PROCEEDINGS OF SCIENCE

Challenges in the extraction of TMDs from SIDIS data: perturbative vs non-perturbative aspects

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We present our recent results on the study of the Semi-Inclusive Deep Inelastic Scattering (SIDIS) cross section as a function of the transverse momentum, q_T. Using the Collins-Soper-Sterman (CSS) formalism, we study the matching between the region where fixed order perturbative QCD can successfully be applied and the region where soft gluon resummation is necessary. We find that the commonly used prescription of matching through the so-called Y-factor cannot be applied in the SIDIS kinematical configurations we examine. We comment on the impact that the non-perturbative component has even at relatively high energies.

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1. Introduction

Collinear perturbative QCD computations allow us to predict the behaviour of the cross section of a hadronic process at high resolution scale Q, in the large $q_T \gtrsim Q$ region. On the other hand, in the low q_T region one must resum the large (double) logarithmic contributions generated by the emission of soft and collinear gluons.

This can be achieved by applying a soft gluon resummation scheme like, for instance, the Collins-Soper-Sterman (CSS) scheme [1], which was originally formulated and extensively tested for Drell-Yan (DY) process, $h_1 h_2 \to \ell^+\ell^- X$ [1, 2, 3, 4, 5]. In the case of Semi-Inclusive Deep Inelastic Scattering (SIDIS) process, $\ell N \to \ell h X$, resummation was studied in Refs. [6, 7, 8]. In the CSS formalism, the resummation is performed in the badly divergent (asymptotic) part of the perturbative cross section σ^{ASY}, which is separated from the regular part (i.e. less singular than $1/q_T^2$) commonly known as the Y-term. In the resummed part, some model-dependence has to be introduced to parametrize the non-perturbative component of the cross section. This model dependence enters in a non-trivial way, since the CSS resummation is done in Fourier space.

A successful resummation scheme should take care of matching the fixed order hadronic cross section, computed in perturbative QCD at large q_T, with the resummed cross section, valid at low $q_T \ll Q$, where large logarithms are properly treated. This matching should happen, roughly, at $q_T \sim Q$ where logarithms are small [1], and is very often realized through the Y-term, which should ensure a continuous and smooth matching of the cross section over the entire q_T range.

In this summary we will describe some specific matching procedures, discuss the delicate interplay between the perturbative and non-perturbative parts of the hadronic cross section and give numerical examples, exploring different kinematical configurations of SIDIS experiments.

2. Resummation in Semi-Inclusive Deep Inelastic Scattering

The starting point for the CSS scheme is the separation of the badly divergent part of the pQCD calculation of the cross section, $d\sigma^{ASY}$, from the regular part, the so called Y-term. Starting from the Next-to-Leading order SIDIS cross section one has

$$\frac{d\sigma^{NLO}}{dx\,dy\,dz\,dq_T^2} = \frac{d\sigma^{ASY}}{dx\,dy\,dz\,dq_T^2} + Y\,. \tag{2.1}$$

Then, one performes the resummation in the asymptotic term alone. In unpolarized SIDIS processes, $\ell N \to \ell h X$, the following CSS expression [6, 7] holds

$$\frac{d\sigma^{total}}{dx\,dy\,dz\,dq_T^2} = \pi\sigma_0^{DIS} \int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2} W^{SIDIS}(x,z,b_T,Q) + Y^{SIDIS}(x,z,q_T,Q)\,, \tag{2.2}$$

where q_T is the virtual photon momentum in the frame where the incident nucleon N and the produced hadron h are head to head, and

$$\sigma_0^{DIS} = \frac{4\pi\alpha_{\rm em}^2}{sxy^2}\left(1 - y + \frac{y^2}{2}\right)\,. \tag{2.3}$$

