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PROBLEM SOLVING COMPETENCE DEVELOPED THROUGH A VIRTUAL LEARNING ENVIRONMENT IN A EUROPEAN CONTEXT

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Abstract*: In modern societies, life itself is a problem solving. OCSE-PISA defined the problem-solving competence as the capacity to engage in cognitive processing to understand and solve problem situations where a method of solution is not immediately obvious. Hence in teaching Mathematics, and in general scientific subjects, it becomes important to adopt new methods centred on student learning, which are able not only to deliver knowledge but also to enhance the competence in problem solving. It is essential to start from simulated real-life problem situations in order to explain new concepts, since it enables students to understand the application of Mathematics as well as to enhance their ability to regulate problem-solving processes. The University of Turin has adopted problem solving digital methodologies using a virtual learning environment integrated with an advanced computing environment, automatic assessment and a web conference system. This set of technologies promote interaction among students and help teachers guide and coach their students in creating, conjecturing, exploring, testing and verifying. In the Erasmus+ Project SMART – Science and Mathematics Advanced Research in good Teaching - the University of Turin shared these methodologies with other European partners (Strategic Partnership Key action). The aim of this exchange of best practices is to obtain Mathematics and Science teachers who are more involved in the methodology and consequently students who are better equipped to develop coherent mental representations of problem situations, more flexible in incorporating feedback and in reflecting. In the paper, examples of interactive material produced and tested in the Project are presented and the results of the experience are discussed.*

Keywords: *Advanced Computing Environment; Mathematics learning; Problem Posing; Problem Solving; Teaching good practices; Virtual Learning Environment.*

I. INTRODUCTION

"All life is problem solving", is the title of a famous book by Karl Popper [1], meaning that the process of knowledge starts from a problem, proceeds with an attempt of solution and ends with the elimination of mistakes. This process affects the evolution of human life, scientific progress, learning, history and

society. Nowadays, strengthening problem solving skills is acknowledged as one of the most important features in the formation of citizens who can contribute to the development of economy and society and who can succeed in life [2]. To be competitive in workplaces, technical skills are not sufficient anymore: employees need to show other attitudes, such as the ability to face the change, the ability to work in team, a strong attitude to problem posing and solving, the ability to use knowledge in real life situations, flexibility in changing roles and learning new skills, new ideas and new approaches, creativity. Duty of the school systems is to produce men and women able to deal with the 21st century challenges. European recommendations encourage national governments to promote disciplinary and cross competences for lifelong learning through curricula [3]. The problem solving competence should be particularly fostered: it is defined by the Programme for International Student Assessment (PISA) as *"an individual's capacity to engage in cognitive processing to understand and resolve problem situations where a method of solution is not immediately obvious. It includes the willingness to engage with such situations in order to achieve one's potential as a constructive and reflective citizen*" [4].

In this European scenery, the Italian Ministry of Education recently proposed several reforms of the school system and supported the demand for innovation in Mathematics curricula through two main National actions: Problem Posing and Solving project (PP&S) [5] and LSOSAlab project. The first one includes a teacher training program for the adoption of innovative methodologies for Mathematics teaching and learning, mainly based on problem posing and problem solving in a Virtual Learning Environment. The second one promotes the laboratory didactics in Science. The National experiences have been shared in a European context through the Erasmus+ Project SMART (Science and Mathematics Advanced Research for good Teaching) [6], in a Transnational Strategic Partnership which includes Italy, Hungary, Germany, the Netherlands and Sweden (www.smart.carloanti.it).

Although SMART involves also Science teaching, this article focuses on the methodologies for Mathematics teaching and learning developed and shared within the project. The SMART working methodology is shown in detail and the results are discussed.

II. REAL-LIFE PROBLEM SOLVING IN MATHEMATICS EDUCATION: STATE OF THE ART

Real-life problem solving has been considered as a significant starting point for learning Mathematics since the Fifties with Polya's work [7]. As Shoenfeld points out [8], learning Mathematics should empower students with the ability to understand quantitative data from the real world, to interpret relationships with analytic spirit and to make judgments; that can be achieved if the teaching strategy is based on problems solving rather than on sterile presentations of patterns and numbers. Learning technologies sensibly affect both problem posing and problem solving: they offer multiple ways of representation, which is a fundamental part of the thinking process; moreover, automatization of computations makes it possible to analyze relevant complex situations and large data without simplifications [9]. Furthermore, there are evidences that the participation to online collaborative activities and discussions promotes critical thinking [10] [11], and online communications are more task-oriented than face-to-face ones while solving problems [12], thus Virtual Learning Environments may be suitable settings for collaborative problem solving.

