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Agency and assemblage in pattern generalisation: a materialist approach to learning

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Abstract In this paper, we draw on the contemporary perspective of inclusive materialism offered by de Freitas and Sinclair to contribute to current discussions on the role of the body in the learning of mathematics. Using the notions of *distributed agency* and *assemblage*, we illustrate the way in which three students engage with a patterning task. We discuss this as an example to show how the mathematics activity involves, besides the students' bodies, other materialities that populate the classroom, and *how* all the human and non-human bodies form a moving assemblage that constantly reconfigures and reorients learning. The inclusive materialism helps us talk about learning as a dynamic assemblage rather than in terms of individual achievements and directs attention to the material learning environment.

Keywords Assemblage · Agency · Body · Generalisation · Inclusive materialism · Pattern

Taking the new materialist perspective proposed by de Freitas and Sinclair (2014), this article contributes to current discussions on the role of the body in learning mathematics. The *inclusive materialism* of de Freitas and Sinclair offers a new way of theorising the nature of embodiment and embodied mathematics, moving away from essentialised views of the body "and towards a more temporal and contingent sense of becoming" (p. 47). It suggests that we must "talk about the 'perceptuo-motor possibilities' of the body while also addressing the social entanglement of bodies", as well as address "the way meaning and matter are entangled" (p. 22). De Freitas and Sinclair propose to focus less on human will and intention and more on distributed agency, so that the human body is *not* conceived as the principal administrator of its own participation or the only centre of activity. Decentralising agency allows us to shift attention to how the material conditions of the classroom partake in mathematical thinking and learning. This approach questions the acquisitionist view that learning occurs through mental schemas or mechanisms that students are expected to acquire,

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thus avoiding representational visions of knowledge. In so doing, it creates space for re-thinking the potentiality of the body in learning, overcoming the mind/body split as well as mechanistic models of cause and effect, while valuing the *material* participation of the environment with which learners are entangled.

Following this perspective, we use the notions of distributed agency and *body-as-assemblage* to look at the students' activity through the material entwinement of bodies that populate the mathematics classroom. Our aim is to show how de Freitas and Sinclair enable us to better understand learning as "an indeterminate act of assembling various kinds of agencies *rather than* a trajectory that ends in the acquiring of fixed objects of knowledge" (p. 52, our emphasis). To this aim, we examine a classroom episode, from a study involving grade 6 students in patterning tasks aimed at introducing algebraic thinking. The study took place in Italy and lasted for five sessions of approximately 2 h each. The tasks used numerical and figural patterns to provide opportunities for students to reason about relations between variables and formulas. The tasks follow the recent lines of mathematics education research that see patterns as a basis for introducing early algebra and for studying processes of generalising (e.g., Ferrara & Sinclair, 2016; Moss & Beatty, 2010; Radford 2010a, 2010b; Rivera, 2011).

In the next section, we discuss in more detail how inclusive materialism re-thinks the body and reframes the provisional nature of agency as distributed. The argument sheds some light on its potential for studying learning processes as not relegated to the bounds of the human body and individual learner, but as engaging the whole material surrounding of learners in an active manner, even offering a new ontology of mathematics.

1 The body-as-assemblage and distributed agency

1.1 Body-as-assemblage

Inclusive materialism problematises some of the ontological tenets that underpin particular conceptions of the human body as the principal administrator of its own participation. Drawing mainly on feminist philosophy and post-humanist theories of subjectivity, de Freitas and Sinclair (2014) propose to *de-essentialise* the body and re-think its potentiality. They redefine the borders and the surfaces of the body by thinking of them as provisional and malleable, so that "the body" exceeds the contours of its skin. They do so by considering the interactionist view according to which materials are not inert but are constantly interacting with each other and with the human body.

Starting with the simple example of a circle drawn by a compass, we see how artefacts become "part of the learner, continually changing the very constitution of their bodies" and how "[h]uman bodies are constantly encountering, engaging and indeed amalgamating with other objects", so that "the limits of our body are extended through these encounters (de Freitas & Sinclair, 2012)" (p. 26). This suggests that the material world, or that which we typically take to be non-human, for instance the compass, is *not* merely passive and inert, but intrinsically part of the learning event. Roth's (2010) materialist phenomenological approach does similar work, examining the multimodal engagement of a child with a cube. This work aims to study how the compass or the cube is not simply taken up and acted upon by the human agent, but rather has its own force and capacity to affect. We need to look at how the cube is "actively involved in the assembling of meaning" and "*itself* becoming-cube through its encounter with the child, shifting its own boundaries in this process of becoming" (de Freitas & Sinclair, 2014, p. 26, our emphasis). Thus, it is not simply that artefacts or manipulatives matter, a common enough claim, but that the very locus of learning shifts from the human to the more than human.