In the CSS resummation scheme, the term $W^{SIDIS}(x,z,b_T,Q)$ in Eq. (2.2) resums the soft gluon contributions, large when $q_T \ll Q$:

$$W^{SIDIS}(x,z,b_T,Q) = \exp\left[S_{pert}(b_T,Q)\right] \sum_j e_j^2 \sum_{i,k} C_{ji}^{\text{in}} \otimes f_i(x,\mu_b^2)\, C_{kj}^{\text{out}} \otimes D_k(z,\mu_b^2), \tag{2.4}$$

where $j = q,\bar{q}$ runs over all quark flavors available in the process, $i,k = q,\bar{q},g$, and

$$S_{pert}(b_T,Q) = -\int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2}\left[A(\alpha_s(\mu))\ln\left(\frac{Q^2}{\mu^2}\right) + B(\alpha_s(\mu))\right] \tag{2.5}$$

is the perturbative Sudakov form factor. In Eq. (2.4), the Wilson coefficients C_{ji}^{in}, C_{kj}^{out} are convoluted with the collinear distribution and fragmentation functions, evaluated at the intermediate scale $\mu_b(b_T) = C_1/b_T$. These Wilson coefficients, as well as A and B in Eq. (2.5), are functions that can be expanded in series of α_s. For our studies, we use Next-to-Leading Log (NLL) accuracy (for more details, see for instance [1, 9, 4, 7]).

The CSS formalism relies on a Fourier integral (2.2) over b_T which runs from zero to infinity. At very large values of b_T, both the Sudakov form factor (S) and the collinear functions f and D in (2.4) involve the evaluation of α_s at low scales. In order to avoid this, one must introduce a prescription to "freeze“ b_T. This can be achieved by making the replacement $\mu_b(b_T) \to \mu_b(b_*)$ in Eq (2.4), where

$$b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}}\,. \tag{2.6}$$

With this replacement, one must also introduce the model-dependent function S_{NP}, in order to parametrize the non-perturbative part "avoided” by the b_*-prescription. Then, one can write the SIDIS cross section as

$$\frac{d\sigma^{total}}{dx\,dy\,dz\,dq_T^2} = \pi\sigma_0^{DIS} \int_0^\infty \frac{db_T b_T}{(2\pi)} J_0(q_T b_T) W^{SIDIS}(x,z,b_*,Q) \exp\left[S_{NP}(x,z,b_T,Q)\right] + Y(x,z,q_T,Q)\,, \tag{2.7}$$

where the angular integral in the Fourier transform has been performed. The predictive power of the b_T-space resummation formalism is limited by our inability to calculate the non-perturbative distributions at large b_T. However, most of these non-perturbative distributions are believed to be universal and can be extracted from experimental data on different processes.

3. Results

Before testing matching prescriptions, we underline the main idea behind Y-term matching. Considering first the expression in Eq. (2.2) the cross section can be written in a short-hand notation as

$$d\sigma^{total} = W + Y = W + (d\sigma^{NLO} - d\sigma^{ASY})\,. \tag{3.1}$$

In the region where $q_T \simeq Q$, the resummed cross section W is expected to be very similar to its asymptotic counterpart, $d\sigma^{ASY}$. Therefore, the cross section in Eq. (3.1) should almost exactly *match* the NLO cross section, $d\sigma^{NLO}$:

$$d\sigma^{total} = W + Y \xrightarrow{q_T \sim Q} d\sigma^{ASY} + Y = d\sigma^{ASY} + d\sigma^{NLO} - d\sigma^{ASY} = d\sigma^{NLO}. \tag{3.2}$$

This matching prescription at $q_T \simeq Q$ only works if $W \simeq d\sigma^{ASY}$ over a non-negligible range of q_T values. Of course, in order to write down the cross section, one must include its non-perturbative component. Then one should ask the question of what impact this model-dependent part has in the calculation, and ultimately in the matching. Model dependence enters through the function S_{NP}, but also through the b_*-prescription. We choose a parametric form for S_{NP} consistent with the one succesfully used in DY processes, namely

$$S_{NP} = \left(-\frac{g_1}{2} - \frac{g_{1f}}{2z^2} - g_2 \ln\left(\frac{Q}{Q_0}\right) \right) b_T^2 \,. \tag{3.3}$$

Fig. 1 shows the impact on the calculation of the resummed cross section, when varying the parameters g_1 and g_{1f} in Eq. (3.3). Fig. 2 displays the effect of changing the parameter b_{max} in expresion (2.6). In these two cases, it is interesting to note that one can observe a mild model-dependence only in the extreme kinematics (left panels of Figures 1 and 2). Of particular interest is the fact that, in the region of current data (HERMES and COMPASS kinematics), at NLL accuracy the model-dependence prevails even at large values of q_T.

