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Francesco Severi and mathematics teaching in secondary schools. Science, politics and schools in the first half of the twentieth century

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Abstract

Francesco Severi (1879 –1961) was, as is well known, a top-level mathematician who made very significant contributions in the field of algebraic geometry as well as in various other areas in mathematics. Less well known are his activities in the field of mathematics education. In this paper we intend to illustrate this work, situating it within the framework of the political and institutional history of the first half of the twentieth century. The aspects we will consider are the following: the reasons which led Severi to become concerned with problems pertaining to mathematics teaching and the influence of Federigo Enriques; his brief involvement in the direction of the Italian association of mathematics teachers Associazione Mathesis; his relationship with Fascism and the conflict with Enriques; his vision of mathematics teaching and its reflections in textbooks. Finally we will attempt to show how Severi's approach to education is characterized by a core set of assumptions whose roots lie in the way of conceiving mathematical research that was common to the Italian School of algebraic geometry.

Introduction

The historical period that provides the backdrop for Severi's scientific and academic life comprises the first half of the twentieth century. The institutional context which frames his commitment to education is characterised, in the first two decades, by the Casati Law (1859), in spite of some attempts at reform which were either unsuccessful, or carried out only in part, as was the case with the important reform project proposed by the Royal Commission (1909). The rise of Fascism and the Gentile Reform (1923) nullified any attempt at renovation in the area of science notwithstanding the battle to restore dignity to mathematics carried out by some mathematicians such as Enriques and Guido

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Castelnuovo.[1] Severi's activities were especially marked by his relations with Fascism.

Born in Arezzo in 1879, Severi graduated in mathematics in 1900 at the University of Torino under the supervision of Corrado Segre, and in 1902 became assistant lecturer to Enriques at the University of Bologna. The scientific collaboration with Enriques resulted, in 1907, in the award of the Prix Bordin of the Académie des Sciences of Paris for their joint research on hyperelliptic surfaces. In the years between 1906 and 1913 Severi received other important awards and honours, such as the Gold Medal of the Società dei XL, the Guccia Medal, and the *Premio Reale* for Mathematics of the Accademia dei Lincei. In 1905 he had obtained the professorship of projective geometry at the University of Parma, but in 1921 he succeeded in transferring to the chair of algebraic analysis at the University of Rome, an important place for scientific research. Although in earlier times he had been anti-Fascist, in 1929 Severi became a member of the Accademia d'Italia, established by the Fascist regime to take the place of the prestigious Accademia dei Lincei. That marked the beginning of his support for government policies, and in 1932 he enrolled in the Fascist Party. In 1938, when the racial laws caused the removal from teaching of all Jewish mathematicians, he assumed the professorship of higher geometry, which had been held by Enriques. The following year he created, in Rome, the Istituto Nazionale di Alta Matematica (INDAM), of which he was president until his death in 1961, turning it into an important centre for research, see (Roghi 2005).

Why Severi became concerned with mathematics education: relationships with Enriques and political agenda

Two factors are of prime importance for fully understanding the reasons which led Severi to become concerned with problems pertaining to mathematics teaching: his relationship, first of collaboration and then of conflict, with Enriques, and his singular political agenda.

The influence of Enriques

Severi's collaboration with Enriques began right after he earned his degree, intensified during the period in which Severi was Enriques's assistant in Bologna, and reached its peak during their joint work on hyperelliptic surfaces. To be sure, the influence of Enriques is one of the principal factors underlying

[1] See Giacardi & Scoth 2014 and the texts of the programmes on the website http://www.mathesistorino.it/?page_id=564.

Severi's interests in mathematical epistemology and teaching. To confirm this, we need only look at the writings and events of the period from 1902 to 1920.[2] In 1903 he collaborated with Enriques and Alberto Conti, the director of *Il Bollettino di Matematica* – a journal addressed mainly to the mathematics teachers in the lower level of secondary schools – to write the report on extensions and limits of the teaching of mathematics in lower and upper levels of secondary schools, which is based on Enriques's pedagogical tenets (formative role of mathematics, reduction of programmes, importance of intuition, usefulness of connecting the teaching of mathematics with that of physics).

