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“Let’s play! Let’s try with numbers!”: Pre-service teachers’ affective pathways in problem solving

Francesca Morselli and Cristina Sabena

University of Turin, Torino, Italy, francesca.morselli@unito.it, cristina.sabena@unito.it

The paper focuses on the intertwining between affect and cognition in pre-service primary teachers, with the twofold goal of studying teachers’ affect when facing problem-solving activities and of exploring paths for teacher professional development. 81 future teachers were proposed problem solving activities with specific “affective documentation” requests. We present here the first results of data analysis, and describe the main affective pathways emerged. Results are then discussed also with respect to methodological issues and to implications for teacher education activities that include the affective factors as relevant variables.

Keywords: Emotions, affective pathways, problem-solving, teacher education.

INTRODUCTION AND THEORETICAL BACKGROUND

This paper originates from our double interest as researchers in mathematics education and as teacher educators and investigates the intertwining between affect and cognition in problem solving processes carried out by future primary teachers.

Starting from the seminal work of Polya (1945), problem solving is a crucial issue in mathematics education (Schoenfeld, 1992), and constitutes also a fertile ground for studying the influence of affective factors in mathematical thinking processes (McLeod & Adams, 1989; Gómez-Chacón, 2000; Furinghetti & Morselli, 2009; Pesonen & Hannula, 2014). In fact, the solving process encompasses cycles, deviations and stops, and is influenced by resources, control, methods, heuristics, and affect (Carlson & Bloom, 2005). In particular, DeBellis and Goldin (2006) speak of the *affective pathways* as “established sequences of (local) states of feeling that interact with cognitive configurations” (ibid, p. 134). They underline also the importance of meta-affect, defined as “affect about affect, affect about and within cognition about affect, and the individual’s monitoring of affect through cognition” (*ibid.*, p. 136). From a methodological standpoint, studying the interplay between affect and cognition is quite a complex issue. Pesonen and Hannula (2014) use screen recordings and emotional states automatic recognition, to study upper secondary students problem solving with dynamic geometry software. Other approaches are based on self reports or self study. Gómez-Chacón (2000), for instance, proposed the Problem Mood Map “for the diagnosis of emotional reactions and the subject’s self-evaluation” (p. 152). In her study, local affect is self evaluated and registered by students during and at the end of the mathematical activity, by means of a special iconic code. The aim of such an instrument is to provide teachers information on students’ affect during problem solving, but also to “foster in the pupil an awareness of his own emotional reactions” (p. 153).

The affective dimension plays a crucial role in future mathematics teachers, because it can endanger the success of their professional education processes and their future teaching processes (Hannula et al., 2007). Many studies have been arguing that what teachers believe and feel have a clear influence on what students believe and feel (e.g., see Hodgen & Askew, 2011). This issue is especially crucial for teachers at the primary school level, since in most countries they are not specialist in mathematics, and may have developed negative experiences with the discipline in their past experiences as students (Di Martino & Sabena, 2011; Lutovac & Kaasila, 2014). Coppola and colleagues (2013) have focused on future teachers’ *attitude towards mathematics and its teaching*, stressing the importance of considering emotional disposition, view, and perceived competence—components of attitude, following Di Martino and Zan (2010)—both with respect to mathematics and to its future teaching. In this research, great attention is given to the link between the *past experiences* of pre-service teachers as students and their *future perspectives* of becoming mathematics teachers: also pre-service teachers with negative past experiences may show positive attitude towards its teaching (Coppola et al., 2013).

In our view, it is crucial promoting in future teachers the reflection on the intertwining between affect and cognition, and to work on the meta-affective dimension, as important steps in their professional development path. In this paper we present the first analysis of one activity conceived within this perspective, i.e. the problem-solving with “affective documentation” requests.

METHODOLOGY

The context

The study was carried out on the university students attending to the first year of the master degree to become kindergarten and primary teachers of the University of Turin (academic year 2013–14), in which the authors are in charge of the Mathematics Education courses. Throughout the math education course, pre-service teachers were asked to engage in various kinds of activities (problem posing, problem solving, analysis of children’s solutions, classroom discussion transcripts analysis, and school textbooks analysis), individually or in group-work, as well as in collective discussions mediated by the teacher educator.

