Vagueness and Quantification

(postprint version)

Journal of Philosophical Logic first online, 2015

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This paper deals with the question of what it is for a quantifier expression to be vague. First it draws a distinction between two senses in which quantifier expressions may be said to be vague, and provides an account of the distinction which rests on independently grounded assumptions. Then it suggests that, if some further assumptions are granted, the difference between the two senses considered can be represented at the formal level. Finally, it outlines some implications of the account provided which bear on three debated issues concerning quantification.

1 Preliminary clarifications

Let us start with some terminology. First of all, the term 'quantifier expression' will designate expressions such as 'all', 'some' or 'more than half of', which occurs in noun phrases as determiners. For example, in 'all philosophers', 'all' occurs as a determiner of 'philosophers', and the same position can be occupied by 'some' or 'more than half of'. This paper focuses on simple quantified sentences containing quantifier expressions so understood, such as the following:

(1) All philosophers are rich

(2) Some philosophers are rich

(3) More than half of philosophers are rich

Although this is a very restricted class of sentences, it is sufficiently representative to deserve consideration on its own.

In the second place, the term 'domain' will designate the totality of things over which a quantifier expression is taken to range. Very often, when a quantifier expression is used, it carries a tacit restriction to a set of contextually relevant objects. For example, on one occasion (1) may be used to assert that all philosophers in a university U are rich, so that 'all' ranges over a set of people working or studying in U, while on another occasion it may be used to assert that all philosophers in another university U' are rich, so that 'all' ranges over a set of people working or studying in U'. In order to take into account such contextual restrictions it will be assumed that, whenever a quantifier expression is used, some domain is associated with its use, that is, the domain over which the expression is taken to range.

One thing that must be clear about this assumption is that it does not settle the question of how the restriction is determined in the context. To appreciate its neutrality, it suffices to think about a debated issue which divides semantic accounts of domain restriction. According to such accounts, domains are represented by some sort of variable or parameter in the noun phrase. But it is controversial where exactly the variable or parameter is located. For example, Westerståhl suggests thay it is in the determiner, while Stanley and Szabo suggests that it is in the noun. The picture sketched in this paper is compatible with both options, as it does not concern the syntactic structure of quantified sentences[1].

Another thing that must be clear about the assumption that quantifier expressions are used in association with domains is that it does not entail that, whenever one uses a quantifier expression, one has in mind a definite set of contextually relevant objects. As a matter of fact, that almost never happens. Most of the time, the use of a quantifier expression involves either a very approximate specification of a set, or no specification at all. In the first case no unique set is specified, in that different sets turn out to be equally admissible. In the second, no set at all is specified, in that nothing is excluded as irrelevant.

The third and last term to be introduced is 'quantifier'. In accordance with an established practice, this term will be used to refer to functions from domains to binary relations. The meanings of 'all', 'some' and 'more than half of' may be defined as quantifiers, that is, as functions *all*, *some* and *more than half of* which satisfy the following conditions for any domain D:

Definition 1. $all_D(A, B)$ *if and only if* $A \subseteq B$.

Definition 2. $some_D(A, B)$ *if and only if* $A \cap B \neq \emptyset$.

Definition 3. *more than half of*$_D(A, B)$ *if and only if* $\mid A \cap B \mid > 1/2 \mid A \mid$

Here A and B are sets whose members belong to D, and the left-hand side is read as 'the relation denoted by the quantifier expression relative to D obtains between A and B'. Note that definition 3 differs from definitions 1

[1] Westerståhl [**?**], Stanley and Szabo [**?**]. For simplicity we will not consider pragmatic accounts of domain restriction, that is, accounts on which the determination of domains is left to pragmatic factors which determine the communicated content as distinct from what is literally said, such as that outlined in Bach [**?**].

and 2 in that it involves a proportional relation that applies to the cardinality of A and B. Accordingly, *more than half of* may be called a *proportional* quantifier[2].

The domain parameter that occurs in definitions 1-3 accounts for the fact that the extension of a quantifier expression may vary from occasion to occasion, even though its meaning does not change: if e is a quantifier expression which means Q, then Q_D is the extension of e relative to D. For example, if D is a set of people working or studying in U and D' is a set of people working or studying in U', 'all' denotes different relations relative to D and D'. So there is a sense in which 'all' means the same thing on both occasions, yet the relations denoted differ. The same goes for 'some' and 'more than half of'.

If the meaning of quantifier expressions is defined in the way outlined, and nominal expressions are taken to denote sets, the meaning of quantified sentences can be obtained by composition. Let A and B be sets denoted by 'philosophers' and 'rich' relative to D. Given definition 1, all_D fixes truth conditions for (1) relative to D, that is, (1) is true if and only if $A \subseteq B$. So the meaning of (1) may be described as a function from domains to truth conditions, which results from the combination of *all* with the meanings of 'philosophers' and 'rich'. The case of (2) and (3) is similar. Assuming that A and B are sets denoted by 'philosophers' and 'rich' relative to D, the meaning of (2) or (3) may be described as a function from domains to truth conditions which results from the combination of *some* or *more than half of* with the meanings of 'philosophers' and 'rich'. More generally, the meaning of a quantified sentence s that contains a quantifier expression e that means Q is obtained by combining Q with the meaning of the nominal expressions in s.

2 Two kinds of indeterminacy

The question of what it is for a quantifier expression to be vague seems to admit two kinds of answers. It is plausible to say that a quantifier expression e (as it is used on a given occasion) is vague if it is possible that a quantified sentence s in which e occurs is neither clearly true nor clearly false - in a way of being neither clearly true nor clearly false which is distinctive of vagueness - and that does not entirely depend on the vagueness of other expressions in s. However, it seems that such unclarity can have two different sources. Roughly speaking, the semantic role of e in s is to specify a certain amount of things which belong to the domain over which e is taken to range. So if

[2]Peters and Westerståhl define quantifiers this way in [?], pp. 62-64. Note that in definitions 1-3 no index is attached to A and B to show that they depend on D, but such effect could easily be obtained with some minor adjustment. For example, the notation adopted in Lappin [?] makes A and B systematically depend on D.

it is unclear whether s is true or false, and this unclarity does not entirely depend on other expressions in s, either there is indeterminacy about the domain over which e is taken to range, or there is indeterminacy about the amount specified.

To illustrate the first kind of indeterminacy, consider (1)-(3). One may easily imagine circumstances in which it is unclear whether (1)-(3) are true or false. Obviously, this is due at least in part to the fact that 'philosophers' and 'rich' do not have a definite extension. But even if 'philosophers' and 'rich' *did* have a definite extension, it could still be unclear whether (1)-(3) are true or false. One source of unclarity is the fact mentioned in section 1 that the use of a quantifier expression may involve only a very approximate specification of a set of contextually relevant objects. For if no definite set is specified, there is a plurality of sets such that it is indeterminate which of them is the intended set. Consider (1). Even if 'philosophers' and 'rich' had a definite extension, it might still be unclear whether (1) is true or false, because it might be unclear what exactly is the domain over which 'all' is taken to range. Suppose that (1) is uttered to assert that all philosophers in U are rich, but that no unique set of contextually relevant objects is specified. In particular, suppose that D is a set of people working or studying in U, and that D' is a proper subset of D which differs from D only in that it does not include a certain person whose affiliation to U is unclear for some reason. If so, it might happen that (1) is neither clearly true nor clearly false. Similar examples can be provided with (2) and (3).