III. SMART PROJECT FOR THE DEVELOPMENT OF PROBLEM SOLVING COMPETENCES

The SMART Project started in September 2014 from a partnership of school, university and ministerial institutions which includes: Italy (Accademia delle Scienze di Torino, Direzione Generale per gli Ordinamenti Scolastici e la Valutazione del Sistema Nazionale di Istruzione, IS Carlo Anti, Risorse in Crescita, Università degli Studi Roma Tre, Università di Torino), Hungary (Pecsi Radnoti Miklos Kozgazdasagi Szakkozepiskola), Germany (St. Thomas-Gymnasium), Sweden (Chalmers Tekniska Hoegskola AB) and the Netherlands (Tecnische Universiteit Delft).

The aims of the Project were the following:

- to improve the professional teachers' competences and to support innovation in the teachers' training through innovative practices based on the new computer and multimedia technologies;

- to provide tools and methodologies to facilitate the acquisition of mathematical competence and basic competences in science and technology through discussion and sharing with European partners and by introducing advanced technological tools to support teaching and learning;
- to develop skills which can be used to contribute to a cohesive society, in particular to increase opportunities for learning mobility through strengthened cooperation between the world of education and training and the world of work, formulating and solving complex problems autonomously, consciously and constructively.

The project relied on the two-year work of a European community of practice which built learning paths after analysing the state and needs of Mathematics and Science teaching among the involved countries. The working methodology can be outlined in 6 points, which will be detailed in the following paragraphs:

1. analysis of educational needs
2. proposal of innovative didactic methodologies
3. implementation of teacher training modules
4. creation of two Open Online Courses
5. experimentation of the learning materials with students of the partner schools
6. evaluation and assessment of learning results

3.1 Analysis of educational needs

The first SMART action was the creation of a questionnaire that was submitted to Italian teachers, with the purpose of collecting data about their educational needs and to define common educational models and possible paths of intervention accordingly. The first part of the questionnaire was intended to verify the status quo both of the perception that teachers have of their teaching practice, and of the quantity and quality of activities practiced in the previous two years for their own professional development; the second part aimed at investigating the areas of interest that the teachers had. From the 669 answers of teachers of Scientific subjects it emerged that they were generally aware that their teaching had to be innovated; in particular, Maths teachers felt that their lessons meet their students' interests less than their colleagues of other subjects (in a Likert scale from 1 to 5 the average value is 3.3 for Maths teachers against 3.5 of other subjects). There was an analogue difference also in the use of learning technologies (average: 3.5 for Maths teachers against 3.8 for other disciplines). Only 9% acknowledged to use a traditional teaching style, even though 71% defined their methodology still partly traditional. About 7% disagreed with the choice of reducing the quantity of specific Mathematical topics in the curriculum in favor of cross-crossing activities, problem solving and applications, however, only 52% totally agreed with this choice. About 65% had spent from 5 to 20 days in the previous 2 years in training activities, and 30% more than 20 days; among the most common topics, new methodologies and ICT stood out. The use of innovative methodologies and the preparation of multimedia and online learning materials stood out also among the desired object of professional development. The favorite training form was training courses/seminars. In conclusion, the portrait that emerged from the results is that of teachers quite open to innovation, aware of their need of renewing their instructional practices, but still partly tied to traditional teaching styles. For the analysis of teachers' needs of other countries, not very different from the ones of the Italian, national reports were used, that were already available.

3.2 Innovative didactic methodologies

Following the survey results, the discussion focused on the didactic methodologies to be proposed. The partners agreed that the ones used in the PP&S project were the most innovative and effective for Mathematics teaching and learning. Part of them, as the advanced computing environment and the automatic assessment system, had already been tested or adopted at Chalmers University and Tecnische Universiteit Delft. In details, they are:

- **Problem Posing and Problem Solving**. SMART embraces the social-constructive insight of problem solving as a learning environment where mathematical knowledge is created in a collaborative discussion starting from a problem. The top-down order traditionally used to study Mathematics is inverted: a real-world situation is analysed and paths to the solutions are outlined, in a constructive approach toward the discipline. Afterward, the solving steps are synthetized and generalized, introducing the typical rigour of Mathematics. Learning technologies are used both for online cooperation and as a mean of representation of the solving process; freed from the burden of calculations, students are asked to focus on the solving strategy, find relationships and better understand the solutions.

