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NLO Standard model effective field theory for Higgs and EW precision data

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A set of constructs, definitions, and propositions that present a systematic view of the Standard Model Effective Field Theory (SMEFT), i.e. how the influence of higher energy processes is localizable in a few structural properties which can be captured by a handful of Wilson coefficients.

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1. Introduction

In these proceedings we provide a consistency proof (not only power counting, but a proof that proves that there are enough Wilson coefficients) of quasi-renormalizability in SMEFT. Theory deals with the well founded theoretical results obtained from first principles, while phenomenology deals with not so well founded effective models with a smaller domain of application. For a definition see Ref. [1].

Mathematics suffers from some of the same inherent difficulties as theoretical physics: great successes during the 20th century, increasing difficulties to do better, as the easier problems get solved. The lesson of experiments 1973 - today: it is extremely difficult to find a flaw in the Standard Model (SM): maybe the SM includes elements of a truly fundamental theory. But then how can one hope to make progress without experimental guidance? One should pay close attention to what we do not understand precisely about the SM even if the standard prejudice is “that’s a hard technical problem, and solving it won’t change anything”.

There is a conventional vision: some very different physics occurs at Planck scale, SM is just an effective field theory. What about the next SM? A new weakly coupled renormalizable model? A tower of EFTs? A different vision: is the SM close to a fundamental theory?

It is possible that at some very large energy scale, all nonrenormalizable interactions disappear. This seems unlikely, given the difficulty with gravity. It is possible that the rules change drastically, it may even be possible that there is no end, simply more and more scales. This prompts the important question whether there is a last fundamental theory in this tower of EFTs which supersede each other with rising energies. Some people conjecture that this deeper theory could be a string theory, i.e. a theory which is not a field theory any more. Or should one ultimately expect from physics theories that they are only valid as approximations and in a limited domain [1, 2]? Alternatively, one should not resort to arguments involving gravity: let us banish further thoughts about gravity and the damage it could do to the weak scale [3].

When looking for ultraviolet (UV) completions of the SM the following remarks are relevant: there are 45 spin $1/2$ and 27 spin 1 dof, only one spin 0? If there are more the present knowledge requires a hierarchy of VEVs which, once again, is a serious fine-tuning problem. Why are all mixings small? Is it accidental or systematic (i.e. a new symmetry)? The real problem when dealing with UV completions is that one model is falsifiable, but an endless stream of them is not.

2. Theoretical framework

Back to the “more and more scales” scenario. Let’s undergo revision (SMEFT) but it is an error to believe that rigour is the enemy of simplicity. On the contrary we find it confirmed by numerous examples that the rigorous method is at the same time the simpler and the more easily comprehended. To summarize: there is a need for a consistent theoretical framework in which deviations from the SM (or NextSM) predictions can be calculated, every 20 bogus hypotheses you test, one of them will give you a p of < 0.05. Such a framework should be applicable to comprehensively describe measurements in all sectors of particle physics: LHC Higgs measurements, past EWPD, etc. Consider the SM augmented with the inclusion of higher dimensional operators and call it T_1; it is not strictly renormalizable. Although workable to all orders, T_1 fails above a certain scale, Λ_1. Consider any BSM model that is strictly renormalizable and respects unitarity (T_2); its parameters can be fixed by comparison with data, while masses of heavy states are presently unknown. Note that $T_1 \neq T_2$ in the UV but must have the same IR behavior. Consider now the whole set of data below Λ_1: T_1 should be able to explain them by fitting Wilson coefficients, T_2 adjusting the masses of heavy states (as SM did with the Higgs mass at LEP) should be able to explain the data. Goodness of both explanations are crucial in understanding how well they match and how reasonable is to use T_1 instead of the full T_2. Does T_2 explain everything? Certainly not, but it should be able to explain something more than T_1. We could now define T_3 as T_2 augmented with (its own) higher dimensional operators; it is valid up to a scale Λ_2. Etc.

2.1 SMEFT

The construction of the SMEFT, to all orders, is not based on assumptions on the size of the Wilson coefficients of the higher dimensional operators. Restricting to a particular UV case is not an integral part of a general SMEFT treatment and various cases can be chosen once the general calculation is performed. If the value of Wilson coefficients in broad UV scenarios could be inferred in general this would be of significant scientific value.