In 1906 Severi published his *Complementi di geometria proiettiva* (1906) as a complement to Enriques's *Lezioni di geometria proiettiva* (1903). The two textbooks were born in symbiosis, and give evidence that Severi accepted the epistemological and didactic vision of his mentor. In the 1914 paper entitled "Razionalismo e spiritualismo" Severi sided with Enriques against the idealism of Benedetto Croce, proclaiming the cognitive and aesthetic value of science and illustrating the harmful consequences of the "movement against science" (Severi 1914, p. 187) in society and education. These and other writings demonstrate an acceptance of many of Enriques's methodological assumptions:

- Knowledge proceeds by successive approximations.[3]
- Geometry is seen as a part of physics.[4]
- Mathematical concepts have a historical and psychological genesis.
- Analogies and inductions play an important role in discovery.
- The experimental and intuitive approach is preferable in mathematics teaching.

The direction of the Associazione Mathesis and first divergences from Enriques

Severi's burning ambition to occupy top-level positions within the mathematics and academic communities inevitably led to his first clashes with Enriques on the academic plane. He himself said, "My will is tenacious to the point of obstinacy" (Severi 1953, p. 69). When Severi became president in 1909 of the Associazione Mathesis, an association of mathematics teachers of secondary schools, he attempted to insert himself into the work of the Italian subcommission of the *Commission Internationale de l'Enseignement mathématique* (later ICMI), whose three delegates – at the time, Castelnuovo, Enriques and

[2] The most important of the papers are: Enriques, Severi, Conti 1903, Severi 1906, Severi 1910, Severi 1914, Severi 1919.

[3] Severi 1914, p. 189: "Every truth is a step along the way to a more profound truth." This and all other translations of quotations from the Italian are by the authors.

[4] Severi 1910, pp. 45-46: "Geometry knows well that of which it speaks: the physical world. It differs from physics only in method: predominantly experimental for the one, deductive for the other. And even the method loses its deductive character when discovery is concerned. At the frontiers of science … one goes forward by dint of fortunate inductions and thought experiments. And there is no lack of cases in which one resorts to genuine physical experiments."

Vailati – were nominated directly by the ICMI Central Committee. The Associazione Mathesis was not officially part of the delegation. To reach his objective, and in particular to carry out an inquiry on the teaching of mathematics in the various kinds of schools in Italy, Severi sought the support of Vito Volterra:

> And since we firmly believe that in a matter as delicate as the one involving methods of teaching, not only useful but necessary and paramount is the counsel of those who are able to treasure everyday experience carried out especially in middle schools, so we intend to conduct the inquiry on our own and report on the outcome, together with the proposals, in a separate report, which will be presented at the Cambridge congress.[5]

He even suggested that Vailati should be encouraged to resign: "Poor Vailati, afflicted as he is by his long illness, might do well to step down … and then much could be put to rights by having a replacement elected by the Mathesis".[6] Severi's attempts to impose himself were not successful because Enriques and Castelnuovo believed that it was important that the subcommission, while collaborating with the Mathesis, maintain its "freedom to act" and not be obliged to conform to the directives of the Association. This first setback was followed by another. During his term as president, Severi sent repeated requests (in January 1909, February and April 1910[7]) to the different ministers for education at the time asking them to consider the proposals put forward by the Mathesis during its national congresses in Florence (16–23 October 1908) and in Padua (20–23 September 1909). These proposals concerned the reform of the Teacher Training Schools (*Scuole di Magistero*), the abolition of the choice between Greek and mathematics beginning in the second year of *liceo*, which had been introduced by the Orlando Decree of 1904, and the reinstatement of the written exam in mathematics for all categories of schools. Despite his efforts, Severi was able to obtain from the Minister only a few general promises, and in all likelihood this drove him to look for different ways to achieve his ends and impose his will on the mathematical and academic communities. Thus on 6 November 1910 he announced his resignation and that of the entire Mathesis executive committee:

> We intend to communicate our decision to the largest daily newspapers, so that public opinion will pause, at least for a moment, to consider whether the slight regard in which cultural Societies, such as ours, are held by executive power, constitutes the most suitable means for stimulating that disinterested attachment to Education, which, despite everything, teachers still show themselves to hold… If with the resignations we are able to achieve the aim of interesting

[5] F. Severi to V. Volterra, Padova, 13 April 1909, in P. Nastasi 2004, pp. 177-178.
[6] F. Severi to V. Volterra, Padova, 20 April 1909 in P. Nastasi 2004, p. 180.
[7] See "Il Consiglio Direttivo dal Ministro della P.I.", *Bollettino della Mathesis* 1909, pp.1-2; "I voti del Congresso di Padova presentati al Ministro della P.I.", *Bollettino della Mathesis* 1910, pp. 1-4; "Il Ministro Credaro e la Mathesis", ibid., pp. 29-30.