- **Virtual Learning Environment (VLE)**. The use of VLEs opens up a great variety of interaction channels among mentors and learners at different levels. Collaborative learning can be fostered through activities for synchronous or asynchronous discussion. Interactivity enhances students' engagement and contributes to increase their motivation. The integrated reporting instruments allow teachers to monitor students' trends, to identify anomalies and to prevent potential problems [13].
- **Advanced Computing Environment (ACE).** The use of an ACE is one of the most innovative and effective methodologies for teaching and learning scientific disciplines. While maintaining the ease and immediacy of use of a common word processor, an ACE enables its users to enhance the capabilities of scientific objects' representation. Students' reasoning benefits from numeric computations, symbolic calculus, geometric visualizations in two and three dimensions, and embedding of interactive components where changing parameters allows to analyse different results.
- **Automatic Assessment System (AAS).** The use of an AAS has clear advantages for teachers, who have the chance to optimize the development of formative assignments while reducing time-consumption for corrections, and for students, who can practice at their own pace obtaining immediate feedback and results to acknowledge their level of preparation. Both questions and assignments created through an AAS can be enhanced by varying them in a random controlled form, inserting parts expressed in a special programming language. This allows a great variety of assessment modalities which strengthen reasoning until it has been mastered: students can obtain different data or graphics at every new attempt, teachers can suggest guided resolutions which depend on the results obtained at each step, feedback and questions can automatically be proposed on the base of previous answers. [14].

The tools adopted in SMART for problem solving, in particular the ACE and the AAS, are integrated in the same VLE. Thence, interactive worksheets and assignments can be made available in the online course pages, with no need to have the other software installed on one's device, and all the results can be consulted by students and teachers in a unique gradebook.

3.3 Teacher training

A VLE has been set up for SMART: it makes use of the integrated e-learning platform Moodle managed by the ICT services of the University of Turin. There, an online training course for the teachers from the schools joining the project has been activated. The same methodologies proposed for teaching have been adopted with teachers: online collaboration, synchronous and asynchronous tutoring, interactive materials delivered via the VLE; an online community of practice has been created. The training course consisted in:

- a series of synchronous meetings held through a web-conference tool integrated in the platform, where participants from all over Europe could interact with a tutor through voice, chat and sharing the screen;
- asynchronous tutoring through forums;
- interactive training materials available in the VLE;
- sample learning material to exemplify the methodologies;
- spaces for the teachers within the VLE where they could autonomously practice with the methodologies proposed and even enroll their students.

After the training, data about teachers' appreciation of the methodologies proposed, their difficulties and experiences have been collected through forums and questionnaires, with the purpose of extending the training methodologies to a community of teachers from around the world.

3.4 Open Online Courses

The methodologies shared and discussed, as well as the teachers' experience with the training and the experimentation in their classes, merged into two Open Online Courses (OOC). Both are available on the SMART platform worldwide: one is intended for Mathematics and the other for Science teachers. The OOC have been opened at the end of the Project in November 2016, whereas a continuous work of adjustment and perfection on the training and learning materials has been carried out during the whole life of the project. They are available at: www.opensmart.unito.it.

The Mathematics OOC – Mathematical Modelling – is organized in two main parts: the first one is an interactive online course which guides teachers through the proposed methodologies; the second one contains the learning materials produced from the sharing of methodologies and experiences during the project and the experimentation with students.

The training part is self-paced, that means that teachers can navigate and use the parts they need when they prefer. It consists of 4 modules dedicated to the 4 methodologies used to innovate the teaching of Mathematics: Problem Posing and Solving, use of a VLE, use of an ACE, use of an AAS. Each module begins with an introductory survey on users' previous experiences with those methodologies and their expectations, then it continues with the interactive and multimedia training materials created and adjusted on the base of the appreciation, comments and observations collected during the teacher training phase. There are activities which exemplify the effectiveness of the methodologies, videos on how to use the technologies to create innovative learning materials, immediate texts and presentations to meet the users through different communication channels. When teachers complete one module they earn a badge, so they are motivated to complete all the modules. Furthermore, they are asked to rate the effectiveness of the training materials with respect to the adoption of the methodologies. Only after completing the module, teachers have access to the world community of teachers who use these methodologies in teaching. They can share ideas and materials and collaborate online.