One way to see that this kind of indeterminacy is correctly described as vagueness is to see how it can be distinguished from context sensitivity. If 'context' is understood informally as a concrete situation in which a sentence is uttered by a speaker, it is realistic to say that the use of a quantifier expression in a context may fail to specify a definite domain. For even if a restricting condition is associated to the quantifier expression - in virtue of contextual features such as the speaker's intentions, the conversational background, and so on - the restricting condition is itself indeterminate. In the example considered, the restricting condition is expressed by 'people working or studying in U', but it may be unclear whether a certain person works or studies in U. Similar examples may be provided with paradigmatically vague expressions: a restricting condition could be expressed by 'bald people', 'thin people' or 'tall people', in which case it would be evident that it involves the kind of unclarity that is distinctive of vagueness. Obviously, one might introduce a finer notion of context by stipulating that a context is an n-tuple of parameters which includes a set of objects as domain. But then one would have to grant the intelligibility of the informal understanding of 'context', and the point would still remain that the use of a quantifier expression in a context informally understood may fail to specify a definite context in the fine sense.

To illustrate the second kind of indeterminacy, consider the following sentences:

(4) Most philosophers are rich

(5) Few philosophers are rich

(6) Many philosophers are rich

It is easy to see that (4)-(6), just like (1)-(3), may be used without specifying a definite set of contextually relevant objects. But in the case of (4)-(6) there is another possible source of unclarity, namely, the fact that a quantifier expression may fail to specify a definite amount of things that belong to a given domain. Consider (4). Even if 'philosophers' and 'rich' had a definite extension, it might still be unclear whether (4) is true, because it might be unclear whether 'most' is to be read, say, as 'more than 1/2' or as 'more than 2/3'. Similar considerations hold for (5) and (6), as 'few' and 'many' admit multiple readings in the same sense. By contrast, 'all', 'some' and 'more than half of' do not admit multiple readings in that sense. This suggests that 'most', 'few' and 'many' are indeterminate in a way in which 'all', 'some' and 'more than half of' are not. While 'all', 'some' and 'more than half' provide a definite specification of a certain portion of the domain, 'most', 'few' and 'many' do not, as they can be understood in more than one way.

Again, one way to see that this kind of indeterminacy is correctly described as vagueness is to see how it can be distinguished from context sensitivity. For it is realistic to say that the use of a quantifier expression in a context may fail to determine a definite reading in the sense just illustrated. More generally, there are two ways in which the use of a quantified sentence in a context may fail to fix definite truth conditions. In the first case, the sentence has no definite truth conditions because no definite domain is fixed. This may be called *domain indeterminacy.* In the second case, the sentence has no definite truth conditions because, given an intended domain, no definite binary relation is fixed on that domain. This may be called *quantifier indeterminacy.*

Both domain indeterminacy and quantifier indeterminacy are plausibly described as linguistic phenomena, that is, as forms of indeterminacy that affect linguistic expressions. For neither of them seems easily reducible to non-linguistic facts. This is not to say that there is indeterminacy only at the linguistic level. More specifically, in the case of domain indeterminacy this is not to rule out the existence of indeterminacy at the metaphysical level. It is consistent with a description of domain indeterminacy as a linguistic phenomenon to suppose that the very things over which a quantifier expression ranges are indeterminate. What such description requires is simply that there is indeterminacy at least at the linguistic level.

Moreover, domain indeterminacy and quantifier indeterminacy are clearly independent of each other. On the one hand, it can be the case that a quantifier expression (as it is used on a given occasion) is indeterminate in the first sense without being indeterminate in the second. For example, 'all' always specifies a determinate portion of the intended domain, even if in some cases it may be indeterminate which is the intended domain. On the other, it is conceivable that a quantifier expression (as it is used on a given occasion) is indeterminate in the second sense without being indeterminate in the first. For example, even assuming that 'most' ranges over a definite domain in a given case, it still makes sense to say that it fails to specify a definite portion of that domain.

In substance, domain indeterminacy and quantifier indeterminacy can be regarded as two ways in which quantifier expressions may be vague. That is, if a quantifier expression (as it is used on a given occasion) is vague, then it is affected either by quantifier indeterminacy, or by domain indeterminacy, or by both. This explains why the question of what it is for a quantifier expression to be vague seems to admit two different kinds of answer.

3 Precisifications of quantifier expressions

As is well known, there are different views of vagueness, because there are different ways to explain its distinctive form of unclarity. But the divergences on the nature of vagueness are to a good extent irrelevant for the purposes of this paper. In what follows it will simply be assumed that the vagueness of a language entails its capacity in principle to be made precise in more than one way. That is,

(VP) If an expression is vague, then it admits different precisifications.

Although (VP) is not universally accepted, it is consistent with more than one view of vagueness. In particular, it is consistent with supervaluationism, epistemicism, and other views that differ both from supervaluationism and from epistemicism. This section suggests that the distinction between quantifier indeterminacy and domain indeterminacy may be understood as a distinction between two kinds of variations in the precisifications of a quantifier expression[3].

To see how domain indeterminacy may be described in terms of precisifications, it suffices to focus on (1)-(3). Let us assume that an *interpretation*

[3]Supervaluationism is consistent with (VP) both in its standard version outlined in Fine [?] and in non-standard versions such as that provided in McGee and McLaughlin [?]. Epistemicism is consistent with (VP) at least in the version advocated in Williamson [?]. Other views consistent with (VP) are those suggested in Braun and Sider [?] and in Iacona [?], which qualify as neither supervaluationist nor epistemicist. Finally, (VP) is consistent with some views according to which vagueness is in rebus, as in Barnes [?] and in Barnes and Williams [?] and [?].

of a sentence s is an assignment of semantic properties to the expressions in s which are compatible with their linguistic meaning and determines definite truth conditions for s. On the assumption that an interpretation of a quantified sentence fixes a domain for the quantifier expression which occurs in the sentence, a case of domain indeterminacy may be described as a case in which a quantified sentence is used in a context, but a plurality of interpretations of the sentence are equally admissible in the context. Each interpretation provides a precisification of the quantifier expression which occurs in the sentence.

To illustrate, suppose that (1) is uttered to assert that all philosophers in U are rich, but that no unique set of contextually relevant objects is specified. In particular, suppose that D is a set of people working or studying in U, and that D' is a proper subset of D which differs from D only in that it does not include a certain person whose affiliation to U is unclear for some reason. Then there are two precisifications p_1 and p_2 such that p_1 assigns D to 'all' and p_2 assigns D' to 'all'. Consequently, it may be unclear whether (1) is true. For (1) might have different truth values in the two corresponding interpretations.

In order to describe quantifier indeterminacy in terms of precisifications, the meaning of 'most', 'few' and 'many' will be defined along the lines suggested in section 1. Even though the definitions that will be adopted may be controversial, since there is no general agreement on the meaning of 'most', 'few' and 'many', nothing essential depends on them. For the present purposes, they may simply be regarded as possible options that illustrate the way in which 'most', 'few' and 'many' differ from 'all', 'some' and 'more than half of'.