In the last few years, the term *assemblage* has been used to describe this new way of thinking about distributed learning. Assemblage is a term that Deleuze and Guattari (1987) introduced in order to capture the structural arrangements of human and non-human that sustain a given social-material configuration or a kind of interaction. Assemblage is also used to describe the system-wide distributed processes that are at work in any ecology or environment (Bennett, 2010; Delanda, 2006; Latour, 2005). Drawing on these various approaches, de Freitas and Sinclair (2014) propose that the body be reconceived as "an assemblage of human and non-human components, always in a process of *becoming* that belies any centralizing control. The body *in/of* mathematics partakes in a 'relational ontology' or 'mutual entailment' that binds the components together in a process of becoming embodied (Barad, 2003, p. 820)." (p. 25, our emphasis). In other words, mathematical concepts are one component in a learning assemblage. The distinctiveness of their use of assemblage theory is in this incorporation of mathematical concepts. The body "of" mathematics refers to how these concepts are at play in the material configurations of the assemblage. Rather than simply speak of human bodies as cyborg-like in their reassembling with technology and artefact, de Freitas and Sinclair show how mathematics itself is part of the living arrangement.

The idea of assemblage is given using the image of "a knot of many different threads, twisting and tangling, composed of loose run away strands and tight little balls of interwoven density", with "no inside or outside, no beginning or end", so that "one is always *in the middle* of the knot, always *moving* along its various threads. The knotted assemblage is composed of diverse elements and vibrant materials of all kinds" (p. 34, our emphasis). Thus, in the process of becoming (not of being) of the body-assemblage, what counts as a body is the *open* set of unstable material relations between the human and non-human components.

The challenge is to put this theory to work so that new insights about learning can emerge. In this paper, we examine classroom data where pattern recognition tasks are explored. In light of the above, our aim is to show how the various numeric patterns are at play in the learning assemblage. This means analysing the data for how "recognising patterns" actually involves *creating* patterns through material-social encounters. This creating, however, following assemblage theory, will have to be sustained collectively rather than individually. In the next section, we unpack in more detail the nature of this kind of distributed agency.

1.2 Distributing agency

Imagine a child holding and feeling a plastic cube, developing a sense of the qualities of cubeness through this bodily encounter: "the human body is not the only agent involved in processes of learning. The matter of the cube and the matter of the mathematical concepts are also agents in this context, as are the teachers and their policy-inflected pedagogical actions." (de Freitas & Sinclair, 2014, p. 24). Thus, the cube is just *one of the many* 'actants' at play in the situation and the child is *not the only one* agent to which we might assign the power and force of an 'I can'. This is to consider the child, the cube, the concept all as partaking in some degree of *agency* and as a source of action: "something", write de Freitas and Sinclair citing Bennet (2010), "that acts or to which activity is granted by others." (p. 30). This vision demands that we associate agency *with* the cube and the concept, instead of associating it only with the human mind, with human will and intention. This also entails re-thinking the concept of agency as spread, dispersed, plural and *distributed* across the learning assemblage.

As Bennet (2010) put it, the fact that "bodies enhance their power in or as a heterogeneous assemblage" suggests for the concept of agency "that the efficacy or effectivity to which that term has traditionally referred becomes distributed across an ontologically heterogeneous field, rather than being a capacity localized in a human body or in a collective produced (only) by human efforts" (p. 23). Agency needs to be reconceived "as operating within the relations of an ever-changing assemblage, a force that flows across the encounters" between artefacts, hands, voice and other bodies (de Freitas & Sinclair, 2014, p. 33). Drawing on Rotman (2008), the body is no longer confined to the flesh borders of the individual person, but it must be conceived in terms of *distributed agency* across a network of interactions, the properties of which are constantly changing. For Rotman, this entails a process of "becoming beside ourselves", which captures the acentred sense of subjectivity that, according to Rotman, emerged in this century, and a network "I" that thinks of itself as permeated by other collectives and assemblages.

According to this perspective, artefacts in the mathematics classroom, including the paper, the pencil, the compass, the digital tools and the diagrams, have some *degree* of agency. They participate in agential relationships with the user so that the user and the artefact mutually constitute each other through interaction (de Freitas & Sinclair, 2013). This post-humanist understanding of agency implies that *subjects* are constituted as dynamic assemblages and that the mathematical subject comes into being through these material and social encounters. The human body and the physical bodies are not alone: the body of the concept also *matters* here. The mathematical concepts (e.g., of cube or circle) are part of the assemblages and engage in a process of becoming that binds them to the actions of the users/mathematicians. Again, concerning the patterning activity, the inclusive materialist vision of agency will help us to draw attention to and explore the force of the pattern, its capacity to affect and be affected, instead of simply talking about how learners think the pattern.

1.3 A new ontology of mathematics

Fundamental in defining an ontology of mathematics consistent with the inclusive materialist perspective is the concept of *virtuality* (Châtelet, 1993/2000). The concept of the virtual is tricky because it seems to point to an invisible aspect or dimension of matter. The term has a long philosophical history, but in this case, it is primarily the work of Deleuze that informs its use. For Deleuze (1994/1968), the virtual and the actual are two dimensions of matter, which mutually presuppose each other. The virtual is immanent to matter—it does not transcend matter like some Platonic ideal form. The virtual is the dynamic indeterminism of matter, its *élan vital*. In fact, it is through the virtual, as distinguished from the logical deductive possible, that mathematics partakes of the material world; "indeed, mathematics and matter are mutually entailed. The mathematical body comes into being through actualizing the virtual—through gestures, diagrams, and digital networks *we become mathematics*; we incorporate and are incorporated by mathematics. The assemblage grows." (de Freitas & Sinclair, 2014, p. 213, emphasis in the original). The virtual dimension of matter is the creative potential of matter and is not to be thought of as only mathematical—it is generative of all meaning. Thus, "[m]atter and meaning are inseparable and do not stand in a relation of exteriority to each other." (p. 49).

