Field reparametrization in SMEFT

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15 December 2021

Field reparametrization in effective field theories

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Abstract

Debate topic for Effective Field Theory (EFT) is the choice of a "basis" for dim $= 6$ operators[2]. Clearly all bases are equivalent as long as they are a "basis", containing a minimal set of operators after the use of equations of motion and respecting the $SU(3) \times SU(2) \times U(1)$ gauge invariance. From a more formal point of view a basis is characterized by its closure with respect to renormalization. Equivalence of bases should always be understood as a statement for the S-matrix and not for the Lagrangian, as dictated by the equivalence theorem. Any phenomenological approach that misses one of these ingredients is still acceptable for a preliminar analysis, as long as it does not pretend to be an EFT. Here we revisit the equivalence theorem and its consequences for EFT when two sets of higher dimensional operators are connected by a set of non-linear, noninvariant, field reparametrizations.

Keywords: Higgs physics; Standard Model Effective Field Theory
PACS: 12.60.-i, 11.10.-z, 14.80.Bn

1. Introduction

The construction of the Standard Model EFT (SMEFT) is based on the fact that experiments occur at finite energy and "measure" an effective action $S^{eff}(\Lambda)$; therefore

① whatever QFT should give low energy $S^{eff}(\Lambda)$, $\forall \Lambda < \infty$;

② one also assumes that there is no fundamental scale above which $S^{eff(\Lambda)}$ is not defined [1] and

③ $S^{eff}(\Lambda)$ loses its predictive power if a process at $E = \Lambda$ requires ∞ renormalized parameters [2].

A question that is often raised concerns the "optimal" parametrization of the dim $= 6$ basis; once again, all sets of gauge invariant, dimension d operators, none of which is redundant, form a basis and all bases are equivalent. For a formal definition of redundancy see Sect. 3 of Ref. [3].

There is no principal obstacle in "extracting" (Wilson) coefficients as defined in a particular basis. However, certain linear combinations of Wilson coefficients in one basis become a single Wilson coefficient in another basis and a mapping of this type that put coefficients and (pseudo-)observables in a one-to-one correspondence may seem more appropriate when considering LO constraints from electroweak precision data (EWPD). Even at this level one should be careful since Wilson coefficients mix under renormalization. We will analyze under which conditions the bijection

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[2] *Besides it is an error to believe that rigour is the enemy of simplicity. On the contrary we find it confirmed by numerous examples that the rigorous method is at the same time the simpler and the more easily comprehended. The very effort for rigor forces us to find out simpler methods of proof.* David Hilbert "Mathematical Problems", Bulletin of the American Mathematical Society (Jul 1902), 8, 44

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November 1, 2016

can be realized. Indeed, there is an important lesson to be remembered: it is impossible to redefine the fields in such a way that the Lagrangian is unaltered, this is a privilege of the S-matrix, as it has been stressed in Sect. II.2.3 of Ref. [4], at least in its original version [5, 6]. There are differences between the Lagrangian and the S-matrix elements; although the Lagrangian plays a central role in any theory, observables are related to S-matrix elements. The correct path is: a) computation of off-shell Green's functions, b) normalization of the sources, c) amputation of Green's functions and (finally) d) on-shell limit for external legs.

With this in mind, one has to understand why certain transformations are used/useful. Field transformations of the form $\Phi' = Z_\Phi \Phi$ (Z_Φ being field independent) are a nice way to derive LSZ [7] factors for the external legs (so-called wave-function normalization factors), as it will be discussed in Eq.(3). They are needed to make sure that residues of propagators are 1 also in the interacting theory: this is what it is meant, for instance, by canonical normalization of kinetic terms in SMEFT.

Remark Local and non-local transformations have been used by 't Hooft and Veltman [8] in their proof of renormalizability. Non-local transforms always require the addition of ghost loops [9]. For local transformations those loops are zero in dimensional regularization (integrals of polynomials). In particular one cannot take a free theory and make it an interacting one by means of field transformations. Equivalently we cannot de-interact (even partially) a Lagrangian, see Sect. 10.4 of Ref. [10] for a complete discussion. In other words, making an interaction term (in the Lagrangian) look like what we want does not change the physics of the problem and that term will keep contributing to every process it was contributing before the transformation; we will discuss a simple example with the help of Fig. 7 in Sect. 2.3.1.

2. Field reparametrizations

Up for debate is imposing non-linear transformations on a gauge invariant EFT basis and the correct interpretation of the Equivalence Theorem (ET) [11–13]. Our notations will be as follows (please note the Pauli metric): the scalar field Φ (with hypercharge $1/2$) is defined by

$$\Phi = \frac{1}{\sqrt{2}} \left(\begin{array}{c} H + 2\frac{M}{g} + i\phi^0 \\ \sqrt{2}\, i\,\phi^- \end{array} \right)$$

H is the custodial singlet in $(2_L \otimes 2_R) = 1 \oplus 3$. The VEV is $v = 2M/g$. Furthermore, $\phi^0, \phi^\pm$ are Higgs-Kibble ghosts, sometimes called Goldstone degrees of freedom.

The typical example that we have in mind is the elimination of $H(\partial H)^2$ terms from the Lagrangian, but the question is more general and concerns the interpretation of interaction terms and the idea of rearranging/eliminating certain contributions from the Lagrangian. For instance, starting from a given set of operators and look for a choice of convenient conventions that should allow to calculate physical observables in a "more transparent way" (e.g. operators orthogonally projected or diagonalization in the space of [Wilson coefficients, SM deviations], isolation of dim = 6 effects in the interaction Lagrangian etc.). The question that we want to answer is: can this be done in general and extended beyond LO? What is the price to pay given the fact that the Lagrangian (sic) is not invariant under field reparametrizations that are not gauge transformations? Is it relevant and/or convenient?

For the time being, we start with a simpler example: consider a Lagrangian

$$\mathscr{L} = \frac{1}{2} \lambda^{-2} \phi \left(\Box - m^2 \right) \phi + g\,\overline{\psi}\,\psi\,\phi + ZJ\phi + \overline{K}\,\psi + \overline{\psi}\,K, \tag{1}$$

where ϕ is a scalar field and ψ a spinor field. Furthermore, we have added the source terms; λ reproduces the effect of higher dimensional operators, e.g. in SMEFT when the Higgs field is replaced by its VEV or when loop corrections are included. The (properly normalized) propagation function for the scalar particle is

$$ZJ \frac{\lambda^2}{p^2 + m^2} ZJ, \tag{2}$$

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fixing $Z = \lambda^{-1}$. The net effect on the S-matrix is that, for each external ϕ line, we have a factor λ. Alternatively, we can define a new field, $\phi = \lambda\,\phi'$: the Lagrangian is now

$$\mathscr{L}' = \frac{1}{2}\,\phi'\left(\Box - m^2\right)\phi' + g\,\lambda\,\overline{\psi}\,\psi\,\phi' + Z'\,J\,\phi' + \dots \tag{3}$$

so that $Z' = 1$. However the S-matrix elements has a factor λ for each external ϕ'-line, e.g. due to the coupling $g\,\lambda\,\overline{\psi}\,\psi\,\phi'$. This simple example proves that the field redefinition is a matter of taste, the crucial point is in the normalization of the source.