> public opinion in the questions of didactics that the Mathesis has defended, we hold ourselves amply compensated for the effort expended for the Society during the past two years.[8]

Severi's mandate was too short to leave a noticeable mark, but in any case he deserves the credit for having put his finger on the two main weaknesses of the Mathesis. On one hand, he called for the reform of the *Bollettino della Mathesis*, the official journal of the Association, which was supposed to be transformed from a simple administrative tool into a journal with articles about science and education. On the other hand, he hoped for a strengthening of the Association's congresses, which were to offer rich programs and, above all, fight absenteeism.[9] His wishes would be carried out by the presidents who succeeded him, first Castelnuovo and then Enriques, both of whom, like Severi himself, were components of the Italian School of algebraic geometry.

In 1914 Croce, in his article "Se parlassero di matematica?", sharply attacked Severi for having invaded territory that did not belong to him – that of philosophy – in the paper "Razionalismo e spiritualismo":

> I have a fervent request of Prof. Severi, who is a cultivated man, and that is not to get involved in discussing concepts that belong to a field he is not in and not to enter into something for which I am not certain he has an aptitude …, but for which he is certainly not prepared. (Croce 1914, p. 80)

Croce's attack contributed to Severi's growing distance from Enriques and in the years that followed the scientific and cultural rivalry with Enriques became gradually more evident. In 1921 Severi brought to light an error of Enriques, leading to a heated polemic that would last over twenty years. That same year, supported by Tullio Levi Civita, Severi had the better of Enriques for the transfer to Rome to the chair of algebraic analysis left vacant by Alberto Tonelli. Enriques would assume the chair in higher geometry in 1923, thanks only to Castelnuovo's renunciation of it.[10] This rivalry, as has been said, led to a genuine "chase" on scientific, academic, educational, editorial and cultural planes, as it will be shown by what follows, see (Faracovi, 2004).

Relationship to Fascism and the conflict with Enriques

Severi's political career was singular: he was a Socialist during the period he was in Padua; as rector in Rome, he resigned after the murder of Giacomo Matteotti; he was a signer of Croce's Manifesto of the Anti-Fascist Intellectuals; he was a supporter of those who opposed the fascistization of the University of

[8] "Dimissioni del CD", *Bollettino della Mathesis* 1910, p. 90.
[9] "Programma del prossimo Congresso sociale", *Bollettino della Mathesis* 1910, pp. 51-52.
[10] See "Il trasferimento di Enriques a Roma", in T. Nastasi 2011, pp. 256-302.

Rome. However, quick to understand the mechanisms of political power and exploit them to his own advantage, following his nomination as a member of the Accademia d'Italia in the spring of 1929,[11] he supported Fascism without reserve. In 1929–1931 he had no qualms about collaborating on the draft of a new form of oath of loyalty to the Fascist party,[12] and, later, about using the racial laws to assume absolute control over Italian mathematics. Thus he began to be involved in the process of the fascistization of culture, contributing to widen the breach between Italian mathematicians and the international mathematics community that was one of the reasons for the ensuing weakening of mathematics research in Italy.[13] When he later became conscious of this process of weakening, he attempted to revitalize Italian research by creating in 1939 the Istituto Nazionale di Alta matematica (INDAM, the National Institute for Higher Mathematics). On this aspect of Severi's personality, Francesco Tricomi wrote:

> Severi … wanted to be (and to a certain extent, was) the 'godfather' of Italian mathematics during the Fascist period. We in any case have the consolation of knowing that – while, as a rule, totalitarian regimes put the worst elements in positions of control, only because they are violent or subservient or both – in the case of Severi, the man was, from a scientific point of view, irreproachable (Tricomi 1967, p. 55).