The inclusive materialism of de Freitas and Sinclair helps us theorise embodiment in the mathematics classroom in new ways, offering an alternative to approaches that over-emphasise the individual who synthesises information in acts of embodied cognition (Lakoff & Núñez, 2000). Similar to enactivist approaches (see e.g., Maturana & Varela, 1992), it offers the vision that doing is knowing and looks for emergent forms of knowing/doing. Thus, this approach looks for how a learning event produces knowledge materially rather than as a mental construct.

The inclusive materialist perspective also dissolves the ontological divide between mind and matter, "between human thought and that which is outside of thought" (p. 30), following Latour (2005) in arguing that the distinction between sentient and non-sentient matter is simply a question of degree, or perhaps of intensity or energy. Even if this kind of ontology might initially seem non-operational in terms of analysing classroom activity and discourse, de Freitas and Sinclair emphasise that "one of the important consequences of this relational ontology is the way it supports new research methods" (p. 115), and one can begin to pursue the vision "through experiments in analysis, describing and studying activity for evidence of this entanglement." (p. 50). In this paper, we discuss one such experiment to study the emergent activity of three students working with patterns. We will look at *how* the learning assemblage evolves and show how the mathematical patterns are at play in the various material activities. Taking the assemblage as our unit of analysis, we will show that learning is *in*separable from the material bodies that are always in motion in the classroom.

2 The study

We undertook our research in the context of a classroom-based intervention with the dual aim of improving classroom practice and addressing problems in the teaching and learning of early algebra. Following Stylianides and Stylianides (2013), these interventions increase the likelihood of understanding the processes and phenomena that underpin the problematics of mathematical practice. In the following section, we present insights from other research on patterns, and describe the participants and context of our research study.

2.1 Mathematics curriculum and research on patterns

The research project involved junior high school students in Italy, where the National K-8 mathematics curriculum considers the topic of patterns as a key element (Ministry of Instruction, University and Research, 2012). The curriculum stipulates that grade 8 students should be able to recognise and describe patterns in numerical/figural sequences, as well as to build, interpret and transform symbolic formulas to express generality. The National assessment in grade 6 mathematics and the International assessment in grade 8 mathematics also require literacy about pattern generalisation. For instance, the TIMSS 2011 mathematics framework has suggested that learners "should be exploring well-defined number patterns, investigating the relationships between their terms, and finding or using rules that generate them" (Mullis, Martin, Ruddock, O'Sullivan, & Preuschoff, 2009, p. 24).

Researchers have investigated how pattern search allows for the successful introduction of basic algebraic thinking in the early years (e.g., Becker & Rivera, 2008; Carraher, Martinez, & Schliemann, 2008; Ferrara & Sinclair, 2016; Moss & Beatty, 2006, 2010; Radford, 2010a, b; Rivera, 2011). Although drawing on different theoretical perspectives, we highlight some of the relevant findings here. For example, grade 4 students show a disposition towards "rule finding" and to understanding explicit functional relationships (Moss & Beatty, 2006). Analysing grade 3 students' generalisations about linear functions involving geometric arrangements, Carraher et al. (2008) have shown that there are good reasons to move back and forth between a recursive approach and a functional (input-output) approach. Indeed, this enables different types of generalisations and representations, as well as to reverse the relation between the variables. Rivera (2011) offers junior high school generalisation activities with numerical/geometric sequences as a way to associate in the long term the generalisation process with the modelling of linear functions.

Others have drawn attention to the forms of algebraic thinking that can be made accessible to 7–8-year-old learners before any conventional use of alphanumeric symbolism (Radford, 2010a, b; Ferrara & Sinclair, 2016), with pedagogical implications "to revisit the genetic relationship between arithmetic and algebraic thinking" (Radford, 2010a, p. 80).

Starting from the relevance of patterns in the K-8 curriculum and early algebraic thinking, our intervention was aimed at promoting generalisation as a way to reason about functions with numerical and figural sequences. In light of our theoretical commitments, we are interested in how generalising a figural sequence emerges from the material surrounding, and in how learning is distributed across all the materialities that are implicated in the classroom. We aim to contribute to the research on pattern by approaching it from a novel theoretical approach.

2.2 Participants and method

The study is part of a classroom-based intervention that involved two grade 6 classrooms and their mathematics teacher. It took place in a junior high school in northern Italy. The students were all between 10 and 11 years old, and the two groups were both heterogeneously composed of males and females. Both the students and their parents were informed about the study and consented to the research process and its method. The students participated in five sessions of approximately 2 h each (for each group), and the activity that we analyse here was carried out in two of these sessions.