We now return to field dependent transformations. Few preliminar facts:

① The scalar manifold is not flat [14], therefore one cannot get rid of $\Phi\,(\partial\Phi)^2$ terms (Φ is the Higgs doublet) in a gauge invariant way.

② Any non-linear transformation of H (the Higgs field) will break the symmetries (including any broken symmetry) of the theory.

③ What to expect from a spin one Lagrangian without symmetries? Note that here we are talking about the Lagrangian, see Sect. 2.1 for details.

2.1. Equivalence Theorem in the literature

It is worth noting that Arzt, in his comprehensive paper [15] is always very careful about invariance, e.g. see

  a) statement at p. 7, *The variable shift we have performed respects the symmetries of the theory ... Because of this the new Lagrangian explicitly retains all the symmetries of the original*;

  b) last paragraph at p. 10 where he is, once again, careful in transforming the scalar doublet and not the Higgs field alone.

Indeed, the Equivalence Theorem has been used to derive the Warsaw basis [16], where everything is manifestly gauge invariant, what to do with non-invariant terms? It seems really trivial, you break gauge invariance and, to get the right result, you have to cure your transformation with something equally non invariant. Cui prodest? Nevertheless we will show how things actually work.

One should also be careful in stating that the proof of the ET is that only renormalized theories can be equivalent and it is rigorous only if in both formulations the theory will not contain any divergences [13]: *The basic defects of the known formal "proofs" of the equivalence theorem are that in all of them one compares and asserts the equivalence of unrenormalized quantities, whereas one should compare renormalized quantities. In addition to the physical considerations there are also mathematical reasons for requiring this; owing to the divergences in the Lagrangian field theory the unrenormalized quantities simply do not exist.*

For instance, dimensional regularization is crucial in proving that ghosts are not needed for local transformations (integrals of polynomials in momenta are zero). As pointed out by Kallosh and Tyutin, it remains to be proved that the renormalized charges are the same in both theories. The conditions for that are only realized in a gauge theory.

For completeness we recall the main argument used in Ref. [16]: given a theory with a gauge invariant Lagrangian $\mathscr{L}(\Phi\,\Phi^{\dagger})$ (e.g. the SM) consider an effective Lagrangian

$$\mathscr{L}_{\mathrm{eff}}^{(2)} = \mathscr{L} + a\,\mathscr{O} + a'\,\mathscr{O}', \qquad \mathscr{O}' - \mathscr{O} = \mathrm{F}\,\frac{\delta\mathscr{L}}{\delta\Phi^{\dagger}}, \tag{4}$$

where F is some local functional of $\Phi, \Phi^{\dagger}$ and their covariant derivatives, $\mathscr{O}$ and $\mathscr{O}'$ are gauge invariant, higher dimensional, operators and a, a' are appropriate powers of $1/\Lambda$. The functional F must transform under gauge transformations such that the r.h.s. of the (second) Eq.(4) is invariant. The effect of $\mathscr{O}'$ on the S-matrix corresponding to

$\mathscr{L}_{\text{eff}}^{(1)} = \mathscr{L} + a\,\mathcal{O}$ is to shift $\Phi^\dagger \to \Phi^\dagger + a'\,\mathrm{F}$ (if the field is real replace with $\Phi \to \Phi + a'\,\mathrm{F}$) at $\mathcal{O}(a')$, i.e. we can use

$$\mathscr{L}_{\text{eff}}^{(2)} = \mathscr{L}\left(\Phi, \Phi^\dagger + a'\,\mathrm{F}\right) + (a + a')\,\mathcal{O}\,. \tag{5}$$

The difference between the two Lagrangians is F times the classical equation of motion (EoM) for $\Phi^\dagger$ and the shift $\Phi^\dagger \to \Phi^\dagger + a'\,\mathrm{F}$ respects the symmetries of the theory [15], i.e. $\Phi^\dagger + a'\,\mathrm{F}$ transforms as $\Phi^\dagger$. Therefore, thanks to the ET (which in this case is trivial), we have equivalence of the S-matrices and gauge invariant Lagrangians, both the original and the shifted one. By repeating the shift the process is continued to all orders in $1/\Lambda$. The advantage, i.e. reducing the number of operators while preserving gauge invariance, is self-evident. More details will be given in Appendix 6.

Even in this case we could do without the elimination of redundant operators. Indeed, it has been pointed out in Ref. [17] that, even if the S-matrix elements cannot distinguish between two equivalent operators $\mathcal{O}$ and $\mathcal{O}'$, there is a large quantitative difference whether the underlying theory can generate $\mathcal{O}'$ or not. It is equally reasonable not to eliminate redundant operators and, eventually, exploit redundancy to check *mrS*-matrix elements, see Appendix 6 for details.

2.2. Equivalence theorem and field reparametrization

Actually it is not clear, at all, why one needs to perform non-linear H transformations (H id not a scalar under the group), or any non-linear, non gauge invariant, transformation in the SMEFT (once EoMs have been used); in any case there is no added value. There are three criteria: wrong, irrelevant and relevant. Let us perform a field transformation on a given set of higher dimensional operators and assume that it is formally correct:

i) the ET tells you that the S-matrices are equivalent, ergo the transformation is irrelevant, as long as propagators have residue one at their poles. If one wants, it is even possible to kill the $H\bar{b}b$ interaction term in the Lagrangian (through a non-local transformation, see Eq.(18)), does that mean that we get read of the decay? Obviously not.

ii) If there are stringent arguments to perform the transformation, be aware of the balance between gain and complexity.

On the fact that restricting to a special gauge and using non-linear transformations makes it extremely hard to go to next-to-leading order (NLO), we hope that there is no discussion (otherwise see below).