To summarize: constructing SMEFT is based on the fact that experiments occur at finite energy and "measure" an effective action $S^{\rm eff}(\Lambda)$; whatever QFT should give low energy $S^{\rm eff}(\Lambda)\,,\;\forall\,\Lambda < \infty$. One also assumes that there is no fundamental scale above which $S^{\rm eff(\Lambda)}$ is not defined [4] and $S^{\rm eff}(\Lambda)$ loses its predictive power if a process at $E = \Lambda$ requires ∞ renormalized parameters [5]. It is remarkable that when constructive proofs are provided, their simplicity always seems to detract from their originality.

2.2 The UV connection

The SMEFT approach is based on the following Lagrangian [6, 7, 8, 9, 10]:

$$\mathscr{A} = \sum_{n={\rm N}}^{\infty} g^n\,\mathscr{A}_n^{(4)} + \sum_{n={\rm N}_6}^{\infty}\sum_{l=1}^{n}\sum_{k=1}^{\infty} g^n\,g_{4+2k}^{l}\,\mathscr{A}_{nlk}^{(4+2k)}\,, \tag{2.1}$$

where we use the "Warsaw" basis [11]. Here g is the $SU(2)$ coupling constant and

$$g_{4+2k} = 1/(\sqrt{2}\,G_{\rm F}\,\Lambda^2)^k = g_6^k\,, \tag{2.2}$$

$G_{\rm F}$ is the Fermi coupling constant and Λ is the scale around which new physics (NP) must be resolved. For each process N defines the dim $= 4$ leading order (LO) (e.g. $N = 1$ for $\rm H \to VV$ etc. but $N = 3$ for $\rm H \to \gamma\gamma$). $N_6 = N$ for tree initiated processes and $N-2$ for loop initiated ones. Single insertions of dim $= 6$ operators defines next-to-leading (NLO) SMEFT. Ex: $\rm H\gamma\gamma$ (tree) vertex generated by $\mathscr{O}^{(6)}_{\phi \rm w} = \left(\Phi^\dagger\,\Phi\right)\,{\rm F}^{a\,\mu\nu}\,{\rm F}^a_{\mu\nu}$, by $\mathscr{O}^{(8)}_{\phi \rm w} = \Phi^\dagger\,{\rm F}^{a\,\mu\nu}\,{\rm F}^a_{\mu\rho}\,{\rm D}^\rho\,{\rm D}_\nu\,\Phi$ etc.

A simple SMEFT ordertable for tree initiated $1 \to 2$ processes is as follows (N.B. g_8 denotes a single $\mathscr{O}^{(8)}$ insertion, g_6^2 denotes two, distinct, $\mathscr{O}^{(6)}$ insertions):

$$\begin{array}{llll}
g\,/\,{\rm dim}\;\longrightarrow & & & \\
\downarrow & g\,\mathscr{A}_1^{(4)} & +\,g\,g_6\,\mathscr{A}_{1,1,1}^{(6)} & +\,g\,g_8\,\mathscr{A}_{1,1,2}^{(8)} \\
 & g^3\,\mathscr{A}_3^{(4)} & +\,g^3\,g_6\,\mathscr{A}_{3,1,1}^{(6)} & +\,g^3\,g_6^2\,\mathscr{A}_{3,2,1}^{(6)} \\
 & \cdots\cdots & \cdots\cdots & \cdots\cdots
\end{array}$$

① $g\,g_6\,\mathscr{A}^{(6)}_{1,1,1}$ defines LO SMEFT. There is also RG-improved LO and missing higher orders uncertainty (MHOU) for LO SMEFT;

② $g^3\,g_6\,\mathscr{A}^{(6)}_{3,1,1}$ defines NLO SMEFT;

③ $g\,g_8\,\mathscr{A}^{(8)}_{1,1,2}$, $g^3\,g_6^2\,\mathscr{A}^{(6)}_{3,2,1}$ give MHOU for NLO SMEFT.

The interplay between integrating out heavy scalars and the SM decoupling limit has been discussed in Ref. [12]. In the very general case the SM decoupling limit cannot be obtained by making only assumptions about one parameter.

