correlated to employment losses when credit conditions deteriorate. It is more difficult to draw causal inferences as to the relationship between financial conditions of firms and employment adjustment. Yet studies reviewed below drawing on exogenous variation in financial conditions of firms and the additional evidence produced in this paper suggest that leverage induces more downsizing during a financial recession. In particular, we assemble a new dataset on firm-level adjustment and financial conditions of firms throughout the Great Recession and implement a new identification strategy based on the involvement of firms in consortia offering third party collateral. We find a sizeable and negative effect of leverage on employment adjustment throughout the Great Recession, mainly operated via the downsizing of firms rather than a slower growth of expanding units.

These documented links between finance and employment adjustment can be better understood in a framework combining financial market and labor market imperfections. The Diamond Mortensen Pissarides (DMP) model is the main paradigm for addressing imperfect labor markets. In the baseline framework, there is no role for finance. All projects that display positive net present values are realized and financial markets are assumed to be perfect. What if financial markets are not perfect? Does a different access to finance influence the firm hiring and firing decisions? These basic questions call for a deeper understanding of the relationship between labor and finance. Among the financial frictions addressed by the literature and reviewed below, this paper exploits the concept of limited pledgeability proposed by Holmstrom and Tirole (2011). The idea is that only part of the entrepreneur's income is pledgeable and can be borrowed upon, either because part of the income is private benefit or because the entrepreneur needs incentives. By adding financial imperfections and borrowing constraints into an otherwise standard equilibrium unemployment model, the paper contributes to the building of an archetype and flexible model of labor and finance.

In our model, firms are financially constrained by limited pledgeability and invest in physical capital within an imperfect labor market. Entering firms attract workers by posting vacancies with wages attached to them and hire up to an endogenously determined size level that depends on the firms' access to finance. Firms anticipate the possibility that new funding will be needed over the lifetime, and that refinancing may not be available in those times. If that happens, the firm must rely on liquid assets for financing the rebuilding of its physical capital. In the absence of such funds, the firm is forced to fire workers and close down its operations. When workers are fired, the firm loses its *search capital*, defined as the cost of attracting and hiring workers. *Ex ante*, firms therefore face a trade-off between investing their limited funds in liquid assets to protect their search capital, or to invest in more capacity and more employees.

Our theoretical model shows that if labor market frictions disappear, so does the motive for firms to hold liquid assets. This implies a fundamental complementarity between labor market frictions and holding of liquid assets by firms that is novel in the literature. In this sense, the paper brings together the work on liquidity by Holmstrom and Tirole (2011) with the traditional Mortensen Pissarides (1994 and 1999) model of equilibrium unemployment.

In section 2 we position our contribution with respect to the earlier theoretical literature and the more recent empirical literature since the Great Recession. In section 3, we present new causal inferences based some on an unexploited data set of European firms. Section 4 presents the environment of our theoretical perspective, the financial contracts and the asset equations. Section 5 derives and solves for the general equilibrium. In Section 6 we extend the basic model to allow for heterogeneous

firms and we reconcile our theory with recent work and evidence on employment and liquidity (Bacchetta et al., 2016). Section 7 discusses our more general implications and reconcile them with the evidence summarized in Section 2, and provided in Section 3. Section 8 concludes.

## 2 Relationship to the existing literature

In this section we relate our theoretical and empirical contributions to the existing literature.

### 2.1 Earlier (mostly theoretical) literature

There exists a substantial literature on financial frictions. The early literature, predating the Great Recession, identified different sources of frictions. In particular, Greenwald-Stiglitz (1993) looked at the risk aversion of firms, Farmer (1985) studied the financing of quasi-fixed costs, and Townsend (1979) proposed the costly verification model. Sticky bank borrower relationships were also emerging in this early literature as a result of asymmetric information with moral hazard (Holmstrom and Tirole, 1997) and adverse selection (Sharpe, 1990). Kiyotaki and Moore (1997) studied the role of capital as collateral. Within a more DMP labor finance literature, the pioneer work was Wasmer and Weil (2004) investigation of the interplay between matching frictions in labor and financial markets. Merz and Yashiv (2007) discussed the relationship between adjustment costs of labor and the value of the firm.

In order to position our contribution within this literature, we need to characterize in some detail the main theories of financial frictions, and their relevant applications. In Townsend's costly verification model (Townsend, 1979), the income of the borrower is private information, and the lender has to incur a cost in order to observe the income. It follows that the cost of borrowing is increasing in the amount borrowed, as it increases the probability that costly verification will take place. In Holmstrom and Tirole's mode (Holmstrom and Tirole, 1997), the entrepreneur can only borrow on a share (less than one unit) of a non-stochastic income flow.<sup>1</sup> This setup fits very well into the Diamond-Mortensen-Pissarides model as the (non-pledgeable) component of income firms cannot borrow upon is simply subtracted from output period by period.

Bernanke and Gertler (1989) analyze an economy in which financial frictions are modeled by Townsend's costly verification framework. In their model, aggregate output is assumed to be stochastic. The impetus for financial fluctuations is linked to individual savings. If the economy is hit by a positive shock, entrepreneurs accumulate more wealth before investing, and this increases the fraction of entrepreneurs who invest. This initial effect persists because the increased investments lead to higher income in the next period, and hence also higher wealth. We study a model without anticipated aggregate shock, hence the effect through initial wealth is absent. Our focus is on the multiplier effect of job creation: more job creation gives more pledgeable income, which in turn allows for more job creation. This mechanism is absent in Bernanke and Gertler, as the size of the projects (ex ante) is exogenous in their model.

In Kiyotaki and Moore (1997), entrepreneurs can only borrow on collateral, not on future income as such. Collateral (land in their setting) is also an input in the production process. A one-period increase in productivity gives rise to multiplier effects as it increases the present and future values of land, relaxing the borrowing constraint of investors (farmers). As already commented upon, firms borrow on future income in our setup, collateral plays no direct role as such. Hence changes in future income flows influence the borrowing constraint directly, not only indirectly as in Kiyotaki and Moore.<sup>2</sup> Our assumption is convenient within a search setting, as physical capital is downplayed

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<sup>1</sup>The underlying information friction is that the entrepreneur must have a sufficiently large stake in the project to be willing to exert effort, and part of the income flow the project generates is therefore non-pledgeable.

<sup>2</sup>Liu, Wang and Zha (2013) argue that there is a positive co-movement between land prices and business investments, and suggest that this may drive the land-price dynamics over the cycle.

in these models. Furthermore, in search models, a part of the investment is upfront investment in search capital, i.e. costs associated with acquiring workers. The search capital per worker hired is endogenously determined in search equilibrium, and it is not clear whether search capital can be used as collateral. This is particularly important in our set-up, as it is the protection of the search capital that motivates firms to hold cash.

## 2.2 The (mostly theoretical) literature after the great recession

After the great recession, a literature on the interplay between financial frictions, liquidity, and employment has developed. A seminal paper is Jermann and Quadrini (2012), who first observe that dividends are pro-cyclical while debt is counter-cyclical, and then construct a model that delivers these facts as an equilibrium outcome. In their model, firms prefer debt financing to equity financing because debt is more favorably treated by the tax system. In addition to long-term financing, firms also have a short-term need for funding of running expenses, including wages. Finally, short-run changes in dividend payments entail convex adjustment costs. Due to financial frictions of the Kiyotaki-Moore type, the firms' total debt cannot exceed a fraction less than one of the value of the firms capital next period, and this fraction is subject to shocks, referred to as financial shocks. A negative financial shock will be partly mitigated through adjustments of the dividend payments. However, due to the convex cost of adjusting dividends, negative shocks also lead to reduced hiring in order to reduce the total wage bill, and hence also the need for borrowing. Finally, Boeri, Garibaldi and Moen (2016) study the interplay between limited pledgeability, job creation and business cycle volatility within the DMP framework.

Bacchetta et al. (2016) study empirically and theoretically the relationship between cash and employment. Empirically they document a negative relationship between firms' cash holdings and employment both on firm level and aggregate level. To explain their findings, they set up a cash-in advance model with similarities to the model of Jermann and Quadrini (2012). Firms hold both liquid assets and cash in order to prepay workers. Firms can only borrow short-term liquid assets up to a share less than one of the firms' gross income in that period. Financial shocks are interpreted as shocks to this share. After observing the shock, firms adjust their cash holdings so that they can meet their obligations to prepay their workers. Moreover, there is a constraint on long-term debt. A negative liquidity shock induces firms to hold more cash and to hire less workers.

Melcangi (2016) uses UK firm-level balance sheet data. He finds that firms increased their cash to total assets ratio during the last recession. Furthermore, he shows that cash-intensive firms cut their workforces by less. He argues that firms that are not currently credit constrained may hoard cash for precautionary reasons. His model follows Jermann and Quadrini (2012) relatively closely, with the extension that firms can hold liquid assets and cash as in Bacchetta et al. (2016). In periods in which the within-period borrowing constraint is tight, firms hoard cash and employ fewer workers.

Garin (2015) builds a model of credit constraints similar to that of Jermann and Quadrini (2012) into a search model with wage bargaining. A positive productivity shock tightens the borrowing constraint of firms and improves the firms' bargaining position. This dampens the cyclicity of wages and increases the cyclicity of employment. In addition, since the borrowing constraint tightens up during a boom, firms initially give priority to investments in capital, which can serve as collateral, rather than increase employment. Hence the adjustment of labor is persistent. Iliopoulos et al (2016) set up a model with similarities to Garin (2016), but with a different specification of the within-period credit constraint. They also find that the firms' bargaining position is cyclical, but through a different channel, and again this leads to more fluctuations in the unemployment rate.

We view Jermann and Quadrini (2012), Bacchetta et al. (2016), Melcangi (2016), Garin (2015), and Iliopoulos et al. (2016) as offering complementary theories to ours. In their papers, liquid assets are needed to prepay worker salaries, and a tightening of short-term borrowing constraints influences the firms' hiring decisions. In our model, the focus is on long-term borrowing, and firms may hoard

liquid assets for precautionary reasons in order to protect their search capital in times of financial distress.

There is also a literature that explores the effects of financial frictions in search equilibrium. An important paper in this literature is Petrosky-Nadeau and Wasmer (2013), who explore the effect on unemployment volatility of adding search frictions in the financial market in addition to search frictions in the labor market. They find that financial frictions create a fixed cost associated with hiring, and this increases the volatility of unemployment. The effect is particularly strong if the sharing rule in the bargaining game between firms and banks deviates from the Hosios sharing rule. In a similar spirit, Petrosky-Nadeau (2014) introduced financial frictions through the financing of vacancy posting. Monacelli, Quadri, and Trigari (2011) analyze the effects of the availability of credit for firms and their hiring decisions. In their model, firms strategically borrow on future income and pay it out as dividends. More debt reduces the *ex post* match surplus, and through wage bargaining, also wages. Hence a higher debt level increases the firms' share of the match surplus, and *ex ante* improves their incentive to enter the market. Firms therefore borrow up to an exogenously given borrowing constraint. By shocking this constraint, the authors obtain fluctuations in the entry-and unemployment rates. Note that the mechanism will be defused if search is competitive or if the firm's debt level is included in the bargaining set. Michelacci and Quadri (2009) study the relationship between financial constraints, firm growth, and wages. Their theoretical innovation is that firms to some extent can mitigate the effects of financial constraints by borrowing from their employees. However, as workers are risk averse and lack access to credit markets, employee borrowing is an imperfect substitute for external borrowing.