Gauge dependence Given that it is impossible to get rid of terms with derivatives in some arbitrary gauge, the situation is as follows: somebody goes to the unitary gauge, transform H with a transformation so to get rid of $\mathrm{H}\left(\mathrm{H}^2\right)\left(\partial\mathrm{H}\right)^2$. Somebody else, wanting to play the same (gauge dependent) game, goes to the R_ξ gauge and gets rid of cubic and quartic terms with, at least, one H derivative. This requires a cubic transformations involving H and ϕ^0. How to compare? Working in the unitary gauge is not as simple as setting the Higgs-Kibble ghosts $(\phi^0, \phi^\pm)$ to zero in the Lagrangian, especially when loops are included. It is not our goal to re-describe the problem, see Sect. 3 of Ref. [9] in order to re-discover the issue.

By the time we accept a Lagrangian which is not invariant a crucial test is given by off-shell Ward-Slavnov-Taylor (WST) identities (which means unitarity) or BRST invariance. Even more, an unstable particle is characterized by its complex pole which requires its self-energy computed off-shell. In that case one has to prove that the latter does not depend on the transformation. Let us forget, for a moment, gauge invariance of the Lagrangian and blindly perform the transformation.

Question time ❶ Are the two Lagrangians (original and transformed) the same? No, they are different;

❷ what about off-shell amplitudes, are they the same? No, not even for a gauge invariant transformation.

Remark To do a correct job one has to take into account the transformed Lagrangian, the Jacobian and the new couplings to the sources (it is the LSZ formalism). To study the effect on mass-shell one needs to renormalize Green's functions, i.e. off-shell Green's functions are changed.

2.3. Scattering matrix vs. Lagrangian

What about the S-matrix? To make them equivalent, accepting that the assumption of the theorem are not violated, would require a very complex set of operations that involve the correct treatment of sources (see below). Extending the calculation for any process beyond LO is remarkably difficult.

It is worth noting that in QCD scaleless bubbles are zero, that is why most people tend to forget LSZ. The core of the exercise is to work out the transformed theory within the framework of the original one and to look at the effect of the "new" vertices. To be precise, diagrams can be partitioned into classes where there is complete cancellation after the transformation (use that the Klein-Gordon operator is minus the inverse propagator) but the treatment of sources is crucial for the rest. Therefore, always assuming that it works, one should transmit a complete information for any new "basis" proposal, both the Lagrangians and the field transformations.

The use of EoM in constructing the "Warsaw" basis leaves the Lagrangian invariant, the ET works in the same way in all gauges and we do not need to bother about details of the transformation, the Lagrangian without redundant operators is the "starting point". A subsequent, non invariant, field reparametrization is a different story: with the (transformed) Lagrangian alone one will never be able to perform a complete calculation, the transformation is needed in order to evaluate the Jacobian (trivial only for a local transformation) and the gauge dependent change in the source factors, i.e. the shift in wave-function renormalization. As a matter of fact, even the "starting point" is not free from arbitrariness in the choice of gauge invariant operators. A classification scheme has been proposed in Ref. [3] where it is shown that the "Warsaw" basis satisfies the criterion dictated by the scheme.

The general statement is that a given effect could be modeled by several different combinations of operators at a fixed order in the SMEFT, depending on the basis. This statement, once again, should always be understood as referring to physical observables, i.e. to the S-matrix. Existing tools are often bound to a given basis choice; any platform designed for supporting the reuse of results derived in the context of one (reparametrized) basis in another (original) basis should take this trivial fact into account. Is it optimal to perform field redefinitions? No. Let us clarify this point: for the sake of simplicity we start with a non-gauge theory,

$$W[J] = \int \left[D\phi \right] \exp \left\{ S(\phi) + \int d^4 x J \phi \right\}, \tag{6}$$

where $S(\phi)$ is the action. Obviously we may change the integration variable without changing the integral, $\phi = \psi + F(\psi)$; it is crucial that F starts at ψ^2. Note that $\phi = a \phi$ is standard renormalization, i.e.. LSZ or wave-function renormalization. We obtain

$$W[J] = \int \left[D\psi \right] \det \left(1 + F' \right) \exp \left\{ S(\psi + F) + \int d^4 x J (\psi + F) \right\}, \tag{7}$$

where $F' = \delta F/\delta \phi$. If we have started with and action $S(\psi + F)$ we would have derived

$$W'[J] = \int \left[D\psi \right] \det \left(1 + F' \right) \exp \left\{ S(\psi + F) + \int d^4 x J \psi \right\}, \tag{8}$$

and W is different from W' (of course $W[0] = W'[0]$), not to mention

$$W''[J] = \int \left[D\psi \right] \exp \left\{ S(\psi + F) + \int d^4 x J \psi \right\}. \tag{9}$$

Remark What do we do if we have an action for which we do not know that it is a somehow transformed other action? A Lagrangian does not unambiguously define correlators. Questionable if you take a non invariant action that is the transformation of an invariant one.

Requiring sources to be physical will not help since there is no restriction on the H source in the SM and in the SMEFT, see Eq. (4.1) - Eq. (4.2) of Ref. [9]. Of course, for a complete discussion, see Sect. 4 of Ref. [13], in particular the discussion on wave-function normalization constants in different gauges. As explicitly shown in Ref. [8] the proof of the equivalence theorem is also the proof of unitarity.

Note that, adding to the functional integral approach, we can provide a purely diagrammatic proof; indeed all manipulations carried out with functional integrals can be explicitly checked using the diagram technique, as discussed in Refs. [15, 18, 19] where the special role of dimensional regularization is emphasized. The diagrammatic proof is based on the following identity: consider the transformation $\phi \to \psi + a \psi^2$ and its effect on the part of the Lagrangian quadratic in ϕ,

$$\frac{1}{2} \phi \left(\Box - m^2 \right) \phi \, . \tag{10}$$

We obtain two "special" vertices, with three and four ψ fields, that can be decomposed according to the rule of Fig. 1. and the propagator is $1/(p^2 + m^2)$; special vertices will be denoted by a black triangle. Of course, there will be

$$\overset{\longleftarrow \ p}{\blacktriangleright \cdots \cdots \cdots} = p^2 + m^2$$

Figure 1: Special vertices in a non-linear field transformation.

additional new vertices coming from $\mathcal{L}_{\text{int}}(\psi)$, to be denoted by a black bullet. The rule is easily generalizable to arbitrary polynomial transformations. For instance, in SMEFT, we will have $H \to H + a H^n$ or $H \to H + P(H, \dots)$ where P is a polynomial in the fields and in their derivatives. Note that the special vertices on the r.h.s. of Fig. 1 do not contribute to the S-matrix if the line is external (no one-particle pole): this is how one should understand elimination of $\dim = 6$ contributions to propagators through EoM. Consequences will be relevant when trying to isolate effects in the interaction, as we will show with the example of Eq.(17) and of Fig. 7.

Shortly: a black triangle pointing towards a line cancels the corresponding propagator (shrinking the line to a point) and changes sign of the diagram, adding the proper combinatorial factor, see Appendix 5.