The "Severi case" has been amply studied by historians,[14] so here I will only mention Severi's overt opposition to Enriques because of its reflections on his activities in education. The most important facts were the following: Severi refused to collaborate with the *Enciclopedia italiana* on the mathematics section, of which Enriques was director, writing: "with a man such as Enriques, … I can no longer have anything in common, much less a relationship akin to subordination".[15] He opposed the request that university chairs be established for history of science, presented by Enriques to the Accademia dei Lincei in 1938. That same year Italy's shameful racial laws were put into effect, which, among other things, excluded people of Jewish extraction from teaching in universities (Israel 2010) and Severi unhesitatingly exploited them in order to rise to a position of absolute predominance in Italian mathematics. He immediately transferred to the chair of higher geometry held by Enriques, and in February 1939 he also assumed the direction of the University School for the History of the Sciences created by Enriques in 1924, leading at last to its

[11] Enriques's name was included on the early lists of candidates of scientific disciplines but was stricken at the last moment; see Capristo 2001.

[12] F. Severi to G. Gentile, Barcelona, 15 February 1929, in Guerraggio & Nastasi 1993, pp. 211-213.

[13] See Guerraggio & Nastasi 1993; "Conclusions" in Brigaglia & Ciliberto 1995, pp. 197-204; Israel 2010, chap. 6.

[14] See footnote 13, Faracovi 2004 and Roghi 2005.

[15] F. Severi to G. Gentile, Arezzo, 24 May 1928, in Guerraggio & Nastasi 1993, pp. 209-210.

closure. As president of the Vallecchi publishing house in Florence, he took advantage of the circular issued by Minister of Education Giuseppe Bottai in August 1938, which ordered school principals to eliminate from use all textbooks written by Jewish authors, to replace the successful geometry textbooks for secondary schools by Enriques and Amaldi with his own textbooks, published by Vallecchi.

Severi's opinion of Fascist school policy

Severi's attitude towards the Gentile Reform was in many respects similar to that of Enriques: he was convinced of the superiority of the *ginnasio-liceo*,[16] because of its frank formative aims, he was in favour of combining mathematics and physics teaching but held that too few hours were dedicated to mathematics, and that the number of hours assigned to teachers (22) was too heavy (Severi 1927–1928, p. 116). Moreover, he conceived of knowledge as a personal conquest and opposed encyclopaedism. There were, however, points where their opinions differed: Severi, in fact, tended to share the nationalistic and autarchic vision of scientific research and only later became aware of the harm that scientific isolation could lead to. Further, Enriques's dialogue with Gentile was on the philosophical plane; the fact is that he did not want to renounce his idea of the fusion of scientific knowledge and humanistic idealism that was the basis of the cultural program to which he had dedicated his whole life (Giacardi 2012, § 3.3). In contrast, Severi's relationship with Gentile assumed a political overtone and he adapted himself to Fascist directives concerning education, as can also be seen in his *Curriculum vitae*, where he states that he "had also contributed with his writings to the most elementary fields of mathematics, to renovate teaching methods" in middle schools, "adapting them to the new lines of knowledge and new pedagogical needs determined by Fascism" (Severi 1938).

Furthermore, when in 1939 the Grand Council of Fascism approved the twenty-nine declarations contained in the *Carta della Scuola* (School Charter) presented by Bottai with the aim of a further fascistization of Italian schools, Severi declared that he agreed "to every single part of it" (Severi 1939, p. 63). He shared the idea of assigning educational value to manual work, and he approved the principle affirmed by Bottai according to which "the humanistic school, be it classical or scientific, as a preparation for the university studies, must be pruned back" (Severi 1939, p. 65). In fact, classical or scientific studies must be directed to those who in future will be the ruling class of the nation; while the other young people will be given the "chance to follow their preferred vocational path" (Severi 1939, p. 65). This, according to Bottai, was an essential condition for the effectiveness of the university and "the prosperity of the

[16] See "Riunione straordinaria promossa dal consiglio direttivo, Roma 11 febbraio 1923", *Periodico di Matematiche* 1923, pp. 156-157.

University must be measured rather with the decrease and not with the increase in the school population" (Severi 1939, p. 65). For example, Severi disapproved of the "combined degrees" (*lauree miste*) in physical and mathematical sciences established in 1921, aimed at qualifying young people to teach these disciplines in secondary schools, because, being easier to award with respect to the degrees in mathematics and in physics, they had attracted "undesirable elements" and "the damage had had repercussions for secondary schools, through the deficient preparation of the teachers, which [had] then … been deleterious to the preparation of the students" (Severi 1941, p. 199).