The regular classroom teacher was present in each session and led the classroom discussions and activities, which he had previously designed in collaboration with the first author. A university student observed the lessons as a visitor to the school, in the context of her master's programme in mathematics education, which she was attending at the time of the intervention. The first author was the student's tutor on her master's degree work but she was not present during the lessons. The groups were video recorded by the university student, who later transcribed all video data and collected the transcripts together with the group and individual write-ups. As co-authors of this paper, we recognise that we are materially implicated in the research process and a part of a "meta-body": the research assemblage that speaks directly to the space of ethics, involving all the relationships expressed above, as well as our entanglements with subjects, data, method and theoretical commitments. From a meta-perspective (e.g., for the reader/researcher), our assembling with the material surrounding discloses relation as "the smallest unit of being and of analysis" (Haraway, 2008, p. 88)—what matters from the vantage of assemblage theory.

The essence of this paper lies in these entanglements. There is of course a tension between responsiveness and responsibility in our being involved in them. But we offer here an interpretation of a classroom episode approached with a sensibility to new materialism.

Thus, we draw attention to the ways of moving, talking (verbally and in writing) and diagramming in the classroom, which the audio, the video, and the written documentation reveal and unfold as the forces and material exchanges that constitute and move the assemblage (our unit of analysis) within the classroom activity. Treating assemblages as units of analysis troubles our own discursive practice, or ways of talking, about the learner and the learning of mathematics. With these considerations, the verbal, the written and the diagrammatic are not seen as reflections of inner knowledge and thinking. Rather, the focus is on the relevance they play in the assemblage of meaning and becoming and is shifted from specific "entities" to mathematical activity.

2.3 The activity

The current activity involves a task in which the students are asked *to explain to a friend of the other class, who has not faced the same activity, what she has to do in order to find exactly the number of circles of any term of the sequence* (Fig. 1).

Previously, the students worked on less complex tasks, like that of exploring terms in specific positions (5th, 6th, 12th, 57th, 100th), through questions concerned with either the way the specific term is made or how many circles make it.

Here we draw attention to a group of three girls. While working on the previous tasks, the three girls had begun to detect how to find the number of circles of a particular term using the Figure number. This is shown for instance in the case of Figure 12 and Figure 57. The process of generalising is present in the two types of calculation that result in 115 circles, that is: $(57\times 2)+1$ and $57+58$ (Fig. 2, left and bottom frames), as well as in the diagram drawn for Fig. 6 (middle of Fig. 2), which adds another example of the same justification. The generalisation started unfolding in the students' use of the lines and the arrows in the diagrams to mark connections, like in the case of Fig. 6.

At this point, reference was still to a specific term as a generic example, and this became more apparent when Elisa, Giorgia and Lucrezia started discussing the new task which demands a method for finding the total number of circles. Indeed, the students wrote on paper: *The figure is made of two rows*, drawing the case of term 3; and *She has to multiply the figure number by 2 and to add 1*, using the generic examples of terms 3 and 4 to reason on the total number of circles based on their spatial disposition (Fig. 3).

In the next section we report a 1-min segment of the activity in which the students were checking the accuracy of their method with respect to the demand of communicating it to a friend. We have chosen to draw on this segment in the lesson, because a distinctive *change* occurred in the dynamic entanglement of the girls, the task and the sequence. Focus on change, or movement, in the activity may bring us to notice mathematically significant practice. Thus, it is relevant in the assembling of learning and enables us to grasp that a change in the assemblage also changes the (meaning of the) pattern. What emerges from movement is a new identification of what constitutes the pattern for the students. The section is divided into two parts: the first will be analysed before moving on to the next.

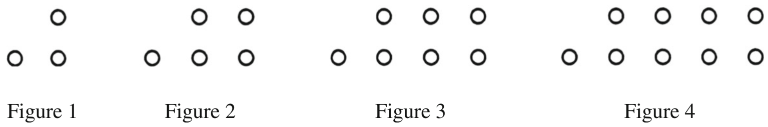

Fig. 1 The first four terms of the sequence given to the students

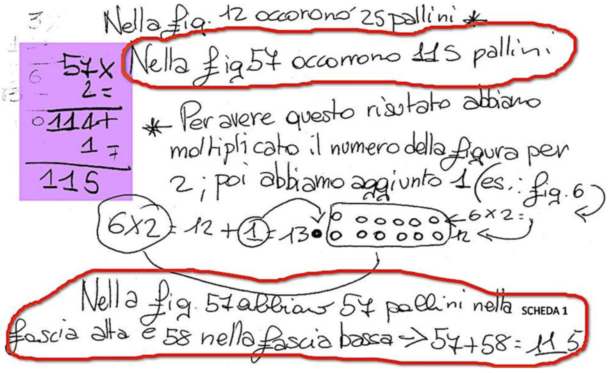

Fig. 2 Answers about Figure 12 and Figure 57. *On top*: *25 circles are needed for figure 12*. Diagrammatic reasoning about figure 6 is in the *middle*. *Top frame*: *115 circles are needed for figure 57*. *Bottom frame*: *In figure 57, we have 57 circles on the top row and 58 on the bottom row → 57 + 58 = 115* (our translation)

3 The moving assemblage

Elisa, Giorgia and Lucrezia were discussing the clarity of their answer. Lucrezia read the task again and posed it in a questioning form. Elisa, changing pitch, read again: "The figure is made of two rows", then she wrote beside the diagram of term 3: "First row, second row, which, the second row (*Right hand index finger running under the corresponding written words*) is equal to the first row, plus 1 (*High pitch*)" (Fig. 4). Giorgia agreed: "Yes, they are all like that".