A preliminary study on prospective teachers’ affect, performed according the model of the attitude towards mathematics and its teaching, suggested that pre-service teachers education should act in two ways: in *continuity* with respect to the need for a personal reconstruction with the discipline (“math redemption” in Coppola et al., 2013) and with the beliefs about the importance of affect in the teaching and learning processes, but also in *discontinuity* with the widespread procedural view of mathematics (Morselli & Sabena, 2014). On the base of these results, we set up a series of three problem-solving activities, coupled with specific “affective documentation” requests.

The affective documentation requests

We set up problem-solving tasks with specific additional questions about the *description of the solving process* and the *description of the emotional states* (affective documentation request) perceived in three different moments: at the reading of the text, during the solution phase, and reconstructed at the end of the activity. To this aim, specific open questions were inserted on the working sheet. The first question was immediately after the text of the problem: *“Describe how you feel when reading the text”*. Then, the solvers were asked to report their solving process *and* emotional state: *“Now try and solve the problem... Don’t use another sheet, try to write down here all your attempts, thoughts and FEELINGS when solving the problem (your reasoning, emotions, blockages…)”*. Finally, once reached a solution, they were asked to write down the story of the solving process: *“Now, tell the story of your solving process, describing as much as possible the emotions you felt during the process”*.

This methodology has some common points with Gomez Chacon (2000)’s instrument, since the final aim is to gather information on local affect and also to promote meta-affect during and after the solving process. Also, our approach has some links with the teacher education interventions performed by Chapman (2008), where self-study has a key role. One way of fostering self-study is asking students to narrate stories of their solving process. Affective documentation can be considered also as a special case of story.

Our approach has the double aim of studying the intertwining between affect and cognition, and of making pre-service teachers more and more aware of the influence of affect during mathematical activities, in order to increase their ability to fruitfully manage it. Thus, asking solvers to document their own thoughts and emotions, although a demanding and non-neutral requirement, was valued also for its meta-affective outcome. It is important to underline that prospective teachers were not used to such a kind of request.

The 1089 task

During the course we presented three problems with an increasing complexity as regards the expected solving processes; also the requests of affective report are made more and more explicit. Here we focus on individual reports to the 1089 problem, the third presented to the future teachers (text inspired by Coles, 2013):

> Pick any three digit number (e.g. 752) with 1st digit bigger than 3rd. Reverse the number (in

> the example, 257) and subtract (in the example, 752-257=495). Reverse the answer and add (in the example 495+594). [*the example is also written in column; the final result, 1089, is written*]
>
> a) Now, try with another three-digit number (with the first digit greater than the third one) and do the same procedure: what result do you get?
>
> b) Can you find a result that's is NOT 1089? Why?

To answer question a) it is sufficient to carry out the same procedure as in the worked example. Question b) is the real challenge. A first exploration on numerical examples may lead to a counterexample (for instance, starting form 423 one gets 198 as final result). Once found the counterexample, the problem is solved; nevertheless, expert solvers could go on with the reflection, in order to see whether it is a sort of "isolated" counterexample or there is a regularity in the set of counterexamples. Indeed, all 3-digits numbers where the first digit is equal to the third digit plus 1 are counterexamples. Moreover, for all the counterexamples the result is 198. One could even come to a general conjecture: if the starting number of the type "first digit is equal to the third digit plus 1" the result is always 198, otherwise the result is always 1089. In the case in which the solver finds all numerical examples that end up always with 1098, she/he is expected to move from an explorative phase to the formulation of a conjecture, and the search for a justification on the general plane. This passage from numerical examples to mathematical argumentation was crucial to the aims and contents of the teacher education course.

Research questions

We explore the potentialities offered by the "affective documentation" requests to research affect in mathematical problem-solving. At the same time, we investigate whether such kind of tasks for pre-service teachers may have professional development value.