Mathematics teaching: methodological assumptions

In spite of their differences, the cornerstones of Severi's methodological and pedagogical vision were nevertheless very close to those of Enriques, although the epistemological considerations upon which they were founded were not as broad and amply illustrated.

Severi dealt with problems concerning the teaching of mathematics in various articles in addition to textbooks.[17] In particular a synthesis of Severi's vision of mathematics teaching appears in the entry "Didattica della matematica" that he wrote for the *Enciclopedia delle Enciclopedie* (Severi 1931), which includes an historical excursus about the teaching of geometry in Italy that goes from the use of the textbooks by Legendre and Bertrand at the beginning of the nineteenth century up to the Gentile Reform.

First of all Severi believed that secondary schools must have an essential formative aim and a "frank humanistic basis", but humanism must not be "disjoined from scientific thought" because "true humanism is integral by nature". Thus it is necessary to transmit to the student a unitary vision of culture, and strictly scientific teachings must be "maintained in the same plane", as historic, literary and philosophic ones (Severi 1940a, p. 70).

To these ends mathematics can play an important role because it trains the faculties of intuition and abstraction and develops an aptitude for "observing, abstracting, and deducing" (Severi 1940a, pp. 72–73). Mathematics teaching should have an intuitive character in lower middle schools and a rational character in upper middle schools, it must proceed by successive approximations from the concrete to the abstract, and allow time for the ideas to "filter slowly through the minds, if it is desired that they leave traces that are useful and lasting" (Severi 1931, p. 365).

In teaching, priority must be given to intuition because it develops in a way that is natural and direct, as a "synthesis of sensations, observations and experiences", almost without any wilful effort at attention on the student's

[17] The principal articles are the following Enriques, Severi, Conti 1903, Severi 1911, 1919, 1927, 1931, 1939, 1940, 1940a, 1951, 1951a.

part", and because only intuition provides the raw material for the logical machine. In his words: "It is necessary to take middle school teaching of mathematics back to its practical and intuitive origins; and this not only for practical reasons […], but above all precisely for the educational goals of secondary studies" (Severi 1931, p. 368).

At the same time he criticizes the pseudo-rigour and incoherence of certain textbooks. He mentions, for example, the introduction of the concept of direction for distinguishing straight line from curves, which "implies that the concept of direction is held to be more intuitive, where instead it descends from the notion of straight line and of tangent at every point of a curve!" (Severi 1927–1928, p. 114). Another aspect often stressed by Severi is the importance of using the utmost parsimony in formulating programs, reducing them for each discipline to things which are truly essential and which have unquestionable educational value. In particular, Severi suggests abandoning the cyclical method by which subjects already treated in an intuitive way in middle schools are repeated and developed in a rational manner in secondary schools, and "bringing teaching closer to the current state of science" (Severi 1940a, pp. 72–73).

In order to give new impetus to teaching by means of continuous and fruitful contact with the real world, it would be useful for teachers to link mathematics teaching to that of physics. From a pedagogical point of view, it is important that they stimulate "the youthful desire for conquest", involve the students in the process of constructing knowledge and exhort them to acquire mathematical truths for themselves, because, "allowing them to find everything nice and ready, does them no good" (Severi 1927, p. V). The role played by teachers in guiding the students in learning is in fact central according to Severi:

> Having discovered the main path [to learning], it is necessary to travel it anew, and to clear away the difficulties that are too serious for non-experts, so that the student can travel them along with us, following us, without excessive effort, in the process of constructing knowledge (Severi 1927, p. V).

Finally, Severi, like Enriques, believed that the history of science can play a significant educational role in facilitating students' comprehension of certain mathematical concepts. For example in introducing real numbers in secondary schools it is preferable to follow the historical path and present them as ratios of magnitudes as Euclid did; later the teacher can gradually arrive to their definition as Dedekind cuts (Severi 1931, p. 365; Severi 1927, p. VI).

Severi himself used history in his lessons at university as well as in the courses of specialisation and advises: "don't forget the masters, because an ingenious idea is worth more in creative power than all of its consequences." (Severi 1955, p. 38).

Severi's vision of teaching of mathematics reflected in textbooks

How this vision of teaching translated into practice emerges above all from the textbooks for lower and upper secondary schools, which constitute Severi's most important and lasting legacy regarding secondary teaching.