### 2.3 The empirical literature after the Great Recession

The macro labor literature that emerged after the financial crisis identified additional channels by which financial markets affect labor adjustment. Some of these papers look at the direct effect of higher interests or larger interest spreads. In particular, Christiano et al. (2015) look at the real effects of borrowing spreads, Hall (2014) considers the effects on employment of a high interest rate, while Keho et al. (2015) the labor market impact of shocks to consumers and firms' discount rate.

After the Great Recession, various empirical papers investigated the interplay between credit shocks and labor market adjustment at the micro-level. In this vibrant literature, findings rarely go beyond simple correlations as concerns of endogeneity and measurement error stand in the way of causal identification. Moreover, studies of firm-level employment variation and financial constraints typically focus on a single country, generally the US, and hence their conclusions are somewhat specific to the labor market institutions and degree of financial deepening of this country.

Campello, Graham and Harvey (2010) use subjective statements of US employers concerning their credit constraints at the outset of the Great Recession to investigate the relationship between financial constraints and various dimensions of firm performance. They find that the average constrained firm in the US planned to reduce employment by more than 10 per cent. Such a dramatic reduction of employment cannot be clearly achieved by simply freezing new hires. Consistently with this finding, Elsby et al (2010) document that there was an increase in the inflow to unemployment from employment during the Great Recession. In previous studies total separation from employment did not appear to change, since the increase in the inflow into unemployment had been camouflaged by a drop in the quit rate, so that the total separation rate did not change so much. When Elsby et al correct for this, they find that an increased lay-off rate contributed significantly to the increase in the unemployment rate, particularly at an early stage of the Great Recession. As they write, *Job loss played a key role in driving increased unemployment in the 2007 recession*. Benmelech Bergman and Seru (2011), drawing on three quasi-experiments in the US, document an important role of financial constraints and the availability of credit on firm-level employment decisions. As they acknowledge, *"given the concerns about the endogeneity of profitability, liquidity, and leverage and the relation be-*

*tween these variables and the economic opportunities available to firms, we are cautious at this stage in arguing for a causal link between financial measures and employment”.*

Pagano and Pica (2012) unveil a dark side of finance: during banking crises employment grows less in the industries that are more dependent on external finance and those located in the more financially developed countries. Their identification strategy applies Rajan-Zingales (1998) in that it uses as instrument the reliance on finance of US listed companies. This comes at the cost of drawing on industry-level data which may conceal a lot of firm-level heterogeneity in access to finance and employment adjustment. Boeri, Garibaldi and Moen (2013) draw on plant-level data and observe a negative correlation between firm-level employment growth and various measure of leverage in the context of the Great Recession. No causality is inferred as leverage is likely to be correlated with demand for the firm’s final product, which itself affects its demand for labor. Bentolila et al (2013) observe that Spanish firms heavily indebted to weak banks before the crisis suffered an additional employment drop. The selection involved in the matching of banks and firms prior to the recession is an issue also in this context. Chodorow-Reich (2014) provides a causality tests on the link of finance to labor and finds that access to credit is associated with employment losses, albeit the relationship is non-linear across firm-size: lender health is negatively and significantly correlated to employment at small and medium firms, but the data cannot reject the hypothesis of no effect at the largest firms. Greenstone, Mas and Nguyen (2014) find that the 2007-2009 lending shocks accounted for statistically significant declines in both small firm and overall employment. Duygan-Bamp et al. (2015) find that financing constraints, notably among small firms in sectors with large financing needs, were one of the drivers of unemployment dynamics in the US at the beginning of the Great Recession. While in normal times small firms appear to be the engine of job creation, *workers in small firms are more likely to be affected by layoffs, primarily those working in firms that depend on bank financing*, as they put it. Cingano, Manaresi and Sette (2016) find that credit shocks are correlated with firm’s employment, and propagate through firms’ trade credit chains. These studies at best analyze net employment variation at the firm-level. Even the studies looking at gross job flows measure them as aggregates of firm-level net employment variation. This is the case, for instance, of Mehrotra and Sergeyev (2016) who detect a negative effect of housing price shocks on job creation.

Thus, there is rather strong evidence of a significant and negative correlation between credit constraints and firm-level employment variation during the Great Recession. These sizeable reductions cannot be achieved in the short-run operating only along the hiring margin and indeed there is evidence that inflows into unemployment increased significantly during the crisis. Job destruction in small firms in sectors with high financial dependence appears to be a key driver of unemployment dynamics. However, this literature rarely controls for the specific institutional conditions and financial development of different countries as firm-level data are typically assembled only at a single country level. Causal effects are also much harder to detect, notably at the firm level.

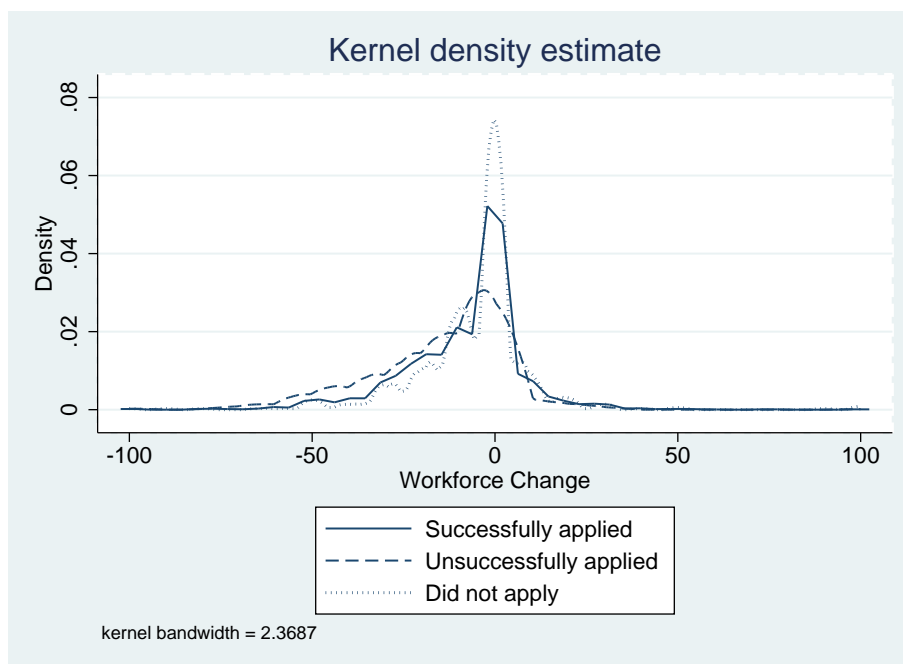
### 3 Evidence from European firm-level data

A dataset that we assembled of firm-level employment adjustment and leverage during the Great Recession can help us in filling some of the gaps of this rich empirical literature. The data cover the period 2007-9 and are obtained by matching data from the EFIGE survey of European firms with information from balance sheets of large firms obtained in the Amadeus archive<sup>3</sup>. Efige samples some 16,000 European firms (3,000 in large countries, such as Germany, France, Italy, Spain and the UK, and 500 firms in smaller countries, such as Austria and Hungary). The data in the matched sample

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<sup>3</sup>Boeri, Garibaldi and Moen (2013) also uses EFIGE data to analyze the effects of the Great Recession on employment adjustment of firms. As pointed out in the survey above, they confined themselves to correlation analyses, without finding appropriate instruments for leverage

Figure 1: Firm-level net employment change, distribution of firms by access to credit



cover mainly large firms (the average firm size in terms of employees is 81).<sup>4</sup>

Our main variable of interest is employment variation.<sup>5</sup> In the appendix we plot the distribution of employment changes using a Kernel density estimator. As our data cover the Great Recession, most firms appear to be downsizing.

Figure 1 plots the Kernel estimates for firms that successfully applied for credit (continuous line), as well as firms that did not apply for credit (dotted line) or that applied, but were not successful (dashed line). The distribution of job losses among those that unsuccessfully applied for credit lies strictly above the other two distributions. This suggests that the firms that were un-successful in refinancing operations were, on average, heavily downsizing (on average by almost 20 %) while the distribution of employment adjustment among successful debtors and firms that did not apply for credit is remarkably similar (in the latter group there is only a larger proportion of firms not experiencing employment variations). The concentration of employment losses (about 30 per cent of the total) among firms experiencing difficulties in refinancing operations is obviously not informative as to causality: it may well be that firms did not obtain credit because they were downsizing and considered viable by banks. Yet, the chart clearly documents a negative relationship between access to credit and employment changes during the Great Recession.

In addition to employment changes, we exploit measures of leverage in 2007, the year before the beginning of the Great Recession. In particular, the *Gearing ratio* is the debt to equity ratio

<sup>4</sup>The questionnaire is very detailed on a number of structural characteristics of firms such as organization, job composition, innovation activities, finance as well as product and labor market strategies. The Amadeus archive provides financial and business data on Europe’s biggest 500,000 companies by assets. Hence, the matched sample covers only the large firms.

<sup>5</sup>We draw on the following question asked to employers at the beginning of 2010: *During the last year (2009) did you experience a reduction or an increase/decrease of your workforce in comparison with 2008?* . For those stating to have changed employment levels, a second question elicited the percentage change in the workforce. We imputed a zero value to firms declaring that they did not experience any change in employment in the first question

measuring the extent to which the firm is using creditor’s vs. owner’s funds, whilst the *solvency* ratio measures the ratio of after tax net profit (excluding non-cash depreciation expenses) over debt, and is a measure of one company’s ability to meet long-term obligations. The appendix reports some descriptive statistics on those data.

To correlate financial leverage to employment changes controlling for firm characteristics, we estimate a regression of changes in employment on firm, sector as well as aggregate country fixed effects, output variations as well as leverage. In particular, Table 1 reports estimates of the following equation

$$\Delta e_{ijc} = \alpha + \alpha_j + \alpha_c + \alpha_j * \alpha_c \beta \Delta y_{jc} + \gamma Lev_{ijc} + \delta S_{ijc} + \epsilon_{ijc} \quad (1)$$

where  $\Delta e$  is the reported employment growth rate *during* the period 2008-9,  $i$  denotes the firm,  $j$  the sector and  $c$  the country,  $S$  is set of size dummies (employment or turnover) and  $Lev$  is the Gearing Ratio, measured *before* the Great Recession (according to 2007 balance sheet data).  $\Delta y_{jc}$  is change in the sectoral output. We also include country and sector dummies as well as interactions between the two sets of dummies. As reported in Columns (1) and (2) in Table 1, the gearing ratio is negatively associated with plant-level employment change, while the Solvency Ratio is positively associated with employment changes.

