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String field theory solution for any open string background

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Abstract: We present an exact solution of open bosonic string field theory which can be used to describe any time-independent open string background. The solution generalizes an earlier construction of Kiermaier, Okawa, and Soler, and assumes the existence of boundary condition changing operators with nonsingular OPEs and vanishing conformal dimension. Our main observation is that boundary condition changing operators of this kind can describe nearly any open string background provided the background shift is accompanied by a timelike Wilson line of sufficient strength. As an application we analyze the tachyon lump describing the formation of a D$(p-1)$-brane in the string field theory of a Dp-brane, for generic compactification radius. This not only provides a proof of Sen's second conjecture, but also gives explicit examples of higher energy solutions, confirming analytically that string field theory can "reverse" the direction of the worldsheet RG flow. We also find multiple D-brane solutions, demonstrating that string field theory can add Chan-Paton factors and change the rank of the gauge group. Finally, we show how the solution provides a remarkably simple and nonperturbative proof of the background independence of open bosonic string field theory.

Keywords: Tachyon Condensation, String Field Theory, Conformal Field Models in String Theory, Bosonic Strings

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1 Introduction

Following Schnabl's analytic solution for tachyon condensation [1], analytic techniques in open string field theory have provided a remarkably clear and beautiful description of the endpoint of tachyon condensation on unstable D-branes [2–4]. However, efforts to extend these techniques beyond the universal sector have been less fruitful. Several solutions describing marginal deformations have been found, especially as a perturbative expansion in the deformation parameter [5–13].[1] But the main question about marginal deformations is whether string field theory can describe the full moduli space of vacua connected to a given D-brane system [21–24], and this question seems out of reach in a perturbative approach. On a different line of thought, there have also been interesting proposals to describe the formation of lower dimensional D-branes by following a given boundary world-sheet RG

[1] A more nonperturbative approach to marginal deformations was provided by the KOS solution [14], which will play a central role in our discussion, and the old identity-based solution for marginal deformations introduced by Takahashi and Tanimoto [15], for which there have been interesting recent developments [16–19]. A solution which aims to unify these approaches was recently proposed in [20].

flow [25, 26]. A success was the computation of the energy and closed string tadpole [26–28], but further work has encountered subtle problems with the equation of motion [27–30], and has been limited by the very few known soluble worldsheet RG-flows.

In light of these difficulties, one particularly attractive proposal was advanced by Kiermaier, Okawa, and Soler (KOS) [14]. By making a gauge transformation of the solution [5, 6] for nonsingular marginal deformations, they managed to construct a solution which could be expressed directly in terms of boundary condition changing (bcc) operators $\sigma, \overline{\sigma}$ relating the perturbative vacuum to the boundary conformal field theory (BCFT) of the D-brane system one wishes to describe. Since the existence of bcc operators relating BCFTs is a generic fact, this suggests a kind of all-purpose string field theory solution which could be used to describe any open string background. For the KOS solution to work, however, the bcc operators must satisfy a rather unusual property:

$$\lim_{s\to 0}\overline{\sigma}(s)\sigma(0) = 1, \qquad (s>0). \tag{1.1}$$

While this is satisfied for backgrounds related by nonsingular marginal deformations, usually bcc operators have nonvanishing conformal weight, and their OPEs are singular. Efforts to generalize the KOS solution to avoid (1.1) have so far been unsuccessful.

In this paper we observe that bcc operators satisfying (1.1) can, in fact, describe any change of boundary condition provided the time component of the X^μ BCFT is unaltered. The idea is as follows. Suppose $\sigma_*, \overline{\sigma}_*$ are bcc operators of weight h satisfying

$$\overline{\sigma}_*(s)\sigma_*(0) \sim \frac{1}{s^{2h}} + \text{less singular}, \qquad (s>0), \tag{1.2}$$

and which act as the identity operator in the time direction. We will construct an analytic solution using a modified pair of bcc operators[2]

$$\sigma(s) = \sigma_* e^{i\sqrt{h}X^0}(s), \qquad \overline{\sigma}(s) = \overline{\sigma}_* e^{-i\sqrt{h}X^0}(s). \tag{1.3}$$

The plane-wave factors $e^{\pm i\sqrt{h}X^0}$ cancel the conformal weight of σ_* and $\overline{\sigma}_*$, and because

$$e^{-i\sqrt{h}X^0}(s)e^{i\sqrt{h}X^0}(0) \sim s^{2h}, \qquad (s>0), \tag{1.4}$$

the modified bcc operators satisfy equation (1.1). The resulting solution will have nontrivial primaries excited in the X^0 BCFT, which *a priori* could effect the physical interpretation of the solution. In fact, the $e^{\pm i\sqrt{h}X^0}$ factors are bcc operators which turn on a Wilson line in the time direction. But since the only physical effect of a Wilson line is through winding modes, and the time direction is noncompact, the timelike Wilson line is physically invisible. In field theory, a constant timelike Wilson line is pure gauge:

$$A_\mu = \lambda\delta_\mu^{\ 0} = e^{i\lambda x^0} i\partial_\mu(e^{-i\lambda x^0}), \tag{1.5}$$

which suggests that the timelike primaries excited by $\sigma, \overline{\sigma}$ could likewise be removed by a gauge transformation in string field theory, though doing this in practice may be nontrivial.

[2] We use $\alpha' = 1$ units.

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The implications of this simple idea are profound. It means that string field theory can provide a closed form description of a far greater range of backgrounds than have been identified in level truncation or analytically, and in fact the bulk of D-brane setups one might care to consider in string theory. After modest generalization of the considerations of KOS, the solution is extraordinarily simple. Finding the energy, the closed string tadpole, and the cohomology are easily reduced to worldsheet computations. Remarkably, the solution even satisfies (a generalization of [3]) the Schnabl gauge condition.

2 Algebra

We begin by quickly reviewing the algebraic ingredients we need to formulate the solution. This is (mostly) standard material; see also the original paper by KOS [14] and Noumi and Okawa [31],[3] and for further explanations of the algebraic formalism we use, see [3, 32–34].

We start with string field theory formulated around some reference D-brane system, described by a boundary conformal field theory BCFT_0. Then we construct a classical solution describing some other D-brane system, described by a boundary conformal field theory BCFT_*. We assume that BCFT_0 and BCFT_* are factorized in the form

$$\mathrm{BCFT}_{c=25}\otimes\mathrm{BCFT}_{X^0}\otimes\mathrm{BCFT}_{bc} \tag{2.1}$$

BCFT_0 and BCFT_* share a common bc ghost factor and a noncompact, timelike free boson X^0 subject to Neumann boundary conditions. The $c=25$ components of the two BCFTs can be different and essentially arbitrary provided they share the same bulk CFT. In this way, the shift between the backgrounds BCFT_0 and BCFT_* can be represented by boundary condition changing operators, as explained in the introduction. For backgrounds not of the form (2.1) we do not have a general construction, though in some cases such backgrounds can be realized.[4]

The solution is formulated within the subalgebra of wedge states with operator insertions [1, 39]. A wedge state [40] is any positive star algebra power of the $SL(2,\mathbb{R})$ vacuum $\Omega\equiv|0\rangle$:

$$\Omega^\alpha,\qquad \alpha\geq 0. \tag{2.2}$$

Here (and in the rest of the paper) we omit the $*$ symbol when multiplying string fields. In the limit $\epsilon\to 0$, the wedge state Ω^ϵ approaches the formal identity of the star algebra, called the identity string field. We write the identity string field simply as 1. The conformal field theory definition of a wedge state is easiest to visualize in the sliver coordinate frame [1, 32, 41, 42], where Ω^α is represented as a semi-infinite, vertical "strip" of worldsheet of width α, as shown in figure 1. The "strip" can be glued to itself or to other "strips" along the

[3] We follow the conventions of [3], in particular we use the "left-handed" star product. KOS use the "right-handed" star product, and the opposite sign for the fields K and B.

[4] The exponential timelike [5, 6, 14, 35, 36] and lightlike [37, 38] rolling tachyon solutions are examples of nonsingular marginal deformations, and therefore are described by boundary condition changing operators of the kind needed to construct the solution. The $\cosh(X^0)$ deformation [35] could be realized by turning on an imaginary Wilson line in a noncompact spacelike direction, if available. These backgrounds are not described by BCFTs of the form (2.1).

