# Private Equity Returns in a Model of Entrepreneurial Choice with Learning*

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**Abstract**

Entrepreneurs invest a large share of their financial wealth in a single business that they personally manage. Despite the large risk implied by this undiversified investment they do not seem to require any extra return on a diversified public equity index. In light of the large public equity premium this fact poses a new asset pricing puzzle. In the present paper I use a quantitative model to explore the issue and find that the choice to be entrepreneurs can be rationalized even with a negative private equity premium when the full return on entrepreneurial investment is properly accounted for.

Keywords: Private equity, learning, portfolio choice, occupational choice, life-cycle.

## 1 Introduction

Entrepreneurs on average invest about 40 percent of their financial wealth in business assets, often in a single firm that they also manage. At the same time the idiosyncratic risk of a single firm is likely to be very large. In light of the large public equity premium one would expect that entrepreneurs require a substantial premium over public equity to hold such undiversified positions in business assets. Yet, in a recent study, Moskowitz and Vissing-Jørgensen (2002) find that the return on an index of private equity they construct does not exceed that on the public equity index. This fact has come to be known as the private equity premium puzzle. [1]The two authors go on suggesting that given heterogeneity, while some entrepreneurs may be earning a proper return to compensate them for the idiosyncratic risk of their business, others must be earning even less, making the choice to become entrepreneurs even more puzzling for them. In the present paper I construct a fully specified life-cycle occupation and portfolio choice model and use it to explore the two issues mentioned above. The main result of the paper is that the choice to become an entrepreneur and hold undiversified portfolios for the typical entrepreneur can be justified in the presence of a very small or even negative excess return on the financial investment in the business over a diversified stock, provided this investment is small and the full return to starting the business, including the one on human capital is properly taken into account. The paper also provides estimates of the return on the value weighted index of private equity, a measure that more closely

[1]Recently Kartashova (2009) performed the same exercise as Moskowitz and Vissing-Jørgensen (2002) on a longer data-set that includes up to the 2007 issue of the Survey of Consumer Finance and found a small positive premium. This is still smaller though than the premium one would expect from standard theoretical considerations.

reflects the behavior of the largest firms. These estimates are in the order of ten percent or more, confirming previous back-of-the-envelope calculations and pointing to the need of further research on this subject.

The model developed here is set in a partial equilibrium framework and is populated by life-cycle agents that go through the stages of working life and retirement. During working life agents make an occupational choice: at each point in time they decide between working for pay and being self-employed. This choice is always reversible so that it is repeated in every period until retirement. Workers supply their labor in exchange for a wage and choose how much to consume and how much to save in a risky financial asset. Entrepreneurs receive earnings from their business, choose their optimal consumption and saving plan and decide how to allocate their wealth between equity in their business and the financial asset. Three features characterize the way entrepreneurial activity is modeled. First I assume that there is a component of the return to a private firm that is fixed in the course of its life and not known at the time the entry decision is made. Agents make the decision to enter self-employment based on a noisy signal of this fixed component. After that they learn about it over time by applying Bayes' rule to the realized stochastic returns. Second, exogenous imperfections in financial markets force the entrepreneur to finance the private business out of his own wealth and a minimum equity requirement is assumed. Finally the process for entrepreneurial earnings is highly correlated with the process for the return on equity invested in the private business; this is meant to capture the fact that most entrepreneurs work in the same business where they invest their wealth.

The model is simulated and its quantitative properties analyzed. The main result of the analysis is that it is possible to rationalize the choice to become an entrepreneur even with a negative expected premium on the financial investment. This occurs for several reasons. First the occupational choice is always reversible and the amount of business assets needed to start a firm is small. This implies that the amount of human and financial wealth that is put at risk upon entry is small, which reduces the required premium. Second, the dynamic structure of the model allows the potential entrepreneur to receive two forms of compensation for this choice that are not directly measured in the financial return on the initial investment. One comes from the fact that a majority of switching workers received a negative earnings shock in the labor market so that becoming self-employed directly improves the return on their human capital. The second one is a consequence of the information structure of the model and the option to quit. The would be entrepreneur does not know the exact quality of the business he may run, however he knows that if it turns out to be very good he will enjoy a substantial return premium on the investment and if it turns out to be bad he has the option to quit. The large dispersion in the fixed unknown component of project quality gives a substantial value to the information about business returns that the agent gets by entering, so that he may choose self-employment even if the expected return on the financial investment is below the one on the stock upon entry and possibly for a few periods on. The model also allows to give an estimate of the return on an index of private equity defined as the value weighted average return to individual firms. The figures obtained range from 8 to 20 percent. This finding stems from the fact that, given the large concentration of business equity, the value weighted return is dominated by the allocation choice of mature entrepreneurs holding large and well established firms. For those entrepreneurs the allocation problem reduces to a standard portfolio choice problem and given the substantial risk of holding private equity this needs to pay a large premium on the financial asset to justify their undiversified choice.

The present paper is closely related to a small number of other papers that in the wake of Moskowitz and Vissing-Jørgensen's (2002) results provided potential explanations to the private equity premium puzzle. These include Polkovnichenko (2002), Hopenhayn and Vereshchagina (2005) and Hintermaier and Steinberger (2005). The latter contribution is the most closely related to the present one since they both construct a model of entrepreneurial choice in a life-cycle framework with idiosyncratic earnings and return shocks. Beyond the surface though the two contributions differ radically in the way the occupational choice is modeled. In fact the setup in Hinteremeier and Steinberger (2005) is that of a limited participation model where entrepreneurship is simply defined as holding of an asset — private equity — with a structure of participation costs and returns that is different from the one of the risky financial asset. On the contrary the present research models the choice between paid and self-employment as a true occupational choice where both the return on human capital and the return on financial capital are governed by different processes in the two occupations. The two models also differ substantially in the information structure since in the current model households face different conditional distributions of project returns prior to entry and slowly learn project quality via Bayesian learning, while in Hintermeier and Steinberger (2005) all households face the same distribution of project return and project quality is learned perfectly after entry. As it will be seen in a later section these two differences imply a radical difference in the way entry into entrepreneurship is obtained. It also implies a more realistic set of characteristics for entrants. Polkovnichenko (2002) uses a static model to assess the size of the premium that is needed to induce the entrepreneurial choice when only a portion of human capital needs to be invested in the business, a realistic assumption when the option to quit is available. He finds that under these assumptions the premium is small so that investment by entrepreneurs could be justified by minor non pecuniary benefits. Hopenhayn and Vereshchagina (2005) construct a model where risky entrepreneurial projects offer the possibility of eliminating non-concavities in the agents' continuation value and provide a valuable alternative to a safe asset even with a negative excess return, especially for poor agents. The main contribution of the present paper with respect to the ones mentioned above is that it includes in a single model a dynamic life-cycle occupational choice between paid and self-employment where this choice is fully specified in the sense that it takes into account both the human and financial capital component of the return jointly. While some of these elements are present in each of those studies, they do not feature all of them in a single setup.

The present paper is also related, although more distantly to the large literature that has studied the choice to become entrepreneurs, both empirically and theoretically but without reference to returns on private equity and the portfolio choice of entrepreneurs. Among the empirical papers we find the works of Evans and Jovanovic (1989) and Hurst and Lusardi (2004) who focus on the impact of liquidity constraints on the decision to become entrepreneur and Evans and Leighton (1989) and Hamilton (2000) that focus on earnings and other demographic and economic characteristics of the potential entrant. In the quantitative literature there has been a recent wave of papers that have studied the decision to become entrepreneur over the life-cycle with a focus on liquidity constraints; examples are Buera (2006), Mondragon-Velez (2006) and Akyol and Athreya (2007). Other models that study entrepreneurial choice in a similar framework with uninsurable earnings risk and credit constraint are Quadrini (2000), Cagetti and De Nardi (2006), Li (2002), Meh (2003) and Yaz (2005). The first two of these papers are focused on studying the impact of entrepreneurship on the concentration of wealth while the last three are more focused on the impact that taxation has on the choice to become entrepreneur and its public policy implications.

The rest of the paper is organized as follows. In Section 2 I describe the model, in Section 3 the choice of parameters, in Section 4 the results and finally in Section 5 I briefly make some conclusions and point to directions for future research. An Appendix contains the description of the numerical methods used to solve the model.

# 2 The Model

## 2.1 Demography and Preferences

Time is discrete and the model period is assumed to be 1 year. Given that time does not enter as a separate state variable from age in the agent's optimization problem age is denoted with $t$ without risk of confusion.[2] Age in the model can range from 1 to $T = 65$ years. Agents are assumed to enter the model at age 25 so that real life age is equal to $t + 25$. Each agent faces an age changing

[2]This is because the model is partial equilibrium and the exogenous price processes are time independent.

conditional probability of surviving to the next period which will be denoted with $p_t$. Surviving agents work the first $S = 40$ years and retire afterwards. All agents start life with zero wealth.