Working in a spontaneously broken gauge theories has consequences related to the duality H−VEV. We recall the concept of (naive) power counting (for a general formulation of power counting see Ref. [13]): any local operator in the Lagrangian is schematically of the form

$$\mathscr{O} = \Lambda^{-n}\, M^l\, \overbrace{\partial^c\, \underbrace{\overbrace{\overline{\psi}^a\, \psi^b}^{\mathrm{N_F}}\, \left(\Phi^\dagger\right)^d\, \Phi^e\, \mathrm{A}^f}_{\mathrm{codim}}}^{\mathrm{dim}}$$

$$\frac{3}{2}\,(a+b)+c+d+e+f+l+n = 4\,. \tag{2.3}$$

where Lorentz, flavor and group indices have been suppressed, ψ stands for a generic fermion fields, Φ for a generic scalar and A for a generic gauge field. All light masses are scaled in units of the (bare) W mass M. We define dimensions according to

$$\mathrm{codim}\,\mathscr{O} = \frac{3}{2}\,(a+b)+c+d+e+f\,, \qquad \mathrm{dim}\,\mathscr{O} = \mathrm{codim}+l\,. \tag{2.4}$$

One loop renormalization is controlled by: $\mathrm{dim} = 6$, $\mathrm{codim} = 4$, $\mathrm{N_F} > 2$. The hearth of the problem: a large number of operators implodes into a small number of coefficients, e.g. there are 92 SM vertices, 28 CP even operators (1 flavor, $\mathrm{N}_\psi = 0,2$).

Debate topic for SMEFT is the choice of a "basis" for $\mathrm{dim} = 6$ operators. Clearly all bases are equivalent as long as they are a "basis", containing the minimal set of operators after the use of equations of motion [11] and respecting the $SU(3)\times SU(2)\times U(1)$ gauge invariance. From a more formal point of view a basis is characterized by its closure with respect to renormalization. Equivalence of bases should always be understood as a statement for the S-matrix and not for the Lagrangian, as dictated by the equivalence theorem, see Refs. [14, 15]. Any phenomenological approach that misses one of these ingredients is still acceptable for a preliminar analysis, as long as it does not pretend to be an EFT. Strictly speaking we are considering here the virtual part of SMEFT; of course, the real (emission) part of SMEFT should be included, see Section 2.5.

2.3 Self energies

Our first step deals with renormalization of self-energies: here $\Delta_{\mathrm{UV}} = \frac{2}{4-n} - \gamma - \ln\pi - \ln\frac{\mu_{\mathrm{R}}^2}{\mu^2}$, n is space-time dimension, the loop measure is $\mu^{4-n}\,d^nq$ and μ_{R} is the renormalization scale.

$$\mathrm{S_{HH}} = \frac{g^2}{16\,\pi^2}\,\Sigma_{\mathrm{HH}} = \frac{g^2}{16\,\pi^2}\,\left(\Sigma^{(4)}_{\mathrm{HH}} + g_6\,\Sigma^{(6)}_{\mathrm{HH}}\right)\,,$$

$$
\begin{aligned}
S^{\mu\nu}_{AA} &= \frac{g^2}{16\pi^2}\,\Sigma^{\mu\nu}_{AA} \qquad \Sigma^{\mu\nu}_{AA} = \Pi_{AA}\,T^{\mu\nu}\,,\\
S^{\mu\nu}_{VV} &= \frac{g^2}{16\pi^2}\,\Sigma^{\mu\nu}_{VV}\,, \qquad \Sigma^{\mu\nu}_{VV} = D_{VV}\,\delta^{\mu\nu} + P_{VV}\,p^{\mu}\,p^{\nu}\,,\\
D_{VV} &= D^{(4)}_{VV} + g_6\,D^{(6)}_{VV}\,, \qquad P_{VV} = P^{(4)}_{VV} + g_6\,P^{(6)}_{VV}\\
S^{\mu\nu}_{ZA} &= \frac{g^2}{16\pi^2}\,\Sigma^{\mu\nu}_{ZA} + g_6\,T^{\mu\nu}\,a_{AZ}\,, \qquad \Sigma^{\mu\nu}_{ZA} = \Pi_{ZA}\,T^{\mu\nu} + P_{ZA}\,p^{\mu}\,p^{\nu}\,,\\
S_f &= \frac{g^2}{16\pi^2}\left[\Delta_f + \left(V_f - A_f\,\gamma^5\right)\,i\not{p}\right]\,. \qquad (2.5)
\end{aligned}
$$