Let us start with 'most'. A basic fact about *most* seems to be that the condition stated in definition 3 must be satisfied for the intended relation to obtain: if one says that most philosophers are rich, one says at least that more than half of philosophers are rich. This may be regarded as a necessary condition on *most*. Yet it is not a sufficient condition. Certainly, we can imagine situations in which 'most' is used as synonymous of 'more than half of'. But if the meaning of 'most' were exhausted by that condition, 'most' wouldn't be indeterminate in the way considered. The meaning of 'most' seems to allow for variation in the proportion between the size of $A \cap B$ and the size of A. Suppose that there are exactly 1.000.000 philosophers on earth, and that exactly 501.000 of them are rich. In that circumstance it might be unclear whether (4) is true, while it is clear that (3) is true. In order to account for this variation, a definition of *most* may be given along the following lines:

Definition 4. $most_D(A, B)$ *if and only if* $\mid A \cap B \mid > n/m \mid A \mid$

Here $0 < n < m$ and $n/m \geq 1/2$. For example, $1/2$ and $2/3$ are equally acceptable values for n/m. In other words, *most* is defined as a class of quantifiers rather than as a single quantifier. Consequently, the meaning of (4) may be described as a class of functions from domains to truth conditions that is obtained by combining *most* with the meanings of ‘philosophers’ and ‘rich’. This means that (4) differs from (1)-(3), in that the determination of its truth conditions involves a parameter other than the domain. Let A and B be sets denoted by ‘philosophers’ and ‘rich’ relative to D. Whether $most_D$ obtains between A and B depends on the values assigned to n and m. For example, if $n = 2$ and $m = 3$, then it obtains just in case $\mid A \cap B \mid > 2/3 \mid A \mid$. In order to determine definite truth conditions for (4), we need both a domain and a value of the additional parameter whose variation is allowed by the indeterminacy of ‘most’[4].

As in the case of ‘most’, the meaning of ‘few’ and ‘many’ may be defined as a class of quantifiers. But there is a significant difference. While ‘most’ is clearly proportional, it is at least *prima facie* acceptable that ‘few’ and ‘many’ behave non-proportionally. Consider *few*. A basic fact about few seems to be that, for an arbitrary D, to say that few_D holds between A and B is to set an upper bound on the size of $A \cap B$. There are at least two ways to express this fact. The first may be called the absolute reading of ‘few’:

Definition 5. $few_D(A, B)$ *if and only if* $\mid A \cap B \mid \leq n$

This reading is called absolute because the upper bound on the size of $A \cap B$ is fixed without reference to the size of A or B. The second reading, instead, may be called the proportional reading of ‘few’, and comes in two versions:

Definition 6. $few_D(A, B)$ *if and only if* $\mid A \cap B \mid \leq n/m \mid A \mid$

Definition 7. $few_D(A, B)$ *if and only if* $\mid A \cap B \mid \leq n/m \mid B \mid$

Here n and m are such that $0 < n < m$. Definition 6 may be appropriate for (5), given that in (5) the number of rich philosophers is said to be small with respect to the number of philosophers. The following sentence, instead, is naturally understood in terms of definition 7:

(7) Few cooks applied

In (7) it is said that the number of applicant cooks is small with respect to the number of applicants, rather than the other way round.

The case of *many* is analogous. A basic fact about *many* seems to be that, for an arbitrary D, to say that $many_D$ holds between A and B is to set a lower bound on the size of $A \cap B$. Again, there are at least two ways to express this fact. The first may be called the absolute reading of ‘many’:

[4]Definition 4 is in line with the suggestion in Barwise and Cooper [**?**], p. 163, and the account in Westerståhl [**?**], pp. 405-406. In the latter work, two readings of ‘most’ are considered. But if definition 4 is adopted there seems to be no reason to do that.

Definition 8. *$many_D(A, B)$ if and only if* $\mid A \cap B \mid \geq n$

This reading is called absolute because the lower bound on the size of $A \cap B$ is fixed without reference to the size of A or B. The second reading, instead, may be called the proportional reading of 'many', and comes in two versions:

Definition 9. *$many_D(A, B)$ if and only if* $\mid A \cap B \mid \geq n/m \mid A \mid$

Definition 10. *$many_D(A, B)$ if and only if* $\mid A \cap B \mid \geq n/m \mid B \mid$

Definition 9 may be appropriate for (6), given that in (6) the number of rich philosophers is said to be big with respect to the number of philosophers. The following sentence, instead, is naturally understood in terms of definition 10:

(8) Many Scandinavians have won the Nobel Prize

In (8) it is said that the number of Scandinavian Nobel Prize winners is big with respect to the number of Nobel Prize winners[5].

The absolute reading and the proportional reading of 'few' and 'many' might be regarded either as two distinct meanings that 'few' and 'many' can take depending on the occasion, or as two different hypotheses about their unique meaning. In any case, the meaning of (5) and (6) is obtained by combining few and $many$ with the meanings of 'philosophers' and 'rich'. Therefore, it may be described as a class of functions from domains to truth conditions[6].

If the meaning of 'most', 'few' and 'many' is defined in the way suggested, quantifier indeterminacy may be described in terms of precisifications. Consider definition 4. The variables n and m which occur in this definition indicate the variability of the proportion between $\mid A \cap B \mid$ and $\mid A \mid$, which constitutes the quantifier indeterminacy of 'most'. Each assignment of values to n and m amounts to a way of sharpening the meaning of 'most'. So it may be assumed that a precisification of 'most' involves such an assignment, in addition to the domain parameter. For example, one precisification of 'most' is that according to which $n = 2$ and $m = 3$, so the condition required is that $\mid A \cap B \mid > 2/3 \mid A \mid$. Definitions 5-10 are similar to definition 4 in this respect. For each of these definitions - no matter whether the reading is absolute or proportional - entails that the quantifier expression defined admits precisifications that differ in the same way. As in the case of domain indeterminacy, the precisifications of a quantifier expression determine interpretations of the quantified sentence in which it occurs.

[5] The examples (7) and (8) are drawn from Peters and Westerståhl [**?**], pp. 213.

[6] The hypothesis that 'most', 'few' and 'many' can be treated along the way suggested is adopted in Barwise and Cooper [**?**] and in Westerståhl [**?**]. Instead, Keenan and Stavi [**?**] and Lappin [**?**] provide differents accounts of 'few' and 'many'.

From what has been said so far it turns out that the distinction between domain indeterminacy and quantifier indeterminacy may be described in terms of two kids of variations in the precisifications of a quantifier expression. On the one hand, if a quantifier expression (as it is used on a given occasion) exhibits domain indeterminacy, then it admits precisifications that involve domain variation. On the other, if a quantifier expression (as it is used on a given occasion) exhibits quantifier indeterminacy, then it admits precisifications that involve quantifier variation. Since interpretations of quantified sentences include precisifications of the quantifier expressions which occur in them, the same distinction may be drawn with respect to interpretations of quantified sentences.

4 Truth conditions and logical form

The foregoing sections draw attention to the distinction between domain indeterminacy and quantifier indeterminacy, and outline an account of the distinction based on some relatively uncontroversial assumptions. This section and the following show how the account outlined may be articulated at the level of logical form. The assumptions that will be adopted are more controversial, so the same goes for the conclusions that will be drawn. But it is important to understand that, even if one is not sympathetic with the line of thought that will be advanced, one may still regard what has been said so far as plausible and interesting in itself.

There are at least two senses in which one may wonder what is the logical form of quantified sentences. One question is how quantified sentences are to be formally represented in order to explain the valid inferences in which they occur. Another question is how quantified sentences are to be formally represented in order to provide a compositional account of their meaning. Although it is often assumed that a unique notion of logical form can provide answers to both questions, it will not be assumed here. In what follows we will focus only on the first question, leaving aside the second. The crucial hypothesis that will be held about the formal explanation of valid inferences is that the notion of logical form it requires is a *truth conditional notion*, that is, a notion according to which logical form is a matter of truth conditions. Since no uniqueness assumption will be made, this is compatible with there being a different notion of logical form that is suitable for the second question. More precisely, it is compatible with the hypothesis that a syntactic notion of logical form - such the notion of LF adopted in linguistics - is to be adopted to answer the second question[7].