Note that, when moving to SMEFT, the transformation $H \to H + P$ is not a gauge transformation, therefore we loose the group property which means that finite transformations (a) do not follow as a consequence of infinitesimal ones (δa).

Proposition 2.1. *The renormalization constants of the two theories, say $\mathcal{L}(\phi)$ and $\mathcal{L}(\psi + F(\psi))$, are related by*

$$Z_\phi = Z_\psi \left(1 + \Delta Z \right)^2 \, , \tag{11}$$

where Z_ϕ and Z_ψ are such that the ϕ, ψ propagators have residue one at their poles, i.e. one has to be careful and write $Z_\phi^{-1/2} J \phi$ etc. in the equations above. At the same time amputated Green's functions satisfy

$$G_\phi^{(k)} = \left(1 + \Delta Z \right)^k G_\psi^{(k)} \, , \tag{12}$$

which is the main result of Ref. [13], proving the theorem.

The ΔZ factor, as discussed in Ref. [13], must be derived in terms of a vertex Γ computed on the mass shell of the ψ line connecting Γ with the amputated Green's function M, see Fig. 2. Here we have shown only one of the terms that contribute, most terms in the list give zero result because they do not exhibit the one-particle pole, e.g. a source emitting n lines directly connected to M. Therefore, it is formally required to start with

$$W[J] = \int \left[D\psi \right] \det \left(1 + F' \right) \exp\{S(\psi + F) + \int d^4x \, (Z_\psi)^{-1/2} J \psi\} \, . \tag{13}$$
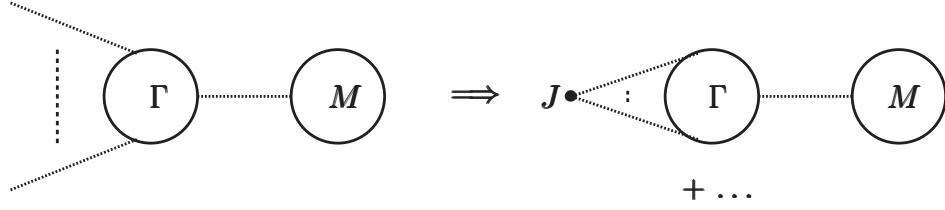
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Figure 2: The vertex Γ needed to construct ΔZ in Eq.(11). M is an amputated Green's function, the lines to the left of Γ correspond to transformed fields ψ, the one connecting Γ with M corresponds to the original field ϕ.

Admittedly, one can neglect the Jacobian if F is local. It is worth noting that Eq.(7) defines the theory in the ϕ framework while Eq.(13) defines the theory in the ψ framework.

Perhaps it is easier to understand the theorem at the diagrammatic level, without having to split the new diagrams into a contribution to the Green's functions or to the source normalization. The ET is based on the fact that diagrams can be allocated to classes within which they cancel, as shown in Fig. 3.
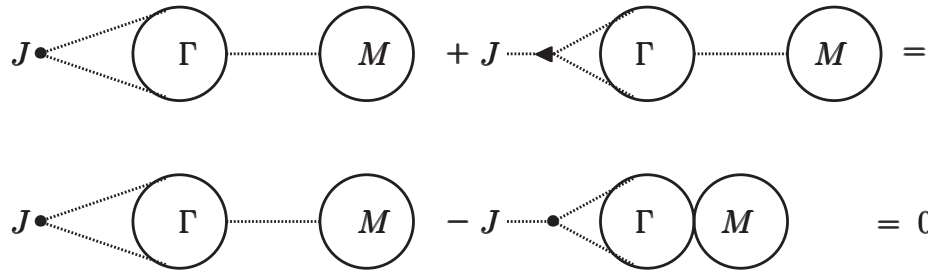

Figure 3: Example of a class of diagrams (exhibiting one-particle poles) where cancellation occurs. The first diagram is part of the change in the source normalization Z while the second gives a change in the Green's function.

2.3.1. The SMEFT case

Generally speaking, one will have to produce a complete account for any pair of bases that are related by a transformation; once again, the Lagrangian alone is not enough. When one takes into account source normalization (e.g. Z_ϕ and Z_ψ) it will be easy to discover that attempting a field redefinition actually makes the whole calculation much, much more involved. Who wants to compute Γ at higher orders? Furthermore, the derivation is completely screwed up if one does not take into account that, now, the source can emit/absorb multi particles (non-linear transformation), something that is not seen from the Lagrangian point of view. We now consider few specific examples.

Off-shell Most of the analyses for off-shell Higgs physics are done (possibly using two different codes) by using off-shell production $\times$ propagator $\times$ off-shell decay. After the transformation the latter, containing diagram b) of Fig. 4, is not gauge invariant, even at LO; in the SM one must go to one loop before seeing the effect of off-shell gauge parameter dependence. See the $pp \to \bar{t}t\bar{b}b$ or the $pp \to HH$ examples, e.g. in $gg \to HH$, as illustrated in Fig. 4 where the two diagrams cancel. In the production $\times$ decay approach it would be wrong to neglect diagram b) using EoM.
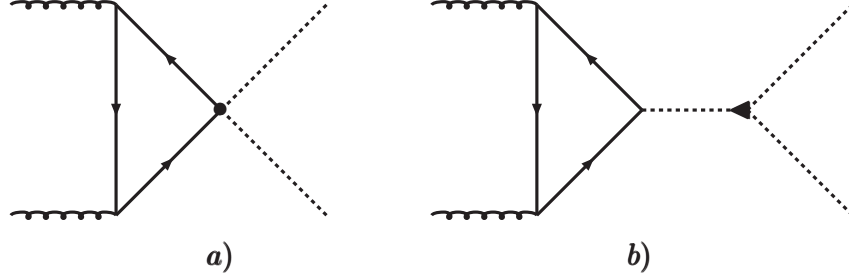
Figure 4: Diagrams contributing to gg → HH after the non-linear transformation, Special vertices are denoted by ● and ◀. The diagram a) is needed to cancel diagram b).