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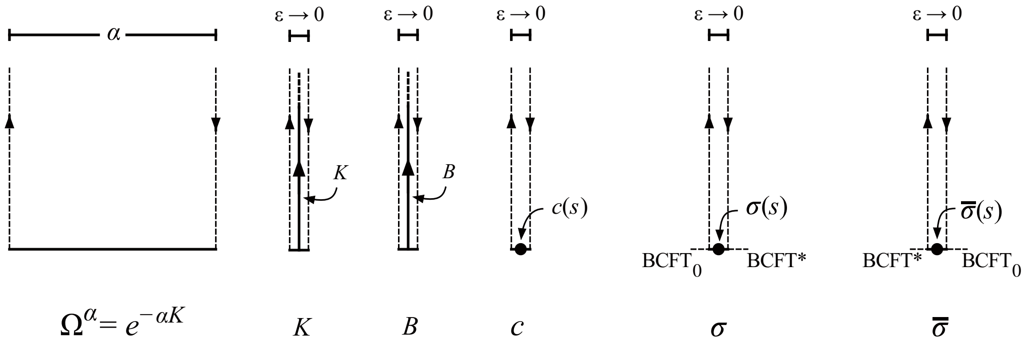

Figure 1. The wedge state Ω^α and the fields $K, B, c, \sigma, \overline{\sigma}$ represented as semi-infinite "strips" with operator insertions in correlation functions on the cylinder. Note that star multiplication of two string fields glues the right half of the first strip to the left half of the second strip.

vertical edges, resulting in worldsheet correlation functions on the cylinder (which can be mapped to the upper half plane). To describe the solution, the "strips" should also contain particular operator insertions, specifically, boundary insertions of the c-ghost,

$$c(s), \tag{2.3}$$

boundary condition changing operators,

$$\sigma(s), \quad \overline{\sigma}(s), \tag{2.4}$$

and vertical line integral insertions of the energy-momentum tensor and b-ghost,

$$K = \int_{-i\infty}^{i\infty} \frac{dz}{2\pi i} T(z), \tag{2.5}$$

$$B = \int_{-i\infty}^{i\infty} \frac{dz}{2\pi i} b(z). \tag{2.6}$$

The operator σ changes the open string boundary condition from BCFT_0 to BCFT_*, and $\overline{\sigma}$ changes the boundary condition in reverse, from BCFT_* back to BCFT_0. We assume that σ and $\overline{\sigma}$ are weight zero primaries constructed by tensoring a primary bcc in the $c = 25$ component of the BCFT with a timelike Wilson line. However, much of our discussion can be generalized to non-primary bccs.

This class of states can be conveniently expressed by taking star products of five "atomic" string fields:

$$K,\ B,\ c,\ \sigma,\ \overline{\sigma}. \tag{2.7}$$

Each string field can be defined as an infinitesimally thin "strip" carrying the respective operator insertion (denoted by the same symbol), as shown in figure 1.[5] The field K generates the algebra of wedge states, in that any positive power of the $\mathrm{SL}(2, \mathbb{R})$ vacuum can be written

$$\Omega^\alpha = e^{-\alpha K}. \tag{2.8}$$

[5]Note that $\sigma, \overline{\sigma}$ are fields of a stretched string between BCFT_0 and BCFT_*. They are not string fields in BCFT_0.

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Of particular importance are the string fields [3]

$$\frac{1}{1+K} = \int_0^\infty d\alpha\, e^{-\alpha}\Omega^\alpha, \tag{2.9}$$

$$\frac{1}{\sqrt{1+K}} = \frac{1}{\sqrt{\pi}}\int_0^\infty d\alpha \frac{e^{-\alpha}}{\sqrt{\alpha}}\Omega^\alpha, \tag{2.10}$$

which are defined via the Schwinger parameterization in terms of a continuous superposition of wedge states. Multiplying a wedge state with $K, B, c, \sigma, \overline{\sigma}$ on the left (right) effectively inserts the corresponding operator on the left (right) edge of the "strip" defining the wedge state. In this way, we can insert all of the operators we need by taking star products of these five basic string fields.

The fields satisfy a number of algebraic relations. First, we have BRST variations:

$$QK = 0; \quad QB = K; \quad Qc = c\partial c; \quad Q\sigma = c\partial\sigma; \quad Q\overline{\sigma} = c\partial\overline{\sigma}, \tag{2.11}$$

where $\partial \equiv [K, \cdot]$. Note that the BRST variation of σ and $\overline{\sigma}$ is exactly like that of a dimension zero matter primary. We also have algebraic relations:

$$\begin{aligned} B^2 = c^2 = 0; \qquad & [K, B] = 0; \qquad & Bc + cB = 1; \\ [\sigma, c] = 0; \qquad & [\sigma, \partial c] = 0; \qquad & [\sigma, B] = 0; \\ [\overline{\sigma}, c] = 0; \qquad & [\overline{\sigma}, \partial c] = 0; \qquad & [\overline{\sigma}, B] = 0. \end{aligned} \tag{2.12}$$

The first three relations are well-known [32], and the last six follow trivially from the fact that σ and $\overline{\sigma}$ represent matter operators. Finally, we have two important relations:

$$\overline{\sigma}\sigma = 1; \tag{2.13}$$

$$\sigma\overline{\sigma} = \text{finite}. \tag{2.14}$$

The first equation follows from the OPE (1.1) discussed in the introduction. The second equation is somewhat surprising, since if $\sigma\overline{\sigma} \neq 1$ we have an "associativity anomaly:"

$$(\sigma\overline{\sigma})\sigma \neq \sigma(\overline{\sigma}\sigma). \tag{2.15}$$

This reflects an ambiguity in the definition of correlators when 3 or more bccs collide. (See appendix A.) This leads to a few subtleties which are important to be aware of. But in practice the product $\sigma\overline{\sigma}$ never appears in all essential computations with the solution, and there is no need to assign it a definite value. So associativity anomalies do not appear.

Still, it is important to understand why $\sigma\overline{\sigma} \neq 1$ in general. Consider the 2-point function of bcc operators on the unit disk:

$$\langle\overline{\sigma}(1)\sigma(e^{i\theta})\rangle. \tag{2.16}$$

For angles in the range $[0, \theta]$ the correlator has BCFT_0 boundary conditions, and outside this range it has BCFT_* boundary conditions. Since σ and $\overline{\sigma}$ are dimension zero primaries,

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this 2-point function is independent of the angular separation. Therefore, in the limit $\theta \to 0^+$ we can use the OPE (1.1) to find

$$\langle\overline{\sigma}(1)\sigma(e^{i\theta})\rangle = g_*, \tag{2.17}$$

where g_* is the disk partition function in BCFT$_*$ (the g-function). Now consider $\theta \to 2\pi^-$. In this limit the correlator should be proportional to g_0 —the disk partition function in BCFT$_0$ —times the coefficient of the identity operator in the σ-$\overline{\sigma}$ OPE. But since the correlator must be equal to g_*, we find

$$\lim_{s\to 0}\sigma(s)\overline{\sigma}(0) = \frac{g_*}{g_0}, \qquad (s>0). \tag{2.18}$$

The disk partition functions will be different if the D-brane configurations have different energies. So in general $\sigma\overline{\sigma}\neq 1$.

Let us explain another puzzle, which in the past seemed to give a compelling argument that the KOS solution could only describe marginal deformations. Since σ and $\overline{\sigma}$ are weight zero primaries with regular OPE, it is natural to define the operator

$$V = \sigma\partial\overline{\sigma}. \tag{2.19}$$

This should be a weight 1 primary, and it naively defines a 1-parameter family of conformal boundary conditions connecting BCFT$_0$ and BCFT$_*$. To see this, consider a wedge state deformed by a V-boundary interaction [14]

$$e^{-(K+\lambda V)}. \tag{2.20}$$

At $\lambda = 0$, this describes the boundary condition of BCFT$_0$. Meanwhile, at $\lambda = 1$ (assuming $\sigma\overline{\sigma} = 1$),

$$e^{-K-\sigma\partial\overline{\sigma}} = e^{-K-\sigma K\overline{\sigma}+\sigma\overline{\sigma}K} = e^{-\sigma K\overline{\sigma}} = \sigma\Omega\overline{\sigma}. \tag{2.21}$$

So we find the boundary condition of BCFT$_*$. Thus it seems that BCFT$_*$ must represent a marginal deformation of BCFT$_0$. The problem is that this argument makes assumptions about the nature of short distance collisions of the σs and V which are not valid in general. We will explain how this happens (for lump solutions) in section 7.

3 Solution

The solution is most easily described by starting with string field theory formulated around the tachyon vacuum. Specifically, we begin with the "simple" solution for the tachyon vacuum, introduced in [3]:

$$\Psi_{\rm tv} = \frac{1}{\sqrt{1+K}}c(1+K)Bc\frac{1}{\sqrt{1+K}}. \tag{3.1}$$

The equations of motion expanded around the tachyon vacuum are

$$Q_{\Psi_{\rm tv}}\Phi + \Phi^2 = 0, \tag{3.2}$$

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where $Q_{\Psi_{\rm tv}} = Q + [\Psi_{\rm tv},]$ is the shifted kinetic operator. To describe the perturbative vacuum BCFT_0, we should take the solution

$$\Phi = -\Psi_{\rm tv}. \tag{3.3}$$

Now suppose we want to describe some other D-brane system BCFT_*. A natural guess would be to subtract the tachyon vacuum in BCFT_*:

$$\Phi \stackrel{?}{=} -\sigma\Psi_{\rm tv}\overline{\sigma}, \tag{3.4}$$

where σ and $\overline{\sigma}$ are needed to translate the degrees of freedom of BCFT_* into BCFT_0. Surprisingly, this almost works. It would be a solution if $Q_{\Psi_{\rm tv}}$ annihilated σ and $\overline{\sigma}$.