The truth conditional notion of logical form stems from the idea that an adequate formalization of a sentence must provide a representation of its

[7]Iacona [?] provides an argument against the uniqueness assumption.

content that exhibits its truth conditions. Let L be a standard first order language with identity. Consider the following sentences:

(9) Aristotle is rich

(10) Aristotle is indeed rich

(11) Plato is rich

Clearly, (9) and (10) have the same truth conditions, because they describe the same object as having the same property, while (9) and (11) have different truth conditions, because they describe different objects as having that property. Therefore, (9)-(11) are adequately formalized in L as Fa, Fa, Fb. On the understanding of adequate formalization that will be adopted, if $\bar{s}$ is an n-tuple of sentences and $\bar{\alpha}$ is an n-tuple of formulas, then $\bar{\alpha}$ adequately formalizes $\bar{s}$ only if the formulas in $\bar{\alpha}$ represent the truth conditions of the sentences in $\bar{s}$ in such a way that two formulas in $\bar{\alpha}$ are logically equivalent if and only if the sentences in $\bar{s}$ to which they are assigned have the same truth conditions[8].

Note that this is only a necessary condition for adequate formalization, so it may not be regarded as a complete account of adequate formalization. When a set of sentences is represented in a formal language, the representation is intended to capture what is said by using these sentences, in some sense of 'what is said' that is relevant for the purpose of formal explanation. So it is reasonable to presume that, for a n-tuple of sentences $\bar{s}$, only some of the n-tuples of formulas that satisfy that condition adequately formalize $\bar{s}$. For example, it is usually taken for granted that Fa is better than $\sim\sim Fa$ or $Fa \wedge (Gb \vee \sim Gb)$ as a representation of (9). Even though $\sim\sim Fa$ and $Fa \wedge (Gb \vee \sim Gb)$ are logically equivalent to Fa, they do not capture what is said by using (9) in the relevant sense of 'what is said'. The underlying thought is that, in order to adequately formalize a sentence, one should choose a formula whose complexity is strictly that required by a correct analysis of the content of the sentence, which means that the formula must have the minimum complexity that is necessary to capture that content. Here 'complexity' is understood in the standard way, as the number of logical symbols that occur in the formula, and 'correct logical analysis' is irreducibly vague and hard to define[9].

On the assumption that sentences have truth conditions relative to interpretations, it seems correct to claim that sentences have logical form relative

[8]This is just a rough characterization and certainly does not settle every issue concerning sameness of truth conditions. In particular, it does not entail that truth conditions are sets of possible worlds. For example, it is consistent with the characterization provided to say that '2 is even' and '3 is odd' have different truth conditions, in that they describe different objects as having different properties, even though they are both true in all possible worlds.

[9]Sainsbury [**?**] suggests a criterion of adequate formalization that rests on the idea that formalization must preserve what is said, pp. 161-162.

to interpretations. Let it be granted that, for an n-tuple of sentences $\bar{s}$, an *interpretation* of $\bar{s}$ is an n-tuple $\bar{i}$ such that each term in $\bar{i}$ is an interpretation of the corresponding term in $\bar{s}$. The criterion of individuation that underlies the truth conditional notion of logical form may be stated as follows:

Definition 11. *$\bar{s}$ has logical form $\bar{\alpha}$ in $\bar{i}$ if and only if $\bar{s}$ is adequately formalized by $\bar{\alpha}$ in $\bar{i}$.*

When $\bar{s}$ has exactly one term, we get that s has logical form α in i if and only if s is adequately formalized by α in i[10].

Definition 11 leaves room for two senses in which a formula α can be said to express the logical form of a sentence s relative to an interpretation i. The first is that in which α, as distinct from some other formula, represents the content of s relative to i, as distinct from some other content. For example, if (9) and (11) are formalized as Fa and Fb, the fact that Fa and Fb contain different individual constants shows that (9) and (11) have different truth conditions because 'Aristotle' and 'Plato' refer to different individuals. The second is that in which α represents the structure of the content of s relative to i in virtue of its being a formula of a certain *kind*. For example, Fa and Fb are both formulas of the form $\Pi\tau$, where Π indicates any unary predicate of L and τ indicates any individual constant of L. In this sense, it is plausible to say that (9) and (11) have the same logical form, although they express different contents.

The account of the meaning of quantified sentences outlined in sections 1 and 3 may be integrated with an analysis of quantified sentences based on the truth conditional notion of logical form. To illustrate, consider (1). As it turns out from section 1, (1) can be understood in more than one way. The simplest case is that in which (1) is used without restriction on the domain. Recall that the assumption that quantifier expressions are used in association with domains does not entail that, whenever one uses a quantifier expression, one has in mind a definite set of contextually relevant objects. It is consistent with that assumption to say that there are contexts in which nothing is excluded as irrelevant. The following formula represents (1) as used in such a context, if P stands for 'philosopher' and Q stands for 'rich':

(12) $\forall x(Px \supset Qx)$

In order to deal with a context in which some things are excluded as irrelevant, instead, P can be read as including the intended restriction. Suppose that (1) is used to assert that all philosophers in U are rich. In this case, (1) can be represented as (12), where P stands for 'philosopher in U' and Q stands for 'rich'. So if two utterances of (1) differ in the intended restriction

[10]Brun [?], p. 27, and Baumgartner and Lampert [?], p. 104, provide some considerations in support of the claim that the logical form of a set of sentences is expressed by an adequate formalization of the set.

on the domain, they may be represented by means of different predicate letters. Suppose that (1) is used on one occasion to assert that all philosophers in U are rich and on another occasion to assert that all philosophers in U' are rich. This difference may be represented in terms of the difference between (12) and the following formula:

(13) $\forall x(Rx \supset Qx))$

Here R stands for 'philosopher in U''. Note that (12) and (13) are analogous to Fa and Fb. On the one hand, (12) and (13) represent different contents insofar as P and R stand for different conditions. On the other, (12) and (13) are formulas of the same kind, in that they differ only for a predicate letter. In this sense it is plausible to say that they express the same logical form.

5 First order definability and first order expressibility

The thesis that quantified sentences can be formalized in L in virtue of their truth conditions has an important consequence which concerns a fact that is usually regarded as crucial for the expressive power of first order logic. The fact is that some quantifiers are not *first order definable*, in the sense that they do not denote quantifiers that satisfy the following condition:

Definition 12. *A quantifier Q is first order definable if and only if there is a formula α of L containing two unary predicate letters such that, for every set D and $A, B \subseteq D$, $Q_D(A, B)$ if and only if α is true in a structure with domain D where the predicate letters in α denote A and B.*

Here 'two' means 'exactly two'. It is easy to see that 'all' is first order definable, in that (12) satisfies the condition required. The same goes for 'some', given that (2) can be represented as follows:

(14) $\exists x(Px \wedge Qx)$

However, 'more than half of' is not first order definable. The same goes for 'most', 'few' and 'many'. Although (3)-(6) are semantically similar to (1) and (2), in that they are formed by expressions of the same semantic categories combined in the same way, there is no formula of L that translates (3)-(6) in the same sense in which (12) and (14) translate (1) and (2)[11].