How to deal with higher orders and renormalization is, yet, another problem. Consider the transformation $\phi \to \psi + F(\psi)$, given

$$< \phi(x)\,\phi(y) > = < \psi(x)\,\psi(y) >_o + < \psi(x)\,F(\psi(y)) >_o + \dots \tag{14}$$

one has to prove that the position of the pole of the ϕ propagator coincides with the position of the pole of the ψ propagator (in a spontaneously broken theory). The index "o" in the r.h.s. denotes that the corresponding quantities are computed in the original theory. We will only indicate the main idea, with the essential details,

Tadpoles Let us consider tadpoles. Taking the Higgs potential and performing the transformation $H \to H + g\,a/M\,H^2$ and using $P = \Phi_{\text{pot}} + 1/2\,M_H^2\,H^2$ (where Φ_{pot} is the scalar potential) we obtain

$$P \to P - a\left[2\,\beta_H\,H^2 + \frac{g}{M}\,\beta_H\,H^3 + \frac{3}{4}\,g^2\,\frac{M_H^2}{M^2}\,H^4 + \frac{1}{8}\,g^3\,\frac{M_H^2}{M^3}\,H^5\right] + \mathcal{O}(a^2)\,, \tag{15}$$

where β_H is fixed order-by-order in perturbation theory to cancel tadpoles. Therefore, neglecting all other fields, at one loop we derive

$$\beta_H = -\frac{3}{64}\,\frac{g^2}{\pi^2}\,\frac{M_H^2}{M^2}\,A_0\,(1+4a)\,, \tag{16}$$

where A_0 is the one point function of argument M_H. This is what we obtain by looking at the Lagrangian, i.e.. by including only part of the diagrams. For instance we included the diagrams of Fig. 5 containing a transformed vertex. Obviously, the correct treatment of β_H makes the whole procedure unnecessarily complex.
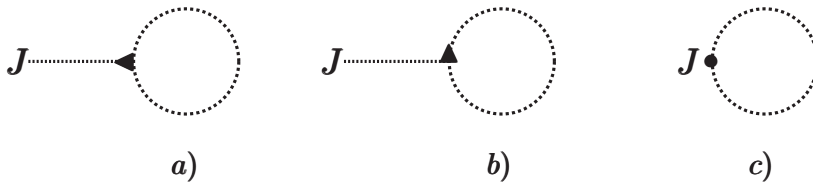

Figure 5: Tadpoles after the non-linear transformation. Diagram b) is zero in dimensional regularization while diagrams a) and c) cancel.

Self-energy The result is also "a" dependent when we carelessly compute the one loop H self-energy (Σ_H), e.g. by including diagram a) and not diagram b) of Fig. 6, plus other, less intuitive, cancellations. Note that Σ_H is needed for renormalization, i.e. for fixing counterterms. Clearly, the road to a unique, renormalized, S-matrix is not simple. N.B. after the transformation, if we perform a generic gauge transformation, there will be tadpoles depending of the parameters of the gauge transformation.
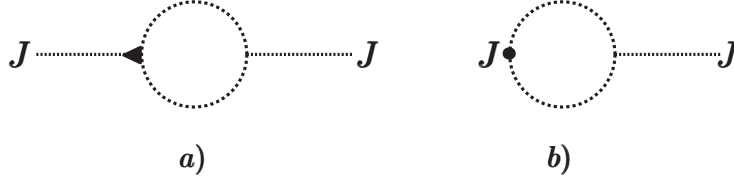
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Figure 6: Example of cancellation in the transformed Higgs self-energy.

2.4. De-interacting/rearranging the Lagrangian?

In this Section we give more details on non-invariant field reparametrizations aimed to modify/rearrange terms in the interaction Lagrangian. Suppose that we want to transform H so that the $\bar{t}tH^2$ term, generated by $\mathcal{O}_{t\phi}$ (see Tab. 2 of Ref. [16]), is eliminated. At $\mathcal{O}(\Lambda^{-2})$ the transformation will be

$$H \rightarrow H + a H^2, \quad a = -\frac{3}{4} \frac{g}{G_F \Lambda^2 M} a_{t\phi}. \tag{17}$$

We can describe the situation with the help of Fig. 7; there is the original diagram a) and the is diagram b) containing a new vertex (•) originating from the interaction Lagrangian after the transformation of Eq.(17). The two cancel by construction. However from the Lagrangian quadratic in H, Eq.(10), we generate diagram c) with a special (◄) vertex (introduced in Fig. 1). Cancellation can be seen both ways, ET is based on the fact that b) and c) cancel, therefore the effect of $a_{t\phi}$ in $\bar{t}t \rightarrow HH$ will survive.

Remark This is precisely the meaning of *the S-matrix does not change*. For the same reasoning the transformation will not change the S-matrix elements for $\bar{t}t \rightarrow HHH$ and $\bar{t}t \rightarrow HHHH$ will remain zero at tree level; after all, $H\Box H^2$ (although generated by $\mathcal{O}_{\phi\Box}$), is not a gauge invariant operator and, more important, cannot be completely eliminated through the EoM. Therefore, if one wants to swap interaction terms this is done at the price of working with a non-invariant Lagrangian even though the S-matrix is the same. The price to pay is a non trivial Z factor. Additional consideration will ge given in Appendix 7.
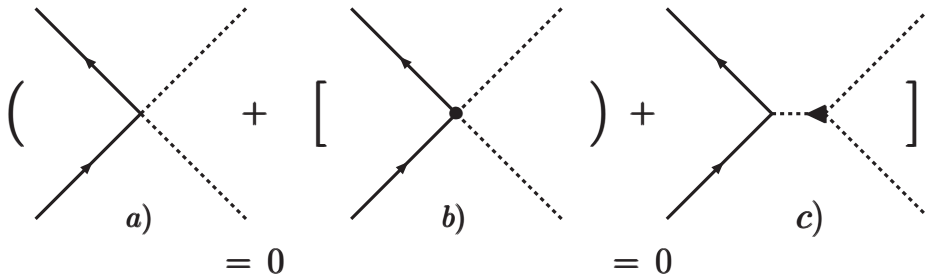

Figure 7: Cancellation, non-cancellation of $\bar{t}tH^2$. The effect of diagram c) is to enforce equivalence at the S-matrix level. Diagrams with special vertices ► on the external legs are not included since they do not contribute to the S-matrix. Therefore, b) + c) = 0.

Non local transformations For completeness we will show that any term of the form HF where F may depend on all the SM fields can always be "cancelled" from the Lagrangian using a non local transformation,

$$H(x) \rightarrow H(x) + i \int d^4y \, \Delta(x-y) F(y), \quad \Delta(z) = \frac{1}{(2\pi)^4 i} \int d^4p \, \frac{\exp\{i\, p \cdot z\}}{p^2 + M_H^2}. \tag{18}$$

It is enough to observe that $(\Box_x - M_H^2)\Delta(x-y) = i\delta^{(4)}(x-y)$. The transformation of Eq.(18) allows us to cancel (in the Lagrangian) all terms, not only those of the form $\Phi^\dagger\Phi$ times a dim $= 4$ operator. In the non local case however, ghosts are required to show equivalence of the S-matrices.