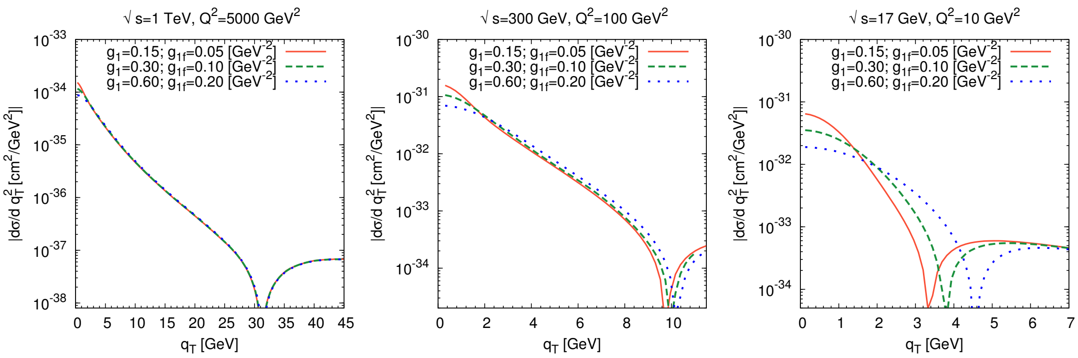

Figure 1: Resummed term of the SIDIS cross section including the non-perturbative contribution S_{NP} in the Sudakov factor, calculated at three different values of g_1 and g_{1f} and corresponding to the three different SIDIS kinematical configurations: on the left panel $\sqrt{s} = 1$ TeV, $Q^2 = 5000$ GeV^2, $x = 0.055$ and $z = 0.325$; on the central panel a HERA-like experiment with $\sqrt{s} = 300$ GeV, $Q^2 = 100$ GeV^2, $x = 0.0049$ and $z = 0.325$; on the right panel, a COMPASS-like experiment with $\sqrt{s} = 17$ GeV, $Q^2 = 10$ GeV^2, $x = 0.055$ and $z = 0.325$. Here $b_{max} = 1.0$ GeV^{-1}.

This model dependence is, quite likely, one of the reasons why Y-term matching is not possible as it is shown in Fig. 3, where it can be seen that the NLL resummed cross section, inlcuding model dependence (and now labeled as W^{NLL}), largely overshoots the asymptotic cross section $d\sigma^{ASY}$ in the region where they are expected to have a similar size. This means that a cancellation like the one shown in Eq. (3.2) cannot happen and in turn, the full NLL cross section $W^{NLL} + Y$ can never be match with the pQCD calculation $d\sigma^{NLO}$. It is interesting to notice that in the kintematics where

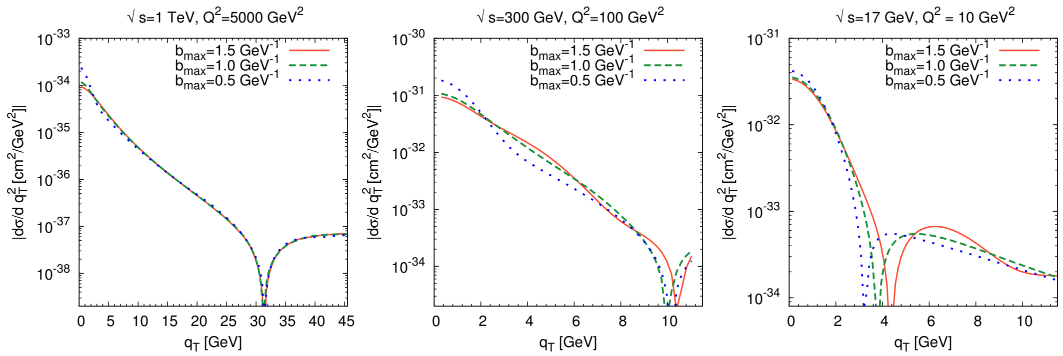

Figure 2: The resummed cross section $W^{NLL}(q_T)$ corresponding to the three different SIDIS kinematical configurations defined in Fig. 1. Here b_{max} varies from 1.5 GeV^{-1} to 0.5 GeV^{-1}, while g_1 and g_{1f} are fixed at $g_1 = 0.3$ GeV^2, $g_{1f} = 0.1$ GeV^2.

data is available, the Y-term actually has a sizeable contribution to the cross section at low q_T. In this region, it is usually expected that the resummed part W^{NLL} accounts for most of the cross section.