It is often taken for granted that this fact constitutes a serious limitation of the expressive power of first order logic. For it is assumed that formalization is a matter of translation, understood as meaning preservation: to

[11] Barwise and Cooper provide a proof of the first order undefinability of *more than half of* in [?], pp. 213-214. Peters and Westerståhl, in [?], pp. 466-468, spell out a proof method that extends to other proportional quantifiers.

say that a quantifier expression is first order definable is to say that L contains some expression that captures its meaning. However, without that assumption there is no reason to think that the first order undefinability of 'more than half of', 'most', 'few' and 'many' rules out the possibility that (3)-(6) are formalized in L. Certainly, it undermines the claim that there are sentences of L that have the same meaning as (3)-(6). But if logical form is a matter of truth conditions, such a claim makes little sense anyway, even in the case of (1) and (2). Instead of asking whether a quantifier is first order definable, one may ask whether it is *first order expressible*, that is, whether it denotes a quantifier that satisfies the following condition:

Definition 13. *A quantifier Q is first order expressible if and only if, for every set D and $A, B \subseteq D$, there is an adequate formula α of L containing two unary predicate letters such that $Q_D(A, B)$ if and only if α is true in a structure with domain D where the two predicate letters denote A and B.*

Again, 'two' means 'exactly two'. The sense in which α is required to be adequate is the same in which a formalization is expected to be adequate, as explained in section 4. That is, α must represent what is said, relative to D, by a sentence which contains a quantifier expression that denotes Q and two predicates for A and B. Of course, adequacy so understood is a vague notion, so it can hardly be phrased in formal terms. However, this does not prevent definition 13 from playing a role analogous to that of definition 12. For if one takes a case in which the notion of adequacy definitely applies, and in which it is provable that the rest of the conditions that constitute first-order expressibility are satisfied, then one can rightfully conclude that definition 13 applies. This is just the kind of case at issue. The formulas that will be considered in our reasoning are assumed to be clear cases of adequacy, so the reasoning itself is to be understood as conditional on that assumption.

To see how adequacy matters, it suffices to think that a trivial proof of the existence of α can be provided if no such condition is imposed on α. For it is easy to find some α that has the required truth value for independent reasons. For example, if $Q_D(A, B)$ and α is a logical truth, then $Q_D(A, B)$ if and only if α is true in the structure. But from what has been said about adequacy it turns out clear that in this case α is not adequate. The same goes for similar trivial proofs of the existence of α. What is not trivial, instead, is to prove the existence of an adequate α. As it will be shown, 'more than half of', 'most', 'few' and 'many' are first order expressible, in that for every D and $A, B \subseteq D$, there is an adequate α containing two predicate letters such that $Q_D(A, B)$ if and only if α is true in a structure with domain D where the predicate letters denote A and B.

Let us start with 'more than half of' and 'most'. In this case the adequacy assumption that underlies the reasoning is that, if what is said by s relative to D is that at least n As are Bs, then a formula of L that contains n

occurrences of $\exists$ followed by n distinct variables and two unary predicates P and Q can provide an adequate representation of s. Such a formula will be indicated as follows:

(15) $\exists_{\geq n} x(Px \wedge Qx)$

A further assumption is that A and B are finite, as it is natural to expect given that 'more than half of' and 'most' are normally used to state relations between finite quantities.

Given these two assumptions, the first order expressibility of 'most' can be proved by showing that, if $A, B \subseteq D$ and $0 < n < m$, there is a k such that $\mid B \mid > n/m \mid A \mid$ if and only if $\mid B \mid \geq k$. The first order expressibility of 'more than half of' follows from this result, as it concerns the special case in which $n = 1$ and $m = 2$.

Theorem 1. *For every D and $A, B \subseteq D$, there is an adequate formula α of L that contains two unary predicate letters such that $most_D(A, B)$ if and only if α is true in a structure with domain D where the two predicate letters denote A and B.*

Proof. First it will be shown that, if $A, B \subseteq D$ and $0 < n < m$, there is a k such that $\mid B \mid > n/m \mid A \mid$ if and only if $\mid B \mid \geq k$. Suppose that $A, B \subseteq D$ and $0 < n < m$. A function F can be defined in the following way. If $j = 0$, then $F(j) = 1$. If $j > 0$ and j is divisible by m, then

$$F(j) = \frac{n}{m} j + 1$$

If $j > 0$ and j is not divisible by m, then $F(j)$ is the smallest integer such that

$$F(j) > \frac{n}{m} j$$

Now let $\mid A \mid = j$ and $k = F(j)$. k is such that $\mid B \mid > n/m \mid A \mid$ if and only if $\mid B \mid \geq k$. Suppose that $j = 0$. Then $n/m \mid A \mid = 0$ and $F(j) = 1$, so $\mid B \mid > 0$ if and only if $\mid B \mid \geq 1$. Suppose that $j > 0$ and j is divisible by m. Then $\mid B \mid > (n/m)j$ if and only if $\mid B \mid \geq (n/m)j + 1$. Finally, suppose that $j > 0$ and j is not divisible by m. Since $\mid B \mid$ is a natural number, $\mid B \mid > (n/m)j$ if and only if $\mid B \mid \geq F(j)$.

Once it is shown that, if $A, B \subseteq D$ and $0 < n < m$, there is a k such that $\mid B \mid > n/m \mid A \mid$ if and only if $\mid B \mid \geq k$, replacing B with $A \cap B$ it turns out that there is a k such that $\mid A \cap B \mid > n/m \mid A \mid$ if and only if $\mid A \cap B \mid \geq k$. Therefore, $most_D(A, B)$ if and only if $\mid A \cap B \mid \geq k$. This means that (15), for $n = k$, can be used to express in L the claim that $most_D(A, B)$. For (15) is true in a structure with domain D where P and Q denote A and B. $\square$

To see that the first order expressibility of 'more than half of' follows from this proof it suffices to think that, once it is shown that there is a k

such that $\mid B \mid > n/m \mid A \mid$ if and only if $\mid B \mid \geq k$, *a fortiori* it is shown that there is a k such that $\mid B \mid > 1/2 \mid A \mid$ if and only if $\mid B \mid \geq k$. Replacing B with $A \cap B$, we get that there is a k such that *more than half of*$_D(A, B)$ if and only if $\mid A \cap B \mid \geq k$[12].

From theorem 1 it turns out that, although *more than half of* and *most* are characterized by a proportional relation, *more than half of*$_D$ and *most*$_D$ fix a non-proportional relation expressible in L for each D. Theorem 1, accordingly, "squeezes" a proportional relation on a set of non-proportional relations. So, for any interpretation, (3) has a logical form representable in L relative to that interpretation, and the same goes for (4).