In this connection, it is worth noting that σ and $\overline{\sigma}$ are killed by the kinetic operator of a different tachyon vacuum solution, namely, a singular tachyon vacuum consisting of a divergent insertion of the c ghost [43]:[6]

$$\Psi_{\rm singular} = \frac{1}{\alpha}c, \qquad \alpha \to 0. \tag{3.5}$$

This is closely related to the boundary string field theory description of the tachyon vacuum [44, 45], in that it naturally leads to a boundary deformation of the worldsheet action given by an infinite constant which sets all correlators to zero. The corresponding kinetic operator leaves σ and $\overline{\sigma}$ invariant in a trivial way since $[\sigma, c] = [\overline{\sigma}, c] = 0$. This suggests we should look for "regularized" analogues of σ and $\overline{\sigma}$ which are left invariant by $Q_{\Psi_{\rm tv}}$:

$$Q_{\Psi_{\rm tv}}\Sigma = 0; \qquad Q_{\Psi_{\rm tv}}\overline{\Sigma} = 0. \tag{3.6}$$

We can guess the needed expressions Σ and $\overline{\Sigma}$ as follows. Since $Q_{\Psi_{\rm tv}}$ has trivial cohomology, we should be able to write Σ and $\overline{\Sigma}$ as $Q_{\Psi_{\rm tv}}$ of some string field. If there is no change of boundary condition, we expect $\Sigma = \overline{\Sigma} = 1$, in which case this string field can be nothing but the homotopy operator for the tachyon vacuum [2, 3], which satisfies

$$1 = Q_{\Psi_{\rm tv}}\left(\frac{B}{1+K}\right). \tag{3.7}$$

If the change of boundary condition is nontrivial, one might guess that the homotopy operator should be accompanied by an insertion of a bcc operator. Thus we are lead to the expressions[7]

$$\Sigma = Q_{\Psi_{\rm tv}}\left(\frac{1}{\sqrt{1+K}}B\sigma\frac{1}{\sqrt{1+K}}\right); \tag{3.8}$$

$$\overline{\Sigma} = Q_{\Psi_{\rm tv}}\left(\frac{1}{\sqrt{1+K}}B\overline{\sigma}\frac{1}{\sqrt{1+K}}\right). \tag{3.9}$$

[6] Plugging into the equations of motion, this is a solution at order $\frac{1}{\alpha^2}$ since $c^2 = 0$. To get subleading orders to work requires a regularization of the solution.

[7] Closely related expressions appear in [20, 25] in the description of the cohomology for marginal deformations.

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By construction, these fields are killed by $Q_{\Psi_{\rm tv}}$. To have a solution to the equations of motion, Σ and $\overline{\Sigma}$ must satisfy the additional property

$$\overline{\Sigma}\Sigma = 1. \tag{3.10}$$

This can be demonstrated as follows:

$$\begin{aligned}
\overline{\Sigma}\Sigma &= Q_{\Psi_{\rm tv}}\left(\frac{1}{\sqrt{1+K}}\overline{\sigma}B\frac{1}{\sqrt{1+K}}\right)Q_{\Psi_{\rm tv}}\left(\frac{1}{\sqrt{1+K}}B\sigma\frac{1}{\sqrt{1+K}}\right),\\
&= Q_{\Psi_{\rm tv}}\left(\frac{1}{\sqrt{1+K}}\overline{\sigma}B\frac{1}{\sqrt{1+K}}Q_{\Psi_{\rm tv}}\left(\frac{B}{1+K}\sqrt{1+K}\sigma\frac{1}{\sqrt{1+K}}\right)\right),\\
&= Q_{\Psi_{\rm tv}}\left(\frac{1}{\sqrt{1+K}}\overline{\sigma}B\frac{1}{\sqrt{1+K}}Q_{\Psi_{\rm tv}}\left(\frac{B}{1+K}\right)\sqrt{1+K}\sigma\frac{1}{\sqrt{1+K}}\right),\\
&= Q_{\Psi_{\rm tv}}\left(\frac{1}{\sqrt{1+K}}\overline{\sigma}B\frac{1}{\sqrt{1+K}}\sqrt{1+K}\sigma\frac{1}{\sqrt{1+K}}\right),\\
&= Q_{\Psi_{\rm tv}}\left(\frac{B}{1+K}\right) = 1.
\end{aligned} \tag{3.11}$$

Taking the product in the opposite order, we also have $\Sigma\overline{\Sigma} = \frac{g_*}{g_0}$, so Σ and $\overline{\Sigma}$ multiply just like σ and $\overline{\sigma}$.[8]

Therefore, after replacing $(\sigma, \overline{\sigma})$ with $(\Sigma, \overline{\Sigma})$, our initial guess for the solution turns out to be correct:

$$\Phi = -\Sigma\Psi_{\rm tv}\overline{\Sigma}, \tag{3.12}$$

Substituting previous expressions for $\Psi_{\rm tv}$, Σ and $\overline{\Sigma}$ we find more explicitly,

$$\Phi = -\frac{1}{\sqrt{1+K}}c(1+K)\sigma\frac{B}{1+K}\overline{\sigma}(1+K)c\frac{1}{\sqrt{1+K}}. \tag{3.13}$$

This is a solution to the equations of motion around the tachyon vacuum. Shifting back to the perturbative vacuum, the solution takes the form:

$$\begin{aligned}
\Psi &= \Psi_{\rm tv} - \Sigma\Psi_{\rm tv}\overline{\Sigma},\\
&= \frac{1}{\sqrt{1+K}}c\left[(1+K) - (1+K)\sigma\frac{1}{1+K}\overline{\sigma}(1+K)\right]Bc\frac{1}{\sqrt{1+K}}.
\end{aligned} \tag{3.14}$$

In the special case $\sigma\overline{\sigma} = 1$, this expression is equivalent to the original solution proposed by KOS [14]:

$$\Psi_{\rm KOS} = -\frac{1}{\sqrt{1+K}}c\partial\sigma\frac{1}{1+K}\overline{\sigma}(1+K)Bc\frac{1}{\sqrt{1+K}}, \qquad (\sigma\overline{\sigma}=1). \tag{3.15}$$

To see this, use

$$\partial\sigma = (1+K)\sigma - \sigma(1+K), \tag{3.16}$$

[8] Note that the derivation (3.11) only requires $\overline{\sigma}\sigma = 1$ and $[B,\sigma] = [B,\overline{\sigma}] = 0$. Therefore, all other relations satisfied by σ and $\overline{\sigma}$ in equations (2.11) and (2.12) are not needed to have a solution to the equations of motion. In particular σ and $\overline{\sigma}$ do not necessarily have to be primaries, but in this case the solution will take a different form from (3.14), and will not satisfy the gauge condition (3.20). In this paper we have a specific realization of σ and $\overline{\sigma}$ in mind, so we will assume all relations in (2.11) and (2.12) without qualification.

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and plug in to the KOS solution to reproduce (3.14). Note that when $\sigma\overline{\sigma} \neq 1$, the KOS solution does not satisfy the equations of motion, whereas (3.14) does.

Let us explain a few technical properties of the solution. It satisfies the string field reality condition,[9]

$$\Psi^\ddagger = \Psi. \tag{3.17}$$

A nice property of the reality condition is that Σ and $\overline{\Sigma}$ are conjugate to each other:

$$\Sigma^\ddagger = \overline{\Sigma}; \qquad \overline{\Sigma}^\ddagger = \Sigma. \tag{3.18}$$

and therefore are analogous to unitary operators. (The analogy is not complete because $\Sigma\overline{\Sigma} \neq 1$ in general). However, for some purposes it is natural to work with a non-real form of the solution [3]:

$$\begin{aligned}\Psi' &= \sqrt{1+K}\Psi\frac{1}{\sqrt{1+K}}, \\ &= c\left[(1+K) - (1+K)\sigma\frac{1}{1+K}\overline{\sigma}(1+K)\right]Bc\frac{1}{1+K}.\end{aligned} \tag{3.19}$$

Now the square root factors do not appear, and the solution requires one fewer Schwinger integration. The non-real solution may be more appropriate for a potential generalization to the superstring, since at present we do not have a controlled solution for the superstring tachyon vacuum which satisfies the reality condition [4]. The solution satisfies a linear gauge condition [3]:

$$\mathcal{B}_{\frac{1}{\sqrt{1+K}},\frac{1}{\sqrt{1+K}}}\Psi = 0. \tag{3.20}$$

This is an example of a so-called *dressed Schnabl gauge*, $\mathcal{B}_{F,G} = 0$, where the operator $\mathcal{B}_{F,G}$ is defined

$$\mathcal{B}_{F,G} \equiv F\frac{1}{2}\mathcal{B}^-\Big(F^{-1}[\,\cdot\,]G^{-1}\Big)G. \tag{3.21}$$

$\mathcal{B}^-$ is the BPZ odd component of Schnabl's $\mathcal{B}_0$ [47] and F, G are any pair of states in the wedge algebra.[10] The Schnabl gauge corresponds to the special case $\mathcal{B}_0 = \mathcal{B}_{\sqrt{\Omega},\sqrt{\Omega}} = 0$. We will discuss the analogous solution in Schnabl gauge in section 6.