3.1 The girl-pattern-diagram assemblage

It is tempting to assert that Elisa noticed the visual configuration of the previous examples and represented them in her diagram, which prompted Giorgia to notice that all the examples have the same configuration. This would be assuming some kind of mental activity that gets expressed through diagram and talk. Even though reference is to the diagram of term 3, the general relationship between the first row and the second row emerges out of the actions of the gesturing hand around the diagram and calls for the specific demand of the task, which is to think of *any* figure of the sequence. The way in which Giorgia agrees makes present all the terms of the sequence, beyond the finite number of terms given on paper by the task, and implicates the pattern as an infinity of "all like that" terms: terms with the same (local) algebraic structure. The girls, the paper, the task, the circles, the pattern and the diagram are part of a mobile incomplete assemblage. In the following, we refer to this provisional developing body as the girl-pattern-diagram assemblage. We do this not to claim that agency

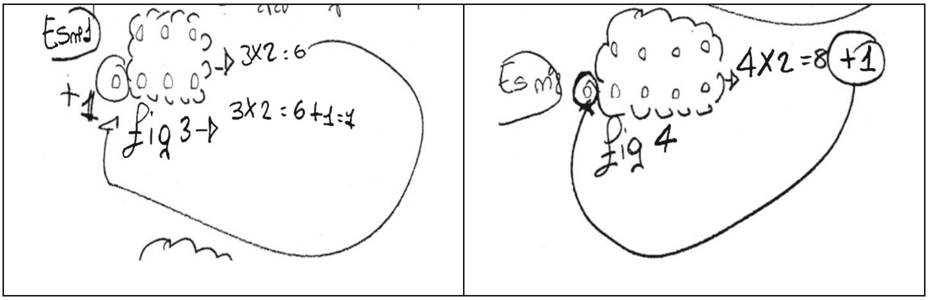

Fig. 3 Terms 3 and 4 as generic examples to justify the answer to the task

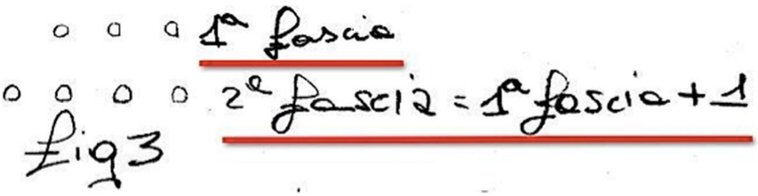

Fig. 4 The relationship between the first and the second row (*on top*: “1st row”; *on the bottom*: “2nd row = 1st row + 1”)

is attributed only to students, sequence and diagrams and to demote other materialities to partaking in the assemblage. Rather, we use a compound name to direct the reader’s attention to the interplay between the bodily and the diagrammatic as constituting the mathematics in the episode.

The diagramming activity of the girls on paper about terms 3 and 4, entwined with the two added operations of *3 times 2 plus 1* and *4 times 2 plus 1*, respectively, already spoke of this assemblage, as well as of the agential partaking of the figures in the assembling of meaning. The clouds, the arrows and the circlets in the diagrams (Fig. 3) brought into being a new (virtual) dimension, forging the connection between the arrangement of the circles and the figure number in the case of the two specific figures. In particular, the arrow—that is about “take all this (whatever it is) and once you’ve done that, add that one that is on the other side”—helps the double row comes out as the first important part of the structure, even though the single circle is first. The clouds, the arrows and the circlets also captured the mobility of the pattern and incarnated the invariance of its structural relationships (e.g. alluding to the distinction between the role of the variable and the role of the constant). But had the task presented a different arrangement of the circles in the given figures, as in the hypothesis of placing them in a single isolated circle plus a unique vertical row, the assembling of matter and meaning would have probably entailed different clouds, arrows and circlets, thus different mobilisations of the diagrams, as well as different ways of writing down the operations, moved by particular gestural configurations and pushing the assemblage to an eventually completely different mode. Similarly, the assembling of matter and meaning implicates the specific requests of the task: speaking about the “total number of circles of any figure” (a claim about circles), speaking about “how to find” this number (a claim about method), even more imagining to speak to a friend who did not experience the same activity (a claim about the explanation of method). These requests, through the claims they entail, reveal the agential relationship of the task designers (the teacher and one of the co-authors) with the other entities in the assemblage. Thus, the teacher and the researcher are also part of the becoming body and, even though not explicitly present in our list above, they are silent actors in the episode, implied by the task. We might say that they “speak” through the request in the task.