We introduce counterterms:

$$
Z_i = 1 + \frac{g^2}{16\pi^2}\left(dZ^{(4)}_i + g_6\,dZ^{(6)}_i\right)\Delta_{UV}\,. \qquad (2.6)
$$

With field/parameter counterterms we can make $S_{HH}, \Pi_{AA}, D_{VV}, \Pi_{ZA}$, V_f, A_f and the corresponding Dyson resummed propagators UV finite at $\mathscr{O}(g^2\,g_6)$, which is enough when working under the assumption that gauge bosons couple to conserved currents. A gauge-invariant description turns out to be mandatory.

2.4 More legs

However, field/parameter counterterms are not enough to make UV finite the Green's functions with more than two legs. A mixing matrix among Wilson coefficients is needed:

$$
a_i = \sum_j Z^{W}_{ij}\,a^{ren}_j \qquad Z^{W}_{ij} = \delta_{ij} + \frac{g^2}{16\pi^2}\,dZ^{W}_{ij}\,\Delta_{UV}\,. \qquad (2.7)
$$

Define the following combinations of Wilson coefficients (where $s_\theta(c_\theta)$ denotes the sine(cosine) of the renormalized weak-mixing angle):

$$
\begin{aligned}
a_{ZZ} &= s^2_\theta\,a_{\phi B} + c^2_\theta\,a_{\phi W} - s_\theta\,c_\theta\,a_{\phi WB}\,,\\
a_{AA} &= c^2_\theta\,a_{\phi B} + s^2_\theta\,a_{\phi W} + s_\theta\,c_\theta\,a_{\phi WB}\,,\\
a_{AZ} &= 2\,c_\theta\,s_\theta\,\left(a_{\phi W} - a_{\phi B}\right) + \left(2\,c^2_\theta - 1\right)\,a_{\phi WB}\,, \qquad (2.8)
\end{aligned}
$$

and compute the (on-shell) decay $\underline{H(P) \to A_\mu(p_1)A_\nu(p_2)}$ where the amplitude is

$$
A^{\mu\nu}_{HAA} = \mathscr{T}_{HAA}\,T^{\mu\nu}\,, \quad M^2_H\,T^{\mu\nu} = p^{\mu}_2\,p^{\nu}_1 - p_1\cdot p_2\,\delta^{\mu\nu}\,. \qquad (2.9)
$$

This amplitude is made UV finite by mixing a_{AA} with a_{AA}, a_{AZ}, a_{ZZ} and a_{QW}

Compute the (on-shell) decay $\underline{H(P) \to A_\mu(p_1)Z_\nu(p_2)}$. After adding 1PI and 1PR components we obtain

$$
A^{\mu\nu}_{HAZ} = \mathscr{T}_{HAZ}\,T^{\mu\nu} \quad M^2_H\,T^{\mu\nu} = p^{\mu}_2\,p^{\nu}_1 - p_1\cdot p_2\,\delta^{\mu\nu} \qquad (2.10)
$$

This amplitude is made UV finite by mixing a_{AZ} with a_{AA}, a_{AZ}, a_{ZZ} and a_{QW}.

Compute the (on-shell) decay $\underline{H(P) \to Z_\mu(p_1)Z_\nu(p_2)}$. How to use it has been explained in Ref. [16]. The amplitude contains a $\mathscr{D}_{HZZ}$ part proportional to $\delta^{\mu\nu}$ and a $\mathscr{P}_{HZZ}$ part proportional to $p^{\mu}_2\,p^{\nu}_1$.

Remark Mixing of a_{ZZ} with other Wilson coefficients makes $\mathscr{P}_{HZZ}$ UV finite, while the mixing of $a_{\phi\Box}$ makes $\mathscr{D}_{HZZ}$ UV finite.

Compute the (on-shell) decay $\underline{\mathrm{H}(P) \to \mathrm{W}^{-}{}_{\mu}(p_1)\mathrm{W}^{+}{}_{\nu}(p_2)}$. This process follows the same decomposition of $\mathrm{H} \to \mathrm{ZZ}$ and it is UV finite in the dim $= 4$ part. However, for the dim $= 6$ one, there are no Wilson coefficients left free in $\mathscr{P}_{HWW}$ so that its UV finiteness follows from gauge cancellations ($\mathrm{H} \to \mathrm{AA}, \mathrm{AZ}, \mathrm{ZZ}, \mathrm{WW} = 6$ Lorentz structures controlled by 5 coefficients).