As an attempt to account for the model-dependence in the matching, we write the SIDIS cross section as

$$d\sigma^{total} = W^{NLL} - W^{FXO} + d\sigma^{NLO}\,, \tag{3.4}$$

where W^{FXO} is the NLL resummed cross section approximated at first order in α_s, and contains the same non-perturbative function S_{NP} as does W^{NLL} (see [10] for a precise definition). In the absence of the non-perturbative function and under some approximations involving the leading power of α_s, it can be shown that $W^{FXO} \to d\sigma^{ASY}$ so that, in this limit [11, 12]

$$d\sigma^{total} = W^{NLL} - W^{FXO} + d\sigma^{NLO} \to W^{NLL} - d\sigma^{ASY} + d\sigma^{NLO} = W^{NLL} + Y\,. \tag{3.5}$$

In this limit this prescription is equivalent to the Y-term matching prescription of Eq. (3.2). It is therefore reasonable to expect to find a region in which $W^{FXO} \simeq W^{NLL}$, allowing to match the SIDIS cross section $d\sigma = W^{NLL} - W^{FXO} + d\sigma^{NLO}$ to the purely perturbative cross section $d\sigma^{NLO}$. Fig. 4 shows the different terms of the cross section in Eq. (3.4), where now W^{FXO} plays the role that $d\sigma^{ASY}$ did in Eq. (3.1). There, one can see that W^{FXO} has a better behaviour, relative to W^{NLL}, than $d\sigma^{ASY}$ did (see Fig. 3). For instance, both W^{FXO} and W^{NLL} become negative at very similar values of q_T. Furthermore, one can see that in the cases shown in Fig. 4, there is a region where these two quantities have the same size. Unfortunately, this does not happen anywhere close to the region $q_T \simeq Q$, where one would expect to match to $d\sigma^{NLO}$. Therefore, no smooth and continuous matching can be performed.

Finally, we would like to point at the fact that the perturbative Sudakov factor plays a central role in the behaviour of the resummed cross section. In fact, the reason why our latter attempt to perform the matching failed can be likely attributed to the problem of expanding S_{pert} to a definite accuracy in powers of α_s (see [10] for a more detailed discussion of this point). In general, S_{pert} is a very intricate quantity that should never be overlooked. In order to ilustrate this, we compare two common approaches to calculate S_{pert}. In the first one, we numerically compute S_{pert} from

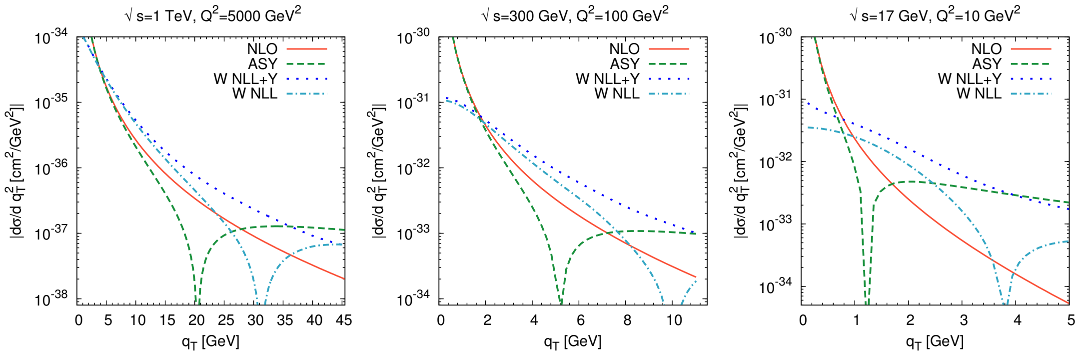

Figure 3: $d\sigma^{NLO}$, $d\sigma^{ASY}$, W^{NLL} and the sum $W^{NLL}+Y$ (see Eq. (3.2)), corresponding to the three different SIDIS kinematical configurations defined in Fig. 1. Here $b_{max}=1.0$ GeV^{-1}, $g_1=0.3$ GeV^2, $g_{1f}=0.1$ GeV^2, $g_2=0$ GeV^2.