The research questions guiding the study are: 1) What affective pathways emerge? How do they intervene in the solving process? 2) Is it possible to establish some link between attitude towards mathematics and affective pathway? 3) What indications for teacher education come from the intertwining of solving and affective pathways?

In this paper, we will face the first question, and try to get some insight on how to direct future research in order to tackle the other (more ambitious) questions.

ANALYSIS

The 1089 task was faced by 81 students (all those who were attending the specific lesson). As usual in the lessons, students were allowed to collaborate in small groups: actually most of them chose the group-work; however, they had to fill the sheet individually and to write down the names of the groupmates. The subsequent qualitative analysis is performed on the individual reports.

The first approach to the problem

Considering the emotional reactions at the reading of the text, 48 future teachers (59%) declare only positive emotions, while 13 (16%) declare only negative emotions. The most quoted emotion is *curiosity*, mentioned in explicit way by 35 students (43%). Curiosity (but also incredulity) comes often along with the *cognitive needs* of understanding and discovering, which push towards undertaking with trust the resolution of the problem. An example is Schirry: "*I feel curiosity and desire of discovering, which push me to engage in solving the problem as a challenge*".

Among positive emotions we remark also *astonishment*, indicated by 11 students. In some cases, astonishment is due not so much to the content of the problem, but to the novelty given by the typology of the problem (Betty: *"I feel astonished because I never focused on these types of problems"*). Among the students that signal mixed emotions (i.e. both positive and negative) we find mostly curiosity and interest combined with puzzlement (FedePeri: "*At the first reading I feel a bit of puzzlement, curiosity about the outcome and desire of trying with other numbers to discover if the results is always the same*"). In these cases, puzzlement, if accompanied with positive emotions such as curiosity, can constitute an important engine for the solving process, pushing towards the exploration phase. Some students welcome the activity even with enthusiasm, underlying its disruptive character with respect to the mathematical activities of their past experience (Amy: "*Let's play! Let's try with numbers! No proof for which it is needed to have studied so much!*"). Though rare (3 cases out of 81), this kind of enthusiastic welcome to a complex problem out of classical schemas can be considered as another sign encouraging us in continuing in this direction, on an intervention plan. On the other hand, also the few students who indicate only negative emotions do undertake the solving process, with some success (possibly due to the collaboration with their mates during the group-work).

Finally, some students, rather than indicating an emotional aspect, underline their cognitive needs when facing the problem. In particular, some of them express the necessity of reading the text several times. Sometimes we can trace the reference to interiorized schemas, which guide the solving process (Lia: "*I had to read the text several times, because during the first reading I try essentially to seize the general sense of the problem*"), and even the description of the kind of performed reading, as G.C.: "*I had to read the text several times lingering especially on the key-words and on the example. Once understood the text, I thought how to solve the problem*". Students who quote the necessity of reading carefully and several times the text, typically show also strong elements of meta-cognitive control over the solution.

All the aforementioned examples refer to an intertwining between text reading (as a first step in the problem solving process) and *emotional disposition* (one component of the construct of attitude). Other examples refer to the role of *perceived competence* (another dimension of attitude) in text reading: a sense of tranquillity is juxtaposed to the awareness of knowing how to face what is requested (perceived competence), while restlessness and discouragement are quoted when the students foresee, by the first reading, not to be able to solve the problem. Some students make an analytical separation between the two requests of the problem, declaring opposite feelings in relation with opposite competences perceived to face them; an example is Chi: "*For what concerns the first part (the first question) I can say I am quite tranquil, because the request is clear and manageable. The problems come out with the second request, because I do not feel I am able to give a valid argument for that*".

The solving process

The specific formulation of the task, providing a precise indication on how to start the solving process, appears to prevent "initial blocks". The first solving steps engage the students a lot, on the one hand thanks to their procedural nature (a kind of mathematical activity in which the students feel at easy, as it emerged from the questionnaires); on the other hand, the task allows for a certain freedom for the exploration. This latter aspect is underlined as a positive factor by some students, as Lavixx94: "*It is a problem which allows us initially a certain freedom in our choices and this can be a positive factor that does not create anxiety in solving it, providing already given numbers to solve. One can so get to solve the passages in a quiet way*". The second request (*Can you find out a number...*) is the actual core of the task. Some students start the exploration without any well-defined goal; some others are instead oriented since the beginning towards finding out a counter-example. As concerns the emotional aspects, we remark that outcomes that are *analogous* from the mathematical point of view are accompanied by very *different* emotional reactions, ranging from tranquillity to frustration, as we detail below.