The second part of the online course is a repository of learning materials which encompasses all the methodologies proposed. It is organized into 4 topics (quantity, space and shape, change and relations, uncertainty) which correspond to the classification adopted for OCSE-PISA survey [15]. The problems start from a real situation and propose one or more solving strategies to get to the solution; at the end, the mathematical content laying behind is presented and discussed. The ACE Maple used to create the problems makes it possible to automatize computations, to display in the same worksheet and in a smart graphical rendering all the ingredients of problems and solutions (texts and comments, data and tables, graphics and animations, computations and results), and to add interactive components which allow students to repeat the reasoning over different data or explore the mathematical models and structures presented. One example of SMART problem solved through the ACE is shown in Figure 1. At the beginning of each problem there is a specification of learning objectives, formulated in terms of:

- Prerequisites, the mathematical knowledge required to understand the problem
- Topic/contents, the Mathematics behind the problem
- Skills/abilities practiced through the solving steps
- Competences developed while solving the problem

Each problem matches with an assignment which aims at verifying whether the learning objectives specified have been achieved. Assignments are automatically graded and they are built using the AAS Maple TA. Question types go far beyond the multiple-choice modality: thanks to the mathematical engine laying behind, the system is able to evaluate mathematical open answers, such as formulas, equations, vectors, graphics and so on. Questions are algorithm-based, so that each attempt is a new exercise. One example of question made with the AAS is shown in Figure 2. The problems and assignments are the result of the European discussion over themes of Mathematics education and they underwent a process of revision by all the partners. This process was carried out via the dedicated VLE. Two aspects of the learning material had to meet the educational needs of different countries: the topics covered and the problem contexts. Regarding the topics, the different school curricula have been analyzed and teachers' requests intersected, so that a list of common mathematical topics acted as a starting point for the creation of problems and assignments. A school level has not been indicated on the materials, so teachers are free to choose the most appropriate/useful materials for their classes and adapt them to their purposes. The flexibility in the use of the materials allows the teachers to organize their teaching in a new way, focusing on the development of problem solving competences. On the other hand, some interesting reflections emerged across Europe over the problems contexts: in order to be relevant for all European students, cultural differences should be taken into account. As an example, a problem on the Blood Alcohol Concentration (BAC) can be interesting for Italian 18-year-olds, as Italian law imposes the BAC limit of 0 for novice drivers and of 0.5 for over 21-year-old drivers. Rules are similar in Germany, while in Sweden there aren't age differences and the BAC limit is 0.2 for everyone; the problem could have very little relevance for students from Hungary, where the BAC limit is 0 for all drivers.