Now let us consider (5) and (6). Although 'few' and 'many' admit both an absolute reading and a proportional reading, the difference between the two readings does not really matter as far as formalization in L is concerned. The two readings certainly differ with respect to first order definability, for 'few' and 'many' turn out first order definable on the absolute reading but not on the proportional reading. However, what matters to formalization in L is first order expressibility. As in the case of 'most', a squeezing argument can be provided to the effect that 'few' and 'many' are first order expressible. In the case of 'many' the adequacy assumption is the same, while in the case of 'few' it is that, if what is said by s relative to D is that at most n As are Bs, then a formula of L that contains n occurrences of $\exists$ followed by n distinct variables and two unary predicates P and Q can provide an adequate representation of s:

(16) $\exists_{\leq n} x(Px \wedge Qx)$

Theorem 2. *For every D and $A, B \subseteq D$, there is an adequate formula α of L that contains two unary predicate letters such that few$_D(A, B)$ if and only if α is true in a structure with domain D where the two predicate letters denote A and B.*

Proof. Let $A, B \subseteq D$. If definition 5 is assumed, $few_D(A, B)$ if and only if $\mid A \cap B \mid \leq n$. Therefore, $few_D(A, B)$ if and only if (16) is true in a structure with domain D where P and Q denote A and B. If definition 6 is assumed, $few_D(A, B)$ if and only if $\mid A \cap B \mid \leq n/m \mid A \mid$. But a result similar to theorem 1 can be proved in similar way, that is, if $A, B \subseteq D$ and $0 < n < m$, there is a k such that $\mid B \mid \leq n/m \mid A \mid$ if and only if $\mid B \mid \leq k$. Therefore, $few_D(A, B)$ if and only if (16) is true in a structure with domain D where P and Q denote A and B. The same goes if definition 7 is assumed. □

Theorem 3. *For every D and $A, B \subseteq D$, there is an adequate formula α of L that contains two unary predicate letters such that many$_D(A, B)$ if and only if α is true in a structure with domain D where the two predicate letters denote A and B.*

[12] A direct proof of the first order expressibility of 'more than half of' is provided in Iacona [**?**]. The theorems presented in this section provide a generalization of that result.

Proof. Let $A, B \subseteq D$. If definition 8 is assumed, $many_D(A, B)$ if and only if $\mid A \cap B \mid \geq n$. Therefore, $many_D(A, B)$ if and only if (15) is true in a structure with domain D where P and Q denote A and B. If definition 9 is assumed, $many_D(A, B)$ if and only if $\mid A \cap B \mid \geq n/m \mid A \mid$. But a theorem similar to theorem 1 can be proved in similar way, that is, if $A, B \subseteq D$ and $0 < n < m$, there is a k such that $\mid B \mid \geq n/m \mid A \mid$ if and only if $\mid B \mid \geq k$. Therefore, we get that $many_D(A, B)$ if and only if (15) is true in a structure with domain D where P and Q denote A and B. The same goes if definition 10 is assumed. □

From theorems 1-3 it turns out that (4)-(6) can be formalized in L. For every precisification of the quantifier expressions that occur in (4)-(6), there is a formula of L that represents the truth conditions of (4)-(6). On the assumption that interpretations of quantified sentences include precisifications of the quantifier expressions which occur in them, this means that for every interpretation of (4)-(6), there is a formula of L that represents the truth conditions of (4)-(6).

6 Two kinds of formal variation

From what has been said so far it turns out that a quantified sentence can be represented by different formulas on different interpretations. But there are basically two ways in which a representation of a quantified sentence can vary as a function of its interpretation. Consider (1) and (3). In section 4 we saw that (12) and (13) can represent (1) on different interpretations. Similarly, section 5 shows how the following formulas can represent (3) on different interpretations:

(17) $\exists_{\geq 3}x(Px \wedge Qx)$

(18) $\exists_{\geq 4}x(Px \wedge Qx)$

In the second case, however, the difference seems more substantial: one thing is to say that more than half of five things have a certain property, quite another thing is to say that more than half of six things have that property.

The difference between these two kinds of variation may be spelled out in terms of a notion of *minimality* based on the understanding of adequate formalization suggested in section 4. As explained in that section, it is plausible to assume that, in order to adequately formalize a sentence s on a given interpretation, a formula must provide a correct logical analysis of the content expressed by s. This assumption leaves room for the possibility that different formulas adequately formalize s on that interpretation. If the differences between formulas that obtain in such a case are called 'minimal', the two kinds of variation in the formal representation of a sentence may be defined as follows:

Definition 14. *A minimal variation in the formal representation of a sentence s is a variation that involves some minimal difference in the formulas assigned to s.*

Definition 15. *A non-minimal variation in the formal representation of a sentence s is a variation that involves some difference in the formulas assigned to s which is not minimal.*

The meaning of 'minimal' can be specified in more than one way. On the one hand, any admissible definition of minimality must entail that certain differences between formulas are minimal, in that they definitely do not affect adequate formalization. Clearly, if two formulas α and β differ only in that β is obtained from α by uniformly replacing some non-logical expression with another expression of the same category, as in the case of Fa and Fb, the difference between them is minimal. The same goes if α and β differ only in that β is obtained from α by applying elementary syntactic transformations that involve simple order, such as that from $Fa \wedge Fb$ to $Fb \wedge Fa$. On the other hand, any admissible definition of minimality must entail that certain differences between formulas are not minimal, in that they definitely affect adequate formalization. Clearly, if α and β are not logically equivalent, it cannot be the case that they both adequately formalize the same sentence in the same interpretation. But there are also intermediate cases in which it is not obvious whether a difference between formulas should be classified as minimal. For example, the transformation from $\forall x(\alpha \wedge \beta)$ to $\forall x\alpha \wedge \forall x\beta$ might be minimal according to one admissible understanding of minimality and not minimal according to another.

However, it is not essential for the purposes at hand that the meaning of 'minimal' is actually specified in this or that way. For the distinction between minimal and non minimal variations is sufficienty clear in our case: the difference between (12) and (13) turns out to be minimal on any admissible definition of minimality, while that between (17) and (18) turns out to be non minimal on any admissible definition of minimality. This means that, given definitions 14 and 15, the former may be described in terms of minimal variation in the formal representation of (1), while the latter may be described in terms of non-minimal variation in the formal representation of (3).

Note that, in accordance with the suggestion provided in section 4, sameness of logical form can be defined in terms of minimal variation in the formal representation of a sentence.

Definition 16. *A sentence s has the same logical form on two interpretations i and i' if and only if the difference between i and i' entails at most minimal variation in the formal representation of s.*

Thus, (1) has the same logical form on the interpretations represented by (12) and (13). By contrast, (3) does not have the same logical form on the interpretations represented by (17) and (18).

Definitions 14 and 15 may be employed to characterize domain indeterminacy and quantifier indeterminacy. In the first place, it seems correct to say that domain indeterminacy entails minimal variation in formal representation. If a quantifier expression e (as it is used on a given occasion) exhibits domain indeterminacy and s is a quantified sentence containing e, then it is indeterminate which is the set of contextually relevant objects over which e is taken to range. That is, there are two sets D and D' such that it is not clear whether e ranges over D or D'. But then there are two precisifications p_1 and p_2 such that the difference between p_1 and p_2 entails minimal variation in the formal representation of s. For different predicate letters must be employed to represent in L the difference between D and D'. Therefore, there are two interpretations which require two minimally different formulas of L[13].

In the second place, it seems correct to say that quantifier indeterminacy entails non-minimal variation in formal representation. If a quantifier expression e (as it is used on a given occasion) is affected by quantifier indeterminacy and s is a quantified sentence containing e, then there are two precisifications p_1 and p_2 such that the difference between p_1 and p_2 entails non-minimal variation in the formal representation of s. For the definition of the meaning of e must include some variables such that, for any domain, different values of those variables determine different binary relations on that domain (no matter whether e is understood as proportional or non-proportional). So if p_1 and p_2 are precisifications that differ in such values, the formulas assigned to s in the corresponding interpretations must differ in non-minimal way[14].