4 Energy and closed string tadpole

We now discuss two important gauge invariant quantities associated with the solution: the spacetime action, and the so-called *Ellwood invariant* [48–52], which is closely related to the closed string tadpole amplitude [52] and the boundary state [53, 54]. Usually the computation of these quantities is a core technical obstacle for an analytic solution. But for us it will require very little work, since the computations almost immediately reduce to those of the tachyon vacuum, which are already described in [3].

[9] The operation $\ddagger$ is defined as the composition of Hermitian and BPZ conjugation [46]. It is formally analogous to Hermitian conjugation of an operator. The fields K, B and c are self-conjugate, $K^\ddagger = K$ etc, while $\sigma^\ddagger = \overline{\sigma}$.

[10] To check the gauge condition, note that $\frac{1}{2}\mathcal{B}^-$ is a derivation of the star product satisfying $\frac{1}{2}\mathcal{B}^- K = B$ and it annihilates all other fields in the algebra.

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Let us start by computing the spacetime action:[11]

$$S = \mathrm{Tr}\left[-\frac{1}{2}\Psi Q\Psi - \frac{1}{3}\Psi^3\right], \tag{4.1}$$

where we use $\mathrm{Tr}[\cdot]$ to denote the 1-string vertex (or Witten integral). Since we consider time-independent configurations, really we are interested in the energy, which is minus the action divided by the volume of the time coordinate:

$$E = -\frac{S}{\mathrm{Vol}(X^0)}. \tag{4.2}$$

For us, the volume of time must be infinite otherwise the timelike Wilson-line alters the physical interpretation of the solution. Still we can compactify time and consider the limit when the volume goes to infinity. This has the effect of normalizing the disk partition function in the timelike component of the X^0 BCFT to unity (for the purposes of the energy computation). Plugging in the solution $\Psi = \Psi_{\mathrm{tv}} + \Phi$ we find:

$$E = \frac{1}{\mathrm{Vol}(X^0)}\left(-\frac{g_0}{2\pi^2} + \mathrm{Tr}\left[-\frac{1}{2}\Phi Q_{\Psi_{\mathrm{tv}}}\Phi - \frac{1}{3}\Phi^3\right]\right), \tag{4.3}$$

where the term $-\frac{g_0}{2\pi^2}$ comes from the energy of tachyon vacuum Ψ_{tv}. Assuming the equations of motion, this simplifies to[12]

$$E = \frac{1}{\mathrm{Vol}(X^0)}\left(-\frac{g_0}{2\pi^2} + \frac{1}{6}\,\mathrm{Tr}\left[\Phi^3\right]\right). \tag{4.4}$$

Plugging in $\Phi = -\Sigma\Psi_{\mathrm{tv}}\overline{\Sigma}$ and using $\overline{\Sigma}\Sigma = 1$, we find

$$\mathrm{Tr}[\Phi^3] = -\,\mathrm{Tr}\left[\Psi_{\mathrm{tv}}^3\right]_{\mathrm{BCFT}_*}, \tag{4.5}$$

where the subscript BCFT_* indicates that the whole boundary in the correlator has BCFT_* boundary conditions. Except for the normalization of the disk partition function, this is exactly the computation of the cubic term in the action for Ψ_{tv}, and by standard manipulations we find

$$E = \frac{1}{\mathrm{Vol}(X^0)}\left(-\frac{g_0}{2\pi^2} + \frac{g_*}{2\pi^2}\right), \tag{4.6}$$

which is the expected energy difference between BCFT_0 and BCFT_*.

Next we compute the Ellwood invariant [52],

$$\mathrm{Tr}_{\mathcal{V}}[\Psi], \tag{4.7}$$

[11] We set the open string coupling constant to unity. This means that the disk partition function in BCFT_0 must be normalized to the volume of the reference D-brane to compute the correct energy.

[12] The validity of the equations of motion contracted with the solution is notoriously subtle in string field theory [32, 58, 59]. In the current context, one might worry about potentially ambiguous collisions of σ and $\overline{\sigma}$. To clarify this question, we considered a regularization of the solution $\Phi \to \Phi\Omega^\epsilon$. We found no evidence of problems in the $\epsilon \to 0$ limit. We confirmed this by explicit computation of the regularized kinetic and cubic terms of the action for the 2-brane solution (8.9), where both the four and six point functions of bcc operators are easily obtained in closed form.

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where $\mathrm{Tr}_{\mathcal{V}}[\cdot]$ is the 1-string vertex with a midpoint insertion of an on-shell closed string vertex operator of the form $\mathcal{V} = c\bar{c}V^{\mathrm{matter}}$. Based on examples and general arguments [52–54, 60], the Ellwood invariant is believed to compute the shift in the closed string tadpole amplitude between BCFT_0 and BCFT_*, or equivalently the shift in the on-shell part of the boundary state [54]:

$$\mathrm{Tr}_{\mathcal{V}}[\Psi] = \frac{1}{4\pi i}\Big(\langle\mathcal{V}|c_0^-|B_0\rangle - \langle\mathcal{V}|c_0^-|B_*\rangle\Big), \tag{4.8}$$

where $|B_0\rangle$ is the boundary state in BCFT_0 and $|B_*\rangle$ is the boundary state in BCFT_*. The contribution from BCFT_0 appears automatically from Ψ_{tv}, and looking at the contribution from Φ,

$$\mathrm{Tr}_{\mathcal{V}}[\Phi] = -\,\mathrm{Tr}_{\mathcal{V}}[\Sigma\Psi_{\mathrm{tv}}\overline{\Sigma}] = -\,\mathrm{Tr}_{\mathcal{V}}[\Psi_{\mathrm{tv}}]_{\mathrm{BCFT}_*} = -\frac{1}{4\pi i}\langle\mathcal{V}|c_0^-|B_*\rangle, \tag{4.9}$$

we get the contribution from BCFT_*. Therefore the solution correctly describes the shift in the closed string tadpole amplitude.

More interesting than the closed string tadpole is the full BCFT boundary state. A rigorous approach would follow the construction of Kiermaier, Okawa, and Zwiebach [14], but for present purposes it is enough to take the more pragmatic route of [54], which requires little more than the above computation of the Ellwood invariant. The key observation of [54] is that we can compute the overlap of the boundary state with any matter primary V provided we tensor with an auxiliary BCFT (with vanishing central charge) and compute the Ellwood invariant with a modified matter vertex operator

$$wV, \tag{4.10}$$

where w lives in the auxiliary BCFT and cancels the conformal weight of V so that the combination is a weight $(1,1)$ matter+auxiliary primary. From the form of the solution, it is clear that the boundary state of the solution $|B_\Psi\rangle$ can be factorized into timelike/spacelike components:[13]

$$|B_\Psi\rangle = |B_\Psi\rangle^{X^0}\otimes|B_\Psi\rangle^{c=25}\otimes|B\rangle^{bc}. \tag{4.11}$$

The matter part of the boundary state can be expressed as a sum of Virasoro Ishibashi states $|V_\alpha\rangle\!\rangle$ associated with spinless primaries V_α in the time/spacelike sectors, with appropriate coefficients:

$$|B_\Psi\rangle = \left(\sum_{\substack{\alpha=\\ X^0\ \text{primaries}}} n_\Psi^\alpha|V_\alpha\rangle\!\rangle\right)\otimes\left(\sum_{\substack{\beta=\\ c=25\ \text{primaries}}} n_\Psi^\beta|V_\beta\rangle\!\rangle\right)\otimes|B\rangle^{bc}. \tag{4.12}$$

The coefficients n_Ψ^α represent disk one-point functions of V^α with the appropriate boundary condition. Here V^α is the dual vertex operator to V_α, so that $\langle V^\alpha|V_\beta\rangle = \delta^\alpha_\beta$. Following the

[13] Note that disk 1-point functions in BCFT_* represent the contraction of a bulk vertex operator with a closed string state which is manifestly space/time factorized on account of the factorization of σ and $\overline{\sigma}$. The ghost factor of the boundary state is universal in bosonic string theory [54].