Agents do not value leisure, hence they derive utility from the stream of consumption they enjoy during their life-time only. Agents have Epstein-Zin preferences so we can define their utility as

$$V_t = \left\{ c^\gamma + \beta p_{t+1} E(V_{t+1}^\alpha)^{\frac{\gamma}{\alpha}} \right\}^{\frac{1}{\gamma}} \tag{1}$$

where $\frac{1}{1-\gamma}$ is the elasticity of inter-temporal substitution, $\alpha$ determines the agent's aversion to risk and $\beta$ is the subjective discount factor.

Agents are assumed to have a kid at the age of 35. No link between members of the same dynasty is assumed, neither in the form of inheritance of parental productivity nor in the form of an active bequest motive. However, unspent wealth in case of early death is passed to the next member of the dynasty in the form of financial assets. Given the timing of birth, the descendant inherits when he is the age of the parent upon death minus 35 and for simplicity it is assumed that he does not anticipate receiving the transfer. Given the focus of the paper on the returns needed to entice households into entrepreneurship this simplification is not relevant. This way of introducing bequests allows some agents to have positive wealth very early in life, hence potentially to start a business, without any extra complication in the computation.

## 2.2 Endowments and Assets

In each period before retirement agents are endowed with a certain amount of units of labor. These units of labor are determined by two components. The first component is deterministic and common to all agents. It is denoted with $G(t)$

and is meant to reproduce the observed hump in life-cycle earnings. The second component is stochastic and specific to the individual. In each period of life the agent can choose between selling her endowment of labor for pay in the market or being self-employed and run a privately held business. The implications of this choice are twofold: first they imply subjecting the deterministic component of the labor endowment to stochastic shocks governed by a different process, second they imply a different set of assets in which the agent can invest. After retirement all agents receive a constant social security benefit $Y^{ss}$ that is equal for everybody. In the next two subsections I will describe the details of the two occupations.

### 2.2.1 Paid Worker Earnings and Investment Choice

Agents who decide to work in the market face idiosyncratic shocks $z_t$ to their endowment of labor units that follow a first order autoregressive process in logarithms:

$$\ln(z_t) = \rho \ln(z_{t-1}) + \varepsilon_t \tag{2}$$

where $\varepsilon_t$ is an i.i.d shock from a normal distribution $N(0, \sigma_\varepsilon^2)$ and is also independent across agents and from all other return processes in the economy. With this we can write the earnings of a paid employed agent as:

$$Y_t^{pe} = G(t) z_t. \tag{3}$$

Agents that choose to be worker in any given period can invest in a single risky financial asset that I call stock. The amount of stock that the household buys in period $t$ and holds up to period $t+1$ is denoted $S_{t+1}$ and is subject to a no short sale constraint

$$S_{t+1} \geq 0. \tag{4}$$

The return on holding the stock from $t$ to $t+1$ is denoted $R^S_{t+1}$.

### 2.2.2 Entrepreneur Earnings and Investment Choice

Entrepreneurs are defined by the fact that they are self-employed and use their human capital to run a business in which they also invest part of their money. Each business is characterized by a fixed parameter $x$ that defines its quality by affecting both the self-employed wage and the return on the financial resources invested in the firm. [3] The human capital invested in the firm is also subject to a shock $\xi_t$ so that the wage earned by the agent as manager of her own firm is defined by the equation:

$$Y^{se}_t = \lambda_1 x G(t) exp(\xi_t). \tag{5}$$

The shock $\xi_t$ is assumed to be normal $N(0, \sigma^2_\xi)$, i.i.d. over time and perfectly correlated with the shock to the return on equity invested in the private firm. This reflects the well documented fact that entrepreneurs typically manage the firm in which they also invest their money so that returns on human and financial capital are highly correlated for them. Finally the constant $\lambda_1$ is a normalization parameter that is equal for all agents and sets the average entrepreneurial earnings in the economy. Entrepreneurs have a richer investment opportunity set than workers since they can invest both in the stock described before and in a second, non-tradable asset called private equity which represents the financial investment in the firm they run and that I denote with $K_{t+1}$. It is assumed that there is a minimum amount of financial resources that the entrepreneur needs

[3] In fact in the course of their lives agents may enter and exit self-employment several times and so have a different $x$ at different times; however I use $x$ instead of $x_t$, that is, I omit the age index to stress the fact that this component of the project's pay-off is fixed for a given firm.

to invest to start up and run a business so that the following inequalities hold:

$$K_{t+1} \geq \underline{K} > 0. \tag{6}$$

As it will become clear later, given that the entrepreneurial technology is assumed to be additive in the agent's endowment of labor and financial investment, this positivity constraint is needed to have positive private equity investment also among entrepreneurs facing a negative expected premium. This fits with the empirical evidence that most private equity is held by individuals who are at the same time the managers of the business. At the theoretical level extensive literature starting with Jensen and Meckling (1976) demonstrates how asymmetric information about managerial actions impose entrepreneurs to commit personal funds to finance their project justifying the assumption made here. The pay-off from the financial investment in the business is given by

$$\Omega_{t+1} = \lambda_2 exp(q_{t+1}) f(K_{t+1}). \tag{7}$$

Here $f(K_{t+1})$ is an increasing and concave function of the amount invested in private equity and $\lambda_2$ is again a normalization constant that is equal for all entrepreneurs. The remaining term $q_{t+1}$ is given by the following equation:

$$q_{t+1} = x + u_{t+1} \tag{8}$$

where $x$ denotes the fixed project quality and $u_{t+1}$ is a normal random variable $U \sim N(0, \sigma_U^2)$ and is i.i.d. over time and independent across firms. Also as it was said before $u_t$ is perfectly positively correlated with $\xi_t$ the shock to the entrepreneur's units of labor. The assumption of decreasing returns to scale in the function describing the pay-off generated by private equity investment is common in models featuring heterogeneous firms like Cagetti and De Nardi

(2006) and Cooley and Quadrini (2001) and can be justified for example by the existence of a fixed factor in production like the managerial talent of the entrepreneur. As the firm grows and the entrepreneur's managerial skills are spread over a larger activity, a marginal increase in investment generates less and less extra income.[4] As it will be seen in the calibration section, at a practical level, the assumption of decreasing returns on the investment in private firms helps slowing the growth of even the firms of the best quality. This in turn is useful to generate an average firms size that is in line with the data. [5]

## 2.3 The Occupational Choice and Learning

The variable $x$ that determines the quality of an entrepreneurial project, defined by its return on human and financial capital invested in it, cannot be directly observed. In each period when an agent is a paid employee she draws a value of $x$ from a distribution $X \sim N(0, \sigma_X^2)$. The agent also draws a realization of the random variable $U$. The agent cannot observe $x$ directly; what she observes instead is a noisy signal of it, that is, the sum $x+u$. Based on this signal and on her prior over the distribution of $x$ which is assumed to coincide with the true distribution $X$, she forms a first estimate of the quality of the project she drew using Bayes' rule. Based on this estimate she decides whether to start running the business and become an entrepreneur or to stay in her current occupation,

[4] The idea of the dimension of the firm being determined by the optimal allocation of managerial talent over resources, or "span of control ", was introduced in Lucas (1978).

[5] An alternative or possibly complementary way to obtain the same result would be to add an AR(1) shock in equation (7) since this would allow even the best projects to turn bad for some periods. Although this way has its own merits it would imply the addition of a further state variable to a problem that has already two continuous state variables plus three discrete ones. For this reason it was chosen not to pursue this idea.

that is, paid employment. If she decides to become an entrepreneur, in the following period she keeps the same initial draw of project quality $x$, observes a new noisy signal $x + u$ and based on this, updates her beliefs. Based on the updated beliefs the occupational choice is repeated in the same fashion until the agent decides to switch occupation. On the other hand if she decides to stay in paid employment, in the following period she receives a new draw of project quality from the distribution $X$ and a new noise $u$ and repeats the same choice again. Successive draws from $X$ are independent over time.[6] Finally, when an agent decides to switch from self-employment to paid employment she draws the initial labor earnings shock in the new occupation from some distribution $Z^s$ that is independent of her past occupational history.