7 Logicality

These last three sections show some implications of the analysis suggested on three debated issues concerning quantification. The first is the issue of logicality. It is generally believed that some quantifier expressions deserve the label of "logical" expressions, in that their meaning has a special significance for logic. So it is natural to wonder whether a principled distinction can be drawn between logical and non-logical quantifier expressions. More

[13] Here it is assumed that contextual restrictions are formally represented in the way suggested in section 4. But note that one would get the same result even if one adopted a formal representation in which a separate predicate letter expresses the restricting condition, because in that case (12) and (13) would be replaced by two formulas $\forall x(Rx \supset (Px \supset Qx))$ and $\forall x(Sx \supset (Px \supset Qx))$ which differ in the first predicate letter.

[14] Note that the converse entailment clearly does not hold. For it may be the case that the sentences containing a quantifier expression e (as it is used on a given occasion) admit non-minimal variation in formal representation even if e does not exhibit quantifier indeterminacy. This is shown by the case of 'more than half of', which does not exhibit quantifier indeterminacy even though (3) may be represented as (17) or (18).

specifically, one may ask whether such a distinction holds for the quantifier expressions that occur in (1)-(6). This section outlines a coherent answer to the latter question. The answer, which implies that logicality and vagueness are independent properties, is intended to apply to the restricted class of sentences considered so far, so it not to be regarded as an attempt to provide a comprehensive account of logicality.

On the one hand, it is seems right to think that not all the quantifier expressions that occur in (1)-(6) must be classified as logical. According to Barwise and Cooper, a distinction must be drawn between logical and non-logical quantifier expressions: 'all' and 'some' belong to the first category, while 'more than half of', 'most', 'few' and 'many' belong to the second. As they have observed, it would be wrong to think that the meaning of every quantifier expression must be "built into the logic"[15].

On the other hand, however, it might be argued that the distinction between logical and non-logical quantifier expressions misses something important, namely, that non-logical quantifier expressions may play some logically interesting role in inferences. Consider the following argument:

$$\text{A}\ \frac{\text{(4) Most philosophers are rich}}{\text{(2) Some philosophers are rich}}$$

Apparently, A is valid, and its validity depends on the fact that 'most' occurs in (4). As Peters and Westerståhl point out, if we switch the predicates in A, we still have a valid inference, while if we switch the quantifier expressions, the entailment is lost. This shows, according to them, that 'most' is constant in a way in which 'philosophers' is not. A worked out and improved version of this notion of constancy is provided by Bonnay and Westerståhl, where it is suggested that, on a suitable understanding of interpretations, a quantifier expression is constant if at least one argument in which it occurs is valid in one interpretation but becomes invalid in another interpretation[16].

As it will be shown, this apparent conflict can be resolved in accordance with the method of formalization adopted here: a distinction can be drawn between logical and non-logical quantifier expressions, without leaving unexplained the inferential role of non-logical quantifier expressions. Given definition 16, logicality may be defined as follows:

Definition 17. *A quantifier expression e is logical if and only if, for every sentence s in which e occurs and for every pair of interpretations i and i' such that i' differs from i in the domain assigned to e, s has the same logical form in i and i'*[17].

[15] Barwise and Cooper [?], p. 162.

[16] Peters and Westerståhl [?], pp. 334-335, Bonnay and Westerståhl [?], section 8.

[17] Note that, given the restriction mentioned in section 1, 'sentence' refers to simple quantified sentences such as (1)-(6).

From definition 17 it turns out that 'all' is logical. Let s be a sentence which contains 'all', and let i and i' be interpretations of s that differ in the domain assigned to 'all'. Since the difference between i and i' is represented by assigning to s two formulas which differ in the first predicate letter, as in the case of (12) and (13), s has the same logical form in i and i'. Similar considerations hold for 'some'. By contrast, 'more than half of', 'most', 'few' and 'many' are non-logical. As it has been shown in the case of (3)-(6), two interpretations that differ in the domain assigned to 'more than half of', 'most', 'few' and 'many' can determine a difference of logical form. This characterization of logicality entails that logicality and vagueness are independent properties. A quantifier expression (as it is used on a certain occasion) may or may not be vague - in either of the two senses considered - independently of whether it is logical or non-logical[18].

Once it is clear how the quantifier expressions that occur in (1)-(6) can be classified as logical or non-logical, it remains to be said how the inferential role of non-logical quantifier expressions can be explained. Consider A. Given definition 11, it is consistent to hold that an argument can have different forms in different interpretations, each of which is valid. This is precisely what happens in the case of A. Since (4) has different logical forms on different interpretations, A has different forms on different interpretations. Suppose for example that (17) and (18) express the logical form of (4) as understood on two different occasions. Then there are two different but equally valid forms for A, that is

$$\mathrm{A}_1 \ \frac{(17)\ \exists_{\geq 3}x(Px \wedge Qx)}{(14)\ \exists x(Px \wedge Qx)}$$

$$\mathrm{A}_2 \ \frac{(18)\ \exists_{\geq 4}x(Px \wedge Qx)}{(14)\ \exists x(Px \wedge Qx)}$$

More generally, the validity of A can be explained in terms of formal validity by using standard principles of first order logic. In this respect, there is no difference between A and any argument that involves logical quantifier expressions.

It is easy to see how other apparently valid arguments can be treated in similar way. In particular, an explanation along the lines suggested seems to hold for a considerably wide class of valid arguments formed by sentences containing either 'most' or 'some'. Note, however, that this does not mean that every argument containing 'most' which is valid in some interpretation must be valid in all interpretations. For example, consider the following:

[18] Instead, there is a straightforward connection between logicality so understood and first order definability. Iacona [?] proves that every logical quantifier expression is first order definable.

$$\text{B}\ \frac{\text{(19) Most beers are cool}}{\text{(20) At least four beers are cool}}$$

If (17) and (18) express the logical form of (19) as understood on two different occasions, then B is valid in some interpretations but invalid in other interpretations. Therefore, the explanation of the validity of arguments such as A is consistent with the hypothesis that 'most' is constant in the sense spelled out by Bonnay and Westerståhl, although the explanation itself does not appeal to constancy so understood. In the perspective adopted here, logicality and constancy may be regarded as distinct properties of quantifier expressions[19].

8 Unrestricted quantification

The second issue that will be addressed is the issue of unrestricted quantification. Although quantifier expressions often carry a tacit restriction to a set of contextually relevant objects, it is legitimate to ask whether they can coherently be used without such restriction, that is, whether it is possible to quantify over absolutely everything. Some uses of quantifier expressions are plausibly interpreted as involving unrestricted quantification. For example, if one uses the word 'everything', which is equivalent to 'all things', to state a general metaphysical claim, presumably one does not want to exclude some things as contextually irrelevant. So, at least *prima facie*, natural language seems to leave room for unrestricted quantification.

Of course, even if it is granted that some uses of quantifier expressions are plausibly interpreted as involving unrestricted quantification, this does not mean that a coherent formal account of unrestricted quantification can be provided. In standard first order semantics, each structure includes a set as its domain, so when formulas are interpreted with respect to the structure, the symbols $\forall$ and $\exists$ are read as restricted to the members of that domain. But according to set theory there is no universal set, that is, there is no set of which everything is a member. The naive idea that there is such a set is proved inconsistent by the Russell paradox. Therefore, in order to provide a formal account of unrestricted quantification, some alternative semantics must be given.

