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# When does Financial Leverage Create

# **Economic Value?**

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#### **Abstract**

This paper addresses an overriding question in the theory of capital structure concerning how external financing contributes to economic value creation. Adjusted Present Value rule for capital-investment decisions (see [5], [6]) is used as performance metric of added value. This frame spotlights the contribution to added value attributable to each financing source. We show that levered project and financial leverage add value, at any debt level, if Net Present Value of the investment project and Net Present Value of debt are both positive. However if Net Present Value of debt is negative, external financing destroys economic value and should be taken at the minimum necessary.

**Keywords**: Financial Leverage; Project finance; Adjusted Present Value; Cost of Equity

### 1 Introduction

An overriding issue in numerous disciplines, including corporate finance, engineering economy and, in general, financial risk management is the theory of capital structure. In this paper, we focus on effects of financial leverage on profitability of levered projects. The economic value created is measured by Adjusted Present Value rule for capital-investment decisions pioneering proposed by [5], see also [6] for a review.

Following [6] we break down economic value creation in two components: (1) one imputable to the profitability of the project; and (2) the other ascribable to the economic value created by external funds. A key factor in evaluation of value creation is the Cost of Equity (COE). Financial leverage turns out profitable only if debt creates value, i.e. Net Present Value (NPV) of debt cash flow is positive. In this circumstance, even an unprofitable project may switch to be advantageous if properly financed by equity and debt mix. Vice versa if the NPV of debt cash flow is negative, financial leverage destroys value and should be taken at minimum as possible.

The remainder of the paper is organized as follows. In Section 2 we introduce basic notation. In Section 3 we discuss conditions under which leverage creates additional economic value. Section 4 concludes.

### 2 Basic notation

Let us consider an economic agent (i.e. a firm) facing the opportunity of investing in a financial project A that promises at time  $t_s$ , with s=0,...,n the cash-flow  $a_s$  with the usual convention that  $a_s < 0$  means that at time  $t_s$  there is a money outflow, while  $a_s > 0$  a money inflow and  $a_s = 0$  no cash movement. For simplicity but without loss of generality, we assume:

- a single project, with initial time  $t_0 = 0$  and the initial project outlay is  $a_0 = -1$ ;
- for an initial unitary borrowing  $f_0 = 1$  at time  $t_0 = 0$ , the paying back requires payments  $f_s \le 0$  at times  $t_s$  where s = 1, ..., n.

Let the investment project is  $\alpha \cdot 100\%$  debt financed and  $(1-\alpha) \cdot 100\%$  equity financed, with  $0 \le \alpha \le 1$ . If  $\alpha = 0$  project is all equity financed; whereas if  $\alpha = 1$  it is all debt financed.

The *debt financing cash-flow* reads  $[\alpha \cdot 1, \alpha f_1, ..., \alpha f_n]$ , where  $f_s \le 0$  for s = 1, ..., n. The equity invested at  $t_0 = 0$  is  $e_0 = a_0 + \alpha f_0 = -1 + \alpha = -(1 - \alpha)$ . So, the *equity cash-flow* generated by the project at time  $t_s$  is

$$e_s = a_s + \alpha f_s$$
 for  $s = 0,...,n$ .

We can now write the Discount Cash Flow (DCF) of:

- the project cash-flow A:  $DCF_A(x) = a_0 + a_1(1+x)^{-t_1} + \dots + a_n(1+x)^{-t_n}$ ;
- the debt cash-flow  $\alpha D$ :  $DCF_{\alpha D}(x,\alpha) = \alpha f_0 + \alpha f_1 (1+x)^{-t_1} + \dots + \alpha f_n (1+x)^{-t_n};$
- the *equity cash-flow*, generated by the  $\alpha \cdot 100\%$  debt financed project:  $DCF_{A+\alpha D}(x,\alpha) = (a_0 + \alpha f_0) + (a_1 + \alpha f_1)(1+x)^{-t_1} + .... + (a_n + \alpha f_n)(1+x)^{-t_n}$ .