In practice then, what an entrepreneur uses to update her beliefs is the sequence of observations $q_t = x + u_t$ independently drawn from a normal random variable $N(x, \sigma_U^2)$ with mean equal to the true value of the parameter that describes the average pay-off to the project. Based on this sequence and the initial prior the agent uses Bayesian updating to form a sequence of posteriors which are themselves normal random variables. The sequential learning problem defined above can be handled by the Kalman filter technique. For the purpose of describing the agent's optimization problem it is sufficient to introduce a

[6]This assumption implies that the experience acquired by running a firm is entirely lost when the business is closed. One consequence is that the model cannot capture the fact that the probability of entry is higher for former business owners than for the general population as reported in Quadrini (2000). An alternative assumption would be that the draw of the project quality is taken from a better distribution if an agent had previously run a business. This assumption would give an extra benefit to the choice of becoming an entrepreneur and induce agents to accept even lower returns to make this choice. This possibility would strengthen the conclusions reached in this paper but is not pursued here since it would further complicate the solution of the model.

variable $\hat{x}_t$ for the mean of the posterior distribution of the fixed project quality parameter. It could be shown that this variable evolves according to the following law of motion:

$$\hat{x}_{t+1,a+1} = (1 - \Lambda_{a+1})\hat{x}_{t,a} + \Lambda_{a+1} q_{t+1} \tag{9}$$

where $\hat{x}_{t,a}$ is the mean of the posterior distribution of an age $t$ agent that has observed $a$ signals about the same project. Given the description above this means that this agent run the business for $a - 1$ periods. The law of motion weights the mean of the current posterior and the new noisy observation to get the mean of the new posterior. The weight $\Lambda_{a+1}$ can be computed using the following recursion:

$$\Lambda_{a+1} = \frac{\Sigma_{a+1}}{\Sigma_{a+1} + \sigma_U^2} \tag{10}$$

$$\Sigma_{a+1} = \frac{\Sigma_a \sigma_U^2}{\Sigma_a + \sigma_U^2} \tag{11}$$

The recursion starts with $\hat{x}_{t,0} = 0$ and $\Sigma_0 = \sigma_X^2$, that is, the mean and variance of the distribution of $X$ which is taken to be the initial prior. The distribution of $\hat{x}_{t+1}$ is then $N(\hat{x}_t, \Lambda_{a+1}\Sigma_{a+1})$. [7]

## 2.4 The Households' Dynamic Programming Problem

With the description of the model given above I can now write the recursive formulation of the household's utility maximization problem during working age. At each age $t$ before retirement the agent's state variables can be described by the 4-dimensional vector $(d_t, z_t, \hat{x}_t, a)$ where $d_t$ denotes currently available resources, $z_t$ is the current labor earnings shock, $\hat{x}_t$ is the mean of the posterior

[7] See Ljungqvist and Sargent (2000) for an introductory treatment or Harvey (1989) for a more complete and formal one.

distribution of the project quality parameter and $a$ is the number of noisy signals observed on a project as of age $t$. As we saw in the last section these last two parameters are needed to form the new posterior that enters the computation of continuation utility. At the beginning of the period and given the states the agent compares the utility of becoming a paid worker with that of choosing entrepreneurship.

The optimal indirect utility of choosing to become a worker in the period for an agent whose state is described by the vector $(d_t, z_t, \hat{x}_t, a)$, is:

$$V_t^{pe}(d_t, z_t, 0, 0) = \max_{c_t, S_{t+1}} \left\{ c_t^\gamma + \beta p_{t+1} E_t \left[ V_{t+1}(d_{t+1}, z_{t+1}, \hat{x}_{t+1}, 1)^\alpha \right]^{\frac{\gamma}{\alpha}} \right\}^{\frac{1}{\gamma}} \tag{12}$$

subject to the following constraints

$$c_t + S_{t+1} \leq d_t \tag{13}$$

$$d_{t+1} = R_{t+1}^S S_{t+1} + Y_{t+1}^{pe} \tag{14}$$

$$\hat{x}_{t+1} = (1 - \Lambda_1) \cdot 0 + \Lambda_1 q_{t+1} \tag{15}$$

and the no short sale constraint given by equation (4). Notice that since agents receive a first signal about a new project while they are still working for pay the variable $a$ is set to 1 in the right hand side of the Bellman equation; for the same reason in the law of motion for $\hat{x}_{t+1}$ I have the subindex on the weight $\Lambda$ taking the value one. Also the second and third argument of $V_t^{pe}$, that is, $\hat{x}_t$ and $a$, are set to 0: this is because when an agent decides to be a worker in the following period, even if he is currently an entrepreneur, his past experience at running a project gets lost.[8] Finally expressions (13) and (14) represent a

[8]Once this point is clear, the more general notation could be used but I preferred to substitute the numerical value of $\hat{x}_t$ and $a$ at the beginning of the recursion on the beliefs about the project quality for the sake of clarity.

standard budget constraint and the law of motion of the resources available to an agent that chooses to be a paid worker.

The utility of becoming an entrepreneur is defined by the following equations:

$$V_t^{se}(d_t, z_t, \hat{x}_t, a) = \max_{c_t, S_{t+1}, K_{t+1}} \left\{ c_t^{\gamma} + \beta p_{t+1} E_t \left[ V_{t+1}(d_{t+1}, z^0, \hat{x}_{t+1}, a+1)^{\alpha} \right]^{\frac{\gamma}{\alpha}} \right\}^{\frac{1}{\gamma}} \tag{16}$$

subject to the following constraints:

$$c_t + S_{t+1} + K_{t+1} \leq d_t \tag{17}$$

$$d_{t+1} = R_{t+1}^{S} S_{t+1} + \Omega_{t+1} + Y_{t+1}^{se} \tag{18}$$

the law of motion of $\hat{x}_{t+1}$ given by equation (9) and the constraints (6) and (4). Some comments are needed to clarify the problem stated above. First notice that in the second argument of the value function on the right hand side of the Bellman equation the notation $z^0$ appears instead of $z_{t+1}$. This is because if an agent chooses to be an entrepreneur in a given period, then when she makes the decision again next period the efficiency units of labor on the market that she will face are not determined by the $AR(1)$ process described by equation (2), they are drawn instead from the distribution $Z^s$. The notation $z^0$ is then a convention to distinguish this case. Second the last state variable in the value function on the right hand side takes the value of $a+1$ because if an agent chooses to be an entrepreneur in the current period then she adds one more noisy observation on the project quality parameter. Following the Bellman equation the first inequality is a standard budget constraint. The second one is the law of motion of the agent's resources: these are the sum of the realized return on the stock investment and on the investment in private equity plus the earnings the agent receives as manager of the firm she owns and manages. These earnings are given by equation (5) substituting $\hat{x}$ for $x$.

Finally the optimal value function of a working age agent aged $t$ is obtained as the result of the optimal occupational choice, that is:

$$V_t(d_t, z_t, \hat{x}_t, a) = \max \left\{ V_t^{pe}(d_t, z_t, 0, 0), V_t^{se}(d_t, z_t, \hat{x}_t, a) \right\} \tag{19}$$

The recursive formulation of the optimization problem during retirement boils down to the following set of equations:

$$V_t(d_t) = \max_{c_t, S_t+1} \left\{ c_t^{\gamma} + \beta p_{t+1} E_t [V_{t+1}(d_{t+1})^{\alpha}]^{\frac{\gamma}{\alpha}} \right\}^{\frac{1}{\gamma}} \tag{20}$$

subject to the budget constraint

$$c_t + S_{t+1} \leq d_t \tag{21}$$

the law of motion of financial resources

$$d_{t+1} = R_{t+1}^S S_{t+1} + Y^{ss} \tag{22}$$

where $Y^{ss}$ is the pension benefit and the no short sale constraint. The problem reflects the fact that upon reaching age 65 all agents in the model are forced to retire and that private equity cannot be held by a non entrepreneur so that the financial asset is the only asset available to the household.

The model is analytically untractable and therefore it is solved by numerical methods. The description of the numerical solution is left to the Appendix.

# 3 Parameter Calibration

In this section I describe the choice of parameters. As it is standard in the literature some parameters are chosen based on other studies while the remaining ones are fixed with the goal of matching some features of the dynamics of self-employment.

### 3.1 Preference and Demographic Parameters

Preferences are described by three parameters. First the subjective discount factor $\beta$ is fixed at 0.96 a value consistent with most macroeconomic studies. Second the elasticity of inter-temporal substitution $\gamma$ is set at 0.5. Finally the values of the risk aversion coefficient $\alpha$ will change across experiments and will be described in the corresponding result sections.

Survival probabilities are obtained using mortality tables for the US male population and are taken from "The Berkeley Mortality Database". [9]

### 3.2 Business Quality

As we saw in the model section it is assumed that there is a population of entrepreneurial ideas characterized by a parameter $x$. Also, the true value of $x$ is not observable because of a random noise $u$. There is no reference in the data about the value of these two abstractly defined random variables so, to pick their variances, I proceed in the following way. First I set the value of $\sigma_U^2$. In the baseline case this value is picked so that the standard deviation of the shock to individual firm private equity return is 32 percent or double the standard deviation of the return on public equity. This is meant to capture the idea that even for a well established firm whose parameter $x$ is estimated precisely the investment in its equity is still substantially riskier than the investment in a diversified stock index. Secondly the value of $\sigma_X^2$ is chosen to obtain a wide

[9] The database is available at the web site http://www.demog.berkely.edu.