2.5. Additional remark

The source J in Eq.(7) behaves by definition as a scalar under reparametrizations of the corresponding field ϕ. In particular the term $J\phi$ is not a scalar under reparametrizations $\phi = \psi + F(\psi)$, it is therefore impossible to make $W[J]$ defined in Eq.(6) and the effective action, $\Gamma[\Phi]$ of Eq.(19), reparametrization-invariant quantities simultaneously.

$$\exp\{\Gamma[\Phi]\} = \int \left[D\phi\right] \exp\left\{S(\phi) + \int d^4x J(\phi - \Phi)\right\}, \tag{19}$$

To go deeper into the subject we observe that Vilkovisky [20] has argued as follows: by requiring additionally that the effective action be invariant under local invertible changes in the choice of basic field variables, one can construct a natural unique gauge-invariant effective action, i.e. 1PI Green's functions are invariant off-shell (in Vilkovisky approach). In other words, a reparametrization-invariant effective action requires the so-called logarithmic map [20, 21].

A more formal study of field diffeomorphisms for free and interacting quantum fields has been performed in Ref. [22] with the result that the theory is invariant if and only if kinematic renormalization schemes are used.

Finally, what happens if we require reparametrization invariance of the SMEFT Lagrangian? As observed in Ref. [23, 24], in the context of heavy quark effective theory, this leads to severe constraints for the couplings in the effective Lagrangian; in any case, neither the form of the Lagrangian nor the form of the reparametrization transformation is unique.

3. Conclusions

In this paper we have examined the relation between two sets of higher dimensional operators that are connected by non-linear, gauge dependent, field reparametrizations. SMEFT is a quantum construct and there is no fundamental scale and no order in perturbation theory above which SMEFT is not defined [1]. That SMEFT loses its predictive power at some $E = \Lambda$ requiring an infinite number of renormalized parameters is another story; we are not focusing on the NLO numerical impact but on the internal consistency of the theory.

The role of the Equivalence Theorem is often underestimated or even misunderstood, oversimplifying a complex situation; we have illustrated the steps that are needed in order to reproduce the correct S-matrix, a non trivial exercise when gauge dependent reparametrizations are involved. In particular the role of wave-function normalization becomes critical. It is highly desirable that any theory of SM deviations presents its predictions in terms of gauge invariant (pseudo-)observables and not in terms of interaction Lagrangians. There is a huge difference between starting from a phenomenological (limited) set of higher dimensional operators and proving a bijection with a basis.

4. Acknowledgments

5. Appendix: The role of the combinatorial factors

In this Appendix we will give more details about the diagrammatic proof of the ET. Consider a Lagrangian

$$\mathscr{L} = \frac{1}{2}\phi \left(\Box - m^2\right)\phi + \lambda \phi^3. \tag{20}$$

There is only one vertex with 3 lines and equal to $3! \times \lambda$. Performing the transformation $\phi \to \phi + a\phi^2$ (up to $\mathscr{O}(a)$) generates a vertex with 4 lines and equal to $3 \times 4! \times \lambda a$. In Fig. 8 we show the diagrams containing "new" vertices. The 6 diagrams in the left part od Fig. 8 cancel the internal propagator and give $6 \times 3! \times (-2\lambda a)$ while the contact term gives $3 \times 4! \times \lambda a$, i.e. they cancel.

Consider now the one-loop three point function. In Fig. 9 we show the diagrams containing "new" vertices. Each diagram in the first row gives $(3!)^2 \times (-2\lambda^2 a)$. Each diagram in the second row gives $\frac{1}{2} \times (3!)^2 \times (-2\lambda^2 a)$, where $1/2$ is the combinatorial factor. The diagram in the last row gives $\frac{1}{2} \times 3! \times 4! \times \lambda^2 a$ where $1/2$ is the combinatorial factor. The total is zero.
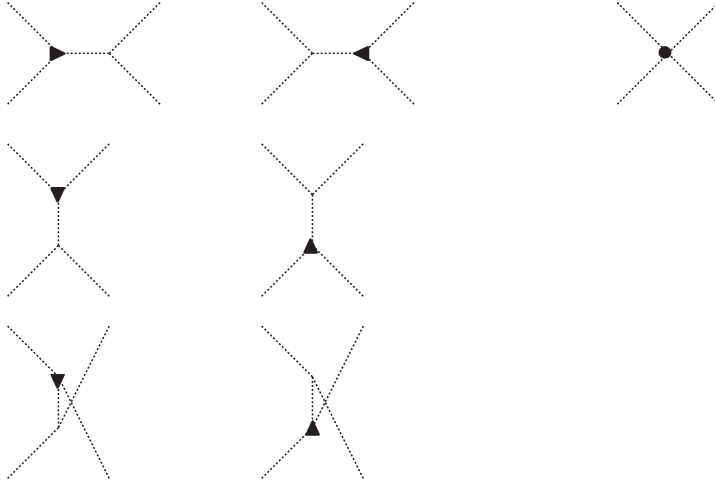

Figure 8: The tree-level four point function in the theory described by the Lagrangian of Eq.(20). The diagrams shown contain vertices generate after the transformation $\phi \to \phi + a\phi^2$

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6. Appendix: More on EoM

The content of Eqs.(4)–(5) deserves additional comments. In Eq.(5) we have two terms, $\mathscr{L}\left(\Phi, \Phi^\dagger + a' F\right)$ and $(a + a')\mathscr{O}$. Does that mean that we can use $\mathscr{L}\left(\Phi, \Phi^\dagger + a' F\right) = \mathscr{L}\left(\Phi, \Phi^\dagger\right)$? Only a posteriori. To study a realistic example, suppose that we enlarge the Warsaw basis by adding a new operator

$$\mathscr{O}_{\phi D}^{(2)} = \left(\Phi^\dagger \Phi\right) \left[\left(D_\mu \Phi\right)^\dagger D^\mu \Phi\right]. \tag{21}$$