Williamson has argued that there is a viable alternative to standard first order semantics. His main point is that, even though a Russell-like paradox can arise if it is assumed that interpretations can be quantified over like other things, that is, with first order quantification, no such paradox can arise if we give up that assumption and recognize that the semantics must

[19] Moss [**?**], section 8.2, provides a complete axiomatization of a class of inferences involving sentences containing either 'most' or 'some'. The explanation suggested here seems to hold at least for that class.

be phrased in an irreducibly second order way. Others, instead, are not convinced by his line of argument and continue to claim that quantification over everything is incoherent. However, this question will not be addressed here. In what follows it will simply be granted that, since at least some uses of quantifier expressions may plausibly be interpreted as involving unrestricted quantification, the possibility of unrestricted quantification must be taken into account[20].

To see that definitions 1-10 are compatible with unrestricted quantification, recall that, as explained in section 1, the use of a quantifier expression may or may not involve an intended delimitation of the domain: there are contexts in which some things are excluded as irrelevant on the basis of some intended condition, and contexts in which nothing is excluded as irrelevant. The distinction between restricted and unrestricted quantification may be understood in terms of these two cases. That is, the contexts of the second kind may be understood as contexts in which the domain is the totality of everything.

Note that, since there is no universal set, if the domain associated to a certain use of a quantifier expression is the totality of everything, that domain is not a set. So it cannot in general be assumed that domains are sets. But this is compatible with definitions 1-10, given that definitions 1-10 do not depend on that assumption. A domain may or may not be a set. All that matters is that, on each domain, a quantifier expression denotes a binary relation over the domain[21].

9 Vague existence

The third issue that will be addressed is the issue of vague existence. Some recent discussions on whether it can be vague if certain things exist hinge on a claim that plays a key role in metaphysical disputes concerning unrestricted mereological composition and four-dimensionalism:

(UP) If 'all' and 'some' are unrestricted, then they are precise.

The main argument for (UP), first sketched by Lewis and then elaborated by Sider, rests on (VP), the assumption about vagueness considered in section 3. The argument is intended to show that, given (VP), it is inconsistent to suppose that 'all' or 'some' are unrestricted and admit different precisifications. Consider 'all'. If 'all' were vague, there would be two precisifications p_1 and p_2 such that, for some x, it is determinately the case that 'all' ranges

[20] Williamson [?], pp. 424-427 and 452-460. Glanzberg [?] argues against Williamson that, for every every domain purporting to contain everything, there are in fact things falling outside the domain.

[21] Peters and Westerståhl, among others, assume that domains are sets, see p. 48. In section 5 the same assumption is adopted for the sake of argument.

over x according to p_1 but not according to p_2. But since 'all' is unrestricted, if there is such an x then 'all' ranges over x. So it is not determinately the case that 'all' ranges over x according to p_1 but not according to p_2. The same goes for 'some'[22].

This argument has been widely discussed. Some find it compelling, others do not. So it is a controversial question whether (UP) is justified. But that question will not be addressed. In what follows it will simply be assumed that (UP) deserves consideration, so that it may be worth to dwell on its relation with the account of quantifier expressions provided in the previous sections. As it will be explained, what has been said so far is consistent with (UP)[23].

First of all, it must be noted that here quantifier indeterminacy is not at issue: since 'all' and 'some' are not vague in that sense, (UP) is clearly safe from quantifier indeterminacy. So the crucial question is whether the fact that 'all' or 'some' can exhibit domain indeterminacy is compatible with (UP). The answer to this question is affirmative, on the assumption that domain indeterminacy arises only in connection with restricted quantification. That is, one may consistently claim that domain indeterminacy concerns the specification of a restricting condition, so that it does not arise if no restricting condition is specified. Thus if (1) is used on a certain occasion and 'all' exhibits domain indeterminacy, that is, there are different precisifications p_1 and p_2 which involve different sets as domains, then its indeterminacy may be understood in terms of different formulas such as (12) and (13) being ascribed to (1) on different interpretations. By contrast, the same kind of ambivalence does not arise when 'all' is used unrestrictedly[24].

At least two interesting corollaries may be drawn from what has been said so far. The first is that, when one deals with vague existence and related metaphysical issues, one must not confuse the quantifier expressions 'all' and 'some', which belong to natural language, with the symbols $\forall$ and $\exists$, which belong to formal languages such as L. In the debate on vague existence, both the advocates of (UP) and their opponents tend to use the two kinds of expressions interchangeably, as if there were a straightforward connection between quantified sentences and their logical form. But according to the method of formalization adopted here, the connection is not so straightforward. Even if 'all' and 'some' may be vague in some sense, in that they may involve domain indeterminacy, there is no sense in which the

[22] Lewis [?], p. 213, Sider [?], pp. 128-129, Sider [?], pp. 137-142.

[23] Lopez De Sa [?] and Sider [?] elaborate and defend the argument. Liebesman and Eklund [?] and Torza [?] argue against it.

[24] Note, however, that it might be unclear whether 'all' is used unrestrictedly, in which case a similar kind of indeterminacy would arise. Note also that, just like 'all' may involve a restriction, the same goes for the general term 'thing' as it occurs in 'all things'. As it is made clear in Lopez de Sa [?], pp. 405-406, (UP) is compatible with recognizing that there might be restricted uses of 'thing' that are vague. For in that case, quantifying over every thing in that sense is not the same thing as quantifying over absolutely everything.

symbols ∀ and ∃ may be vague.

The second corollary is that domain indeterminacy, unlike quantifier indeterminacy, is a property that concerns the *use* of a quantifier expression, rather than the expression itself. In other terms, while quantifier indeterminacy is an intrinsic property of quantifier expressions, domain indeterminacy is an extrinsic property of quantifier expressions, in that it arises only in connection with restricting conditions that may be associated with them. Perhaps one might be tempted to conclude that domain indeterminacy is not a genuine property of quantifier expressions, namely, that the only sense in which quantifier expressions may be vague is that in which they may involve quantifier indeterminacy. But much depends on what 'genuine property' is taken to mean. In any case, even if domain indeterminacy were not classified as genuine because it is not an intrinsic property of quantifier expressions, its existence could hardly be denied. All that matters for this paper is that there is a kind of indeterminacy which arises in connection with the use of quantifier expressions and differs from quantifier indeterminacy in the way suggested.

A different question that might be raised in connection with this second corollary is the following: if the domain indeterminacy that affects a quantifier expression e as it is used on a certain occasion depends on the restricting condition associated with e on that occasion, doesn't it follow that domain indeterminacy is reducible to indeterminacy of expressions *other* than e, the expressions that are tacitly taken to fix that condition? The answer to this question is that strictly speaking it doesn't follow. At least two further issues seems relevant to the justification of such conclusion. One is whether every restriction is fixed - or can in principle be fixed - by some description. The other is the issue mentioned in section 1, that is, whether the restriction depends on some variable or parameter in the determiner or in the noun. Since neither of these two issues need be addressed here, the reducibility question may be left unsettled. In any case, nothing important hinges on that question. Again, all that matters is that there is a kind of indeterminacy which arises in connection with the use of quantifier expressions and differs from quantifier indeterminacy in the way suggested[25].

[25]I presented the material for this paper in talks at the University of Milan (spring 2014), at the University of Barcelona (spring 2015), and at the University of L'Aquila (fall 2014). The paper has benefited enormously from the questions, objections, and suggestions I have received on those occasions. Special thanks go to Dan López De Sa, Sven Rosenkranz and Elia Zardini. I also owe much to two anonymous referees for their sharp and accurate comments.