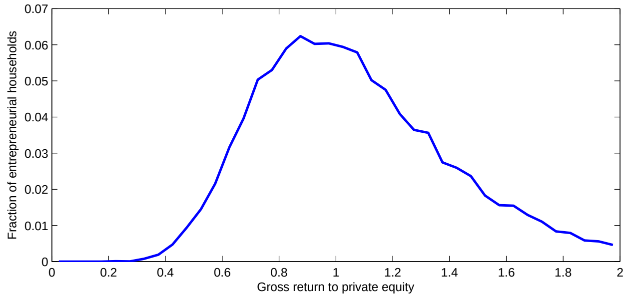


Figure 1: Cross section of private firm returns: Baseline calibration.

distribution in the cross section of firm returns, given the standard deviation of the shock $u$. The resulting distribution of cross-sectional firm returns for the baseline case is depicted in Figure 1.

This distribution reproduces the main features of the one reported in Moskowitz and Vissing-Jørgensen (2002), in particular the wide range of returns, extending from - 80 percent to more than a 100 percent, the fatter right tail of the distribution and the modal values centered slightly to the left of a 0 percent net return.[10] In the sensitivity analysis an alternative case is considered where the ratio between $\sigma_U^2$ and $\sigma_X^2$ is kept constant and the standard deviation of $U$ is reduced to 20 percent.

### 3.3 Earnings

Earnings, both for the paid employed and for the self employed incorporate a deterministic component $G(t)$ that is a function of age and is equal for all agents. This component is represented by a third order polynomial with the parameters taken from the estimates by Cocco, Gomes and Maenhout (2005). For the paid

[10]See Moskowitz and Vissing-Jørgensen (2002), Figure 2 on page 770.

employed this deterministic component is hit by AR(1) shocks in logarithms with the autocorrelation coefficient $\rho$ equal to 0.95 and the standard deviation of the innovation $\varepsilon_t$ equal to 0.158. Both values are close to the ones normally found in the macroeconomic literature.[11] In the case of the self-employed we need to fix two more parameters: the variance of the i.i.d shock $\xi_t$ and the normalization constant $\lambda_1$. The value of $\sigma_\xi$ is set at 0.54 to reflect the well document fact that entrepreneurs face substantially more volatility in their earnings than workers. [12] It is also assumed that this shock is perfectly correlated with the shock to the return on private equity $u_t$ to reflect the fact that entrepreneurs manage the firm in which they invest their money. The constant $\lambda_1$ determines the average level of earnings for the population of self-employed agents. To understand how this parameter is chosen, observe that in any period when an agent is a worker a key factor in the decision to enter into entrepreneurship is the comparison between the expected earnings in the two occupations. By looking at equation (5) we can see that $\lambda_1$ will determine the threshold values of the estimated $x$ that prompt entry into self-employment for any given value of the current shock $z_t$ to the earnings the agent obtains in paid-employment. We can then fix its values so that given all the other parameters we match the entry rate into self-employment of 2.4 percent reported in Evans and Leighton (1989). The description of the earnings processes in the economy is completed by the distribution $Z^s$ that an agent faces when she switches occupation from self to paid-employment. For simplicity this is assumed to be a 3 point distribution.

[11] See, to mention two examples, Hubbard, Skinner and Zeldes (1994) and Huggett and Ventura (2000).

[12] This fact together with some estimates is reported for example in Heaton and Lucas (2000) or Rosen and Willen (2002).

This distribution is important in determining both entry and exit rates, thus also the average share of entrepreneurs, because it determines endogenously the value of the alternative option for an agent that is currently an entrepreneur, that is, going back to paid employment. For simplicity I pick the realizations of $Z^s$ on the same grid used for the distribution of $z$ and fix the probabilities of being in each state so that the model is able to match the average share of the population that are entrepreneurs, given the entry rate. The resulting distribution features an expectation that is below the expected earnings for a continuing worker with the lowest earnings realization. This implies that in the model there is an opportunity cost of leaving work in the form of missing labor experience on top of the wage lost.

Finally I set the social security benefit $Y^{ss}$ at approximately 40 percent of average earnings in the economy, a value that is used for example by De Nardi (2004).

## 3.4 Assets

As it was said in the model description section there are two assets in the economy. The financial asset is thought to have a mean return of 7 percent and a standard deviation of 16 percent in line with the historical first two moments of the return on the US stock market as reported for example in Mehra and Prescott (1985).

The second asset is private equity whose pay-off is defined by equation (7): the parameters defining the distributions of $X$ and $U$ were described before in subsection 3.2. The function $f(K)$ is taken to be of the following form:

$$f(K) = \begin{cases} K & \text{if } K \leq K^* \\ (K - \tilde{K})^\nu + b & \text{if } K > K^* \end{cases}$$

that is, the function is concave with a linear stretch close to the origin and a strictly concave one to the right of the threshold value $K^*$. Both this value and $\tilde{K}$ depend uniquely on $b$ and $\nu$ and are determined so that the function $f$ is continuous and with continuous first derivative. The parameters to calibrate are then $\nu$ which determines the strength of decreasing returns to scale in private equity investment and $b$ which determines the amount of private equity where decreasing returns kick in. The first of the two parameters is chosen based on external studies. Basu and Fernald (1997) estimate returns to scale at the industry level and find values that are slightly below 1 both for the overall economy and for manufacturing only. Cooley and Quadrini (2001) based on the study mentioned above use a value of 0.975, while Cagetti and De Nardi (2006) find a value of 0.88 from calibrating a macroeconomic model with private entrepreneurs. Based on these studies I set $\nu$ to 0.94. Once that parameter is set the value of $b$ is chosen so that in the baseline calibration the model can approximately match the ratio of average private firm equity to the average earnings of working age households. The values for these two variables are taken from Moskowitz and Vissing-Jørgensen (2002) and from Budría-Rodríguez et al. (2002) respectively.[13]

The parameter $\lambda_2$ is a normalization constant which is crucial in the experiments performed on the model. It is fixed so that the average un-weighted share

[13]Since the value of $\nu$ is not pinpointed exactly in the data, I also experimented with values close to 0.94 and readjusting $b$ accordingly. Results, which are not reported to economize space, are not different from the ones presented here in an important way.

of wealth invested by entrepreneurial households in private equity matches the one in the data. Moskowitz and Vissing-Jørgensen (2002) and De Nardi et al. (2007) report values of this variable. As a target I choose the one in De Nardi et al. (2007) which refers to the ratio of business assets to wealth and as such is closer to the definition in our models that features no liabilities by assumption. Averaging across several issues of the Survey of Consumer Finances they report a value of about 33 percent.

Finally, investment in private equity in the model is subject to a minimum requirement $\underline{K}$. Hurst and Lusardi (2004) report figures about equity size of firms: for example in the 1987 National Survey of Small Business Finances the median amount of capital needed to found a business was \$32500 and about 25 percent founded a firm with less than \$7500 (in 1998 dollars). In the 1989 Panel Study of Income Dynamics about 38 percent of entrepreneurs had less than \$6500 in business equity, and among those who become entrepreneurs in the interval 1989 to 1994 over 75 percent had less than \$27500 in business equity at the end of the period. Overall this suggests that the amount of equity needed to run a business is small. Based on those figures I set the baseline value of $\underline{K}$ to be equivalent to 50 percent of average annual wages or about \$20000 based on the figures for average earnings in Budría Rodríguez et al. (2002). A sensitivity analysis on this parameter is performed as well.

# 4 Results

In the present section I report the results of the quantitative experiments. The presentation is organized in three subsections: the first one is devoted to an extensive analysis of a baseline case, the second one considers the consequences of removing learning from the model and the third one is dedicated to a set of sensitivity analysis. The focus of the analysis is on the decision to become entrepreneur investing a large share of personal assets in the business and the premium that private equity needs to pay above public equity to obtain that allocation. Also a number of statistics about the dynamics of entry and exit in and out of self-employment over the life-cycle are reported to lend support to the model. To address the main question of the paper the following experiment is performed. The value of the constant $\lambda_2$ is chosen so that the target average allocation to private equity is matched. The resulting value of the average excess return on private equity above the financial asset is taken as the measure of the private equity premium needed to rationalize entrepreneurial choice. The question is then similar to the one in Heaton and Lucas (2002) and Polkovnichenko (2003) who, for a given excess risk generated by the idiosyncratic nature of private equity, look at the premium that must be paid to induce agents to make that investment. The difference lies in the fact that here the size of the investment is not fixed but can be adjusted subject to a minimum equity requirement, so that I look at the excess return on private equity that generates the empirically observed cross-section of private capital in the portfolio of entrepreneurs. I report two different measures of the return on private equity. In order to define them I first need to define the yearly return to an individual firm. This is given by the ratio of the value of capital invested in the business at the end of the period — including both capital appreciation and the dividend — and the initial investment itself, that is, using the notation described in the model section:

$$R_{i,t}^{K} = \frac{\Omega_{i,t}}{K_{i,t}}. \tag{23}$$

With the definition of firm's $i$ return at hand we can define two different measures of aggregate returns. Omitting for notational simplicity the time index, the first one is a simple equally weighted average return:

$$R^{PE}_{un-weighted} = \frac{\sum R^K_i}{n_{ent}} \tag{24}$$

where $n_{ent}$ is the number of entrepreneurs. The second one is a value weighted average return or equivalently the return on the index of private equity, $R^{PE}_{index}$ and is defined as the average of individual ex-post realized firm returns — the $R^K s$ — weighted by the firms' share in total private equity, that is:

$$R^{PE}_{index} = \sum \left\{ R^K_i \frac{K_i}{\sum K_i} \right\}. \tag{25}$$

This latter measure is the model counterpart to the index return constructed by Moskowitz and Vissing-Jørgensen (2002), while the former is more relevant when studying the individual decision to become entrepreneur.