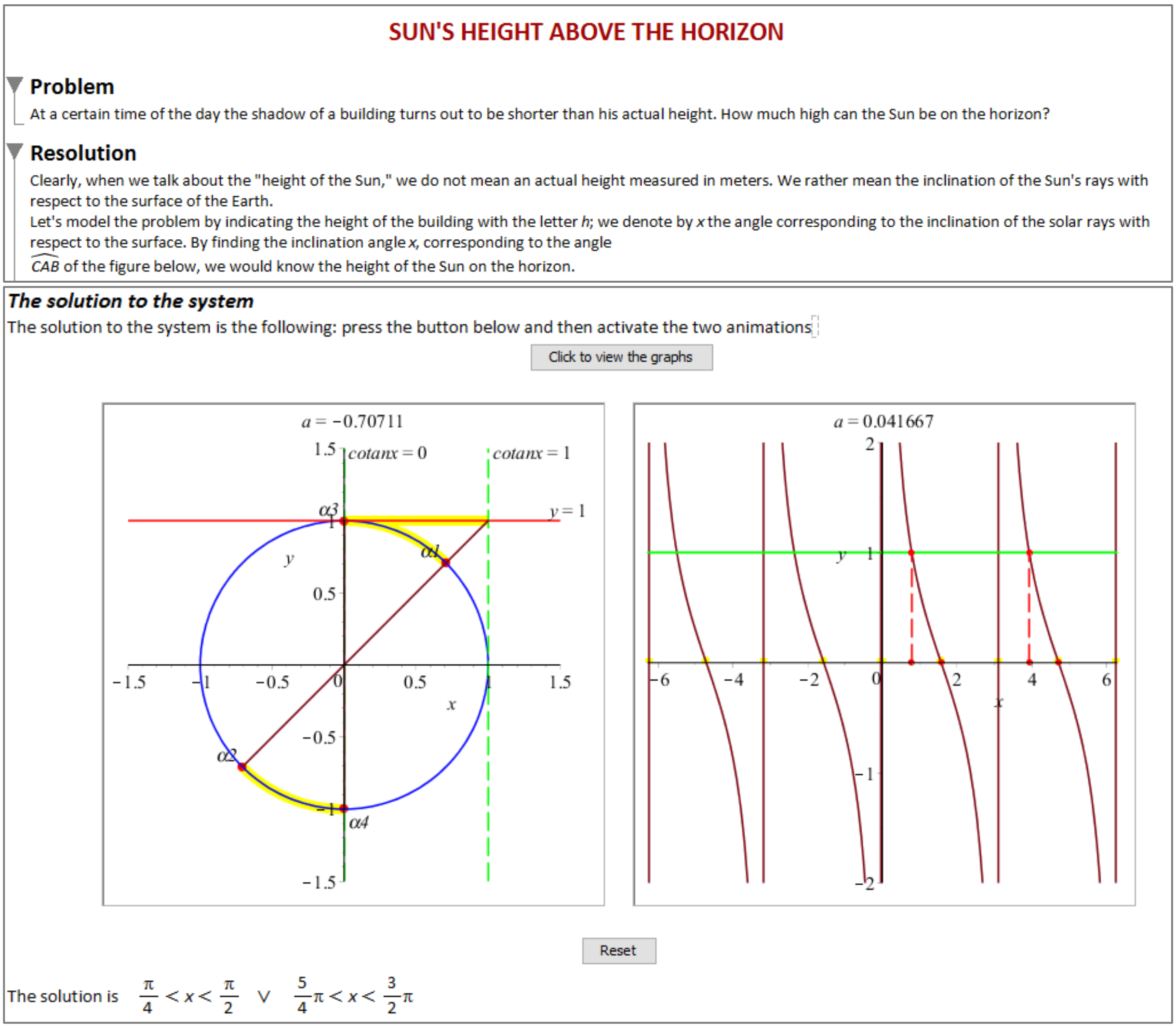

Figure 1. Text of a SMART problem and part of the suggested solution

3.5 Testing with students

The methodologies proposed and the learning materials created have been tested on the students of the three high schools that joined the project (respectively from Italy, Hungary and Germany). Teachers, after the training, could select the problems of their interest and adapt them to their classes. Materials were not translated but proposed in English as common language, according to the CLIL methodology (Content and Language Integrated Learning) [16]. After the experimentation, the teachers had to fill a questionnaire to evaluate the methodology and the materials. Questions were asked about the modalities of testing with the class (number and age of students, place where the activity took place) and how relevant and interesting the activity has been for students and their curriculum. Data collected from the survey have been taken into account to improve problems and assignments. The testing activity is actually a continual action, even now that the OOC has been published: similar questions are asked to all the teachers that use the materials proposed, and their answers are used to continually improve the materials.

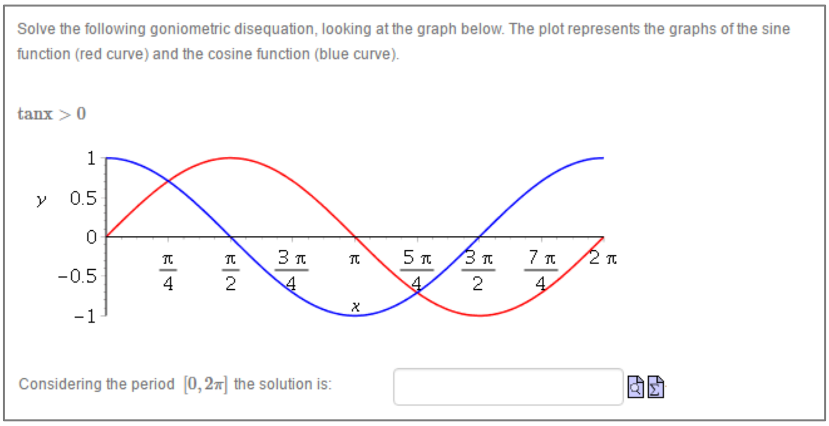

Figure 2. One question of the assignment associated to the problem "Sun's height above the horizon".