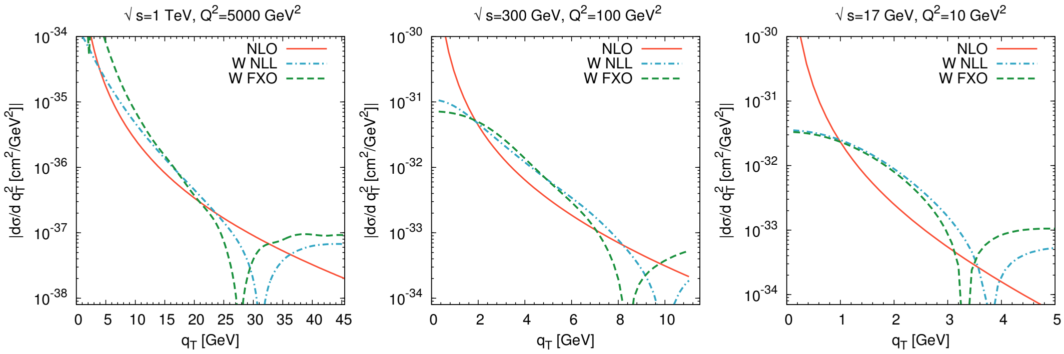

Figure 4: $d\sigma^{NLO}$, W^{NLL} and W^{FXO} (see Eq. (3.4)), corresponding to three different SIDIS kinematical configurations. Here $b_{max}=1.0$ GeV^{-1}, $g_1=0.3$ GeV^2, $g_{1f}=0.1$ GeV^2, $g_2=0$ GeV^2.

Eq. (2.5). In the second one, we use an analytic expression obtained in Ref. [7], for which the replacement

$$\log(Q^2/\mu_b^2) \to \log(1+Q^2/\mu_b^2)\,, \tag{3.6}$$

was made in Eq. (2.5). This replacement leads to a modified, better behaved S_{pert} as $b_T \to 0$ [13, 14]. Fig. 5 shows both the standard and modified S_{pert}. For the extreme kinematics, in the left panel, one can see that it is only in the region of large b_T where significant differences arise. Large b_T behaviour is commonly asociated to non-perturbative physics, which should be accounted for via S_{NP}, so at these kinematics, both the standard and modified S_{pert} seem equally suitable for calculate the cross section. As seen in the right panel of Fig. 5, this is not the case at low energies (compatible to those of COMPASS and HERMES), or even a moderately high energies (central panel). In both cases, there is a sizeable difference even in the low-b_T regime. Interestingly enough, at kinematics similar to available data by COMPASS and HERMES, the modified S_{pert} would have almost no effect in the calculation of the resummed cross section, i.e. $exp(-S_{pert}) \simeq 0$.

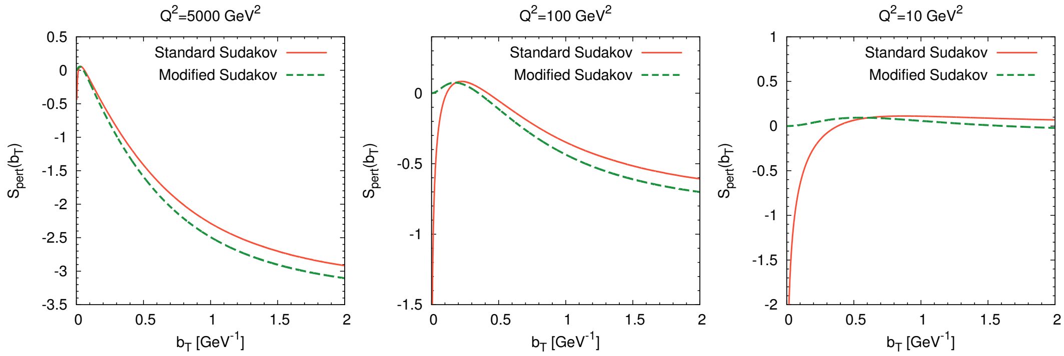

Figure 5: Sudakov factor as given by Eq. (2.5) (solid line), and its modified form given in Eqs. (44)-(47) of Ref. [7] (dashed line), for three different values of Q^2.

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