## 4.1 Baseline Model

In this experiment I take the coefficient $\alpha$ which determines agents' risk aversion to be 2 for one half of the population and 5 for the remaining half. The choice to have two types of agents, one with low risk aversion and the other with high risk aversion follows Gomes and Michaelides (2005). The two authors show that this choice is able to rationalize both the average stock market participation rate and the average stock share for participants in a standard portfolio choice model with reasonably low participation costs. This choice is meant to insure that the model here would be consistent with the allocation between stock and bonds if the latter asset was added to the model. The results are organized in two subsections: the first one is focused on the main subject of the paper, that

Table 1: Entrepreneurial portfolio and returns

| | |
|---|---|
| Std. of returns | 36.4 % |
| Public equity return | 7.0 |
| Private equity return (equally-weighted) | 8.3 |
| Private equity return (value-weighted) | 24.1 |
| Portfolio share of private equity: (equally-weighted) | 32.5 |
| Portfolio share of private equity: (value-weighted) | 45.6 |

is, the predictions of the quantitative model regarding private equity returns. In the second subsection the patterns of entry and exit in and out of self-employment generated by the model are described. While not central to the main result, this latter section supports the present model and calibration as a reasonable description of entrepreneurial choice over the life-cycle and therefore its predictions about the private equity return.

#### 4.1.1 Entrepreneurial Portfolio and Returns

In this section the focus is on the portfolio allocation of the self-employed and the premium that private equity needs to pay to obtain that allocation. Results are reported in Table 1. In the top line we see that the standard deviation of the shock to returns, which determines the risk of entrepreneurial firms is approximately 36 percent, that is, twice the historical volatility of the Standard and Poor 500 index. In the fifth line we see that the average entrepreneur holds about 33 percent of his wealth in a business as does the average American entrepreneur: this is no surprise since this was a calibration target. We can see from the last line of the table that, when the share of wealth invested in private equity is weighted by firm size, the average is 45.6 percent. The fact that value weighted allocations to private equity are higher than simple averages conforms with the empirical evidence in Moskowitz and Vissing-Jørgensen (2002). The second line reports the return on the stock investment which was set exogenously at 7 percent. The third and fourth lines report the two measures of the return on equity in private business. The first one is the equally-weighted average. As we can see this average is only 8.3 percent, slightly more than one percentage point above the return on the public stock. The second measure is the value weighted average computed according to equation (25). This measure is the return on the index of private equity. What we see is that this return is 24.1 percent or approximately 17 percentage points above the return on the stock. This number is 7 percentage points higher than the 10 percent figure that Moskowitz and Vissing-Jørgensen (2002) report as a back-of-the-envelope calculation — based on a number of previous studies — of the premium needed to induce an agent to take the idiosyncratic risk of a single entrepreneurial project. In what follows we will explore better the working of the model to interpret these findings.

First we want to explore the difference between the two measures of returns. This is done by way of Figure 2 which reports the average return of existing firms as a function of age. For comparison the figure also reports the return on the stock represented by the flat dotted line. As we can see the lines representing the two measures of private equity returns are both increasing. This reflects learning and selection of poor quality firms out of the market. Recall here that agents make the decision to become entrepreneur based on a noisy signal of project quality: while some of the entrants do run good firms many of them enter "by mistake ". With time the latter businesses will close down, hence the average

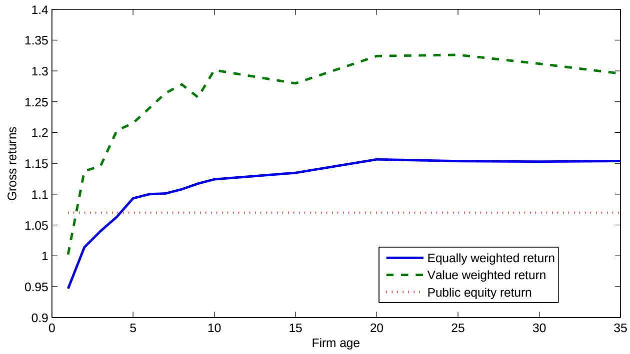


**Figure 2: Average returns by age of the firms.** Data are plotted yearly for firms less than five years old and by 5 years groups for older firms.

quality of firms and the average return improve with the age of the cohort of firms. Although qualitatively similar, the two curves differ quantitatively. They start relatively close to each other and more than 7 percentage points below the return on public equity; afterwards, the curve of value-weighted returns climbs up much faster reaching a level of about 30 percent while the one of equally-weighted returns never exceeds the public equity return by more than 8 percentage points. This result follows from the fact that firms of better quality, hence with higher average return on their equity, experience a faster rate of growth. This explains the finding that the return on the aggregate index of private equity is so high: the index is overwhelmingly made by the equity of the high return, fast growing firms.

Second as one may infer from the two curves many firms — especially recently created ones — do face an expected return that is below the public equity index, yet households are willing to undertake the investment. [14] I explore this issue with the help of Table 2. In the model the choice to become entrepreneur offers two additional benefits that are not measured in the return on the investment in private equity. First, an agent can receive a bad labor earnings shock while in paid employment. Because of persistence this translates into worse expectations about future paid-employment earnings. Switching to self-employment then, may give a direct benefit in terms of improved return on human capital. A look at the upper panel of Table 2 confirms this fact: the probability of switching into self-employment is 8.9 percent for agents with the lowest earnings shock, 0.2 percent for those with the intermediate shock and virtually 0 for those with the highest labor shock. The idea that entry into self-employment is more likely among households with negative performance in the labor market is consistent with the findings of Evans and Leighton (1989). Second entering entrepreneurs face substantial uncertainty about the average pay-off to their firm. If this turns out to be high they may end up earning a substantial premium on their investment. In the opposite case they can quickly switch back to paid employment. The possibility of exit reduces the losses in case the business project proves to be bad. To the extent that the potential benefit in case of success outweighs the cost in case of failure and exit, the option of undertaking entrepreneurship is valuable and the household will be willing to make the initial investment in private equity even if in the first few years it pays a negative premium on public equity in expectation. [15] This can

[14] To see why, observe that individual firm's returns are hit by shocks with mean one. If all firms had the same $x$ the average return marked on the continuous curve would coincide with the expected return of all firms. With heterogeneous $xs$ the expected return on their private equity will be above the curve for some firms and below it for others.

[15] The idea that there is an option value of learning, that is, that an agent may want to

Table 2: An analysis of entry

| Low shock | Intermediate shock | High shock |
|---|---|---|
| Frequency of entry by earnings shock | | |
| 8.9% | 0.2% | 0.0% |
| Percentage of entrants with expected income loss | | |
| 20.5 | 62.7 | 18.1 |

be seen in the bottom panel of Table 2 that reports the percentage of entrants that expect a net loss from choosing to become entrepreneurs in the period this choice is made. This quantity can be easily computed using the simulated data which include the variables $z$ and $\hat{x}$. With these values it is possible to compute the difference in expected earnings in the two occupations plus the difference in expected return on the two investments (the stock and private equity) multiplied by the optimal private equity choice if entrepreneurship is selected. This quantity forms the expected net income gain/loss from entrepreneurship upon making the entry decision. As it can be seen in Table 2, 20.5 percent of those who decide to become entrepreneur when they face the lowest labor earnings shock and 62.7 percent of those with the intermediate shock expect a net income loss at the time the decision is made. This confirms the fact that the mechanism described above and induced by learning can be a quantitatively relevant factor in determining the choice to become entrepreneur.

Summarizing, with respect to the two issues raised by the empirical findings of Moskowitz and Vissing-Jørgensen (2002) we can say that the present model provides an explanation for the fact that households may decide to become

run a project with negative NPV while he waits for new information about a possible higher return in the future is also discussed in a theoretical paper by Miao and Wang (2007).

entrepreneurs even in the face of a negative expected premium of private equity compared to public equity. On the other hand, with respect to the index return the estimate of the premium over public equity provided by the model is in the range of 15 percentage points, thus confirming the puzzling features of the data.