Significantly, beginning in 1926 he directed the book series entitled *Collezione di testi di matematica per le scuole medie* for the Vallecchi publishing house in Florence. The series included his own textbooks for geometry, arithmetic, algebra (with trigonometry, financial mathematics and infinitesimal analysis) for the different types of secondary schools (*ginnasio-liceo, scuole tecniche, istituti tecnici, scuole professionali femminili*, *istituti magistrali,* …).[18] which were often written in collaboration with two teachers, his niece Maria Mascalchi,[19] who taught at the *liceo classico* Massimo d'Azeglio in Turin, and Umberto Bini, teacher at the liceo scientifico Cavour in Rome.

The distinguishing features of these books are: the use of an intuitive approach, which does not exclude due attention to rational aspects, suitably adapted according to school level and type of school; some use of history of mathematics; questions to facilitate learning; a great number of exercises; clarity and precision. Moreover, Severi was a fervent supporter of the need for brevity of treatment, stripping it of anything that is not essential to the comprehension of the structure of a mathematical theory, and of making room for more modern topics (Severi 1934, I, p. V).

The best known of Severi's textbooks is entitled *Elementi di geometria* (2 vols, 1926, 1927, adapted for the various types of schools. This text is distinguished by its particular approach to the principal topics of geometry (congruence, equivalence, parallel theory, theory of proportions), as well as for the methodological framework dictated by the concern that "the intuitive underpinnings of each notion introduced does not escape the students" (Severi 1939, pp. 9–10), and that the programs be slimmed down by eliminating superfluous subjects. About this Severi claims:

> The experience of the decade that has passed since the Gentile Reform has shown the necessity of thinning out and simplifying in order to lighten the load on students, without harming the formative function of mathematics teaching, and in particular of geometry. I have been a tenacious advocate of these

[18] The list of Severi's textbooks – all published by Vallecchi – can be found in the website http://www.mathesistorino.it/?page_id=886.

[19] Maria Mascalchi (Lucca 1902 – Torino 1976), recalled by her students (among whom Primo Levi) as a Fascist of no great charm, graduated in mathematics from the University of Torino in 1919 and in 1928, after having taught at the Istituto Tecnico in Venice, was appointed to the professorship of mathematics and physics in the Liceo Classico D'Azeglio in Torino. Here she saw to the adoption of the textbooks by Severi, and after the issue of the Fascist School Charter in 1939, she directed the Laboratory for wood and metal working (Archivio del Liceo Massimo d'Azeglio, Torino, Fascicolo insegnanti, 123/1).

> reductions, and once translated into act, I held it my duty to adapt to them (Severi 1934, I, pp. V–VI).

It was Gentile who wrote the preface of the 1926 edition:

> I am pleased to see that books such as these by Prof. Severi are beginning to be published for the study of mathematics in secondary schools. […] And to me these books seem to correspond wonderfully to our desire that these subjects, which always run the risk of ending up in one of two opposite extremes, either stiffening into abstruse abstraction, or falling into intolerable triviality, also be presented in the most suitable form for beginners: the heuristic form of the concept arrived at by means of intuitions that are concrete, evident and attractive (Severi 1926, p. V).

Without going into details about all the topics treated, we will mention only Severi's handling of congruence, parallels theory and real numbers.

With regard to congruence, Severi turns to Euclid's approach, that is, the use of movement, but he frames it in a complete logical structure. As a primitive he assumes the notion of congruent line segments, and defines movement as the one-to-one correspondence that transforms each segment into an equal segment, adopting, however, from the very beginning, "the language of physical movement". In fact he states that "the concept of congruence can never be detached from that of movement, because the two concepts are indissolubly linked in the mind" (Severi 1926, p. XI). This approach is linked to Severi's firm belief that geometry is a "chapter of physics" and its teaching must be brought closer to that of physics. For this reason Severi criticizes those authors, such as Enriques and Amaldi, who adopted Hilbert's approach to the congruence theory, which is irreproachable from a logical point of view, but

> besides the serious didactic drawback of forcing the assumption as a postulate of one of the cases of the congruence of triangles … it offers others of no less seriousness. The student cannot in fact understand why one has gone to such lengths of reasoning to prove the congruence of certain figures, which he would be able to verify immediately through superposition …, it leads further to an artificial and harmful hashing of the concept (Severi 1926, p. XI, XII).