# 3. Financial leverage and extra economic value creation

For all-equity financed projects the conventional method to quantify the created wealth is the *Net Present Value* (*NPV*) (see [1] and [2] among others). Specifically, denoted with *i* the *discount rate*, let us consider the following cases:

• if  $\alpha = 0$ , all-equity financed project A generates economic value given by

$$NPV_{A}(i) = a_0 + a_1(1+i)^{-t_1} + \dots + a_n(1+i)^{-t_n}$$

• if  $0 < \alpha \le 1$ , the project A is  $\alpha \cdot 100\%$  debt financed. Following the seminal ideas of [5], we measure the economic value created by the Adjusted Present Value (APV). As argued by [6], APV is useful in project financing, where debt is issued in one-shot specifically for the project and paid down on a predetermined schedule. APV is defined as NPV of the equity cash-flow:

$$APV_{A+\alpha D}(i) = (a_0 + af_0) + (a_1 + af_1)(1+i)^{-t_1} + \dots + (a_n + af_n)(1+i)^{-t_n}$$
$$= \sum_{s=0}^{n} a_s (1+i)^{-t_s} + \alpha \sum_{s=0}^{n} f_s (1+i)^{-t_s}$$

Re-writing the above formula we get

$$APV_{A+\alpha D}(i) = NPV_A(i) + \alpha NPV_D(i)$$
, with  $0 \le \alpha \le 1$ , (1)

where A stands for the investment project and D for the unitary debt cash stream<sup>1</sup>.

<sup>&</sup>lt;sup>1</sup>  $NPV_D(i)$  is the NPV generated by the discounted cash-flow generated by an unitary debt  $f_0 = 1$ 

Clearly, if  $\alpha = 0$  the project is all-equity financed and  $APV_{A+0\cdot D}(i)$  reduces to  $NPV_A(i)$ ; and if  $\alpha = 1$  the project is all-debt financed and  $APV_{A+1\cdot D}(i)$  reduces to the sum  $NPV_A(i)$  and  $NPV_D(i)$ .

The discount rate i plays a crucial role in the APV evaluation and the appropriate discount rate is the Cost of Equity  $(COE)^2$  for the period from 0 to  $t_n$ . Discounting with the weighted average cost of capital (WACC) may lead to severe distortions because the debt cost is already incorporated in the debt financing stream  $\left[\alpha,\alpha f_1,...,\alpha f_n\right]$ , (see [4], [3]).

From now on we use the shorthand notation  $NPV_A = NPV_A(COE)$  and  $NPV_D = NPV_D(COE)$ . A *sufficient and necessary* condition guaranteeing that additional debt creates extra-positive economic value is simply given by

$$NPV_D > 0$$
.

In words, debt conditions must be profitable. Nevertheless, the above condition is *not sufficient* to guarantee that the levered project  $A + \alpha \cdot D$  adds economic value, i.e.  $NPV_{A+\alpha \cdot D} > 0$ . Let us consider the following cases:

- 1.  $NPV_D > 0$  and  $NPV_A > 0$ . As seminally pinpointed in [7], if both addenda in (1) are positive then that is *sufficient and necessary* to guaranteeing the levered project  $A + \alpha \cdot D$  profitability. Then the higher the leverage the higher the economic value, at *any leverage level*  $\alpha$ .
- 2.  $NPV_D > 0$  and  $NPV_A = 0$ . The unlevered project A neither creates nor destroys economic value; on the other hand debt creates value. So, any levered project creates positive value and the optimal leverage is obtained by debt financing the project as much as possible.
- 3.  $NPV_D > 0$  and  $NPV_A < 0$ . Although the *unlevered project A is not profitable* and destroys value, the use of external funds may create economic value depending on the debt repayment conditions. Let us distinguish the following cases:

with discounting rate *i*. Then the NPV of debt is  $NPV_{Debt}(i) = \alpha \cdot NPV_D(i)$ .

<sup>&</sup>lt;sup>2</sup> This interest rate *i* must be seen as the interest rate at which equity could be invested outside the project and at which reinvestments will be made for the (net) cash inflows earned by the project. If *COEs* are variable over the period, the APV approach can be extended to non-flat structure via the GAPV (generalized APV) pionerred by [7].

- 3.a) if  $\alpha > -NPV_A/NPV_D$ , the positive economic value generated by the external funding (i.e.  $NPV_D > 0$ ) compensates the negative value induced by A (i.e.  $NPV_A < 0$ ). It follows that although the project is unprofitable, leverage adds economic value;
- 3.b) if  $\alpha \le -NPV_A/NPV_D$ , the positive value generated by the external funding is lower (or equal to) than value loss imputable to A. Levered projects make negative or null value.
- 4.  $NPV_D = 0$ . Leverage has no effects on value creation, so the debt is sizable according to the desired target;
- 5.  $NPV_D < 0$ . Debt destroys value. As a consequence the use of external funds should be taken at the minimum necessary.

### 4. Conclusion

Properly balancing equity with debt is a core question in corporate finance. The aim of this short note is to discuss the debt impact on economic value creation measured by APV method. APV result is broken down in two components, one spotlights the added value made by project, the other the added value created by the debt. Sufficient and necessary conditions to create positive economic value for any leverage, are stated. We show that even a non-profitable unlevered project may turn to profitable if properly levered. However if debt NPV is negative leverage destroys value and should be due to the minimum necessary.

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