#### 4.1.2 Entrepreneurial Dynamics

In this section I turn to the implications of the model for entrepreneurial dynamics. Results are reported in Table 3. In the first row we see that the entry rate is 2.4 percent in the model, matching exactly the value that can be found in Evans and Leighton's (1989) study about self-employment. The fifth line reports the average fraction of the population that is an entrepreneur at a point in time. This figure is 11.0 percent in the data according to Moskowitz and Vissing-Jørgensen (2002) and is 10.7 percent in the model. The two statistics reported above were calibration target so not surprisingly they are matched quite well by the model. To check the ability of the model to reproduce the dynamics of entrepreneurial choice I report four more statistics. In the second and third row we can see the survival rate at 5 and 10 years. The survival rate at five years is 38.5 percent in the data and it is a very close 38.9 percent in the model. At a ten year horizon the difference between model and data is slightly larger with the survival rates being respectively 23.6 and 29.5 percent. The exit rate is 14.0 percent in the model and is 21.6 percent in the data, according to what reported in Evans and Leighton (1989) based on the Current Population Survey. Finally the average firm age is 10.3 years in the data reported by Moskowitz and Vissing-Jørgensen (2002) and it is 9.4 years in the model. The last four statistics reported above were not calibration targets. The fact that

Table 3: Entrepreneurial dynamics

| | Data | Model |
|---|---|---|
| Average entry rate | 2.4 | 2.4 |
| Survival rate (5 years) | 38.5 | 38.9 |
| Survival rate (10 years) | 23.6 | 29.5 |
| Exit rate | 21.6 | 14.0 |
| Fraction of entrepreneurs | 11.0 | 10.7 |
| Average firm age | 10.3 | 9.4 |

they are close to their empirical counterparts then suggests that the learning model adopted here represents a reasonable approximation to the dynamics of the self-employment decision in the US data.

## 4.2 More on the role of learning and the minimum equity requirement

In this subsection I further explore the role of learning by solving a version of the model that starting from the baseline choice of parameters simply assumes that project quality is already known at the time the decision to become entrepreneur is made. Secondly I explore the effects of changing the size of the minimum equity requirement in the baseline model with learning. Results are reported in Tables 4, 5 and 6.

As we can see from the first line of Table 6 eliminating learning leads to an increase in the entry rate from 2.4 to 3.3 percent. While at first sight this may seem to contradict the fact that learning is an important mechanism explaining the choice to become entrepreneur, a more careful analysis of the results will show that this is not the case. First, the reason why the entry rate jumps up is that the distribution of expected project quality, that in the model without learning coincides with the distribution of true project quality is more dispersed.[16] For any given threshold level of estimated project quality that triggers entry this would generate a higher entry rate. In the model without learning, though entry can occur only if the true quality of the project is good. This is reflected in the statistics reported in Table 4. We can see that the equally weighted return on private equity jumps up from 8.3 percent to 18.9 percent and the equally weighted share invested in business assets by entrepreneurs increases from 32.5 to 53.8 percent. Another notable result reported in the table is that the difference between the equally and value weighted return on private equity is substantially smaller than in the model with learning: it is 5 percentage points in the model without learning (the two returns are 18.9 and 23.9 percent respectively) while it is 16 percentage points in the model with learning (the two returns are in this case 8.3 and 24.1 percent). This difference reflects the fact that in the model without learning the only source of heterogeneity in individual firms' return is related to true differences while in the model with learning there is an added source related to the initial uncertainty about project quality. This acts towards reducing the equally weighted return by determining the entry of many firms of poor quality which are later selected out. Summarizing, while there is more entry in the model without learning, this is determined by returns on private equity that are higher and exceeding those on the stock since the early years of a firm's life.

This is confirmed by Table 5. The lower panel of the table reports the fraction of entrants that expect a net income loss upon making the decision

[16] This result can be proved analytically, see Harvey (1989).

Table 4: Entrepreneurial portfolio and returns

| | Baseline | No learning | Low $\underline{K}$ |
|---|---|---|---|
| Std. of returns | 36.4 % | 36.1 % | 35.9 % |
| Public equity return | 7.0 | 7.0 | 7.0 |
| Private equity return (e-w) | 8.3 | 18.9 | 7.0 |
| Private equity return (v-w) | 24.1 | 23.9 | 26.2 |
| Share of private equity: (e-w) | 32.5 | 53.8 | 29.9 |
| Share of private equity: (v-w) | 45.6 | 47.7 | 46.1 |

Note: e-w stands for equally weighted, v-w stands for value weighted.

to become entrepreneurs in the model without learning. What we can see is that among agents with the lowest labor earnings shock only 1.8 percent decide to enter expecting a net loss — compared to the alternative occupation — in the first year of business and among agents with the median earnings shock the figure is 3.5 percent, down from 20.5 and 62.7 percent in the model with learning. [17] Overall this result confirms the intuition that learning plays an important role in determining entry: once it is removed from the model the choice of becoming entrepreneur with the expectation of an immediate income loss over paid-employment becomes a very unfrequent outcome.

The fact that there is a non negligible fraction of agents with the highest earnings shock that are willing to become entrepreneurs even if they expect an immediate income loss in less easy to interpret. One possibility is that since these agents are likely to also have substantial financial wealth, they can reap the diversification benefits from having access to a second asset. This points to

[17] The comparison of this figure for agents with the highest labor earnings shock is not meaningful because in the model with learning there are so few entrants that the figure reported in Table 2 in this case is statistically unreliable.

Table 5: An analysis of entry

| Low shock | Intermediate shock | High shock |
|---|---|---|
| Frequency of entry by earnings shock: No learning | | |
| Low shock | Intermediate shock | High shock |
| 6.4% | 2.7% | 1.9% |
| Percentage of entrants with expected income loss | | |
| 1.8 | 3.5 | 28.3 |
| Frequency of entry by earnings shock: low $\underline{K}$ | | |
| Low shock | Intermediate shock | High shock |
| 12.9% | 0.2% | 0.0% |
| Percentage of entrants with expected income loss | | |
| 14.9 | 58.6 | 50.0 |

a complementary explanation to the choice of becoming entrepreneur that I do not further explore here because it is outside the scope of the paper. Finally the model without learning produces very counterfactual results in terms of firm dynamics: as it can be seen in Table 6, the survival rates are 95 percent or more at both horizons, the exit rate is only 3.8 percent and the average fraction of entrepreneurs is 32.9 percent, all figures that are very far from their data counterparts.

Next I examine the effects of changing the minimum equity requirement. This is done because the size of equity in reality is endogenous while in the model this requirement makes the amount of equity exogenous for those firms with an expected return below the stock. Analyzing the effects of a change in the value chosen for this parameters is meant to insure that the main results of the model are robust with respect to this choice. I consider a calibration that is like the baseline model except that $\underline{K}$ is reduced to half its original value or

Table 6: Entrepreneurial dynamics

| | Baseline | No learning | low $\underline{K}$ |
|---|---|---|---|
| Average entry rate | 2.4 | 3.3 | 3.3 |
| Survival rate (5 years) | 38.9 | 98.0 | 40.6 |
| Survival rate (10 years) | 29.5 | 94.6 | 31.5 |
| Exit rate | 14.0 | 3.8 | 12.9 |
| Fraction of entrepreneurs | 10.7 | 32.9 | 15.0 |
| Average firm age | 9.4 | 13.9 | 10.3 |

equivalently from 20000 to 10000 dollars. Intuitively, everything else equal, this reduces the initial opportunity cost of becoming entrepreneur and should draw in the occupation agents with projects offering lower returns. This intuition is supported by the results. As we can see in the third column of Table 6 the entry rate goes up to 3.3 percent, while the survival rates at 5 and 10 years slightly go up to 40.6 percent and 31.5 percent. Looking at Table 4 one can see that the equally weighted return on private equity is slightly reduced to 7.0 percent (down from 8.3 percent in the baseline case) and that a conditional share in private equity of 29.9 percent is supported by that premium. The value weighted return is 26.2 percent corresponding to a premium of about 19 percent over the public equity index. A look at Table 5 shows that 12.9 percent of agents with the lowest earnings shock decide to enter, while this percentage drops to 0.2 percent among agents with the intermediate shock and to virtually zero for agents with the highest shock. The percentage of entrants that expect a net income loss from the choice to become an entrepreneur is 14.9 percent among those with the lowest paid employment earnings shock and 58.6 percent among those with the median shock. Overall all the results obtained when $\underline{K}$ is reduced are very similar to the baseline case confirming that the particular choice of the value of the minimum equity requirement is not driving the results.

### 4.3 Sensitivity Analysis

In the present section I describe the results of two sensitivity analysis conducted respectively on the risk aversion parameter of the agents and on the risk entailed by private equity. The experiments involved in these two sensitivity analysis consist of changing the parameter under study and then readjusting the constants $\lambda_1$ and $\lambda_2$ so that the model still matches the entry rate into entrepreneurship and the average equally weighted share invested in private equity conditional on an agent being an entrepreneur. In this way we can still interpret the return on private equity produced by the model as the one needed to explain the decision to become self-employmed and the associated investment choice observed in the data.