At the same time he also criticises those textbooks where movement is introduced, but not placed within a complete logical framework.[20]

Instead, for the theory of parallels Severi distanced himself from Euclid, whose definition of parallels "presupposes an integral concept of the plane". Since the student can only ever utilise a part of the plane it is necessary that the geometry that he is taught be "realisable in a drawing" (Severi 1931, p. 367) and he thus defines as parallel two equidistant straight lines, postulating that "in a

[20] See, for example, the textbook *Elementi di geometria* (Venezia 1878) by Aureliano Faifofer (1843 –1909), which Severi nevertheless considered "the first good Italian treatise of elementary geometry" (Severi 1926, p. XI).

plane, the locus of the points located by a part of a given straight line and having from this a given distance, is still a straight line" (Severi 1926, pp. 111–112).

With regard to the real numbers, Severi states "In our schools, for decades and decades, ever since Faifofer transported Dedekind's theory into elementary teaching, the real numbers have become the thing most abstract and indigestible" (Severi 1927–1928, p. 113). For this reason, he introduces the real numbers in the upper level of secondary schools, in the way that he understood them to have emerged historically, that is, as relations among homogenous magnitudes and thus starts by considering the approximate values of the ratio of two incommensurable magnitudes, gradually arriving at the definition by means of Dedekind cuts (Severi 1927, Chap. IV).

To complete his text, at the end of each chapter Severi introduces numerous problems (almost 500), the most complex of which are accompanied by hints towards the solution. For the best students he inserts various complements: continuous fractions (Severi 1927, p. 23); conic sections (ibid., pp. 202–203); area of the ellipse (ibid., p. 204); spherical triangles (ibid., pp. 218–219); the Pappus-Guldinus theorem (ibid., p. 239); the graphic representation of functions (ibid., chap. XV), and more. For teachers he adds appendices to clarify the logical layout of the treatment (Severi 1926, pp. 173–184, Severi 1927, pp. 263–271).

The same didactical tenets, adapted for youngsters from 11 to 14 years old, characterise the textbook co-authored with Maria Mascalchi, *Nozioni di Aritmetica per le scuole secondarie e di avviamento professionale* (1935, 8th rpt. 1938). Here again the teaching of the discipline is accompanied by empirical observations, and that didactical requirement is compensated by the rigour of exposition and sobriety of language. The rules are given after suitable explanations and examples and are sometimes accompanied by "observations" that aim at either clarifying critical points or highlighting possible errors. As the authors say:

> The rules are actually almost always accounted for by examples and an embryo of reasoning, … the formulation has been limited and retouched to permit the greatest possible brevity and clarity; the fundamental concepts are introduced without exclusion of methods, in order to reconcile with a minimum of effort the necessities of teaching and respect for logic (Prefazione, 1938, p. VII).

For example, the concept of fractions and the related properties are made to descend from both the division of the magnitudes into equal parts and from the consideration of fractions as "operating symbols of a potential multiplication and division" (1938, pp. 57–58).

Algorithms are illustrated step by step by numerical examples, and particular attention is paid to approximations and to those arguments (interest, discounts and so forth) that are especially useful for beginning a trade, the book being

aimed at those who will not pursue further study. Before addressing this kind of problems, the authors dedicate a section to the solution of first-degree equations, also explaining the conditions for solvability (1938, pp. 108–110).

Some parts of the text are devoted to illustrating methods of rapid calculation (1938, pp. 25–27, 77–78) and mental calculation, while others present problems that arouse curiosity (1938, pp. 144–146), partly drawn from the book *Giochi di aritmetica e problemi interessanti* (1924) by Giuseppe Peano, and from *Matematica dilettevole e curiosa* (1st ed. 1913, 3rd ed. 1929) by Italo Ghersi.

The textbook presents a rich selection of problems (about 600) of various kinds: some require the simple application of the rules; others are drawn from real life experiences; others are connected to simple notions of physics; still others require "reasoning" on the basis of notions presented.

The treatment is enriched by short digressions (the monetary system, daylight saving time, longitude and latitude, systems of numeration other than base 10) and by a few historical notes on the origin of the decimal-based metric system and the calendar. Numerical tables of the primes from 2 to 3 000, of squares and square roots conclude the volume.