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proposal of [54] we can compute these coefficients with the Ellwood invariant:[14]

$$n_\Psi^\alpha = 2\pi i\,\mathrm{Tr}_{\mathcal{V}^\alpha}[\Phi], \qquad \mathcal{V}^\alpha = c\overline{c}(w^\alpha V^\alpha). \tag{4.13}$$

From (4.9) it is clear that this computes the disk one-point function in BCFT$_*$ (provided $\langle w^\alpha\rangle_{\mathrm{disk}} = 1$), and therefore we recover the expected boundary state. One important point, which we can now address in a more explicit manner, is the extent to which the timelike Wilson line effects the physical interpretation of the solution. For this we need to investigate the coefficients n_Ψ^α for the timelike factor of the boundary state. Evaluating the vacuum correlator for the spacelike components and mapping to the upper half plane, we find

$$n_\Psi^\alpha = \text{const.} \times \left\langle \exp\left[\sqrt{h}\int_{-\infty}^{\infty} ds\, i\partial X^0(s)\right] V^\alpha(i,\overline{i})\right\rangle_{\mathrm{UHP}}^{X^0,\mathrm{BCFT}_0}, \tag{4.14}$$

where, by evaluating the correlator in the timelike component of BCFT$_0$, we bring out the Wilson line boundary interaction with coupling $\sqrt{h}$ given by the conformal weight h of the spacelike bcc operators. Since $i\partial X^0$ is a chiral operator, we can regularize the boundary interaction by simply deforming the contours away from the boundary [61]. Deforming the contours to surround the bulk insertion, we potentially pick up a residue from a pole in the OPE if $i\partial X^0$ with V^α. The only primary with such a pole is a plane wave

$$i\partial X^0(z)e^{ikX^0}(w,\overline{w}) \sim \frac{k}{2}\frac{1}{z-w}e^{ikX^0}, \qquad z\to w, \tag{4.15}$$

but momentum conservation in the 1-point function forces $k = 0$. Therefore the $i\partial X^0$ contours close without hitting a pole, and we find:

$$n_\Psi^\alpha = \text{const} \times \langle V^\alpha(0,0)\rangle_{\mathrm{UHP}}^{X^0,\mathrm{BCFT}_0}. \tag{4.16}$$

The timelike component of the boundary state is unchanged by the solution. This is consistent with the expectation that the timelike Wilson line is pure gauge.

5 Cohomology and background independence

The physical excitations of the solution Ψ are described by the cohomology of the shifted kinetic operator,

$$Q_\Psi = Q + [\Psi,\cdot]. \tag{5.1}$$

This cohomology should be the same as the cohomology of Q in BCFT$_*$. With a few qualifications, we will show that this is indeed the case.

Let us start by assuming that the solution Ψ is a marginal deformation, since here the argument is uncomplicated. In this case the disk partition functions are equal $g_0 = g_*$ and Σ and $\overline{\Sigma}$ are inverses in both directions:

$$\Sigma\overline{\Sigma} = \overline{\Sigma}\Sigma = 1, \qquad \text{(marginal case)}. \tag{5.2}$$

[14] A potentially subtle point in this approach is that it requires a definition of the solution in an enhanced BCFT which includes the auxiliary factor. For the solution (3.14), this only requires taking $K \to K + K^{\mathrm{aux}}$, where K^{aux} represents an insertion of the energy-momentum tensor in the auxiliary BCFT.

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Note also the relations[15]

$$Q_{\Psi 0}\Sigma = 0; \qquad Q_{0\Psi}\overline{\Sigma} = 0. \tag{5.3}$$

For example the first can be demonstrated as follows:

$$\begin{aligned} Q_{\Psi 0}\Sigma &= Q\Sigma + \Psi\Sigma, \\ &= Q\Sigma + \Psi_{\rm tv}\Sigma - \Sigma\Psi_{\rm tv}\overline{\Sigma}\Sigma, \\ &= Q\Sigma + \Psi_{\rm tv}\Sigma - \Sigma\Psi_{\rm tv}, \\ &= Q_{\Psi_{\rm tv}}\Sigma = 0. \end{aligned} \tag{5.4}$$

and the second follows similarly. With these ingredients, we can define an isomorphism between states in BCFT_0 and BCFT_*:

$$\varphi_0 = \ f(\varphi_*) \ = \Sigma\varphi_*\overline{\Sigma}; \tag{5.5}$$

$$\varphi_* = f^{-1}(\varphi_0) = \overline{\Sigma}\varphi_0\Sigma, \tag{5.6}$$

satisfying

$$f\circ f^{-1} = 1_{\mathrm{BCFT}_0}; \qquad f^{-1}\circ f = 1_{\mathrm{BCFT}_*}. \tag{5.7}$$

where φ_0 and φ_* are suitably well-behaved states in BCFT_0 and BCFT_*, respectively. Furthermore, (5.3) implies that f and f^{-1} satisfy

$$Q_\Psi f(\varphi_*) = f(Q\varphi_*); \qquad Qf^{-1}(\varphi_0) = f^{-1}(Q_\Psi\varphi_0), \tag{5.8}$$

so we have an isomorphism of cohomologies. In summary, if Ψ is a marginal deformation, the cohomology of Q_Ψ in BCFT_0 is identical to the cohomology of Q in BCFT_*.

The non-marginal case is more subtle. Here, we still have equations (5.3) and (5.8), but since $\Sigma\overline{\Sigma}\neq 1$ equation (5.7) is replaced with

$$f\circ f^{-1} = \left(\frac{g_*}{g_0}\right)^2 \times 1_{\mathrm{BCFT}_0}; \qquad f^{-1}\circ f = 1_{\mathrm{BCFT}_*}. \tag{5.9}$$

Thus composition of f and f^{-1} would seem to be non-associative, and it is not clear that we have a well-defined isomorphism. This is an indication that we need to be more careful about domains. To start, let $\mathcal{H}_0$ denote the state space of BCFT_0 and $\mathcal{H}_*$ the state space of BCFT_*. Consider a subspace of "perturbative" states in BCFT_*,

$$\mathcal{H}_*^{\rm pert}\subset\mathcal{H}_*. \tag{5.10}$$

Loosely speaking, $\mathcal{H}_*^{\rm pert}$ consists of string fields which produce no collisions with bcc operators upon multiplication with Σ and $\overline{\Sigma}$. This includes, for example, perturbative Fock states. Mapping $\mathcal{H}_*^{\rm pert}$ using f in (5.5) defines a subspace of states in BCFT_0:

$$f\circ\mathcal{H}_*^{\rm pert}\subset\mathcal{H}_0. \tag{5.11}$$

[15]We define $Q_{\Phi_1\Phi_2}A\equiv QA+\Phi_1 A+(-1)^A A\Phi_1$. This is the kinetic operator for a stretched string between classical solutions Φ_1 and Φ_2.

Using the inverse map f^{-1} in (5.6), it is clear that this subspace in BCFT_0 is isomorphic to the subspace $\mathcal{H}_*^{\mathrm{pert}}$ in BCFT_*. Therefore, if we look for the cohomology of Q_Ψ within $f\circ\mathcal{H}_*^{\mathrm{pert}}$, it will be the same as the cohomology of Q in $\mathcal{H}_*^{\mathrm{pert}}$.

This may not appear to be fully satisfactory. While $\mathcal{H}_*^{\mathrm{pert}}$ represents a fairly generic class of states in BCFT_*, $f\circ\mathcal{H}_*^{\mathrm{pert}}$ are rather peculiar states in BCFT_0. Our main reason for restricting domains is to have a well-defined isomorphism between the state spaces, but this is probably more than is needed to prove the isomorphism of cohomologies. Let us explain this with a degenerate example. Consider the tachyon vacuum, where $\Sigma=\overline{\Sigma}=0$. In this case, $f\circ\mathcal{H}_*^{\mathrm{pert}}$ is the zero vector, which of course is consistent with the absence of cohomology. But the tachyon vacuum kinetic operator has trivial cohomology not just when computed on the zero vector, but also when computed for fairly arbitrary states in BCFT_0.[16] In a similar way, for general backgrounds the cohomology of Q_Ψ may be correct even when computed outside $f\circ\mathcal{H}_*^{\mathrm{pert}}$, though at present we will not attempt to make this statement precise.