The results of the first sensitivity analysis are reported in Table 7. In the first two lines the model standard deviation of individual firms' equity returns and the return on the stock are reported. These are respectively about 35 percent and 7 percent reflecting the fact that they match their calibration target. For the same reason in the fifth line the value of the equally weighted conditional share invested in private equity is about 32 percent in all the three risk aversion cases. The main result of the sensitivity analysis is expressed in the third line. In the second column we can see that the average equally weighted return on private equity when all agents have a risk aversion coefficient of 2 is 2.9 percent; this corresponds to 4 percentage points less than the return on the stock so in this case we find that the entrepreneurial choice can be justified even with

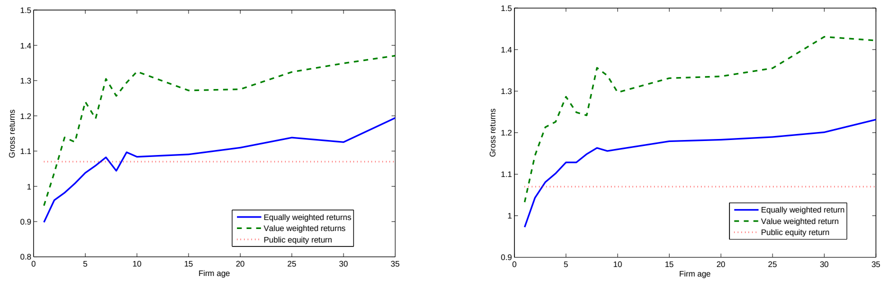


**Figure 3: Return by age of the firm:** Low risk aversion (Left panel), high risk aversion (right panel). Baseline risk of private equity. Data are plotted yearly for firms less than five years old and by 5 years groups for older firms.

a negative private equity premium for the average entrepreneur. When risk aversion is raised to 5 instead, the equally weighted average return to private equity reaches a value of 11.3 percent, thus exceeding the one on the stock by 4.3 percentage points. Although this is a sizeable excess return, still it does not invalidate the main result about entrepreneurial choice. This can be seen by looking at the right panel of Figure 4.3. The figure reports average returns — equally and value-weighted — on private equity by age of the firm. The right panel shows that even in the case where all agents have a risk aversion of 5, the average returns on firms aged 3 or fewer years is still below the one on the diversified stock. This implies that for the "typical entrant"the expected return on the financial investment is below the one on public equity. The fourth and last line of the table report the value weighted private equity return and average allocation to business asset respectively. As we can see the return on the index of private equity goes down to 22.8 percent when risk aversion is 2 for all agents and goes up to 27.4 when it is 5. These numbers imply an excess return above the stock of about 15 and 20 percentage points. The associated conditional

Table 7: Entrepreneurial portfolio and returns: Changing risk aversion.

| | Baseline | RA=2 | RA=5 |
|---|---|---|---|
| Std. of returns | 36.4 % | 35.0 % | 37.4 % |
| Public equity return | 7.0 | 7.0 | 7.0 |
| Private equity return (equally-weighted) | 8.3 | 2.9 | 11.3 |
| Private equity return (value-weighted) | 24.1 | 22.8 | 27.4 |
| Share of private equity: (equally-weighted) | 32.5 | 32.4 | 31.8 |
| Share of private equity: (value-weighted) | 45.6 | 51.1 | 38.7 |

weighted allocation to private equity are 51.1 percent when risk aversion is 2 and 38.7 percent when risk aversion is 5.

Table 8 reports the results of the other sensitivity analysis, the one on the risk of holding private equity. In particular here we assume a lower value of the standard deviation of the returns on private equity. This is reflected in the first line of the table where we can read a standard deviation of firms' returns of about 24 percent. As usual in the fifth line the equally weighted share of private equity in entrepreneurial portfolio is showed and once again the value is at its 32 percent calibration target. The equally weighted average return on private equity is 4 percent when half of the agents have risk aversion of 2 and half have risk aversion of 5, it is 1.4 percent when all agents have risk aversion of 2 and it is 6.9 percent when they all have risk aversion 5. This implies that a negative average premium of private over public equity is consistent with the pattern of entry and un-diversified investment in business assets observed in the data. Clearly, as it was pointed out before, this also means that for a large number of entrepreneurs entry and duration in the occupation do occur in the face of a negative expected premium of business assets compared to the diversified stock.

When we look at value weighted figures we see that the value weighted return on private equity — equivalently the return on the index of aggregate private equity — ranges from 14.3 percent when all agents have low risk aversion to 17.6 percent when they all have high risk aversion, implying a premium over the return on the financial asset between 7 and 10 percentage points. The associated value-weighted shares of portfolio allocated by entrepreneurial households to assets in their business range from 44.8 to 58.6 percentage points. Table 9 reports the fraction of agents that decide to become entrepreneur by earnings shock. As it can be seen, in all cases that probability is close to 9 percent for agents with the lowest earnings shock, it is between 0.1 and 0.2 percent for agents with the intermediate shock and it is virtually 0 for agents with the highest earnings shock. Table 10 reports the percentage of entrants that decide to do so even if they expect a net loss from becoming an entrepreneur compared to working for pay. This percentage is significant in all cases considered: among agents with the lowest earnings shock — that form the vast majority of entrants — it ranges from 10.7 percent in the low risk, low risk aversion case to 25.8 percent in the high risk, high risk aversion case, among agents with the intermediate earnings shock it is above 45 percent in all cases.

Summarizing, the sensitivity analysis confirms the main results obtained under the baseline parametrization, that is, entry into self-employment even in the face of a negative expected premium on the assets invested in the business compared to public equity can be easily rationalized. On the other hand the assessment of the private equity premium gives figures that are quite large, always above 7 percent, leaving unexplained this finding of Moskowitz and Vissing-Jørgensen's (2002) paper. Entry is concentrated among agents with the lowest

Table 8: Entrepreneurial portfolio and returns: Reduced risk of private equity.

| | Baseline | RA=2 | RA=5 |
|---|---|---|---|
| Std. of returns | 23.9 % | 23.5 % | 24.4% |
| Public equity return | 7.0 | 7.0 | 7.0 |
| Private equity return (equally-weighted) | 4.0 | 1.4 | 6.9 |
| Private equity return (value-weighted) | 15.5 | 14.3 | 17.6 |
| Share of private equity: (equally-weighted) | 32.2 | 31.8 | 31.9 |
| Share of private equity: (value-weighted) | 53.0 | 58.6 | 44.8 |

Table 9: Frequency of entry by earnings shock

| | High risk | | |
|---|---|---|---|
| | Baseline | RA = 2 | RA = 5 |
| Low shock | 8.9% | 8.6% | 9.0% |
| Intermediate shock | 0.2 | 0.2 | 0.2 |
| High shock | 0.0 | 0.0 | 0.0 |
| | Low risk | | |
| Low shock | 9.1 | 9.3 | 9.1 |
| Intermediate shock | 0.1 | 0.1 | 0.1 |
| High shock | 0.0 | 0.0 | 0.0 |

labor earnings shock, however a non negligible fraction of those who decide to become entrepreneurs do so even if they expect a net loss in the first year of business operation, suggesting that the option value created by the learning mechanism is a relevant feature in the decision.

## 4.4 Discussion

In this section I will briefly discuss two issues. First I will compare the present research to the one that is more closely related to it, that is, the one by Hin-

Table 10: Percentage of entrants with expected net loss

| | High risk | | |
|---|---|---|---|
| | Baseline | RA = 2 | RA = 5 |
| Low shock | 20.5% | 12.4% | 25.8% |
| Intermediate shock | 62.7 | 53.9 | 58.9 |
| High shock | 18.1 | 66.7 | 33.3 |
| | Low risk | | |
| Low shock | 11.3 | 10.7 | 12.2 |
| Intermediate shock | 58.3 | 77.8 | 44.8 |
| High shock | 25.0 | n.a. | 66.7 |