3.2 The becoming of the assemblage

The action of writing down of the connection between the first and the second row makes a new operation emerge in the activity as a new configuration of the assemblage. This change in activity, or movement of the assemblage, is expressed by a focus on the local difference between the two rows of circles instead of concentrating on the global number of circles, which was captured by the previous configuration centred on the ways of diagramming and gesturing around terms 3 and 4. The new configuration of the assemblage is a new way of seeing the pattern, a new identification of the pattern. The local connection also works for any figure whatsoever, not only for the term 57 (as described in Fig. 2), no matter whether it is associated to term 3 of the sequence (Fig. 4). In fact, the equality explicitly involves *the first row* and *the second row*, even though it suggests the link between the numbers of circles and the presence of one circle more in the second row with respect to the first. It is as if now the girl-pattern-diagram assemblage contracted to a mode in which counting the circles recedes, being no longer important for looking at the pattern, while the recognition of a difference between the rows emerges.

The limited surface of the paper has also been implicated in this movement, directing the material practice to the blank space next to the diagram of term 3 and pushing the writing down of the new operation in correspondence with the two rows (Fig. 4).

The nature of the pattern emerged out of the actions of the diagramming hands that occurred before the 1-min interaction and of the writing gesturing hand during the interaction. The pattern is from moment to moment reconfigured in terms of the total number of circles (through the operation of multiplying the specific term number by 2 and adding 1) or in terms of the local connection between the two rows. In this back-and-forth movement, the role of the term number as a variable is lost, while that of the constant still remains in the new operation. It is the case that what connects the two modes of the assemblage (or identifications of the pattern) is not yet explicit: the fact that the number of circles in the first row corresponds to the term number. Once the students read their answer again, this generated confusion about the method in the second part of the interaction:

E: She has to multiply the figure number by 2 and to add 1 (*Reads what is written on the paper*), and it's what we wrote
L: But for, you have to say to her: For having the next figure (*Mimes* "*the next*" *turning her right hand's index finger and thumb in the space in front of E, just over the surface with the diagrams of terms 3 and 4*; Fig. 3), otherwise…
G: For me
L: For having the next figure (*Repeats twice the previous gesture. Looks at E and G*), you need to multiply by 2 and add 1
E: No! (*Shakes both hands inside the pocket of her sweatshirt, for disagreeing. Looks puzzled at L*) But no!
L: Then, write: You need to (*Looks at term 3 on the paper*)
G: But, but in this way it seems that you multiply this figure (*Moves her right thumb in a circlet around term 3 drawn on the paper*; Fig. 4), entirely (*Emphasises the tone, while E looks at term 3*)
L: Hm, no: You have to multiply… (*Points with both index fingers to the term*)
G: And you have to say: Removing 1 (*Rotates her right arm on the left to indicate subtraction. Looks at L, who remained in the last position*) and adding it later (*Rotates the arm back to indicate addition, while E looks at her*; Fig. 5a), because otherwise (*Looks at E and recedes*)
L: That's it, because this is what we (*Brings her hands together back to her breast*) think, but the one who doesn't (*Extends the right hand towards the paper again*), that is, a student of another class (*Points with the right arm to and looks at the door, to refer to the other class*) doesn't…
E: I didn't understand (*Turns the paper to better look at it*)

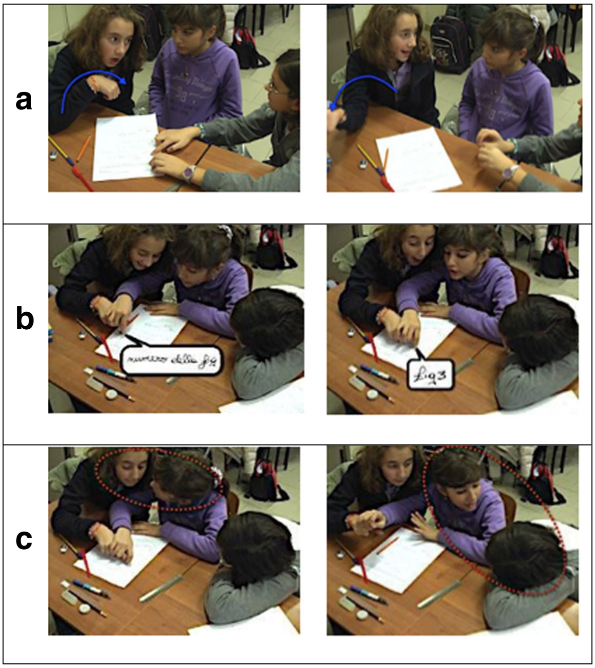

Fig. 5 **a** Giorgia subtracting and adding 1. **b** Elisa moving Giorgia's finger to indicate on the paper. **c** Elisa's gaze towards her group mates