While these correlations are significant, leverage is clearly endogenous. In order to identify causal effects of access to finance on employment, we instrument leverage by a dichotomic variable capturing firms that can use third party collateral as a result of being part of a consortium of firms. The underlying identification assumption is that the presence of this collateral affects the (equilibrium) level of leverage prevailing before the financial crisis while it does not directly affect employment variation during the Great Recession. Economic theory predicts that, in presence of frictions (e.g., informational asymmetries), availability of collateral increases access to credit. Thus, we expect that, *ceteris paribus*, access to third party collateral increases the probability that firms can draw on external finance. However, firms are not randomly allotted to these consortia: there is a selection into the consortium where the economic conditions of the applicant are likely to play an important role as other firms are reluctant to accept in the consortium non-viable firms. Thus, if firms in the consortium are among the healthier in our sample, our proposed instrument may also exert a direct effect on employment variation. This should bias downwards our results, that is, instrumented estimates of the effects of leverage on employment variation should be less negative (or more positive) than uninstrumented estimates. Another problem with our identification assumption is that involvement in consortia of firms, in addition to capital mobility, may also involve labor mobility within the consortium. This may allow firms to downsize at lower costs as the other firms in the consortium may absorb redundant workers. However, labor is less mobile than capital, and it is particularly less so under a generalized shock like a recession. Under the Great Recession the possibility to absorb shocks via reallocations of workers across firms in the consortium has been particularly small due to the pervasiveness of the crisis.

With the above caveats in mind, columns (3) to (6) of Table 1 display 2-stages least squares estimates in which leverage is instrumented by access to third party collateral in the base year. The first-stage results point to a significant and positive (negative) effect of third party collateral on leverage (solvency). In the second stage we still find a negative and statistically significant effect of leverage and solvency on firm-level employment adjustment. All regressions are based on standard errors clustered at the country level. The effects of leverage on employment adjustment is larger (in modules) than in the uninstrumented regressions and non-negligible: bringing, say, a typical Austrian firm to the average gearing ratio of a German firm involves additional employment losses of the order of 3 per cent during a financial recession; increasing by 10 basis points the solvency ratio (like moving an average Italian firm to France) involves a 6 per cent increase of employment. As shown by the bottom row of Table 3, the 2SLS estimates have substantially less observations than the OLS estimates. This is because there are many missing values in the question about third party collateral.<sup>6</sup>

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<sup>6</sup>We did run regressions replacing missing values with 0, but did not find substantial differences



As stressed above, it is possible that access to third party collateral is correlated with other characteristics of firms we cannot control for, and that could be themselves correlated with employment variation over the Great Recession. To the extent that consortia of firms are typically devices allowing medium-sized firms to improve their access to capital markets, they typically involve firms that are in rather good financial conditions. The control at entry is rather tight as there is a clear perception of the negative externalities that could be exerted over the entire consortium by firms in financial distress. We expect firms in bad financial conditions to have downsized during the Great Recession. This should bias our results reducing the effect of leverage on downsizing, while we find the opposite. Notice further that our regressions include size and sector dummies as well as interactions between the two, capturing potential key factors of heterogeneity across firms.

Where do these effects come from? Columns (5) and (6) display estimates of equation (1) when only firms downsizing or only firms up-sizing are considered. The focus is on the gearing ratios, but the results are not too different when we consider solvency ratios. They suggest that after the financial crisis the effect of leverage on firm-level employment adjustment is driven by firms that are downsizing. For upsizing firms the second-stage coefficient is negative, but not statistically significant<sup>7</sup>. Thus, the effects of financial leverage on employment changes during a financial crisis are concentrated in downsizing firms.

Overall, our results suggest that leverage matters for employment adjustment during a financial recession and operates mainly along the gross job destruction margin. *Ceteris paribus*, more leveraged firms destroy more jobs than firms with a higher solvency ratio.

## 4 Model

Our theoretical perspective is a directed search model of the labor market, where entrepreneurs pay a fixed cost of entry and may potentially hire many workers in a labor market with frictions, and where the initial investments are financed by borrowing in an imperfect financial market. Section 4.1 goes through the building blocks of the theory. First, we describe in some details the environment and the timing of the model. In section 4.2 we focus on the labor market friction and the search process. Section 4.4 describes the financial contract while section 4.3 presents the asset value equations. Section 4.5 presents the financial decision in partial equilibrium. General equilibrium and the key propositions are derived in Section 4.

### 4.1 The environment

We study a model of risk neutral workers and firms, who discount the future at the same rate  $r$ . Workers are infinitely lived. Entrepreneurs set up firms, finance their investment in a credit market with frictions, and hire workers in a labor market that also contains frictions.

Entrepreneurs set up a firm at cost  $F$ . We assume that  $F$  is an effort cost, and hence does not need financing. Then entrepreneurs invest in  $A$  units of capacity ( $A$  machines), where  $A$  is decided endogenously by the firms. The unit cost of capacity is normalized to 1. The maximum capacity of firms is irreversible and cannot be increased later on. The production technology is Leontief in workers and machines, hence the firm hires  $A$  workers. Output is linear in the number of jobs with

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<sup>7</sup>We also run regressions including firm-level output growth (rather than the average growth rate at the sectoral level) as right-hand-side variable. Such a specification clearly creates a problem of endogeneity, but potentially captures idiosyncratic shocks unrelated to the financial recession. Also in this case, there is still an effect of leverage on employment growth. As a further robustness check, we run regressions putting on the left-hand-side a categorical variable (0 for downsizing, 1 for firms keeping the same employment level, 2 for those upsizing) in order to cope with measurement error, notably heaping in the reporting of employment adjustment. There is still a statistically significant effect. Coefficients are remarkably stable across these different specifications

Table 1: Leverage and Employment Adjustment

	(1)	(2)	(3)	(4)	(5)	(6)
Variables	$\Delta e(\%)$	$\Delta e(\%)$	$\Delta e(\%)$	$\Delta e(\%)$	$\Delta e(\%)$	$\Delta e(\%)$
Sample	All firms	All firms	All firms	All firms	Downsizing	Upsizing
Method	OLS	OLS	IV	IV	IV	IV
Second stage						
$\Delta \bar{y}$	1.11*** (0.03)	1.05*** (0.03)	-57.31*** (8.97)	98.56 (54.1)	95.87** (10.73)	33.53 (858)
Gearing	-0.01*** (0.00)		-0.03** (0.01)		-0.34* (0.01)	-0.03 (0.219)
Solvency		0.04*** (0.01)		0.60*** (0.25)		
Constant	-8.12*** (0.07)	-10.73*** (0.18)	-13.09*** (0.79)	-13.19 (0.69)	-24.75 (3.37)	0.98 (132.1)
Country	YES	YES	YES	YES	YES	YES
Sector	YES	YES	YES	YES	YES	YES
Country*Sector	YES	YES	YES	YES	YES	YES
Size	YES	YES	YES	YES	YES	YES
First stage						
Third party collateral			Gearing 108.24*** (16.48)	Solvency -6.85*** (1.69)	Gearing 88.37*** (21.31)	Gearing 31.11*** (68.12)
Observations	8596	9649	2358	2900	1195	307

Standard errors in parentheses

Standard errors are clustered at the country level

\*\*\* p&lt;0.01, \*\* p&lt;0.05, \* p&lt;0.1

marginal productivity equal to  $y$ , so that the total output is  $f(A) = yA$ . The workers stay with the firm until their job is destroyed.

After the workers are hired, a firm produces until it is hit by a negative shock, which arrives at probability rate  $\lambda$ . We refer to this as a  $\lambda$ -shock. The shock destroys the machines, and makes all contracts void. The firm, if it gets refinancing, can reinvest, buy new machines, and retain the workers. If it does not get refinancing, it has to rely on its own internal financial resources for reinvestments. If production continues, the wage contracts are renegotiated, and in this renegotiation game the firms have all the bargaining power. Hence, after refinancing, the employees will receive their outside option. This assumption will be relaxed in Section 6.

The process of matching vacant jobs to unemployed workers is characterized by a standard aggregate matching function. Firms post costly vacancies to attract workers. Attached to each vacancy, the firm posts a present discounted wage of  $W$ . In this sense, our model is coherent with the competitive search specification a-la-Moen. Search is directed, and firms maximize profits given the life-time value of unemployment,  $U$ . As will be clear below, firms attract workers immediately by paying an upfront search cost. In this respect, the specification and characterization of the search process used in our paper is coherent with the fixed cost approach taken by Blanchard and Gali (2010), and represents a microfoundation of their assumption in terms of competitive search.

The entrepreneurs have no financial wealth of their own, and finance their investments through credit. The entrepreneurs receive an exogenous income flow  $y_0$ , independently of production levels. In addition, they obtain income by the production process. As in Holmstrom and Tirole, the financial friction in our model is that an entrepreneur cannot commit to repay her entire future income obtained by the production process. More specifically, we assume that the entrepreneur can commit to repay her exogenous income  $y_0$  (the private income is necessary in order to get any borrowing at all) plus a fraction strictly less than one of the income that the project is expected to generate. There may be several reasons why the entrepreneur cannot borrow on the entire future income flow. First, part of the gain from running a business may be a private, non-pecuniary, benefit which cannot be easily transferred to the creditor. Second, in order to provide incentives to the entrepreneur to make the right decisions, taking properly care of the machines and so on, the entrepreneur must have a sufficiently large stake in the project. We refer to the part of the income that the entrepreneur can commit to repay as the entrepreneur's pledgeable income.

The time horizon of the initial contract with the financier is up to the point where the  $\lambda$ -shock hits. At this point, the collateral, i.e., the machines, are gone, and the contracts are void. Hence, at the point of firm creation, the firm cannot borrow on incomes (neither  $y_0$  nor the output flow) that accrue after the shock.

A key feature of our model is that the firm may want to build financial reserves prior to the  $\lambda$ -shock in order to be able to self-finance the reinvestment costs if it cannot get refinancing through the financial system. The size of the financial reserves is denoted by  $I$ ,  $I \leq A$ . The financial reserves are liquid assets. They may consist of cash, treasuries or liquid stocks. Potentially, even real estate could be considered, even though we prefer to think of  $I$  in terms of financial assets.<sup>8</sup> In order to simplify the exposition, and avoid tedious and complicated tracking of the accumulated interests on the reserves when the  $\lambda$ -shock hits, we assume that the firm places the assets  $I$  at an intermediary. The intermediary promises to pay the firm an amount  $I$  when the  $\lambda$ -shock hits, at an *ex ante* cost of  $\tilde{\lambda}I$ , where  $\tilde{\lambda} = \lambda/(r + \lambda)$ .<sup>9</sup>

To summarize, the life cycle of a firm is as follows:

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<sup>8</sup>In section 6 we discuss the empirical evidence on firm liquidity around the Great Recession.