As shown in Ref. [16] this operator is redundant and is equivalent, through the EoM, to a linear combination of $\mathscr{O}_\phi$, $\mathscr{O}_{\phi\square}$, $\mathscr{O}_{f\phi}$ and dim $= 4$ operators. The best way of dealing with the dim $= 4$ operators is to perform a shift in the parameters such that the quadratic part of the Lagrangian, after shifting the fields, coincides with the dim $= 4$ SM Lagrangian, see Refs. [25, 26]. If we compute vertices with 3 fields, one of them being H, we observe that $a_{\phi D}^{(2)}$ can be absorbed by shifting

$$a_{\phi\square} \to a_{\phi\square} + \frac{1}{2} a_{\phi D}^{(2)}, \quad a_{u\phi} \to a_{u\phi} - \frac{1}{2} a_{\phi D}^{(2)}, \quad a_{d\phi} \to a_{d\phi} + \frac{1}{2} a_{\phi D}^{(2)}, \quad a_\phi \to a_\phi + \frac{1}{4} \frac{M_H^2}{M^2}. \tag{22}$$
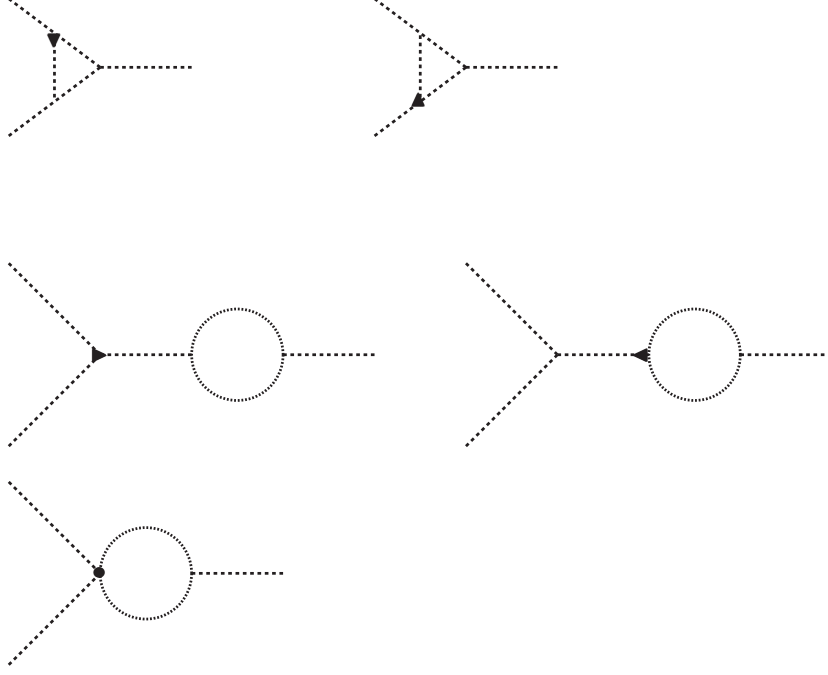
Figure 9: The one-loop three point function in the theory described by the Lagrangian of Eq.(20). The diagrams shown contain vertices generate after the transformation $\phi \to \phi + a\phi^2$

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The request that the functional forms of $\mathscr{L}_2(\text{SMEFT})$ and $\mathscr{L}_2(\text{SM})$ are the same (where $\mathscr{L}_2$ is the quadratic part of the Lagrangian) implies, among other things, the shift

$$M_{\text{H}} \to M_{\text{H}} \left[1 + \frac{g_6}{4} \left(\overline{a_{\phi\text{D}}} - 4\overline{a_{\phi\square}} + 24 \frac{M^2}{M_{\text{H}}^2} \overline{a_{\phi}} + 3 a_{\phi\text{D}}^{(2)} \right) \right], \tag{23}$$

where $g_6 = 1/(G_{\text{F}}\Lambda^2)$. Consider now vertices with 4 fields; as an example, we consider the process $\text{W}_{\mu}^{+}(p_1) + \text{W}_{\nu}^{-}(p_2) \to \text{H}(p_3) + \text{H}(p_4)$. After shifting the Wilson coefficients as in Eq.(22) we have a contact diagram

$$\text{V}_{\text{WWHH}} = g^2 g_6 \delta_{\mu\nu} \frac{1}{4} \left(\overline{a_{\phi\text{D}}} - 4\overline{a_{\phi\square}} + 3 a_{\phi\text{D}}^{(2)} \right) + \dots \tag{24}$$

where the shifted parameters of Eq.(22) have benn denoted with a bar. Therefore, there are terms, proportional to $a_{\phi\text{D}}^{(2)}$, that survive. This is exactly the role played by $\mathscr{L}\left(\Phi, \Phi^{\dagger} + a'\text{F}\right)$. The HWW vertex is

$$\text{V}_{\text{HWW}} = -gM\delta_{\mu\nu} + g^2 g_6 \delta_{\mu\nu} \frac{1}{4} M \left(\overline{a_{\phi\text{D}}} - 4\overline{a_{\phi\square}} \right) + \dots \tag{25}$$

with a HHH vertex

$$\text{V}_{\text{HHH}} = -\frac{3}{2} g \frac{M_{\text{H}}^2}{M} + g g_6 \left[\frac{9}{8} \frac{M_{\text{H}}^2}{M^2} \left(\overline{a_{\phi\text{D}}} - 4\overline{a_{\phi\square}} \right) - \frac{1}{4} \frac{P^2 + M_{\text{H}}^2}{M} \left(\overline{a_{\phi\text{D}}} - 4\overline{a_{\phi\square}} - 3 a_{\phi\text{D}}^{(2)} \right) \right.$$
$$\left. - \frac{1}{4} \frac{(p_3^2 + M_{\text{H}}^2) + (p_4^2 + M_{\text{H}}^2)}{M} \left(\overline{a_{\phi\text{D}}} - 4\overline{a_{\phi\square}} - 3 a_{\phi\text{D}}^{(2)} \right) \right] + \dots \tag{26}$$

where $P = p_1 + p_2$. By adding the contact diagram to the s-channel diagram we obtain the total, explicit, $a_{\phi\text{D}}^{(2)}$-dependent part of the process at $\mathscr{O}(g^2 g_6)$,

$$-\frac{3}{4} g^2 g_6 a_{\phi\text{D}}^{(2)} \delta_{\mu\nu} \frac{1}{P^2 + M_{\text{H}}^2} \left[\left(p_3^2 + M_{\text{H}}^2 \right) + \left(p_4^2 + M_{\text{H}}^2 \right) \right], \tag{27}$$

which does not contribute to the S-matrix. Once again, the Lagrangian is explicitly $a_{\phi D}^{(2)}$ dependent even after the shift of Eq.(22), only the S-matrix is independent. In going beyond LO one should always remember that Green's functions are divergent and it is not until renormalization is performed and S-matrix elements are computed that we can keep ET at bay. Given $\mathscr{L}_{\text{eff}}$ of Eq.(5), the correct statement is

$$\frac{\partial}{\partial a'} S\left[\mathscr{L}_{\text{eff}}\right]\Big|_{a+a'=\text{fixed}} = 0, \tag{28}$$

where $S[\mathscr{L}]$ is the S-matrix associated to $\mathscr{L}$. Note that the whole procedure is manifestly invariant.