Let us point out an important consequence of our construction. Consider the action expanded around the solution Ψ:

$$S=\frac{g_0-g_*}{2\pi^2}+\mathrm{Tr}\left[-\frac{1}{2}\Phi_0 Q_\Psi\Phi_0-\frac{1}{3}\Phi_0^3\right]. \tag{5.12}$$

Setting $\Phi_0=\Sigma\Phi_*\overline{\Sigma}$ this becomes

$$S=\frac{g_0-g_*}{2\pi^2}+\mathrm{Tr}\left[-\frac{1}{2}\Phi_* Q\Phi_*-\frac{1}{3}(\Phi_*)^3\right]_{\mathrm{BCFT}_*}. \tag{5.13}$$

Thus we have recovered the string field theory formulated around BCFT_*. This gives an astonishingly simple proof of background independence in open string field theory.[17]

6 Schnabl gauge solution

The solution we have been working with is simple, but it is close to being singular from the perspective of the identity string field [66] and for some purposes it may be necessary to work with a more regular solution. Ideally, we would like to find an analogue of (3.14) in Schnabl gauge [1]:

$$\mathcal{B}_0\Psi_{\mathrm{Sch}}=0. \tag{6.1}$$

The expectation is that in Schnabl gauge the solution should be built from wedge states Ω^α with α strictly greater than one. By contrast, the original solution (3.14) is built from wedge states all the way down to the identity string field.

There is a simple transformation relating solutions in different dressed Schnabl gauges:

$$\Psi_F=\sqrt{F/f}\,\Psi_f\frac{1}{1+B\,\frac{1-F/f}{K}\,\Psi_f}\sqrt{F/f}, \tag{6.2}$$

[16] For Schnabl's solution and related solutions, the absence of cohomology is clear for reasonably well-behaved states [2]. However, this remains a subtle question. There are indications that cohomology at exotic ghost numbers appears for the Siegel gauge condensate [55], and for the identity-based tachyon vacuum solution of Takahashi and Tanimoto [15, 56, 57].

[17] Previous analysis of this problem can be found in [25, 62–65].

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where Ψ_f is a solution in $\mathcal{B}_{\sqrt{f},\sqrt{f}}$-gauge and Ψ_F is a solution in $\mathcal{B}_{\sqrt{F},\sqrt{F}}$-gauge. This is a version of the Zeze map, introduced in [67]. In the current situation, we want to map from a solution satisfying $\mathcal{B}_{\frac{1}{\sqrt{1+K}},\frac{1}{\sqrt{1+K}}} = 0$ to Schnabl gauge $\mathcal{B}_0 = \mathcal{B}_{\sqrt{\Omega},\sqrt{\Omega}} = 0$, and the transformation becomes[18]

$$\Psi_{\rm Sch} = \sqrt{\Omega(1+K)}\Psi\frac{1}{1+B\Delta\Psi}\sqrt{(1+K)\Omega}, \tag{6.3}$$

where Δ is the string field

$$\Delta \equiv \frac{1-\Omega}{K} - \Omega. \tag{6.4}$$

The field Δ has a special interpretation. Given any $\mathcal{B}_{\sqrt{F},\sqrt{F}}$-gauge there are two distinguished elements of the wedge algebra: the "security strip" F, which surrounds the operator insertions in the solution, and the "homotopy field" $\frac{1-F}{K}$, which appears (for example) in the homotopy operator which trivializes the cohomology around the tachyon vacuum [2, 68]. The simplest gauge $\mathcal{B}_{\frac{1}{\sqrt{1+K}},\frac{1}{\sqrt{1+K}}} = 0$ has the special property that the security strip and homotopy field are equal. Therefore, the field Δ characterizes the failure of Schnabl gauge to be "simple." Substituting (3.14) we find a more explicit expression for the Schnabl-gauge solution:

$$\Psi_{\rm Sch} = \sqrt{\Omega}c\frac{1}{1+\left(1-(1+K)\sigma\frac{1}{1+K}\overline{\sigma}\right)\Delta}\left(1-(1+K)\sigma\frac{1}{1+K}\overline{\sigma}\right)(1+K)Bc\sqrt{\Omega}. \tag{6.5}$$

We would like to define this as a power series in Δ. Computing the Δ^n correction in the Fock space requires knowledge of a $2n+3$-point correlator with a test state in BCFT_0 and $2n+2$ bcc operators. Such correlators would be difficult to compute in general, and the original solution (3.14) is certainly simpler.

One immediate question is whether a power series in Δ converges. We do not know the answer to this question in general, but if our goal is to regulate the identity-like nature of the solution (3.14), we can choose any number of dressed Schnabl gauges where the analogue of Δ can be taken to be as small as we like, and presumably the power series can be made to converge. Still, in the case of Schnabl gauge we can get some insight into the nature of convergence by looking at the the case $\sigma = \overline{\sigma} = 0$. This gives Schnabl's solution for the tachyon vacuum, expressed in the form

$$\Psi_{\rm Sch} = \sqrt{\Omega}c\frac{1+K}{1+\Delta}Bc\sqrt{\Omega}. \tag{6.6}$$

Actually, for illustrative purposes we can ignore the ghosts and look at the ghost number zero toy model [1]:

$$\frac{1+K}{1+\Delta}\Omega. \tag{6.7}$$

[18] This formula actually takes the same form in transforming $\mathcal{B}_{\frac{1}{\sqrt{1+K}},\frac{1}{\sqrt{1+K}}}$ gauge to any $\mathcal{B}_{\sqrt{F},\sqrt{F}}$ gauge, with the replacement $\Omega \to F$.

To see convergence in powers of Δ, consider the coefficient of $L_{-2}|0\rangle$, which can be computed by the formula [1, 69]

$$\begin{aligned} L_{-2}|0\rangle \text{ coefficient } &= -\frac{1}{3}+\frac{4}{3}\int_0^\infty dK\ Ke^{-K}\left(\frac{1+K}{1+\Delta}\Omega\right), \\ &= -\frac{1}{3}+\frac{4}{3}\int_0^\infty dK\ Ke^{-K}\left(\frac{K\Omega}{1-\Omega}\right), \\ &= -3+\frac{8}{3}\zeta(3). \end{aligned} \tag{6.8}$$

Now expand this in powers of Δ:

$$L_{-2}|0\rangle \text{ coefficient } = -\frac{1}{3}+\frac{4}{3}\sum_{n=0}^\infty(-1)^n\int_0^\infty dK\ K(1+K)e^{-2K}\Delta^n. \tag{6.9}$$

Using the method of steepest descent, the nth contribution to this sum for large n can be estimated as

$$(-1)^n\int_0^\infty dK\ K(1+K)e^{-2K}\Delta^n = \sqrt{\frac{2\pi\Delta(\gamma)}{n|\Delta''(\gamma)|}}\Delta(\gamma)^n\gamma(1+\gamma)e^{-2\gamma}+\dots, \tag{6.10}$$

where $\Delta(\gamma)\approx 0.298426$ is the maximum value of Δ as a function of K, $\Delta''(\gamma)\approx -0.0736153$ is the second derivative of Δ at its maximum, and $\gamma\approx 1.79328$. Thus the nth term in the expansion in powers of Δ is exponentially suppressed, and the series converges fairly quickly. In fact, convergence is much faster than standard definition of Schnabl's solution as a power series in Ω:

$$L_{-2}|0\rangle \text{ coefficient } = -\frac{1}{3}+\frac{4}{3}\sum_{n=0}^\infty\int_0^\infty dK\ K^2e^{-(2+n)K}. \tag{6.11}$$

where the nth term contributes as $2/n^3$.

The power series expansion in Δ has another interesting property: it gives a definition of Schnabl's solution without the phantom term.[19] To see that this is the case, compute the Ellwood invariant:

$$\mathrm{Tr}_{\mathcal{V}}[\Psi_{\mathrm{Sch}}] = \mathrm{Tr}_{\mathcal{V}}\left[c\frac{1+K}{1+\Delta}Bc\Omega\right] = \sum_{n=0}^\infty(-1)^n\,\mathrm{Tr}_{\mathcal{V}}\left[c(1+K)\Delta^nBc\Omega\right]. \tag{6.12}$$

Using the well-known formula [71]

$$\mathrm{Tr}_{\mathcal{V}}[cFBcG] = -F(0)G'(0)\frac{1}{4\pi i}\langle\mathcal{V}|c_0^-|B_0\rangle, \tag{6.13}$$

for F,G states in the wedge algebra, we find

$$\mathrm{Tr}_{\mathcal{V}}[\Psi_{\mathrm{Sch}}] = \sum_{n=0}^\infty(-1)^n\Delta(0)^n\frac{1}{4\pi i}\langle\mathcal{V}|B_0\rangle. \tag{6.14}$$

[19] For some relevant discussions of the phantom term in Schnabl's solution and other solutions, see [1, 3, 29, 32, 58, 68, 70].

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Since $\Delta(0) = 0$, the sole contribution to the Ellwood invariant comes from the zeroth order term in the power series in Δ. Dropping a BRST trivial piece, this is simply the zero momentum tachyon:

$$\sqrt{\Omega}c\sqrt{\Omega} = \frac{2}{\pi}c_1|0\rangle. \tag{6.15}$$

The source for the closed string does not come from a sliver-like phantom term.