<sup>9</sup>To see that this is a fair price, note that

$$\begin{aligned} r\tilde{\lambda}I &= \lambda(I - \tilde{\lambda}I) \\ \tilde{\lambda} &= \frac{\lambda}{r + \lambda} \end{aligned}$$

1. The entrepreneur sinks a fixed effort cost  $F$  in order to create the firm.
2. The entrepreneur signs a contract with a financial intermediary, and invests in machines  $A$  and financial assets  $I$ .
3. Job creation and hiring takes place.
4. The firm produces until a  $\lambda$ -shock hits. The shock destroys the machines, and renders all contracts void, and the firm has to reinvest in new machines.
  - (a) With probability  $1 - \tau$ , the firm gets refinancing. A new machines are bought at full cost, and workers are retained and paid their outside option that we indicate with  $U$ .
  - (b) With probability  $\tau$ , the firm does not get refinancing. The firm uses its financial reserves to buy  $I \leq A$  machines at full cost.  $I$  workers are retained, and  $A - I$  workers are displaced. Both the displaced and retained workers get their outside options  $U$ .
5. The firm continues to produce until a second adverse  $\lambda$  shock concludes the life of the firm.

## 4.2 Search

Since all the agents in the economy have linear preferences, the interest rate  $r$  is independent of  $\tau$ .

Employees are costly to acquire due to search frictions. Gali and Blanchard (2010) assume that firms attract workers immediately, but have to pay a hiring cost per worker equal to  $C(\cdot)$ , where  $C$  is a function of aggregate variables that each firm takes as given. With financial frictions, it is convenient that firms attract workers without a time lag. We will now show how this can be obtained endogenously in a way that is fully coherent with the Diamond-Mortensen-Pissarides framework, by allowing firms to post many vacancies per job slot. Furthermore, the hiring cost  $C$  will only be a function of  $U$ , the expected net present value of the future income flow of an unemployed worker.

We assume that search is competitive (Moen (1997), Shimer (1996), and Mortensen and Wright (2002)).<sup>10</sup> At the aggregate level, a constant return to scale matching function  $x(u, v)$  maps the stocks of searching workers  $u$  and firms with vacancies  $v$  into a flow  $x$  of new hires. To simplify some of the expressions we assume that the matching function is Cobb-Douglas, i.e., that  $x(u, v) = u^\beta v^{1-\beta}$ . Let  $p(\theta)$  denote the job finding rate of searching workers and  $q(\theta)$  the arrival rate of workers to searching firms, where  $\theta = v/u$  is labor market tightness.

A firm  $i$  posts  $v_i$  vacancies, and each vacancy comes with a rent  $R_i = W_i - U$ , where  $W_i$  is the net present value of the future income flow of a worker hired in firm  $i$ . The firm can post as many vacancies as it wants at cost  $c$  per vacancy. The instantaneous probability rate of finding a worker when  $v_i$  vacancies are posted is  $v_i q(\theta)$ , where  $\theta$  depends on the rent  $R_i$  being offered. Since the expected time to fill a single vacancy when the firm has  $v_i$  vacancies open is  $\frac{1}{v_i q(\theta)}$ , the time necessary to fill a vacancy tends to zero as  $v$  becomes infinitely large. Furthermore, as the value of a filled job exceeds the expected search cost (this must be true in equilibrium, since the firm has to capitalize the fixed cost  $F$ ), the firm has an incentive to post as many vacancies as possible. As  $v_i$  goes to infinity, the flow cost  $\gamma v_i$  also goes to infinity. In the limit, the worker is obtained immediately, hence we can ignore discounting. The search cost per worker is thus equal to  $\lim_{v_i \rightarrow \infty} \frac{\gamma v_i}{q(\theta) v_i} = \frac{\gamma}{q(\theta)}$ . In words, this means that as  $v_i$  goes to infinity, the firm receives a worker immediately at search cost  $\gamma/q(\theta)$ .

From a worker's standpoint, the rent  $R$  and the job finding probability  $p(\theta)$  are connected through the lifetime value of unemployment, as

$$rU = z + \theta q(\theta)R \tag{2}$$

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<sup>10</sup>For a large-firm application (as in the present paper) see Garibaldi, Moen and Sommervold (2016) and Kaas and Kircher(2015).

where  $R$  is the equilibrium worker rents offered by firms. In equilibrium, workers receive the same expected income independently of which firms they search for. The relationship between  $\theta$  and  $R$  into equation (2), is akin to an unemployed indifference curve, and highlights the workers' tradeoff between  $R$  and market tightness  $\theta$ .

We define hiring cost for the firm as the costs associated with employing the worker, over and above his opportunity cost  $rU$  of working in the firm. Total hiring cost in firm  $i$  is thus  $A(\gamma/q(\theta) + R_i)$ , and total hiring cost per worker is  $C = \gamma/q(\theta) + R_i$ .<sup>11</sup> As will be clear below, the firm has an incentive to minimize total hiring cost per worker, and hence solves the problem

$$C = \min \left[ \frac{\gamma}{q(\theta)} + R_i \right] \quad \text{S.T.} \quad rU = z + \theta q(\theta) R_i \quad (3)$$

Total hiring cost per worker is a function of  $U$ , and we write  $C = C(U)$ . Below we refer to  $C(U)$  as the *search capital* associated with having one worker on board. In the appendix we show that the total hiring cost function is

$$C(U) = \frac{\gamma}{q(\theta(U))} \frac{1}{1 - \beta} \quad (4)$$

where

$$\theta(U) = \frac{rU - z}{\gamma} \frac{1 - \beta}{\beta}$$

It follows that  $C'(U) > 0$  and  $\theta'(U) > 0$ . With a Cobb-Douglas matching function, one can show that  $C''(U) < 0$ .

Another issue is the time profile of wages. We assume that the worker is paid a fixed wage  $\bar{w}$  up to the  $\lambda$ -shock hits. The fixed wage is given by

$$\bar{w} = rU + (r + \lambda)R \quad (5)$$

which gives an expected rent of  $R$ . Alternatively, the firm could pay the entire rent upfront. As will be clear shortly, the time profile of the wage is irrelevant, even in the presence of financial frictions.

### 4.3 Asset values

Let  $M(A, I)$  denote the joint revenue obtained by a firm of size  $A$  with financial resources  $I$ . Since wage payments are a transfer between the firm and the workers, they do not enter in the joint income. Furthermore, recall that the worker after a  $\lambda$ -shock obtains his outside option, independently of whether he is retained or fired. The asset value equation for joint income,  $M(A, I)$ , thus reads

$$rM(A, I) = yA + \lambda \{UA + J(A, I) - M(A, I)\} \quad (6)$$

where  $J(A, I)$  is the value of the firm after the shock. The left-hand-side is the asset returns of a firm of value  $M$ . The right-hand-side is the sum of the flow output  $yA$  and the capital gain (or, in this case, capital loss) associated with the  $\lambda$ -shock. The latter consists of firm profit  $J(A, I)$  and the outside option of the work force  $U$ , less  $M$ .

We now need to specify the firm profit  $J(A, I)$ . When a  $\lambda$ -shock occurs, the firm will receive external funding with probability  $1 - \tau$ . In this case, the firm borrows  $A - I$  units. With probability  $\tau$ , the firm does not get funding, and invests its  $I$  units of reserves. The firm can produce until a second  $\lambda$ -shock occurs, which destroys the firm. It follows that we can write  $J(A, I)$  as

$$J(A, I) = \tau I \frac{y - rU}{r + \lambda} + (1 - \tau) \left[ A \frac{y - rU}{r + \lambda} - (A - I) \right] \quad (7)$$

<sup>11</sup>In the literature, hiring costs are often paid upfront. In our setting, a part of the hiring cost  $c/q$  are paid upfront, while  $R_i$  may be paid at the start of or during the employment relationship.

The total surplus from a firm  $S(A, I)$ , is defined as the joint income net of the outside option of the worker, so that  $S(A, I) = M(A, I) - AU$ . Using equation (6), this implies

$$S(A, I) = \frac{y - rU + \lambda J(A, I)}{r + \lambda} A \quad (8)$$

Firms maximize profits. The initial cost of obtaining  $A$  machines and workers, and financial resources  $I$ , is equal to  $A(1 + C(U)) + \tilde{\lambda}I$ . Total profits thus read

$$V(A, I) = S(A, I) - A(1 + C(U)) - \tilde{\lambda}I$$

Inserting from equations (7) and (8) and upon some manipulations we obtain

$$V(A, I) = \underbrace{\left[ \frac{y - rU}{r + \lambda} - 1 \right] \left( 1 + \tilde{\lambda}(1 - \tau) \right) A}_{\text{Capacity}} + \underbrace{\left[ \frac{y - rU}{r + \lambda} - 1 \right] \tau \tilde{\lambda} I}_{\text{Liquid Asset}} - \underbrace{C(U)A}_{\text{Hiring}} \quad (9)$$

There are three components of profits. The first component shows the net present profit of investing in capacity. It is the product of three factors. The first factor is the net present value of the return from investing in one machine, with the cost of labor equal to the opportunity cost  $rU$ . The second factor shows the discounted number of times each unit of capacity will be utilized in the absence of reserves, taking into account that there is a probability  $1 - \tau$  that the firm can reinvest, and this has to be weighted down by  $\tilde{\lambda}$  since the investment happens in the future, and the firm discounts the future. The last factor is total capacity  $A$ . The second component of profits is the net present value of investing in liquid assets. It is again the product of three terms. The first factor is once more the net present value of the return from investing in one machine. The factor  $\tau$  is the probability that the reserves are needed, while the factor  $\tilde{\lambda}$  captures discounting. The last factor is the size of financial reserves. The third component of profits is the total hiring costs  $C(U)A$ , and it is just the product of hiring costs per worker times the number of employees.

#### 4.4 Financial constraints

In order to invest in capacity, the firm needs finance, which is obtained in a financial market with frictions. The fixed cost  $F$  implies that there are increasing returns to scale in production. Hence the firm wants to become as large as possible. However, the expansion is curbed by the availability of finance. As we anticipated in Section 4.1, we follow Holmstrom and Tirole and assume that at the initial stage the entrepreneur cannot commit to repay her entire future income to a creditor. It can only repay its pledgeable income. While we already discussed the various theoretical motivations behind limited pledgeability, we stress that the entrepreneur cannot save her non-pledgeable income.<sup>12</sup>

Formally, we assume that the non-pledgeable income is proportional to the number of machines that the entrepreneur controls, i.e., it is equal to  $xA$ , where  $x$  is a parameter.<sup>13</sup> Let  $\tilde{p}$  denote the pledgeable income flow if the firm invests  $A$  units of capacity. Then

$$\tilde{p} = y_0 + (y - \bar{w} - x)A \quad (10)$$

<sup>12</sup>Note, however, that the assumption is easily rationalized if the non-pledgeable income is private benefits. However, the assumption is made for convenience. As long as accumulated retained earnings is likely to be less than the reinvestment needed, accumulated savings will only influence the size of the liquid asset, not whether the firm will have one or not (due to the linear structure of the model, as explained in the text).