IV. RESULTS AND DISCUSSION

The teacher training has been followed by 27 Mathematics teachers from the three countries and the learning methodologies have been tested on 178 students. The high teachers' appreciation of the innovative practices shared in SMART is shown by their answers to the questionnaire after using them with their classes, shown in **Errore. L'origine riferimento non è stata trovata.Errore. L'origine riferimento non è stata trovata.**.

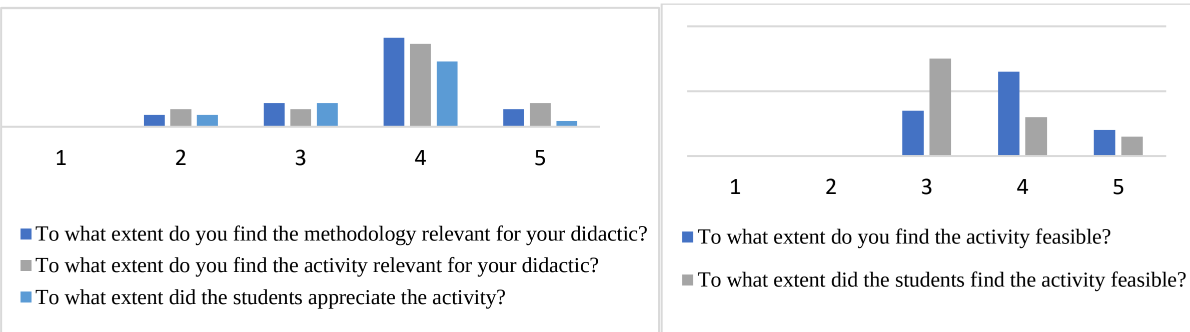

Figure 3. Teachers' answers after testing the materials with their classes.

General appreciation for the methodologies proposed has also been expressed during the periodical meetings of the working group and in the online discussions. The project has also been analyzed by "Red Team", an agency composed by representatives of the industrial world, company consultants and vocational teachers, who have evaluated the impact of SMART on the world of work. They monitored the e-learning platform, the topics report, the competence report and the report on teachers' needs. The overall evaluation expressed by the agency was positive: in particular, they stated that the e-learning platform is user friendly and the contents are clear and relevant to the objectives of the project, in particular for developing problem solving competence. The methodologies and the exercises are functional and clear. The questionnaire used to assess the teachers' needs is consistent with the project aims and the conclusions are clear and exhaustive. According to Red Team, the competences considered in SMART are consistent with the needs of a working life and the methodology to develop them is clear and effective. SMART methodologies have been correlated to the companies' needs, as shown in Table 1: the methodologies that SMART proposes for Mathematics teaching and learning could really be effective to educate and train the future society to be competitive and successful in work, society and life.

SMART methodologies	Workplace needs
Problem posing and solving	New approaches to face problems
Collaborative activities	Ability to work in a team
Experimental methodology	Ability to use knowledge in a real situation
Down-top approach	Attitude to learn new skills

Table 1. Correlation between SMART features and workplace needs

V. CONCLUSIONS

The end of the project, along with the awareness that the methodologies proposed are relevant for the formation of the future generations of European citizens, emphasized a willingness to further extend the work and the experiences made. The fruitful international collaboration made us consider the proposal to experiment a *European class,* composed of students from different schools and teachers of different subjects working together with the same methodology. By virtue of the collaboration of experts in different disciplines, cross-cutting modules based on problem-solving could thus be created and tested in several contexts.

Similar modules can also be extended to university and upper technical institutes: the demand for the development of problem solving competences has indeed reached also tertiary education. Collaborative problem-solving tasks, carried out in English and in a VLE, encompassing several subjects, could be an effective way to achieve deep understanding of the disciplines themselves and to prepare students to face working situations.

The database of materials in the OOC Mathematical Modelling can be used by Mathematics teachers for lessons in the CLIL methodology, which is recommended for a non-language subject in the last school year of Italian high schools. Problems can be assigned to groups of students who are asked to discuss and solve them together. Beyond problem solving and disciplinary competences, it is a way to practice also written and spoken communication skills in a foreign language.

Moreover, the modules intended for the teacher training can also be used for the training of novice teachers and for the professional development of teachers in charge.

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