7 Tachyon lump

Having completed the general discussion of the solution, we turn our attention to a specific (and fundamental) example: the tachyon lump, describing the formation of a D$(p-1)$-brane in the string field theory of a Dp-brane. Previous numerical constructions of the tachyon lump in Siegel gauge are discussed in [54, 72–74]. We will describe formation of the lump along a direction X^1 which has been compactified on a circle of radius R. The bcc operators describing this background are the Neumann-Dirichlet twist operators $\sigma_{\rm ND}, \overline{\sigma}_{\rm ND}$ of weight $\frac{1}{16}$, described for example in [75–77]. Tensoring with a Wilson line gives:

$$\sigma(s) = \sigma_{\rm ND}e^{iX^0/4}(s); \qquad \overline{\sigma}(s) = \overline{\sigma}_{\rm ND}e^{-iX^0/4}(s). \tag{7.1}$$

For our computations, the most important piece of information we need to know about the Neumann-Dirichlet twist operators is the three-point function with a plane wave, computed in [78]:

$$\langle e^{inX^1/R}(s_1)\sigma_{\rm ND}(s_2)\overline{\sigma}_{\rm ND}(s_3)\rangle^{X^1}_{\rm UHP} = \frac{2\pi 2^{-2\frac{n^2}{R^2}}e^{\frac{ina}{R}}}{|s_{12}s_{13}|^{\frac{n^2}{R^2}}|s_{23}|^{\frac{1}{8}-\frac{n^2}{R^2}}}, \tag{7.2}$$

where the Dirichlet boundary condition is fixed to a position a along the circle and $s_{ij} = s_i - s_j$.[20]

In light of earlier discussion, one immediate question about the proposed lump solution is why it does not represent a marginal deformation. To understand this we must determine the fate of the marginal operator (2.19):

$$V = \sigma\partial\overline{\sigma}. \tag{7.3}$$

Finding V requires knowledge of the subleading structure of the $\sigma_{\rm ND}$-$\overline{\sigma}_{\rm ND}$ OPE. The leading term is proportional to the identity operator, and the next to leading term must be proportional to the first cosine harmonic on the circle. The precise coefficients can be derived from the 3-point function (7.2):[21]

$$\sigma_{\rm ND}(s)\overline{\sigma}_{\rm ND}(0) = \frac{1}{s^{1/8}}\cdot\frac{1}{R} + \frac{1}{s^{1/8-1/R^2}}\cdot\frac{2^{-2/R^2+1}}{R}\cos\left(\frac{X^1-a}{R}\right)(0) + \ldots, \quad (s>0). \tag{7.4}$$

[20] When X^1 has Neumann boundary conditions, we will normalize the disk partition function in the X^1 BCFT to the spacetime volume $2\pi R$. This means that when X^1 has Dirichlet boundary conditions, the disk partition function must be normalized to 2π to obtain the correct ratio of tensions. This is the origin of the factor of 2π in (7.2).

[21] The OPEs (7.4), (7.5), and (7.6) are correct for $R > 1/\sqrt{2}$, otherwise the contribution from the first cosine harmonic is subleading to descendents of the identity and $i\partial X^0$. In addition, (7.5) and (7.6) assume $R < \sqrt{2}$ otherwise the contribution from $i\partial X^0$ is subleading to second or higher cosine harmonics.

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The bcc operators used in the solution must therefore have the OPE

$$\sigma(s)\overline{\sigma}(0) = \frac{1}{R} + s^{1/R^2} \cdot \frac{2^{-2/R^2+1}}{R} \cos\left(\frac{X^1 - a}{R}\right)(0) + s \cdot \frac{i}{4}\partial X^0(0) + \ \ldots, \qquad (s > 0). \quad (7.5)$$

The marginal operator is obtained by taking the derivative with respect to s and considering the limit $s \to 0$:

$$\sigma(s)\partial\overline{\sigma}(0) = s^{1/R^2-1} \cdot \frac{-2^{-2/R^2+1}}{R^3} \cos\left(\frac{X^1 - a}{R}\right)(0) - \frac{i}{4}\partial X^0(0) + \ \ldots, \qquad (s > 0). \quad (7.6)$$

The fate of the $s \to 0$ limit depends on the compactification radius R:

- $R > 1$ (Relevant deformation): the "marginal operator" is infinite, or more specifically, a divergent constant times the relevant matter operator $\cos\left(\frac{X^1-a}{R}\right)$. Since the marginal operator does not exist, there is obviously no corresponding family of conformal boundary conditions connecting BCFT_0 and BCFT_*.

- $R = 1$ (Marginal deformation): in this case we have a marginal operator

$$V = -\frac{1}{2}\left[\cos(X^1 - a) + \frac{i}{2}\partial X^0\right]. \qquad (7.7)$$

  Since this operator has regular self-OPE, it can be used to construct a solution for nonsingular marginal deformations in Schnabl gauge [5, 6] or following KOS [14]. Ignoring the timelike Wilson line, this operator obviously represents the cosine marginal deformation on the circle at self-dual radius [79]. In our conventions, the moduli space of the cosine deformation $\lambda\cos(X^1 - a)$ is periodic with the identification $\lambda \sim \lambda + 1$. $\lambda = 1/2$ represents the critical value where the boundary condition becomes Dirichlet, which is why an overall factor of $1/2$ appears in V.

- $R < 1$ (Irrelevant deformation): in this case we have the marginal operator

$$V = -\frac{i}{4}\partial X^0. \qquad (7.8)$$

  This operator turns on a timelike Wilson line, but all information about the formation of the D$(p-1)$-brane has been lost. In this case, the solution is more naturally understood in the T-dual picture $R \to 1/R$, where it represents the reverse process of formation of a higher dimensional Dp-brane in the string field theory on a D$(p-1)$-brane. While we are able to construct a marginal operator, because it has singular self-OPE the state $e^{-(K+V)}$ assumed to exist in (2.20) is divergent. We could renormalize the boundary interaction to create the Wilson line, but the formal argument of (2.21) connecting BCFT_0 to BCFT_* will no longer apply.

Therefore, the fact that the tachyon lump is not a marginal deformation does not pose a contradiction for the solution. As a consistency check on this picture, we verify the σ-$\overline{\sigma}$ OPE using the four-point function of Neumann-Dirichlet twist fields in appendix A.

One important thing to compute from the solution is the position space profile of the tachyon field. This gives a concrete (but gauge dependent) spacetime picture of the solitonic lump describing the lower dimensional D-brane. To construct the tachyon profile, we expand the solution in the Fock space basis and focus on the tachyon state $|T\rangle$, which can be further expanded in plane wave harmonics on the circle

$$|T\rangle = \sum_{n\in\mathbb{Z}} t_n |T_n\rangle, \qquad |T_n\rangle \equiv c\, e^{inX^1/R}(0)|0\rangle, \tag{7.9}$$

where t_n are Fourier coefficients. The tachyon profile is defined by the function

$$t(x) = \sum_{n\in\mathbb{Z}} t_n\, e^{i\frac{nx}{R}}. \tag{7.10}$$

Define a "test state" $|\tilde{T}_n\rangle$ dual to the nth tachyon harmonic:[22]

$$|\tilde{T}_n\rangle = -\frac{1}{2\pi R}\, c\partial c\, e^{-inX^1/R}(0)|0\rangle. \tag{7.11}$$

By construction, this satisfies $\langle \tilde{T}_m, T_n\rangle = \delta_{mn}$. The tachyon coefficients t_n can be computed from the contraction

$$\begin{aligned} t_n &= \langle \tilde{T}_n, \Psi\rangle, \\ &= -\frac{1}{2\pi R}\left(\frac{2}{\pi}\right)^{-1+n^2/R^2} \mathrm{Tr}\Big[\sqrt{\Omega}(c\partial c e^{-inX^1/R})\sqrt{\Omega}\Psi\Big]. \end{aligned} \tag{7.12}$$

In the last step we mapped the test vertex operator to the sliver coordinate frame (using $f_{\mathcal{S}}(z) = \frac{2}{\pi}\tan^{-1}(z)$ [1, 42]) where we can compute the contraction as a correlation function on the cylinder. To have simpler formulas we will use the non-real form on the solution (3.19), which eliminates the square roots and places the "security strip" $\frac{1}{1+K}$ completely to the right of operator insertions. Furthermore, it is convenient to rewrite the solution in a form which isolates the zero momentum contribution:

$$\Psi = \left(\frac{R-1}{R}\, c(1+K)Bc\frac{1}{1+K}\right) - \left(c\partial\sigma\frac{B}{1+K}\overline{\sigma}(1+K)c\frac{1}{1+K}\right). \tag{7.13}$$

We recover the solution expressed in (3.14) by substituting $\partial\sigma = [1+K,\sigma]$ and using $\sigma\overline{\sigma} = \frac{1}{R}$. The first term gives the sole contribution at zero momentum, and is proportional the the "simple" tachyon vacuum, while the second term is the KOS solution (3.15). Note that when the lump is marginal at $R=1$, the tachyon vacuum term disappears and we are left with the KOS solution, as expected. Since the tachyon coefficient of the "simple" tachyon vacuum was already computed in [3], we immediately obtain

$$t_0 \approx \frac{R-1}{R} \times 0.2844. \tag{7.14}$$

[22]For the purposes of this computation, we will compactify all directions besides X^1 on circles of unit circumference, so that the norm of the $SL(2,\mathbb{R})$ vacuum is $2\pi R$. Strictly speaking, the time direction is noncompact so the vacuum should be delta function normalized. Then the dual test state $|\tilde{T}_n\rangle$ should include a superposition of plane waves in the time direction which creates an eigenstate of the zero mode of X^0. The tachyon coefficients computed in this way turn out to be the same as when compactifying time.