<sup>13</sup>In an earlier version of the paper we showed that all the results also go through if we instead write the non-pledgeable income as a fraction of output net of the opportunity cost of workers,  $\rho(y - rU)$ , where  $\rho$  is a constant.

where  $y_0$  is the exogenous income flow, and  $\bar{w}$  is given by (5). The NPV of the pledgeable income,  $\tilde{P}$ , writes<sup>14</sup>

$$\begin{aligned}\tilde{P} &= Y_0 + A \frac{y - \bar{w} - x}{r + \lambda} \\ &= Y_0 + A \frac{y - x - rU}{r + \lambda} - AR\end{aligned}\tag{11}$$

where  $Y_0 = y_0/(r + \lambda)$ , and  $R = \frac{\bar{w} - rU}{r + \lambda}$  from (5). If the firm borrows  $\tilde{P}$ , it pays back all its pledgeable income until the machine is destroyed and the contract is terminated.<sup>15</sup>

The firm can use its available financial resources to invest in machines and search, or build financial reserves. The financial constraint that the firm faces can be written as  $\tilde{P} \geq (c/q + 1)A + \tilde{\lambda}I$ . Inserting from (11) gives

$$Y_0 + A \frac{y - rU - x}{r + \lambda} \geq (C(U) + 1)A + \tilde{\lambda}I\tag{12}$$

since  $C(U) = c/q + R$ . We denote the left-hand-side of the equation by (12) by  $P$ . Hence, the borrowing constraint reads

$$P \geq (C(U) + 1)A + \tilde{\lambda}I\tag{13}$$

Note that although worker rent is paid after the worker is employed, it is total hiring cost  $C(U) = c/q + R$  that emerges on the right-hand-side. Hence direct search costs  $c/q$  and workers rents  $R$  tap equally much into the firm's financial resources. It follows that the financial frictions do not give the firm incentives to twist their choice of  $R$  in order to ease the financial constraint. Hence, as already anticipated, the firm will always choose to set the workers' wage so as to minimize the total hiring cost  $C(U)$ . Even though the wage payment occurs later than the direct search cost, it is subtracted one to one from the pledgeable income, hence it creates the same financial burden as upfront investments in search costs. We refer to this as a *decoupling* between the firm's wage policy and the financial friction it faces.

**Proposition 1** *Decoupling between wages and finance: financial frictions do not directly influence the firm's wage setting*

The decoupling result rests on two assumptions. The first is that the rent  $R$  is paid to the worker before the  $\lambda$ -shock hits. If the firm could postpone payment of the rent until after a negative  $\lambda$ -shock hits, that part of the rent payment would not tap into the firm's financial resources. In this case the firms would have an incentive to tilt the hiring expenditures in the direction of higher wages paid after the  $\lambda$ -shock. The second assumption is that wages are subtracted dollar for dollar from the pledgeable income, i.e., that the pledgeable income flow is  $y - w - x$ . If employees as insiders are better at disciplining the entrepreneur than an external bank, this may no longer be the case. If pledgeable income is of the form  $y - aw - x$ , with  $a < 1$ , the firm would again have an incentive to tilt hiring expenses in the direction of wages. The opposite holds if employees are less able to discipline the entrepreneur than an external bank.

## 4.5 Financial decisions in partial equilibrium

We will now analyze the firms' financial decision, that is, its choice of size  $A$  and financial reserves  $I$  in partial equilibrium, for a given value of  $U$ . To simplify notation, we shall suppress the endogenous variables' dependence of  $U$ . The firms' financial decision solves

<sup>14</sup>We assume that  $y - rU - x > 0$ . This will be true in equilibrium.

<sup>15</sup>Note that *ex ante*, the wage-tenure profile does not matter for  $\tilde{P}$ . What matters is only the expected net present value of wages until the  $\lambda$ -shock hits.

$$\begin{aligned}
V(U) &= \max_{A,I} V(A,I) \\
\text{s.t.} & \quad \tilde{I}\tilde{\lambda} + (1+C)A - P \geq 0 \\
& \quad 0 \leq I \leq A; A \geq 0; I \geq 0
\end{aligned} \tag{14}$$

Solving for  $A$ , and assuming that the borrowing constraint binds, gives

$$\begin{aligned}
A &= \frac{Y_0 - \tilde{\lambda}I}{1 + C - \frac{y-rU-x}{r+\lambda}} \\
&= k(Y_0 - \tilde{\lambda}I)
\end{aligned} \tag{15}$$

We refer to  $k$  as the *investment multiplier*, and it is a function of  $U$ . It shows the maximum units of capacity the firm can finance per unit of exogenous income  $Y_0$  the entrepreneur is in possession of. It follows that

$$\frac{dI}{dA} = -\frac{1}{k\tilde{\lambda}} \tag{16}$$

so that the borrowing constraint is just a negatively sloped line in a  $(I, A)$  space.

The firm's objective function is a linear function of  $A$  and  $I$ , and the financial constraint is also linear in  $A$  and  $I$ . Hence the firm's maximization problem generically has a corner solution. Either the firm will go for maximum capacity, or it will hold liquid assets so that it can refinance all the machines. We call the latter, the *Liquid Asset Equilibrium* (LA equilibrium, hereafter), when formally firms set  $I = A$ . Conversely, we call the former a *No Liquid Asset Equilibrium* (NoLA equilibrium, hereafter) and all firms set  $I = 0$ . By substituting the borrowing constraint into the objective function, and taking derivatives, we find that the firms will choose to hold liquid assets if

$$\left[ \frac{y-rU}{r+\lambda} - 1 \right] (1 + \tilde{\lambda}(1-\tau)) \leq C + \left[ \frac{y-rU}{r+\lambda} - 1 \right] \frac{\tau}{k}. \tag{17}$$

The left-hand-side shows the gain from hiring one more worker. The right-hand side shows the gain from having  $1/k$  more units in liquid assets, including the search cost savings of not expanding capacity today.

Let  $D$  denote the difference between the right-hand-side and the left-hand-side in (17). We say that a high value of a parameter pushes the firm toward liquid assets if  $D$  is increasing in the parameter around the bliss point  $D = 0$ . We say that a high value of the parameter pushes the firm toward more capacity if  $D$  is decreasing in the parameter.

**Lemma 1** *In partial equilibrium, for a given  $U$ , the following holds*

1. *A high probability of distress,  $\tau$ , pushes the firm toward liquid assets*
2. *A high value of the pledgeability parameter  $x$  (large financial frictions) pushes the firm toward liquid assets*
3. *A high value of the search cost  $\gamma$  pushes the firm toward liquid assets*

An increase in  $\tau$  increases the probability that the financial resources are needed, hence they are more valuable to the firm. A reduction in pledgeable income (an increase in  $x$ ) reduces the multiplier  $k$ , and hence reduces the shadow cost of investing in the liquid asset. This pushes the firm toward liquid assets. An increase in the search cost  $\gamma$  increases  $C$  (for a given  $U$ ). This has two effects. First, it reduces the multiplier. Second, it makes it more expensive to expand capacity. Both effects increase the right-hand-side of (17) and push the firm toward liquid assets.



## 5 General equilibrium

In general equilibrium, firms enter the market up to the point where the value  $V(U)$  of entering is equal to the cost  $F$  of entering. Let  $V^A(U)$  denote the NPV value of a firm that maximizes capacity and holds no financial reserves ( $I = 0$ ). Similarly, let  $V^I(U)$  denote the value of a firm that holds financial reserves and sets  $I = A$ . Clearly,  $V(U) = \text{Max}[V^A(U); V^I(U)]$ . Hence, we can define general equilibrium as follows<sup>16</sup>

**Definition 1** *The general equilibrium of the model is a vector  $(A^*, I^*, U^*, C^*)$  that satisfies*

1. *Optimal search behavior by firms:  $C^*$  is the solution to (3)*
2. *The firms' choice of capacity  $A^*$  and cash holdings:  $I$  solves (14)*
3. *Free Entry of firms:  $V(U^*) = F$*

From the envelope theorem it follows directly that  $V^I(U)$  and  $V^A(U)$  are strictly decreasing in  $U$ . It is also straight-forward to show that  $V(U) \equiv \max\{V^I(U), V^A(U)\}$  is continuous and strictly decreasing in  $U$ . Existence and uniqueness thereby follow more or less directly.

**Proposition 2** *The general equilibrium exists if*

$$\frac{y - z}{r + \lambda} > F$$

*Generically, the equilibrium is unique*

Note that for any given  $U$ , the firms choose one of the corners  $I = A$  or  $I = U$ , except in the non-generic case with  $V^A(U) = V^I(U)$ , in which case the choice of  $A$  and  $I$  are indeterminate. Furthermore, since  $V(U)$  is strictly decreasing in  $U$ , the zero profit condition ensures a unique equilibrium value of  $U$ . To understand why, note that the only market parameter that influences firm profits and the financial decision is  $U$ . For a given  $U$ , a firm's financial choice is unaffected by the other firms' financial choices.

### Liquid assets or firm capacity in general equilibrium

Parallel with our definition in partial equilibrium, we say that an increase in a parameter  $\psi$  (where  $\psi$  can be any parameter in the model) pushes the equilibrium toward liquid assets if, from an initial situation where firms are indifferent between holding liquidity or not ( $U^A = U^I$ ), an increase in  $\psi$  implies that all firms hold liquid assets.

It is not trivial to see how parameters change the liquidity-size trade-off, as shifts in parameters typically have several countervailing effects. In particular, studying the effects of parameter changes on the inequality (17) is a difficult route, as partial and general equilibrium effects tend to go in opposite directions.

Note, however, that at the point where firms are indifferent between holding liquid assets or not,  $V^I = V^A = V = F$ . In particular, the zero profit condition for no-cash firms reads (from (9) and (15))

$$Y_0 k \left[ \left( \frac{y - rU^*}{r + \lambda} - 1 \right) (1 + \tilde{\lambda}(1 - \tau)) - C(U^*) \right] \equiv F \quad (18)$$

Let  $V(I; U^*)$  denote the value of the firm that has  $I$  units of liquid assets. Insert (15) into the expression for the value of a firm, (9), to get

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<sup>16</sup>We do not specify unemployment rates and employment in new and old firms. See the appendix for details on labor stocks

$$V(I; U^*) = (Y_0 - \tilde{\lambda}I)k \left[ \left( \frac{y - rU^*}{r + \lambda} - 1 \right) (1 + \tilde{\lambda}(1 - \tau)) - C(U^*) \right] + \tau \tilde{\lambda}I \left( \frac{y - rU^*}{r + \lambda} - 1 \right)$$

Taking derivatives gives

$$\frac{\partial V(I; U^*)}{\partial I} = -\tilde{\lambda}k \left[ \left( \frac{y - rU^*}{r + \lambda} - 1 \right) (1 + \tilde{\lambda}(1 - \tau)) - C(U^*) \right] + \tau \tilde{\lambda} \left( \frac{y - rU^*}{r + \lambda} - 1 \right)$$

Inserting from (18) gives

$$\frac{\partial V(I; U^*)}{\partial I} = -\tilde{\lambda} \frac{F}{Y_0} + \tau \tilde{\lambda} \left( \frac{y - rU^*}{r + \lambda} - 1 \right) \quad (19)$$

At the point of indifference, where  $V^A(U^*) = V^I(U^*)$ , by definition we have that  $\frac{\partial V(I; U^*)}{\partial I} = 0$ . The next lemma follows immediately

**Lemma 2** *An increase in a parameter  $\psi$  pushes the equilibrium toward liquid assets if and only if it increases the right-hand-side of (19).*

The lemma is very convenient in order to establish how the demand for liquid assets is linked to aggregate variables. The following proposition follows almost immediately:

**Proposition 3** *The following two results hold*

- *Increased search costs  $\gamma$  push the equilibrium toward liquid assets, and in a frictionless market with  $\gamma = 0$ , firms do not invest in liquidity.*
- *An increase in  $y$  and in  $\tau$  pushes the equilibrium toward liquid assets. An increase in  $x$  and in unemployment income  $z$  pushes the equilibrium away from liquid assets. size.*

The proposition follows more or less directly from lemma 2. A formal proof is offered in the appendix.