Proposition 2.1. *This is the first part in proving closure of NLO SMEFT under renormalization.*

Remark Mixing of $a_{\phi D}$ makes $\mathscr{D}_{HWW}$ UV finite.

Remark Compute the (on-shell) decay $\underline{\mathrm{H}(P) \to \mathrm{b}(p_1)\overline{\mathrm{b}}(p_2)}$. It is dim $= 4$ UV finite and mixing of $a_{d\,\phi}$ makes it UV finite also at dim $= 6$.

Remark Compute the (on-shell) decay $\underline{\mathrm{Z}(P) \to \mathrm{f}(p_1)\overline{\mathrm{f}}(p_2)}$. It is dim $= 4$ UV finite and we introduce

$$\begin{aligned} a_{l\,W} &= s_\theta\, a_{l\,WB} + c_\theta\, a_{l\,BW} \qquad & a_{l\,B} &= s_\theta\, a_{l\,BW} - c_\theta\, a_{l\,WB}\,, \\ a_{d\,W} &= s_\theta\, a_{d\,WB} + c_\theta\, a_{d\,BW} \qquad & a_{d\,B} &= s_\theta\, a_{d\,BW} - c_\theta\, a_{d\,WB}\,, \\ a_{u\,W} &= s_\theta\, a_{u\,WB} + c_\theta\, a_{u\,BW} \qquad & a_{u\,B} &= c_\theta\, a_{u\,WB} - s_\theta\, a_{u\,BW}\,, \end{aligned} \tag{2.11}$$

$$\begin{aligned} a^{(3)}_{\phi l} - a^{(1)}_{\phi l} &= \frac{1}{2}\,(a_{\phi\,l\,V} + a_{\phi\,l\,A})\,, \qquad & a_{\phi l} &= \frac{1}{2}\,(a_{\phi\,l\,A} - a_{\phi\,l\,V})\,, \\ a_{\phi\,u\,V} &= a^{(3)}_{\phi q} + a_{\phi u} + a^{(1)}_{\phi q} \qquad & a_{\phi\,u\,A} &= a^{(3)}_{\phi q} - a_{\phi u} + a^{(1)}_{\phi q}\,, \\ a_{\phi\,d\,V} &= a^{(3)}_{\phi q} - a_{\phi d} - a^{(1)}_{\phi q} \qquad & a_{\phi\,d\,A} &= a^{(3)}_{\phi q} + a_{\phi d} - a^{(1)}_{\phi q}\,, \end{aligned} \tag{2.12}$$

and obtain that

$\mathrm{Z} \to \bar{\mathrm{l}}\mathrm{l}$ requires mixing of $a_{l\,BW}, a_{\phi\,l\,A}$ and $a_{\phi\,l\,V}$ with other coefficients,

$\mathrm{Z} \to \bar{\mathrm{u}}\mathrm{u}$ requires mixing of $a_{u\,BW}, a_{\phi\,u\,A}$ and $a_{\phi\,u\,V}$ with other coefficients,

$\mathrm{Z} \to \bar{\mathrm{d}}\mathrm{d}$ requires mixing of $a_{d\,BW}, a_{\phi\,d\,A}$ and $a_{\phi\,d\,V}$ with other coefficients,

$\mathrm{Z} \to \bar{\nu}\nu$ requires mixing of $a_{\phi\nu} = 2\,(a^{(1)}_{\phi l} + a^{(3)}_{\phi l})$ with other coefficients.

At this point we are left with the universality of the electric charge. In QED there is a Ward identity telling us that e is renormalized in terms of vacuum polarization and Ward-Slavnov-Taylor (WST) identities allow us to generalize the argument to the full SM. We can give a quantitative meaning to the the previous statement by saying that the contribution from vertices (at zero momentum transfer) cancels those from (fermion) wave function renormalization factors. Therefore, compute the vertex $\mathrm{A}\bar{\mathrm{f}}\mathrm{f}$ (at $q^2 = 0$) and the f wave function factor in SMEFT, proving that the WST identities can be extended to dim $= 6$; this is non trivial since there are no free Wilson coefficients in these terms (after the previous steps); the (non-trivial) finiteness of $\mathrm{e}^+\mathrm{e}^- \to \bar{\mathrm{f}}\mathrm{f}$ follows.