The re-reading became part of an activity of discussing the accuracy of the method, which can be seen as a new movement of the developing assemblage, which includes again the demand of the task (involving a specific addressee, a friend of the other class) and focus on the diagram of term 3 (drawn on paper before). This provoked further gesturing, in the air and on the writing surface. In this movement, new relations came to the fore, leaving previously central agents (like the diagrams of the generic examples of terms 3 and 4, with their arrows, clouds and circlets) fade away while new ones emerged out of the activity and became part of the moving body (the next figure, the operations around the desk, the imaginary listener). The insistent turning gesture over the desk actualised the recursive way in which the pattern is still part of the assembling of meaning. The imperatives expressed in words ("you have to say", "write") actualised the need for clarity of the method. The verbal instructions ("you need to", "You have to"), the pointing arm and the eye towards the door made present the imaginary friend, who does not know the patterning activity, as the interlocutor. The pointing and gazing to the diagram of term 3 called for the role of the specific term as a generic example. The specifically not neutral role of the term number in the pattern is implicated in this movement, which makes the body of the girl-pattern-diagram goes back and forth between the actualisation of a recursive and a functional reading of the pattern, both ways virtually present in the pattern. Indeed, according to the inclusive materialist perspective, the pattern participates in the same material plane as the girls and the paper, emerging out of the mobility of the situation. The pattern is in motion throughout the continual reconfiguration of the assemblage, being from moment to moment the local connection between two rows, the local relation between term number and number of circles in a row or the connection between the term number and the total number of circles. The extent to which the pattern could be considered as a mathematical structure starts being dictated by these material contractions and expansions, which constitute the activity of the three girls entangled with the task and the pattern.

In particular, the verbal and gestural actions of subtracting and adding 1 (Giorgia) brought into being the role of the constant as well as the term number as that which has to be multiplied by 2 in the method. Focus was shifted again to the spatial arrangement of the particular term, possibly contracting for a moment the new assemblage to the arrows, clouds and circlets of its previous configuration and expanding it again to the dimension of the new operation connecting the two rows of a figure. The removing of the constant unravelled the direct relationship between the term number and the total number of circles in the pattern, and the material environment was redistributed eventually extending the original multiplication by 2 to its virtual inverse. However, these mobilisations of the developing body implied new confusion (Elisa), and the activity changed in the third part of the interaction, with the return to the specific case of term 3:

> G: So this figure, for instance, (*Indicates with her right index finger term 3*) 4 (*Points to the bottom row*) and 3 (*Points to the top row*)
> E: But I said: (*High pitch. Takes with her right hand Giorgia's right index finger*) I have to multiply the nuuumber (*High pitch and prolonged* "*u*". *Presses with Giorgia's finger the verbal expression* "*the number of the figure*") of the figure (*Presses again with Giorgia's finger the expression*; Fig. 5b left). Which number is this figure? (Fig. 5b right)
> L: 3!
> E: 3 (*Looks at Giorgia first and then at Lucrezia*; Fig. 5c) times 2 plus 1
> G: She's right! Ah, I understood!

The pointing hand began to reconfigure the activity again, redirecting focus to the figure number as a variable and redistributing agency, which makes the pattern now emerge out of the gesturing on the diagram, as a functional relationship between term number and total number of circles. A new mode of the assemblage appeared, including the spontaneous action of one of the girls of taking her classmate's finger and moving it through the working space ("this figure") and time ("I said"), as well as the flight of her voice to stress the "I" in intonation ("I said", "I have to") first and the prolonged "u" later, and the gazes among the three girls. The hand, the finger, the voice, the gaze, and the words are all in motion, in the assembling of meaning. The collective of pattern-diagram-operation-answer is also part of this movement. The many gestures traced the movement back to the working surface where the diagram of term 3 and the answer to the task were both placed, and to the time when the method was initially discovered and written, then justified. The high pitch and the sound of the "u", along with the rhythm and repeated pressure of the finger gesture, are affective forces, which affirmed the agential partaking of the figure number in the moving body, actualising "what" has to be multiplied by 2 and deleting the previous confusions. The assemblage moved to incorporate the direct rule, implicating the shift from the pronouncing of "the number of the figure" to the specific diagram of term 3, which became part of the mobility as a generic example ("Which number is this figure?"), and to all its virtual characterisations in the old and new diagrams. Speaking of the relation between term and term number, the expression "Which number *is*", instead of "Which number *has*", referred to the figure, introduced the term and the term number in the assemblage as no longer distinct, almost as the same entity. The gestures that followed the answer ("3!") are not external representations of something here, but a re-doing that changed the pattern again. In this new configuration, the pattern was identified with the function that gives the direct link between term number and total number of circles, in relation to the previous diagramming activity: the term number (*3*) *times 2 plus 1*. The kind of concert and empathy between the three students, which the exchanging and gazing eyes made apparent, together with the satisfying sounds of the voice, became part of the affective forces in the assemblage, which spoke directly to the acceptance of the method description and moved the final configuration to the emergence of the term number as a variable that runs the method.

4 Discussion

The episode above points out how the three girls, Elisa, Giorgia and Lucrezia, engaged with a given pattern and with their material surrounding to solve the task of determining the total number of circles at any position in the pattern.

We have shown that the entanglements of the students' bodies with the pattern, the paper, the task, the circles and the diagrams constitute the body-assemblage, which is developing in the classroom (which we have called the girl-pattern-diagram assemblage). The hand, the arm, the finger, the eye, the voice, the paper, the task, the mathematics, etc. are all in motion, in continual reconfiguration. Throughout the moment-to-moment activity, some of them fade away and others emerge in the activity, so that they eventually become part of the moving assemblage. They move and are moved by the assemblage. This mobility gives rise to new modes or configurations of the assemblage, which are also new identifications of the pattern (e.g., the local connection between two rows, the local relation between term number and number of circles in a row, the local connection between one term and the next, the relation between term number and total number of circles).