The first result states that there is a complementarity between financial frictions and labor market frictions. In the presence of financial frictions, a firm's desire to hold liquid assets is created by search frictions. Without search frictions, there is no search capital to protect, and the firms will not hold liquid assets. Furthermore, as higher search frictions increase the search capital, increased search frictions push the equilibrium toward liquid assets.

Higher output means a tighter labor market, and this increases the value of search capital. Hence, under higher productivity, firms have stronger incentives to protect the search capital by holding liquid assets.

Recall that  $\tau$  reflects how frequently a firm cannot get refinancing, and hence can be considered as a measure of the quality of the financial system, with a low value of  $\tau$  reflecting a high-quality financial system. The more likely it is that the financial system will fail, the stronger are the incentives to hold liquid assets. Also the parameter  $x$  reflects the quality of the financial system. A higher  $x$  increases the shadow cost of holding liquid assets. Again, a higher quality of the financial system favors size, and self-financing through liquid assets becomes less attractive.

For changes in  $F$  and  $Y_0$ , proposition 3 (or lemma 2) gives us no guidance. The direct and indirect effects (through  $U^*$ ) in (19) have different signs. Hence we are unable to derive general results on whether changes in  $F$  or  $Y$  favor liquidity or not.

## Comparative statics within regimes

We can easily derive various comparative static results summarized in the following proposition:<sup>17</sup>

**Proposition 4** *In equilibrium, the following holds*

1. *A marginal increase in the difficulty of obtaining refinancing (an increase in  $\tau$ ), has no effect on the LA equilibrium, while it reduces welfare  $U$  in the NLA equilibrium*
2. *Increased pledgeability (reduced  $x$ ) increases the value of unemployment and the market tightness and reduces equilibrium unemployment in both types of equilibria.*
3. *An increase in firm productivity ( $y$ ) increases the value of unemployment, market tightness and reduces equilibrium unemployment in both types of equilibria.*
4. *An increase in the entry cost,  $K$ , reduces the value of unemployment, market tightness and increases equilibrium unemployment in both types of equilibria.*

The proofs are straightforward and omitted.

## 6 Extensions

### Heterogeneous firms in steady state

In our framework all firms are identical, and hence face the same trade-off regarding investment in liquid assets versus physical capacity. In order to get cross-sectional differences, we have to introduce firm heterogeneity. This extension allows us to reconcile our theoretical perspective with the empirical evidence on the links between leverage, liquidity and employment provided in Section 3 as well as by Bacchetta et al. (2016)

Specifically, we now suppose that  $\tau$  is stochastic, and realized upon entry after  $F$  is sunk.<sup>18</sup> Conditional on entry,  $\tau$  is drawn from a discrete distribution  $\Omega$  with support over the interval  $[\tau_l, \tau_m]$  with  $\tau_l \geq 0$  and  $\tau_m \leq 1$ . Let  $V(U, \tilde{\tau})$  denote the value of a firm with a realization of  $\tau$  equal to  $\tilde{\tau}$ . The zero profit condition of the firm then reads  $E^{\tilde{\tau}}V(U^*, \tilde{\tau}) = F$ .

**Lemma 3** *Suppose that in general equilibrium, the firms prefer liquid assets if  $\tilde{\tau} = 1$ . Then there exists a unique  $\tau^*$ ,  $0 < \tau^* \leq 1$ , so that firms hold liquid asset if and only if  $\tilde{\tau} \geq \tau^*$*

Note that  $V^A(\tilde{\tau}; U^*)$  is strictly decreasing in  $\tilde{\tau}$ , while  $V^I(\tilde{\tau}; U^*)$  is independent of  $\tilde{\tau}$ . Furthermore, it is always true that  $V^A(0; U^*) > V^I(0; U^*)$ , while by assumption,  $V^A(1; U^*) \leq V^I(1; U^*)$ . Hence there exists a unique value  $\tau^*$  such that  $V^A(\tau^*; U^*) = V^I(\tau^*; U^*)$ .

This equilibrium specification with the threshold  $\tau^*$  allows us to obtain cross sectional predictions in terms of various firm-level variables. The distribution of firm size is degenerate and it is characterized by two mass points on  $A^{NoLA} = Y_0k$  for firms with no liquid assets ( $\tau \leq \tau^*$ ) and  $A^{LA} = Y_0k/(1 + k\tilde{\lambda}) < A^{NoLA}$  for firms that hold liquid assets ( $\tau > \tau^*$ ), see equation (15) (the dependence of  $U$  is suppressed). Firms that hold liquid assets are thus smaller.

In our model, firms borrows  $\tilde{P}$  given by (11). Since  $\bar{w}$  is independent on  $\tau$  in the cross section, it follows immediately that  $\tilde{P}^{LA} < \tilde{P}^{NoLA}$ . Thus firms with no liquid assets have more debt than firms with liquid assets. The cash ratio is defined in the empirical literature as the ratio of liquid assets to total assets, or generically, as

$$Cash\_Ratio = \frac{I}{\tilde{P} + F + Y_0}$$

<sup>17</sup>With endogenous machine prices, increased job creation may increase the price of machines. This may dampen (but not overturn) the expansionary effects of a shift in for instance productivity  $y$ .

<sup>18</sup>Note that we could just as well impose heterogeneity in terms of  $x$  or  $y$  rather than  $\tau$

where the denominator is total assets, or the sum of investments and the value of the firm.

Note the following: firms that do not hold liquid assets are large, with size  $A^{NoLA}$ , and their cash ratio is zero. There are also firms that do hold liquid assets (and have low debt level  $\tilde{P}$ ) and hence have a strictly positive cash ratio. They are also smaller, with size  $A^{LA} < A^{NoLA}$ . Finally, there are small firms that had liquid assets initially, but have been hit by a negative shock and used their liquid assets to buy machines. They have no liquid assets. Since there are only small firms that hold liquid assets, the following proposition holds:

**Proposition 5** *In the model with heterogeneous refinancing shocks  $\tau$ , there is a negative cross sectional relationship between firm level employment and the cash ratio.*

The proposition reconciles our theory with the work of Bacchetta et al. (2016), who find a negative cross-sectional relationship between cash ratio and firm size.

### Heterogeneous firms across steady-states

In the previous section we analyzed the model with heterogeneous refinancing shocks  $\tau$  drawn from a discrete distribution  $\Omega(\tau)$  with support contained in the interval  $[t_l, t_m]$ . In this subsection, we investigate the effects on aggregate employment and liquidity of shifts in this distribution. The goal is to reconcile the properties of our model with the negative relationship between aggregate employment and liquidity reported by Bacchetta et al. (2016). To this end, define  $\Omega^\Delta(\tau)$  with support contained in the interval  $[t_l + \Delta, t_m + \Delta]$  as  $\Omega^\Delta(\tau) \equiv \Omega(\tau - \Delta)$ , where  $\Delta$  is the aggregate shifter.

For any  $\Delta$  there may be a subset of firm types that hold cash and a subset of firms that do not hold cash, as above. Denote by  $\tau^*(\Delta)$  the threshold value of  $\tau$  above (below) which firms hold (do not hold) cash. In steady state, workers employed in firms that do not hold cash transit into unemployment at rate  $\lambda/(2 - \tau)$  while workers employed in firms that hold cash transit into unemployment at rate  $\frac{\lambda}{2}$ . In aggregate, inflows to employment are equal to outflows from employment. The distribution can be obtained by simple balance flow conditions.

Consider a positive shift in  $\Delta$ . The effect on total employment is negative, since the equilibrium value of unemployment falls. The effect on total liquidity is more complicated, as there are two forces at work. On the one hand, all firms will be smaller since the equilibrium value of unemployment will fall. It turns out that this tends to reduce aggregate liquidity. On the other hand, a higher proportion of firms hold cash, and this tends to increase aggregate liquidity. The question is thus numerical.

We thus turn to numerical simulation. Boeri et al. (2016) calibrate a related model without financial shocks, and we take our parameter values from them. In addition we assume that  $\Omega$  is uniform with support contained in the interval  $[\.65, \.909]$  with 20 mass points, and that  $\Delta = \.1$ . See Table 2 for details. The results are plotted in Figure 2. The Figure clearly shows that there is a negative relationship between the aggregate cash ratio and aggregate employment.

Table 2: Parameters For Heterogeneous Model Across Steady State

Parameter	Notation	Value
Pure Discount Rate	$r$	0.010
productivity	$y$	1.000
unemployed income	$z$	0.500
pledgeable income	$x$	0.100
destruction shock	$\lambda$	0.053
Matching (Cobb Dougals) elasticity	$\alpha$	0.500
Matching constant	$M$	1.027
Search Cost	$c$	0.552
Entry Fixed	$F$	1.898
<i>Heterogeneous Values</i>		
Number States	$n$	20.000
Minimum	$\tau_l$	0.650
Maximum	$\tau_m$	0.900
Uniform Distribution	$\delta_i$	0.050
<i>Macro Shifts</i>		
Minimum Shift in refinancing	$\Delta\tau_i$	0.000
Maximum Shift in refinancing	$\Delta\tau_i$	0.100
<i>Source: Authors' calculation</i>		