Proposition 2.2. *This is the second part in proving closure of NLO SMEFT under renormalization.*

2.5 The IR connection

Consider the decay $\mathrm{Z} \to \bar{\mathrm{l}}\mathrm{l}$, where the amplitude is

$$\mathscr{A}^{\text{tree}}_{\mu} = g\,\mathscr{A}^{(4)}_{1\,\mu} + g\,g_6\,\mathscr{A}^{(6)}_{1\,\mu}\,, \tag{2.13}$$

$$\mathscr{A}^{(4)}_{1\,\mu} = \frac{1}{4\,\mathrm{c}_\theta}\,\gamma_\mu\,\left(\mathrm{v_l} + \gamma^5\right)\,, \qquad \mathscr{A}^{(6)}_{1\,\mu} = \frac{1}{4}\,\gamma_\mu\,\left(\mathrm{V_l} + \mathrm{A_l}\,\gamma^5\right)\,, \tag{2.14}$$

$$\begin{aligned} \mathrm{V_l} &= \frac{\mathrm{s}^2_\theta}{\mathrm{c}_\theta}\,\left(4\,\mathrm{s}^2_\theta - 7\right)\,a_{\mathrm{AA}} + \mathrm{c}_\theta\,\left(1 + 4\,\mathrm{s}^2_\theta\right)\,a_{\mathrm{ZZ}} + \mathrm{s}_\theta\,\left(4\,\mathrm{s}^2_\theta - 3\right)\,a_{\mathrm{AZ}} \\ &+ \frac{1}{4\,\mathrm{c}_\theta}\,\left(7 - \mathrm{s}^2_\theta\right)\,a_{\phi\,\mathrm{D}} + \frac{2}{\mathrm{c}_\theta}\,a_{\phi\,\mathrm{l}\,\mathrm{V}}\,, \\ \mathrm{A_l} &= \frac{\mathrm{s}^2_\theta}{\mathrm{c}_\theta}\,a_{\mathrm{AA}} + \mathrm{c}_\theta\,a_{\mathrm{ZZ}} + \mathrm{s}_\theta\,a_{\mathrm{AZ}} - \frac{1}{4\,\mathrm{c}_\theta}\,a_{\phi\,\mathrm{D}} + \frac{2}{\mathrm{c}_\theta}\,a_{\phi\,\mathrm{L}\,\mathrm{A}}\,. \end{aligned} \tag{2.15}$$

After UV renormalization, i.e. after counterterms and mixing have been introduced, we perform analytic continuation in n (space-time dimension), $n = 4 + \varepsilon$ with ε positive.

Proposition 2.3. *The infrared/collinear part of the one-loop virtual corrections shows double factorization.*

$$\Gamma\left(\mathrm{Z} \to \bar{\mathrm{l}} + \mathrm{l}\right)\,|_{\text{div}} = -\frac{g^4}{384\,\pi^3}\,M_{\mathrm{Z}}\,\mathrm{s}^2_\theta\,\mathscr{F}^{\text{virt}}\left[\Gamma^{(4)}_0\,\left(1 + g_6\,\Delta\Gamma\right) + g_6\,\Gamma^{(6)}_0\right]\,. \tag{2.16}$$

Proposition 2.4. *The infrared/collinear part of the real corrections shows double factorization.*

$$\Gamma^{\text{app}}\left(\mathrm{Z} \to \bar{\mathrm{l}} + \mathrm{l} + (\gamma)\right) = \frac{g^4}{384\,\pi^3}\,M_{\mathrm{Z}}\,\mathrm{s}^2_\theta\,\mathscr{F}^{\text{brem}}\left[\Gamma^{(4)}_0\,\left(1 + g_6\,\Delta\Gamma\right) + g_6\,\Gamma^{(6)}_0\right]\,. \tag{2.17}$$

Proposition 2.5. *The total = virtual + real is IR/collinear finite at $\mathscr{O}(g^4\,g_6)$.*