7. Appendix: More on non invariant reparametrizations

In this Appendix we elucidate the question of calculating physical observables in a "more transparent way" by means of field reparametrizations. As we have already seen in Sect. 2.4 it is possible to swap an interaction term $\bar{t}tH^2$ in favour of H^3 and of $H\square H^2$, at the price of having a non invariant Lagrangian, i.e. we are not dealing with a rescaling of the Wilson coefficient for $\mathscr{O}_{\phi\square}$.

When we compute $\bar{t}t \rightarrow HH$ (at LO) in the original theory there are 4 diagrams, the s-channel H exchange, a contact interaction and t,u-channel t exchange. Killing the contact term still leaves 3 diagrams and the same S-matrix element. Moving NLO requires a careful treatment of the H wave function.

Another example concerns the transformation

$$H \rightarrow H - \frac{1}{8}\frac{g g_6}{M^2}\left(a_{\phi D} - 4a_{\phi\square}\right)\left(MH^2 + H^3\right), \tag{29}$$

that changes 3H and 4H vertices by killing terms with two derivatives. The argument goes as before, the number of diagrams in $HH \rightarrow HH$ remains the same and so does the S-matrix. To be precise there are s,t,u-channel diagrams and a contact interaction; one subtlety is the following: the Lagrangian contains a term linear in H whose coefficient β_H, is fixed, order-by-order, by the request of cancelling H tadpoles [27]. After Eq.(29) this term will contribute to $HH \rightarrow HH$, with no effect at LO since β_H starts at $\mathscr{O}(g^2)$ but with sizable complications starting at NLO. Another non negligible complication (even in the unitary gauge) is the presence of 19 new terms induced by the transformation. No "more transparent" road here, one gets the same answer with the same amount of work, not having to jeopardize gauge invariance. It is worth noting that we have neglected ϕ^0 and $\phi^\pm$; to eliminate all terms with two H derivatives in the R_ξ gauge we need a cubic transformation mixing the four fields, $H, \phi^0, \phi^\pm$. Terms with two drivatives of the Higgs-Kibble ghosts will remain, as expected since the scalar manifold is not flat [14].

The message to be taken is clear: if special (◄) vertices are not included the result is wrong since not all of them cancel at the S-matrix level, as shown in Fig. 7; if they are included then the LO computation of a process is not "more transparent" and becomes considerably involuted at NLO. As an example consider the contribution to the Z-factor for external H lines due to the diagrams of Fig. 10; they obviously cancel against similar diagrams involving ► vertices, as long as one is aware of their existence, something that cannot be read from the Lagrangian alone without knowing the transformation. The possibility of "absorbing" terms is not limited to H tansformations. For instance, guided by EoM, we could transform Z in order to "absorb" $HZ\bar{b}b$. This is achieved by using

$$Z^\mu \rightarrow Z^\mu + i g g_6 \frac{c_\theta}{2M^2}\left[\left(a_{\phi q}^{(1)} - a_{\phi q}^{(3)}\right)\bar{b}\gamma^\mu\gamma_+ b + a_{\phi d}\bar{b}\gamma^\mu\gamma_- b\right]. \tag{30}$$

After cancelling the $HZ\bar{b}b$ terms the transformation has produced the following additional terms:

$$\mathscr{L}_{\text{add}} = -\frac{i}{2}\frac{g g_6}{M^2}c_\theta\left(a_{\phi q}^{(1)} - a_{\phi q}^{(3)}\right)\bar{b}\gamma^\mu\gamma_+ b\left(\square - M_0^2\right)Z_\mu$$
$$-\frac{i}{2}\frac{g g_6}{M^2}c_\theta a_{\phi d}\bar{b}\gamma^\mu\gamma_- b\left(\square - M_0^2\right)Z_\mu$$

Figure 10: The wave function factors that are requested when using the transformation of Eq.(29).

$$-\frac{1}{24}\frac{g^2 g_6}{M^2}(1+2c_\theta^2)\left[(a_{\phi q}^{(1)}-a_{\phi q}^{(3)})\bar{b}\,\gamma^\mu\,\gamma_+\,b\,\bar{b}\,\gamma_\mu\,\gamma_+\,b+a_{\phi d}\,\bar{b}\,\gamma^\mu\,\gamma_+\,b\,\bar{b}\,\gamma_\mu\,\gamma_-\,b\right]$$

$$+\frac{1}{12}\frac{g^2 g_6}{M^2}s_\theta^2\left[(a_{\phi q}^{(1)}-a_{\phi q}^{(3)})\bar{b}\,\gamma^\mu\,\gamma_-\,b\,\bar{b}\,\gamma^\mu\,\gamma_+\,b+a_{\phi d}\,\bar{b}\,\gamma^\mu\,\gamma_-\,b\,\bar{b}\,\gamma_\mu\,\gamma_-\,b\right],\tag{31}$$

where $\gamma_\pm=1\pm\gamma^5$ and $M_0=M/c_\theta$.

Remark Special vertices (◄) have been generated but they do not always cancel, only when referring to external Z lines (e.g. in the $Z\to\bar{b}b$ decay). Indeed, they are needed since Eq.(30) has generated four-fermion interaction terms and there is no contribution to the scattering $\bar{f}f\to\bar{f}f$ from $\mathscr{L}_{add}$, as it should be. Furthermore, also the decay $H\to Z\bar{b}b$ remains unchanged. Once again, cancellations are there as long as all terms are kept and no evident advantage is seen at LO with increasing complexity at NLO, even in QCD; loop corrections to the contact $HZ\bar{b}b$ diagram become loop corrections to a diagram with a special vertex and there is a substantial complication with respect to the treatment of Z factors, now the Z source can emit/absorb a $\bar{b}b$ pair. The resurgent contact diagram is depicted in Fig. 11; the ◄ in the second diagram, acting on the internal Z line, shrinks it to a point and reproduces the first diagram (including QCD corrections): transformation does not kill the Phoenix.
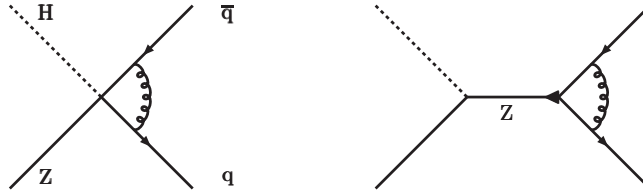

Figure 11: The Phoenix: resurgent contact diagram in $H\to Z\bar{q}q$, even including QCD corrections. The second diagram is equal to the first.

Proving off-shell WST identities is, of course, another story. Even this scenario is not complete since we have taken only a subset of terms, essentially those involving H,Z and $\bar{b}b$. The situation is more involved since we have all the other terms involving a Z, among them the Z coupling to Faddeev-Popov ghosts (R_ξ gauge).

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