### Macro elasticity

In this subsection we analyze how the firms' holding of liquid assets influence the responsiveness of the equilibrium to unanticipated productivity shocks. To this end we go back to the representative firm framework, and study the elasticity of the value of unemployment (which is a welfare measure) to productivity  $y$  under two equilibrium configurations, when firms hold and do not hold liquid assets, labeled  $\eta^I$  and  $\eta^A$ , respectively. Taking a first order log-approximation of the steady state,  $\eta^A$  reads

$$\eta^A = \frac{\frac{y}{Y_o(r+\lambda)} + \frac{1}{F} \left( \frac{1+\bar{\lambda}(1-\tau)}{r+\lambda} \right)}{\frac{\beta r U}{rU-z} C \left( \frac{1}{F} + \frac{1}{Y_o} \right) + \left( \frac{y}{Y_o(r+\lambda)} + \frac{1}{F} \frac{1+\bar{\lambda}(1-\tau)}{r+\lambda} \right) rU} \quad (20)$$

while  $\eta^I$  can be written as

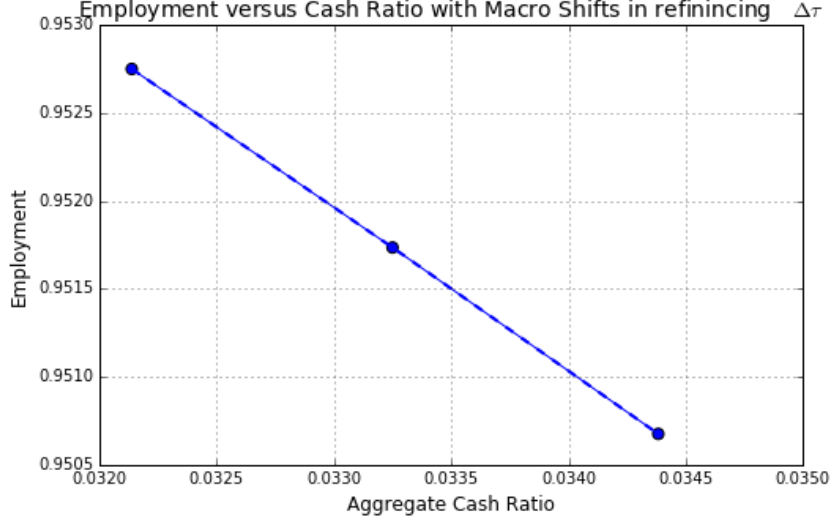
$$\eta^I = \frac{\frac{y}{Y_o(r+\lambda)} + \frac{1}{F} \left( \frac{1+\bar{\lambda}}{r+\lambda} \right)}{\frac{\beta r U}{rU-z} C \left( \frac{1}{F} + \frac{1}{Y_o} \right) + \left( \frac{y}{Y_o(r+\lambda)} + \frac{1}{F} \frac{1+\bar{\lambda}}{r+\lambda} \right) rU} \quad (21)$$

In these expressions,  $U$  is of course endogenous, and depends on parameters. We want to compare the elasticities  $\eta^A$  and  $\eta^I$  at the bliss point  $\tau^*$  at which the equilibrium constellation switches. At this point,  $U$  in the two expressions are identical. It follows directly that at this point,  $\eta^A < \eta^I$ . Hence the elasticity of  $U$  is higher in the equilibrium when firms hold liquid assets than when they do not. The intuition is that when firms hold liquid assets, they are smaller, and investment in liquid assets play a similar role as a fixed cost. We know from Pissarides (2009) that fixed costs in the hiring process increases the responsiveness of  $U$  to shocks to  $y$ . For values of parameters coherent with Boeri et al. (2016), and with  $\lambda = 0.05$ , the elasticity  $\eta^I$  at the bliss point is approximately 2 percent higher when firms hold liquid assets.

### Unanticipated financial crises

We define a financial crisis as a situation in which a subset of firm creditors require that firms repay an amount  $T < P$  immediately. We may think of this as credit facilities (credit lines) suddenly drying up. We assume that the crisis only lasts for an instance, so that  $U$  is not affected. Finally, we assume that the crisis is unanticipated.

Figure 2: Aggregate Employment and Aggregate Cash Ratio with heterogeneous refinancing probability  $\tau$



For firms with cash, the forced repayment shock does not create problems, as they can use their liquidity to repay  $T$ . Furthermore, since the shock lasts for an instant, the probability that a refinancing shock occurs during the crisis is zero.

In order for a firm without liquid assets to repay, it has to sell off its machines. Suppose that the scrap value of a machine is  $\kappa$ ,  $\kappa > \frac{y-rU-x}{r+\lambda}$ . Hence the value of the machine is higher than the net present value of the pledgeable income flow that the machine creates.<sup>19</sup> In order to repay the loan, the firm sells machines and lays off workers. It will have to lay off a total of  $H/\kappa$  workers.

**Proposition 6** *Suppose that a financial crisis hits, in the form of a repayment shock  $H$ . This has no effect on firms with liquid assets. Firms without liquid assets fire  $H/\kappa$  workers, and the unemployment rate increases.*

If the firm has to pay a firing cost to the replaced workers, this will increase the amount of firing the firm has to undertake. If the firing tax is  $t$ , the firm has to fire a total of  $H/(\kappa - t)$  workers.

Note that as long as  $\kappa < 1$ , resources are lost when the firm fires the workers. For each unit of capacity it sells, the firm only pays back  $\kappa < 1$  units of debt. When funds again are available, the amount the firm can borrow is smaller than before the crisis, hence the firm cannot scale up the loan to the pre-crisis level. It can be shown that each unit of capacity that the firm scraps has the same effects as reducing  $Y_0$  with  $1 - \kappa$  units in terms of borrowing potential when funds are again available (under the assumption that the laid off workers can be called back at no cost).

### Wage bargaining after the shock

Suppose now that the workers, after the financing shock occurs, bargain over the wage, and receive a share  $\alpha$  of the surplus. The surplus per worker after the shock is  $\frac{y-rU}{r+\lambda}$ , hence the worker who is retained gets a surplus of  $\alpha$  times this. Note that the reinvestment cost is not a part of the surplus

<sup>19</sup>If this was not the case, selling off machines would make the firm insolvent

the agents bargain over. We assume that the worker's bargaining power is not too high so that firms will invest after the shock if funding is available.

We assume that when a job is advertised, it is informed whether the firm has financial resources so that the job can continue after a shock. If this is the case, we say that the job is funded. Hence workers, when evaluating the attractiveness of the job, take into account whether the job is funded or not. At this stage, the net present value to the worker of the rents obtained through bargaining after a negative shock,  $R_2^i$  ( $i \in \{0, 1\}$ ,  $i = 1$  if the vacancy is funded), is given by

$$R_2^i = \tilde{\lambda}\alpha\left[\frac{y - rU}{r + \lambda}((1 - \tau) + \tau I[i = 1])\right] \quad (22)$$

where  $I[i = 1]$  is an indicator function, equal to 1 if the job is funded and 0 if it is not.

The firm's trade-off between rents and search costs is not altered by renegotiation *ex post*. In particular, the firm still minimizes  $C(U)$  defined by (3) with solution (4). Let  $\tilde{C}^i(U)$  denote the part of  $C(U)$  that is paid out before the shock hits, i.e.,  $\tilde{C}^i(U) = C(U) - R_2^i$ . Hiring a worker now taps  $\tilde{C}(U)$  from the firms' financial funds, not  $C(U)$  as before.<sup>20</sup> Giving bargaining power to the workers after the shock reduces the wage the firm has to pay before the shock, and eases the financial constraint of the firm.

It follows that (12) reads

$$Y_0 + A\frac{y - rU - x}{r + \lambda} \geq (\tilde{C}^0 + 1)A - \tilde{\lambda}(1 - \tau\alpha\frac{y - rU}{r + \lambda}) \quad (23)$$

Define  $\hat{\lambda} = \tilde{\lambda}(1 - \tau\alpha\frac{y - rU}{r + \lambda})$ , which clearly is less than  $\tilde{\lambda}$ . It follows that the multiplier,  $\hat{k}$ , is defined as above, with  $C$  substituted out by  $C^0$ . It follows that  $\hat{k} < k$ .

The *ex ante* asset value of the firm is unchanged. The reduction in profit *ex post* profits after the shock is reduced, as workers receive some of the surplus, however this is exactly matched by the reduced cost of attracting the worker before the shock,  $C(U) - \tilde{C}(U)$ . We can now rewrite equation (17) as

$$\left[\frac{y - rU}{r + \lambda} - 1\right] (1 + \tilde{\lambda}(1 - \tau)) \leq C + \left[\frac{y - rU}{r + \lambda} - 1\right] \frac{\tilde{\lambda}\tau}{\hat{\lambda}\hat{k}}. \quad (24)$$

Clearly, it is more attractive to hold cash now, since  $\hat{\lambda} < \tilde{\lambda}$  and  $\hat{k} < k$ . Hence, if the workers have bargaining power in the *ex post* wage renegotiation game, the firm is more likely to hold liquid assets. It also follows that Lemma 2 still holds.

Let us then turn to general equilibrium. Since the asset values are unaffected, it follows that (19) can be rewritten as

$$\frac{\partial V(I; U^*)}{\partial I} = -\hat{\lambda}\frac{F}{Y_0} + \tau\hat{\lambda}\left(\frac{y - rU^*}{r + \lambda} - 1\right) \quad (25)$$

It follows trivially that proposition 3 still holds.

## 7 Discussion and implications

In this paper we integrate limited pledgeability with labor market imperfections in an archetype model framing the interplay between labor and financial imperfections. This framework is sufficiently flexible to be extended in several directions and can offer guidance to future empirical research on labor-finance interactions.

<sup>20</sup> Attracting workers by setting a high wage, or posting many vacancies, still tap equally much financial resources at the margin, hence *ex post* bargaining does not change the trade-off between the two.

Our theory offers four key insights in this respect.

First, we uncover a key complementarity between firms holding liquid assets and labor market imperfections. In our model the corporate sector holds liquidity as a way to protect its search capital. The latter is defined as the total hiring cost created by labor market imperfections. The model predicts also that firms do not hold liquid assets when labor market frictions disappear. While we are aware that the precautionary motives for firms holding cash and liquid assets are many, the complementarity between liquid assets and labor market imperfections is novel and should be investigated in future empirical work.<sup>21</sup> If we take literally the structure of our model, the larger are the labor market frictions, the larger should be the amount of cash held within the firms. We thus expect that firms operating within very tight labor markets will be more prone to hold liquid assets. Future empirical research may evaluate the relevance of this relationship in detail.

Second, our theory predicts that during financial recessions firms that hold more liquid, and hence are less leveraged, are more protected to adverse shocks hitting their lender. The empirical evidence reviewed in section 2 suggests that job destruction, and hence inflows into unemployment play a key role in the adjustment to a financial shock. Our own results on the Efige sample of European firms also suggest that more leveraged firms experienced stronger employment reductions, and that this adjustment occurred via a stronger downsizing of firms rather than a decline of hiring in expanding units.

Third, our theoretical results on wage bargaining indicate that empowerment of workers may have consequences for the financial decisions of firms. Our model, in particular, indicates that empowerment of workers may ease the financial constraints of the firms, as it will act as a commitment device, akin to allowing the firm to borrow from its workers' future earnings (after the shock). This is consistent with evidence that firms under financial stress tend to rely more on wage reductions than firms hit by other types of shocks (Boeri and Jimeno, 2016). Furthermore, giving bargaining power to workers also changes the trade-off between liquid assets and size which is framed in our model in the direction of liquid assets: the marginal firm indifferent between holding cash or expanding capacity at the initial equilibrium, when workers have bargaining power, for a relatively large probability of a refinancing shock, will be induced to hold liquid assets at the new equilibrium.