Let us consider first those who obtain always 1089 (32 out of 81). Some of them are comforted from the regularity of the result, and as a consequence pushed to exploring and understanding. Others are satisfied with finding out other examples giving 1089, but feel stuck in proving it. In few cases (5) the reference to the need of a proving phase is completely missing, while a sense of amazement is the final step of the process. It is the case of Elis94, who checks only one example and comments "*Amazement, we did not think that it turned indeed into 1089!*", but also of Estestest, who explores with six numerical examples. A third category is formed by those who feel sad, frustrated, or even angry for not finding out any counter-example (Martitea93: "*I remain astonished because I cannot realize of not being able of finding out a starting number that does not give 1089. I felt so to say also some anger"*). In the last two categories we can signal a lack of *knowledge at a meta-level on the mathematical activity*, which would lead to recognize a regularity in the absence of the counter-example, and hence to searching for a general explanation. Probably those who feel frustrated are too much focused on the question "Can you find out...", as if it were a rhetoric question ("*Surely I have to find it out!*"). In this case, the emotional reaction would be strictly related to the *problem formulation* or to the *problem interpretation*: this appears to us a viable route to work on with the students, in an intervention perspective.

The majority of the students (59 out of 81) found out one or more counter-examples (leading to 198). An analogous range of emotional reactions, from surprise to puzzlement, can be detected. In other terms, we want to stress that it is not the result *per se* to provoke a certain emotional reaction, rather its interpretation within the mathematical activity, which in particular is determined by one's own *expectations on the problem* and *meta-mathematical knowledge*.

Dually, the same emotion may be linked to very different behaviours in the solving process. The most meaningful case relates to *astonishment*: it is sometimes indicated as final emotion, not stimulating further reflections (especially in students who obtain always 1089), whereas in other cases it is the engine for questioning and continuing the mathematical work (DadiLuca: *"With this example I don't find 1089 and I feel a sense of astonishment. Will it maybe be because the 3rd digit is less than the 1st only of 1?"*).

Generally, we can observe that the positive emotions, when are the *only* mentioned emotions, are not necessarily associated with a good solving process or high quality mathematical activities. In our data we can trace a serious *dark side of positive emotions as the only emotions*: students indulge on their positive feelings, do not receive boosts to further verify their results, to question their process, to check it. Their solving process burns out early, with little control at meta-cognitive level. Furthermore, it often happens that the negative emotion is felt later, when the students talk with their mates or the teacher, and realize their low performances.

On the contrary, those who indicate only negative emotions during the solving process do not always carry out poor or uncorrected mathematical activities. In many cases, an initial *negative* reaction is followed by a *positive* reaction associated with a discovery. For instance, Kika narrates in this way the story of her solution: *"At first we took numbers at random, seeing that it always came out 1089, hence we were resigning ourselves to the fact that it always came 1089. Then we tried to take numbers in sequences (e.g., 421-422-423) and we noticed that the results changed with 423, where the last digit was smaller than the first of 1 only. Hence we chose other two numbers with this structure (e.g. 524 and 726) and we transformed the hypothesis in a theory! What a satisfaction!"*).