termeier and Steinberger (2005). Second I will mention the omission of a third safe asset from the menu of available investment options. With respect to the first issue Hintermeier and Steinberger (2005) present a life-cycle occupational and portfolio choice model with idiosyncratic earnings and private equity risk to study the issue of the decision to invest in a private business. Although this is similar to the current research the two follow radically different approaches to model the occupational decision problem. In Hintermeier and Steinberger (2005) the earnings process is the same in the two occupations so that indeed the occupational choice problem is reduced to a portfolio choice problem where the two risky assets differ in their return process and participation costs: a one-time monetary entry cost for private equity and a very large per period participation cost for the stock. Also in that model all agents, conditional on a given labor earnings realization face the same distribution of private equity returns which, if they decide to enter is then immediately revealed. In the current model, on the contrary, the earnings process is different and independent across the two occupations so that the employment decision is a true occupational choice, not a simple asset market participation decision. Moreover expected business pay-offs at the time the decision to enter is made differ across agents because different agents receive different signals. The structure of the model implies that in Hintermeier and Steinberger (2005) all agents that have sufficiently high earnings realizations and enough wealth to pay the entry cost will decide to become entrepreneurs. In the current model this is not the case: there are wealth and/or earnings rich households who do not consider starting a business and poorer ones that do. The threshold level of wealth and earnings will depend on how good the signal is. Secondly since in Hintermeier and Steinberger (2005) the shock to earnings also enters as a component of the private equity return it will be the households with the best labor earnings realizations that will quit paid employment to move to self-employment. In the current model it is the reverse: households with lower earnings realizations will require lower signals to enter, hence they will do so more often. Under both of these dimensions the current model is then closer to the empirical evidence: the second fact is supported for example by work of Evans and Leighton (1989) and Blanchflower and Meyer (1994). With respect to the first fact Hurst and Lusardi (2004) found that the probability of becoming an entrepreneur is increasing in wealth but only slightly so: it is neither very high even for the wealthiest nor zero at low levels of wealth. The different assumptions also have different implications for the mechanism that generates entry even in the face of a negative excess return on business assets compared to the stock. In Hintermeier and Steinberger (2005) this is obtained thanks to the large per period stock market participation cost. This cost is set at 17 percent of annual average consumption, thus making the choice of the stock unviable except for very wealthy agents so that for a majority of households the choice is indeed between the bond and private equity and the latter does pay a premium on the former. In the current model instead this trick is not available and all households, no matter what their wealth is, do face the choice between investing in undiversified private equity and the diversified stock. The key elements that explain the result are the minimum equity requirement, the properties of the occupation specific earnings process and the information structure implied by the learning model. The minimum equity requirement prevents households from trying at entrepreneurship without investing financial resources in it. At the same time the fact that the earnings process is different for the two occupations implies that an agent that received a bad earnings shock as a worker expects better earnings in the following period if he switches to self-employment. Second there is an option value of learning: the agent may wait in business even in the face of a temporarily poor return on the investment because staying is needed to collect new information about the project which could potentially be profitable.

As in Polkovnichenko (2003) and Hopenhayn and Vereshchagina (2003) but unlike Hintermeier and Steinberger (2005) the model presented here abstracts from a safe asset. The main results of the paper though would not change if a third asset was introduced. This is because such an asset would be available to households in both occupations, hence it would not affect the relative benefits of the two choices. Moreover it was shown that the choice to become entrepreneur despite a negative expected premium of undiversified business equity over public equity was obtained under a broad set of risk aversion parameters. This set included, as a baseline, one with heterogeneous risk aversion that Gomes and

Michaelides (2005) showed it is able to explain the observed choice between bonds and stock in a similar life-cycle portfolio allocation model with reasonable stock market participation costs. As a result the findings of the model do not depend on assumptions about risk aversion that would be inconsistent with dimensions of the household financial choices that are omitted here. Although not relevant for the results concerning the decision to become self-employed, the introduction of a risk free bond would clearly allow to study in more details the portfolio choices of entrepreneurs but this is beyond the scope of the present research.

## 5 Conclusions

In the present paper I have constructed a life-cycle occupational and portfolio choice model where agents can be paid or self-employed and conditional on the latter decide how much to allocate between business equity and a risky stock. Self-employment is characterized by running a project whose quality is ex-ante heterogeneous and unobservable. The uncertainty about project quality is reduced over time by observing the realized returns. The model is used to explore the relationship between private equity returns and entrepreneurial choice in light of the empirical findings of Moskowitz and Vissing-Jørgensen (2002) and subsequent research that tried to rationalize them. The model shows that entry and private equity allocation for the majority of entrepreneurs can be rationalized even with negative expected premia on individual business investment. The paper also provides an estimate of the required premium the index should display to rationalize the behavior of the top of the private equity distribution that makes the largest part of the index itself and finds that this premium should be in the order of 10 percent thus confirming the puzzling nature of Moskowitz and Vissing-Jørgensen's (2002) result.

The model has some limitations, of which the most important is perhaps the fact that our knowledge of the actual risk entailed by entrepreneurial investment is quite limited due to the poor quality of the data, typically survey based. For example Moskowitz and Vissing-Jørgensen (2002) suggest a 60 percent standard deviation of individual private equity returns.[18] However that figure is based on the average daily volatility of publicly traded stocks while private companies are by definition non traded and there are no direct estimates of their return volatility. In the calibration we tried to make the cross section of returns similar to the one in the data but that does not tell us how much of it is business risk and how much comes form fixed differences across firms. This suggests a potentially fruitful, yet challenging, area for future research.

[18]This value is based on work by Campbell at al. (2001).

# Appendix

In this appendix I describe the numerical methods used to solve the model. Computation of the model requires two separate steps: first the numerical solution to the household's dynamic programming problem and second the simulation of the artificial economy.

**The Numerical Dynamic Programming** The optimization problem during working age has 4 state variables. One of them, that is, the number of signals on project quality is discrete by nature, while the remaining three are continuous. These are the paid employment labor earnings shock $z$, the variable describing the mean of the posterior distributions of project quality $\hat{x}$ and assets. The labor earnings shock is discretized using the method in Tauchen (1986) and taking a 3 point grid. With respect to the variable $\hat{x}_t$ I evaluate the value function at 7 points. For any of these points, the law of motion (9) and the recursive formulas (10) and (11) tell us the distribution of $\hat{x}_{t+1}$. The integration of the RHS of the value function with respect to this distribution is performed using 5 point Gauss-Hermite quadrature. Since the integration points do not fall on the grid where the value function is computed I resort to cubic spline approximation to evaluate the integral. Finally with respect to assets I use a 65 point grid that is finer close to the origin and coarser away from it. This is obtained by dividing the domain $[\underline{d}, \overline{d}]$ into a small interval close to the origin $[\underline{d}, d_m]$ and a large interval $[d_m, \overline{d}]$. In the interval $[\underline{d}, d_m]$ I lay 25 uniformly spaced points. As for the interval $[d_m, \overline{d}]$ I first take the cubic root of its extreme points, then I lay 40 uniformly spaced points in the interval thus obtained and finally I take the third power of those points. In this way I can make the intervals grow smoothly in size as we move away from the origin. Interpolation between grid points is performed again by cubic splines.

The maximization of the RHS of the Bellman equation at each state space point must be performed twice, once to compute the value of being an entrepreneur $V_t^{se}(d_t, z_t, \hat{x}_t, a)$ and then to compute the value of being a paid employee $V_t^{pe}(d_t, z_t, 0, 0)$. Then once this is done for all the state space points the upper envelope is taken as the value function. The computation of the two functions $V_t^{se}$ and $V_t^{pe}$ implies the solution of a two variable and a one variable maximization problem respectively. The one variable maximization problem is performed using Brent's parabolic interpolation method while the two variable maximization is performed by exploiting the fact that for any function $g(x_1, x_2)$ we can write that $\max_{x_1,x_2} g(x_1, x_2) = \max_{x_1} \left\{ \max_{x_2} g(x_1, x_2) \right\}$ and then applying again Brent's method along each direction. The advantage of using Brent's method is that its convergence is super-linear, hence it is faster than bisection, but it does not require concavity of the objective function as Newton-type methods, a property that is violated here because of the kinks introduced by the occupational choice.[19] The solution to the dynamic programming problem delivers decision rules for occupation, consumption and holding of each of the two assets.

**The Simulation** The economy is simulated as an Overlapping Generation model. This means that I take a set of 10000 agents and simulate the evolution of the economy for 1100 periods. As agents reach real age 35 a new agent that we may think as the descendant is introduced in the economy. Since a few agents do not reach the age of 35 some new agents are introduced in each period to

[19]See Brent (1973) for an illustration of the method.

keep the population constant. The first 600 periods are discarded, then every 25 periods I extract a cross section and compute the relevant statistics on that cross section. Results presented in the paper are obtained by averaging over the 20 cross section thus sampled.[20]

An important issue in the simulation is the way actual decisions are computed. Normally the most straightforward way of performing the simulation is to use piecewise linear interpolation of the decision rules. In the case of the present model though the need to treat the variable $\hat{x}$ as a continuum introduces some complications. In particular there will be numerous instances where the occupational decision switches when moving from the grid point $\hat{x}_i$ to the next one $\hat{x}_{i+1}$. Given that the distribution of the $\hat{x}$ variables is log normal there is no obvious way how to randomize and assign arbitrarily the participation decision in these cases. For this reason when such cases occur I compute the simulated decisions by resolving the optimization problem using the value function obtained in the dynamic programming stage. When instead the occupational decision does not switch between two state space points I resort to the usual piecewise linear interpolation. This method, although safe, slows the program quite considerably, to the point that the time of a single simulation is in the order of the hours. This must be taken into account when considering some of the simplifying modeling choices adopted in the paper and the fact that the calibration of parameters chosen to match some key statistics of the US economy does not occur through a systematic search over a grid but by a rougher trial and error procedure.[21]

[20] To check the stability of results the model was also simulated for 2000 periods discarding the first 1500 and delivered results that are almost identical to the one reported in the paper.

[21] Observe that in the model there are 4 parameters that are calibrated to match some

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targets. Even assuming a grid with only 5 points in each dimension this would imply 625 simulations, hence running times in the order of 100 days.

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