Fourth, our theory can also rationalize the recent empirical evidence on cash and employment. Bacchetta et al. (2016) show that there is a negative association between holding of cash and total employment both across firms and at the aggregate level. When our model is allowed to feature heterogeneous firms, as we do in Section 6, we show that there is a negative cross-sectional correlation between cash ratio and firm-level employment. We also show that the model implies a negative aggregate correlation between cash and employment, due to a composition effect. When there are aggregate shifts in the refinancing probability, there are two forces at work. On the one hand, all firms will be smaller since the equilibrium value of unemployment will fall. This effect tends to reduce aggregate liquidity. On the other hand, there is likely to be a shift in the proportion of firms that are willing to hold cash, so that more firms hold cash in equilibrium. In the numerical example provided in Section 6, the latter *composition* effect can dominate the first effect and aggregate employment and cash ratios move in opposite directions across steady states.

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<sup>21</sup>Opler et al. (1999) argue that in general there are precautionary and transaction motives for the firms holding cash. First, the firm saves transaction costs to raise funds and does not have to liquidate assets to make payments. Second, the firm can use the liquid assets to finance its day-by-day activities if other sources of funding are not available. Armenter and Hnatkovska (2012) argue that firms accumulate cash holdings in order to avoid being financially constrained in the future. In their paper, firms operate within a perfect labor market and must resort to costly equity every now and then. It turns out that the value function is strictly concave even if their utility is linear.



## 8 Final remarks

This paper contributes to the flourishing literature on labor/finance interactions by integrating the traditional Diamond-Mortensen-Pissarides model with the limited pledgeability friction developed by Holmstrom and Tirole. The result is a tractable micro-founded model of labor-finance interactions that generates a demand for liquid assets inside the firm. The model is flexible and can be extended on a variety of dimensions. It also yields a number of testable implications. The most relevant in the context of the Great Recession is that highly leveraged firms should experience larger employment losses during a financial crisis. Recent empirical evidence reviewed in the paper as well as micro data on employment adjustment and balance sheets of European firms indicate that highly leveraged firms are characterized by higher job destruction rates during financial recessions. A causal interpretation of this link going from financial conditions to employment adjustment of firms is not falsified by our empirical results.

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## Theoretical annex

### Derivation of optimal search equations

The constraint implicitly defines an indifference curve  $\theta = \theta(R, U)$  where  $U$  is the given value of unemployment. Further

$$\frac{d\theta}{dR} = -\frac{\theta q(\theta)}{q(1-\beta)R}$$

where  $\beta$  is the absolute value of the elasticity of  $q(\theta)$ , independent of  $\theta$  under a Cobb-Douglas specification of the matching function. Total search cost define implicitly an isocost and the equilibrium is going to be a tangency condition between the isocost  $C$  and the indifference curve  $U$ .

Formally, the first order condition for a minimum- once we use the indifference curve is thus

$$\frac{\gamma q'(\theta)}{q^2} \frac{\theta q(\theta)}{q(1-\beta)R} = 1$$

or

$$R = \frac{\gamma}{q} \frac{\beta}{1-\beta}$$

Total hiring cost is thus

$$C = \frac{\gamma}{q} \frac{1}{1-\beta}$$

Over and beyond the rent, the firm pays the worker a flow value  $rU$  per period employed, as we further discuss at the end of this section. Finally,  $\theta$  is given by

$$\theta q(\theta) = \frac{rU - z}{R} = (rU - z) \frac{1-\beta}{\beta} \frac{q}{\gamma}$$

hence

$$\theta(U) = \frac{rU - z}{\gamma} \frac{1-\beta}{\beta} \quad (26)$$

## Worker flows and stocks

To complete the specification of the economy we have to account for the aggregate labor flows. In the economy there is a measure 1 of workers who can be employed in new firms or firms that already experienced the first  $\lambda$  shock. We label respectively  $n_1$  and  $n_2$  the share of workers employed in the two types of firms. In the war chest equilibrium, conditional on a  $\lambda$  shock firms do not fire any worker and continue with their cash holdings. Let  $\omega$  be an indicator function that takes value 1 if the economy is in a no-cash equilibrium. The general balance flow conditions reads

$$\begin{aligned} \theta q(\theta)u &= \omega \lambda n_2 + (1-\omega)(\lambda \tau n_1 + \lambda)n_2 \\ \omega \lambda n_1 + (1-\omega)(\lambda(1-\tau))n_1 &= \lambda n_2 \\ u + n_1 + n_2 &= 1 \end{aligned}$$

The first equation is simply the outflows from unemployment and inflows into unemployment, where the latter involve also the share of workers in type 1 firms that do not find refinancing in the no cash equilibrium. The second condition is the flow into  $n_2$  from type 1 firms and outflows out of  $n_2$ . Again, in the no cash equilibrium only the surviving employed enter the type 2 state. The last condition is the aggregate labor market condition. Solving for the stock yields

$$\begin{aligned} u &= \omega \frac{\lambda}{\lambda + 2\theta q(\theta)} + (1-\omega) \frac{\lambda}{\lambda + (1 + (1-\tau))\theta q(\theta)} \\ n_1 &= \omega \frac{\theta q(\theta)}{\lambda + 2\theta q(\theta)} + (1-\omega) \frac{\theta q(\theta)}{\lambda + (1 + (1-\tau))\theta q(\theta)} \\ n_2 &= \omega \frac{\theta q(\theta)}{\lambda + 2\theta q(\theta)} + (1-\omega) \frac{\theta q(\theta)(1-\tau)}{\lambda + (1 + (1-\tau))\theta q(\theta)} \end{aligned} \quad (27)$$

### Proof of proposition 3

*Proof:* It is straightforward to show that  $U^*$  is decreasing in  $\gamma$ . It follows that an increase in  $\gamma$  increases the right-hand side of (19), and hence makes liquidity more favorable. Furthermore, in the limit, as  $\gamma \rightarrow 0$ , one can easily show that  $C \rightarrow 0$ ,  $\theta \rightarrow 0$  and  $R \rightarrow 0$ . The labor market is competitive with a wage  $w = rU < y$ .<sup>22</sup> Equation (17) then reads

$$(1 + \tilde{\lambda}(1 - \tau)) \leq \frac{\tau}{k}$$

where  $k = \frac{1}{1 - \frac{y-w-x}{r+\lambda}} > 1$ . As the left-hand-side is strictly greater than one, while the right-hand-side is strictly less than one, the result follows.

An increase in  $y$  increases  $y - rU$ . Suppose not. Then it follows from (9) that profits per worker fall strictly, and from (15) that the financial constraint tightens. Hence profits fall, a contradiction. It follows that  $y - rU$  decreases, and hence that cash is more likely. An increase in  $\tau$  reduces  $V^A$  while it does not influence  $V^I$ . An increase in  $\tau$  therefore makes cash more likely. Finally, an increase in  $x$  increases  $U^*$ , and hence reduces the left-hand-side of (19).

### Equilibrium when $\gamma \rightarrow 0$

When  $\gamma \rightarrow 0$ , it follows that  $w \rightarrow rU < y$  and that  $C = 0$  (the firms still have finite size and have to capitalize the investment  $F$ , hence  $w < y$ ). It follows from (17) that the firm will hold cash whenever

$$\left[ \frac{y-w}{r+\lambda} - 1 \right] (1 + \tilde{\lambda}(1 - \tau)) \leq \tilde{\lambda}\tau \left[ \frac{y-w}{r+\lambda} - 1 \right] \frac{1}{k(w)\tilde{\lambda}} \quad (28)$$

This immediately simplifies to

$$k(w)(1 + \tilde{\lambda}(1 - \tau)) \leq \tau \quad (29)$$

Since  $k(w) \geq 1$  (it is 1 if the firm does not borrow from the bank), the inequality is always satisfied.

## Statistical annex

Figure 3 plots the distribution of employment changes across firms in the EFIGE survey, using also a Kernel density estimator (blue line) to characterise the distribution. As data refer to a global recession year, most firms appear to be downsizing: the median is 0, the mean is -6. In addition to the mode at 0, there are also some spikes at -10, -20 and -30. This may indicate that respondents answered doing some rounding. Some of our estimates below take into account of such heaping problems.

Table 3 provides some descriptive statistics on the measures of leverage which are used in the empirical analysis in 2007, the year before the beginning of the Great Recession. In particular, the *Gearing ratio* is the debt to equity ratio measuring the extent to which the firm is using creditor's vs. owner's funds. As shown by table 3, there is significant cross-country and within country (across sectors) variation in these measures. At the same time, there are large differences in the average size of firms across countries, which confirms that data are not cross-country comparable.

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<sup>22</sup>Even in the limit, firms don't grow infinitely due to the borrowing constraint, hence wages must be below productivity in order for the firms to capitalize on  $K$ .

Figure 3: Firm-level net employment change, Distribution of firms

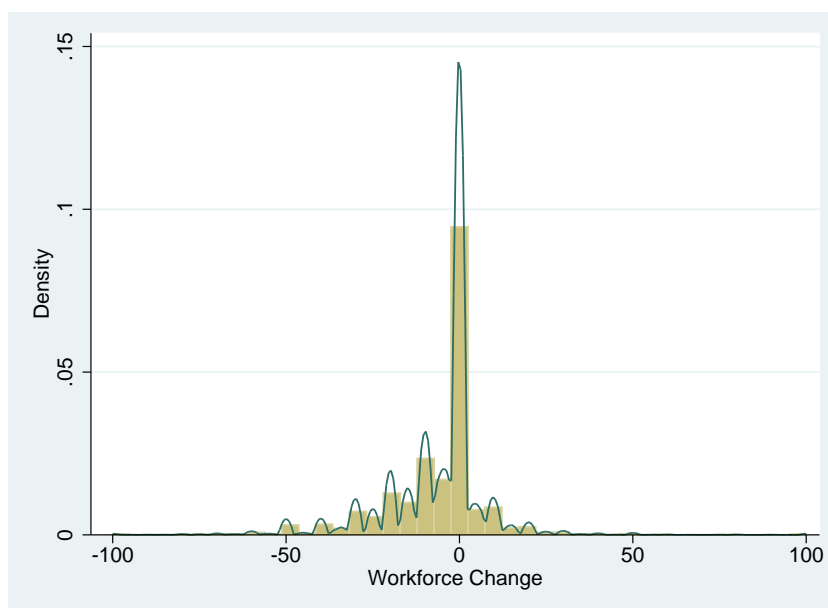


Table 3: Measures of Leverage, Descriptive Statistics

Country	Number of Firms	Revenues growth 2008-2009		Size of Firms (Employees 2007)	
		Average	Standard Deviation	Average	Standard Deviation
AUT	443	63.5%	0.740	100	33
FRA	2,973	-8.3%	0.010	50	8
GER	2,935	-5.2%	0.008	96	11
HUN	488	-12.4%	0.015	68	9
ITA	3,021	-18.6%	0.005	40	2
SPA	2,832	-16.8%	0.015	45	3
UK	2,067	0.1%	0.032	20	773

Variable	Average	Min	Max	Standard Deviation
Gearing Ratio (2007)	1.20	0.00	997.53	175.46
$\Delta e$	-6.18	-100	100	15.16
$\Delta \bar{y}$	-0.09	-0.30	5.64	0.39
Size of Firms (2007)	116.65	0.00	365,630	3,595.00
Third Party Collateral	0.04	0	1	0.21