The cases with more complete and effective solving processes are also those that document *quick up-and-downs of emotions* linked to the different solving moments (e.g. quick sequences of discomfort-new exploration-discomfort- astonishment). An example is given by Giuly who writes down three examples, finding 1089 in all cases and commenting "*Astonishment, wow, happy*", then finds an example with 198 ("*Ops I did something wrong*"), finds another case of 198 and writes "*OK, maybe I understood the rule, we feel yet more accomplished".*

This emotional swing ends up sometimes with a negative emotion, especially frustration or dissatisfaction, which may be linked to three different reasons:

1) feeling unable to communicate properly a certain conjecture, explanation, or argument, as Rubinorosso: "*curiosity-get lost-illumination-frustration in not being able to express well our discoveries*";

2) feeling unable to understand why there is a certain result, as Vios:*"But we cannot understand why it is always 198"*;

3) feeling unable to prove a certain result, as Chi: *"The discomfort is due to the fact that I am not able, in logical-mathematical terms, to prove the reason why, considering a number with the first digit greater than the third one, and the second and third the same, I got a number different from 1089"*

Generally, the final dissatisfaction is indeed related to greater mathematical and meta-mathematical competences. The typical cases are given by those pre-service teachers that express a global sense of success (for being able to find out examples) but also regret, delusion and resignation for not finding a general rule to distinguish the two sets of cases 1089 and 198. Asking where the counter-example comes from, and why only with some classes of numbers you find 198 is a typically mathematical curiosity: as future teachers educators we interpret in a very positive way these final students' dissatisfactions.

DISCUSSION AND CONCLUSIONS

A preliminary reflection concerns our method for studying the affective pathways in problem solving, i.e. the series of affective documentation requests. Even if prospective teachers were asked to report their emotions after text reading, during the solving process, and once solved the problem, many of them did not report emotions *during* the solving process, they rather commented immediately after having accomplished the task, or wrote twice the same comments. As a consequence, we organized our analysis in two sections (emotions after text reading and affective pathways in problem solving), without distinguishing between documentation within and after the process. Actually, writing down emotions and feeling when solving the problem is a demanding task, since the solver should stop the process in order to comment it. In light of these findings, in future interventions we would skip the request of documentation during the process and just ask for a report after the solution: in terms of meta-affect, this should be a valuable request as well. Also, we did not focus on the possible effect of group work on the affective and solving pathways. Future research could be organized so to distinguish between the two levels of “individual emotions” (reported when working individually) and “group emotions” (reported when working in group).

Concerning the results of the study, data analysis confirms the deep intertwining of affective and cognitive factors in problem solving, and reveals some patterns in the affective pathways (first research question): rich and complex problem solving processes are characterized by emotional pathways with a “continuous swinging” of emotions. Such emotional states are both cause and effect of the exploration, conjecture and proving steps. On the contrary, “fixed” emotional pathways, with a more stable (positive or negative) emotional state that persists throughout all the process, occur when solving processes are poor and non efficient. In order to interpret the different affective behaviours, we need more research on the possible link between awareness of the emotional pathway, meta-affect and attitude towards mathematics and its teaching.

As a provisional answer to the third research question, we discuss some possible routes for teacher education. A first result concerns the generally positive emotional disposition at the first reading of the text. It confirms the opportunity of engaging prospective teachers in such a kind of activity, which may be seen also as an occasion to experience new ways of doing mathematics. Another reflection comes from the reported emotions at the end of the solving process. We highlighted that different solvers report the same emotions in different situations and that, finally, it is not matter of positive or negative states. For instance, dissatisfaction and frustration may be linked to the need of better understanding or to the awareness of difficulties at epistemic and/or communicative level. Thus, negative emotional states may be read in terms of competence at meta-mathematical or mathematical level. Conversely, positive emotions such as satisfaction may be linked to a lack in meta-mathematical competence. The 1089 problem is a highly demanding task for pre-service primary teachers, also because of its theoretical nature: the core is constituted by conjecturing and proving, which we value as fundamental mathematical activities. However, pre-service teachers with low competencies at mathematical and meta-mathematical level happened to live the experience of solving the 1089 problem—with correlated positive emotions—and to discover their failure only later through the confrontation with more expert mates or the teacher educator: in this case, a negative emotion is likely experienced, risking to be the most persistent result of the overall activity. The crucial point, for us as teacher educators, is to find out efficient ways to intervene on the latter cases. This also opens the discussion on what kind of mathematical activities are worth to be proposed, and how to push the reflection at meta-level. We need more research on the design of suitable tasks for promoting prospective teachers’ awareness of their emotional pathways during problem solving.

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