Assembling everything gives (terms in red give the SM answer)

$$\begin{aligned} \Gamma^{\mathrm{l}}_{\text{QED}} &= \frac{3}{4}\,\Gamma^{\mathrm{l}}_0\,\frac{\alpha}{\pi}\,\left(1 + g_6\,\Delta^{(6)}_{\text{QED}}\right)\,, \quad \Gamma^{\mathrm{l}}_0 = \frac{G_{\mathrm{F}}\,M^3_{\mathrm{Z}}}{24\,\sqrt{2}\,\pi}\,\left(\mathrm{v}^2_{\mathrm{l}} + 1\right) \\ \Delta^{(6)}_{\text{QED}} &= 2\,\left(2 - \mathrm{s}^2_\theta\right)\,a_{\mathrm{AA}} + 2\,\mathrm{s}^2_\theta\,a_{\mathrm{ZZ}} + 2\,\left(\frac{\mathrm{c}^3_\theta}{\mathrm{s}_\theta} + \frac{512}{26}\,\frac{\mathrm{v_L}}{\mathrm{v}^2_{\mathrm{L}} + 1}\right)\,a_{\mathrm{AZ}} \\ &- \frac{1}{2}\,\frac{\mathrm{c}^2_\theta}{\mathrm{s}^2_\theta}\,a_{\phi\,\mathrm{D}} + \frac{1}{\mathrm{v}^2_{\mathrm{L}} + 1}\,\delta^{(6)}_{\text{QED}}\,, \\ \delta^{(6)}_{\text{QED}} &= \left(1 - 6\,\mathrm{v_l} - \mathrm{v}^2_{\mathrm{l}}\right)\,\frac{1}{\mathrm{c}^2_\theta}\,\left(\mathrm{s}_\theta\,a_{\mathrm{AA}} - \frac{1}{4}\,a_{\phi\,\mathrm{D}}\right) \\ &+ \left(1 + 2\,\mathrm{v_l} - \mathrm{v}^2_{\mathrm{l}}\right)\,\left(a_{\mathrm{ZZ}} + \frac{\mathrm{s}_\theta}{\mathrm{c}_\theta}\,a_{\mathrm{AZ}}\right) + \frac{2}{\mathrm{c}^2_\theta}\,\left(a_{\phi\,\mathrm{l}\,\mathrm{A}} + \mathrm{v_l}\,a_{\phi\,\mathrm{l}\,\mathrm{V}}\right) \end{aligned} \tag{2.18}$$

2.6 Next steps

The W-decay series is almost completed; next, inclusion of triple/quadrupole gauge couplings, last stop before renormalizability? This brings us to gauge anomalies and anomaly cancellation; d'Hoker-Farhi [17], (Wess-Zumino [18]) terms required? Extra symmetry? Severe problems are expected; perhaps, a deeper understanding of SMEFT, a low-energy limit of an underlying anomaly-free theory?

Proposition 2.6. *SMEFT anomalies are UV finite (it is good for renormalizability), restoring gauge invariance order-by-order by adding finite counterterms, i.e. it is possible to quantize an anomalous theory in a manner that respects WSTI [5] and local. The latter is good for unitarity, another tiny step forward.*

3. Conclusions

NLO results have already had an important impact on the SMEFT physics program. LEP constraints should not be interpreted to mean that effective SMEFT parameters should be set to zero in LHC analyses. It is important to preserve the original data, not just the interpretation results, as the estimate of the missing higher order terms can change over time, modifying the lessons drawn from the data and projected into the SMEFT. The assignment of a theoretical error for SMEFT analyses is always important. Considering projections for the precision to be reached in LHC RunII analyses, LO results for interpretations of the data in the SMEFT are challenged by consistency concerns and are not sufficient, if the cut off scale is in the few TeV range. If the scale is below experimental sensitivity we are in trouble, but let's push constraints to the experimental limit consistently. Unfortunately, ideas that require people to reorganize their picture of the world provoke hostility.

To conclude, the journey to the next (and next-to-next) SM may require crossing narrow straits of precision physics. If that is what nature has in store for us, we must equip ourselves with both a range of concrete models as well as a general theories. However, each paradigm will be shown to satisfy more or less the criteria that it dictates for itself and to fall short of a few of those dictated by its opponent.

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