The texbook was updated after the introduction of the *scuola media unica* by the minister Bottai. In the revised 1941 edition (F. Severi, M. Mascalchi, *Nozioni di Aritmetica pratica con cenni storici per il 1° e il 2° anno della scuola media*), the graphic aspect is more refined, and questions are often introduced to verify the student's comprehension and solicit an active learning. Historic notes (concerning numbers, fractions, calendar, and so forth) are introduced at the end of each chapter. In contrast to the textbooks by Mascalchi for the third, fourth and fifth classes of elementary school, which presented drawings, problems and observations that were clearly Fascist propaganda,[21] here the only references to Fascism appear in three exercises (pp. 8, 34, 60)[22] that introduce the "Balilla", the Fascist youth organization, and seem to be inserted opportunistically, inasmuch as the phase of fascistization of the schools was in full swing.

Conclusions

Our examination of Severi's commitment to questions regarding mathematics teaching allows us to discern a core set of didactical assumptions shared by the members of the Italian School of algebraic geometry, which consist in a common way of conceiving mathematical research, and constitute an ulterior indicator of the appropriateness of the term "School" in speaking of the Italian geometers: attributing an educational value to mathematics, in hopes of attaining a scientific *humanitas*; preferring to use the faculty of intuition and the

[21] On the topic of Fascist propaganda in elementary school books, see Luciano 2013-2014.
[22] There were also three references to Fascism in the 1938 edition: pp. 10, 29, 56.

heuristic procedures in teaching; aiming at rigour in substance, rather than formal rigour; establishing connections between mathematics and other sciences; and giving importance to the history of mathematics in teaching and in research.

These tenets show that when we use the word School speaking about the group of Italian researchers in algebraic geometry, we are referring not only to a group of researchers trained by the same *maestri*, from whom they draw topics of investigation, methodologies, approaches to research and a particular scientific style, and a place where talents are developed and contacts made, but also an environment in which a common way of viewing and conveying mathematical knowledge, directed their activities in education, in spite of the fact that their motivations and even the strategies they employed sometimes followed different channels.

For Corrado Segre, the father of the Italian School of algebraic geometry it was above all the intimate connection that he saw between teaching and research that led him to become concerned with questions regarding mathematics education. Instead, Castelnuovo's motivation was mainly social, see (Giacardi 2010). What led Enriques to become interested in problems of education were his strong philosophical, historical and interdisciplinary interests, and especially the studies on the foundations of geometry. He adopted a wide range of strategies and worked on different fronts: institutional, publishing (journals, book series, textbooks), and cultural. Further, he addressed his activities to different categories – secondary school teachers, researchers, philosophers, scientists, people of culture – inviting their cooperation.

As we have seen, Severi's itinerary was of yet a different nature: his interest in problems concerning the secondary teaching of mathematics was inspired both by his relationship, first of collaboration and then of rivalry, with Enriques, and by political reasons. After his unsuccessful attempt to insert himself into ICMI, and the sparse results as president of the Mathesis Association, in the course of about a decade Severi marshalled his ideas into line with the school policies of the Fascist regime, while holding to the pedagogical tenets of the Italian School of algebraic geometry. The route he favoured for improving mathematics teaching was the publishing of textbooks, a choice which reflected his political attitude towards Fascism, but which, as we have tried to show, was certainly a mirror of his conviction in the high and formative role of mathematics.[23]

[23] After the fall of the Fascist regime, Severi was accused by the commission charged with the purge of university personnel of having carried out activities in defence of Fascism and of having collaborated with the Republican Fascist government. After a first deliberation (23 December 1944) that resulted in Severi's dismissal from service, he presented an appeal in the form of a lengthy, detailed document in his defence. After various vicissitudes and following testimony in his favour, the commission arrived at the following conclusion: "Severi did not receive from Fascism anything more than what he merited; he did however consent that his famous name, his

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moral rectitude, his high scientific merits, and his past as an anti-Fascist intellectual be used, not only for the good of Italy, but also for the political aims of the regime. [This is] A much more serious fault, in that this adhesion, in reference to a personage such as Severi, effectively constituted, for the regime, a noteworthy reinforcement". Ultimately, since the activity carried out by Severi was of a predominantly scientific nature, it was not deemed sufficiently serious to lead to a declaration of his being "unworthy of serving the State", and the commission sentenced him to a lesser penalty, that is, censure (Archivio Centrale dello Stato, Roma: *MPI, Professori epurati*, B 31 *Francesco Severi*, N. 372, Seduta del 9 maggio 1945, Sezione I).

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