From moment to moment, the pattern emerges out of the students' activity as a new assemblage, which corresponds to a particular distribution of agency across the material relations among bodies (each body partakes of degrees of agency). Thus, the pattern is also mobile and implicated in a process of change and alteration. Briefly speaking, the body of the pattern engages with the bodies of the three students, and a new kind of body-assemblage comes into being. In this movement, the diagrammatic, the gestural, the verbal, the written and the seen are not external representations or codes of internal processes or of knowledge capacity. Instead, they are a re-making that, in the learning assemblage, changes the pattern again and again. So, for example, in the last part of the episode, the many gestures over the desk and on the specific diagram of term 3, alongside the spoken words and the gazes, reconfigure the activity, giving rise to the final configuration, which makes the pattern emerge as a direct relationship between the term number and the total number of circles.

We have also seen that the *task*, which demands to explain to someone else, plays a fundamental role in shifting the goal and mode of activity for the students. It needs accuracy for the method that is to be addressed to the imaginary learner. It entails that the friend of the other class becomes part of the developing body, as a virtual presence in discourse. The three girls speak with her, through the spoken word "you", used for the instructions, which transforms the previous "She" and "her" and their gesturing in the air, inside their space, as well as towards the door, outside this space. The final gesture/diagram interplay, with the collective Elisa-Giorgia-Lucrezia-friend acting as a single student, introduces a significant change in the activity: a new configuration of the assemblage, for which the term number becomes the variable that determines the method.

A second relevant aspect that we want to underline is concerned with the absence of the *teacher*'s body. In fact, there is no teacher physically involved in the episode, even though, as said above, he is silently implicated in the assemblage, alongside the researcher, by the request of the task. In a similar episode, Radford (2010b) studies some grade 2 students' investigation of the first terms of the same pattern (where squares are used instead of circles). In particular, the kids are required to discuss whether or not the drawing of term 8 by Monique—an imaginary grade 2 student—is correct. Even if reference to an imaginary character is present, there is a difference with respect to our example. In Radford's episode, the teacher *has* to help the students attend (both through speech and gesture) to the relationship between the term number and the numbers of squares, perceiving a general structure behind the pattern. Radford speaks of this in terms of a transformation of the students' perception "into a theoretical cultural form of perception required to tackle generalizing questions" (p. 3). He argues that the teacher, by mobilising key semiotic resources, created the conditions of possibility for the students to perceive certain things in certain ways.

While Radford's example sheds light on the fact that the teacher makes possible the students' encounter with the emerging mathematical structure, the mobilised semiotic resources at hand are given by her pointing gestures, words and rhythm. For the students, it is a moment of individual learning, in which the teacher's purposeful actions shift our attention away from the agential relations at play. We are not saying that the teacher should not become a significant operator in an assemblage. However, the sociocultural signs that she privileges need not diminish the presence and force of the material. In our episode, the various reconfigurations of the assemblage, by changing the pattern all the time, speak back to the authors of the task.

As stated in the introduction, the paper contributes to the current discussion on the role of the body in learning mathematics. Through the notions of assemblage and distributed agency, we can re-think the potentiality of the body and the learning of mathematics. The analysis of the episode shows how we can think of mathematics learning in terms of the moving assemblage of human and non-human components and decentres the human learner from being the only source of mathematical activity. We can also give justice to the materiality of the task, the paper and the pattern, for example, recognising to them the degrees of agency. The materials, the pattern, are not just performed or acted upon, nor do they merely fulfil some purpose. They are in motion and partake in learning.

5 Conclusions

Figural patterning tasks require not only just recognising the spatial arrangement of the figures in a pattern but also looking for relationships between adjacent terms of the pattern and, in particular, between each term and its position in the pattern. The last type of connection entails a layer of generality that allows for the development of early algebraic thinking.

In this paper, we have pursued the inclusive materialist approach to study the ways in which three students solved a figural patterning task, which involved their explanation of the pattern to an imaginary friend. The episode that we have discussed shows how the students' activity develops through the *assemblage* of heterogeneous relations between the girls, the task, the pattern, the mathematics and the diagrams (limiting the scene), which form a collective *body* of human and non-human agents, always in an indeterminate process of becoming that reconfigured learning and knowing. All the materialities in the classroom are incorporated and implicated in the assembling of meaning. The assemblage was mobilised in a decisive manner, towards the pattern as a functional way of relating term number and total number of circles, through the collective of Elisa-Giorgia-Lucrezia-friend operating across the working surface, with gestures and diagrams. In the episode, agency was diffused across the material relations of the changing assemblage. The pattern and the mathematics were also mobilised in a way that "both the subject and object of learning" were "in the midst of an ontological process of change or alteration, each moving away from that which they were and towards something entirely new." (de Freitas & Sinclair, 2014, p. 226).

Focusing on this process, the inclusive materialism helps us to shift from an emphasis on the semiotic resources to a different way of seeing activity in the classroom, which gives a different theorising of the doing and the doer of mathematics, while also re-thinking the body in learning mathematics and the body *of* mathematics. In so doing, we are able to talk about learning in terms of the agential relations that constitute the moving assemblage of the materialities implicated in the situation, a movement from which learning cannot be separated.

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