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The remaining tachyon coefficients come from the KOS solution. In [14], KOS gave the general form of the contraction of the solution with any state of the form $\phi = -c\partial c\phi^{\rm m}$, with $\phi^{\rm m}$ a matter primary of weight h:

$$\langle\phi,\Psi\rangle = C_\phi\times g(h). \tag{7.15}$$

Here C_ϕ is the 3-point function of $\phi^{\rm m}$ with the two bcc operators:

$$C_\phi \equiv \langle\overline{\sigma}(\infty)\phi^{\rm m}(1)\sigma(0)\rangle^{\rm matter}_{\rm UHP}, \tag{7.16}$$

and $g(h)$ is a universal function which depends only on the weight h of ϕ^m, and not on the details of the boundary conformal field theories in question. For us, the function $g(h)$ takes a somewhat different form than originally written by KOS since we use the non-real solution, and most importantly, the formulas written in [14] assume the existence of a marginal operator $V = \sigma\partial\overline{\sigma}$ which turns out to be divergent for relevant deformations. After some computation we find[23]

$$g(h) = g_1(h) + g_2(h), \tag{7.18}$$

where

$$g_1(h) = -h\int_0^\infty ds\, e^{-s}\left(\frac{4}{L}\cot\theta_{\frac{1}{2}}\right)^{h-1}\left(\frac{1}{L}-\frac{1}{\pi}\sin 2\theta_s\right), \qquad (L=s+1); \tag{7.19}$$

$$g_2(h) = 2h\int_0^\infty ds\int_{1/2}^\infty dy\frac{e^{-L+1}\sin\theta_{s+\frac{1}{2}}}{L\sin^2\theta_{\frac{1}{2}}}\left(\frac{2\sin\theta_s}{L\sin\theta_{\frac{1}{2}}\sin\theta_{s+\frac{1}{2}}}\right)^{h-1}(\theta_{\frac{1}{2}}\cos\theta_{s+\frac{1}{2}}-\cos\theta_s\sin\theta_{\frac{1}{2}}),$$
$$\left(L=\frac{1}{2}+s+y\right). \tag{7.20}$$

The angular parameters in these integrals are defined

$$\theta_\ell \equiv \frac{\pi\ell}{L}, \tag{7.21}$$

where L appears in parentheses accompanying the respective integral. Therefore when $n\neq 0$ the tachyon coefficients can be computed

$$t_n = \frac{2^{-2n^2/R^2}}{R}g(n^2/R^2), \tag{7.22}$$

where we substituted

$$C_{\tilde{T}_n} = \left\langle\overline{\sigma}(\infty)\frac{1}{2\pi R}e^{-inX^1/R}(1)\sigma(0)\right\rangle^{\rm matter}_{\rm UHP} = \frac{2^{-2n^2/R^2}}{R}, \tag{7.23}$$

which follows immediately from (7.2). We center the lump at the origin by taking the constant a in (7.2) to be zero. With this definition, the tachyon coefficients satisfy $t_{-n} = t_n$.

[23]The two terms come from further reexpressing the KOS solution in the form

$$\Psi_{\rm KOS} = -c\partial\sigma\frac{B}{1+K}c\overline{\sigma} - c\partial\sigma\frac{B}{1+K}\overline{\sigma}\partial c\frac{1}{1+K}. \tag{7.17}$$

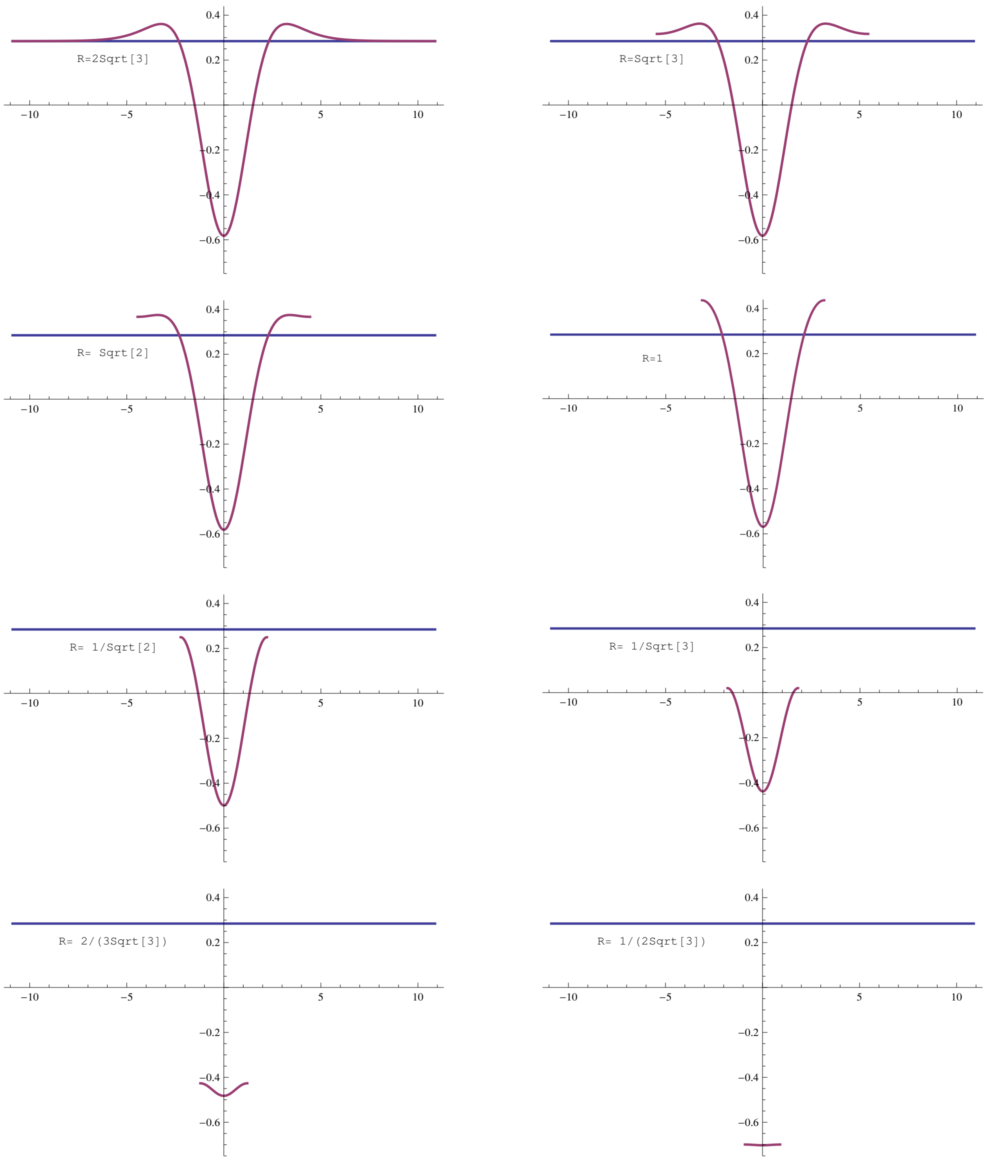

Figure 2. Lump profiles plotted, starting from the top left, for radii $R = 2\sqrt{3}, \sqrt{3}, \sqrt{2}, 1, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{2}{3\sqrt{3}}$ and $\frac{1}{2\sqrt{3}}$.

Now we are ready to plot the tachyon profile, which we show for various values of the compactification radius R in figure 2. For $R \gg 1$, the profile takes a fixed form with negligible differences for different values of R. Similar behavior was observed for Siegel gauge lumps in [72]. The largest radius shown in figure 2 is $R = 2\sqrt{3} \approx 3.5$, which is already representative of the profile at larger radius. As R approaches 1 from above, the tail of the lump begins to feel the finite size of the box, but the core is mostly unaffected. As we cross the marginal threshold $R = 1$ we enter uncharted territory since (at the time of writing) no solution has been identified in level truncation. We find that for $R < 1$ the depth of the lump rapidly decreases, while its spatial average falls to negative values. The smallest radius shown in figure 2 is $R = \frac{1}{2\sqrt{3}}$, where the profile is almost completely

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flat with $t(x) \approx -.7$. This behavior can also be seen in the explicit form of the tachyon coefficients, as shown in table 1.

Curiously, negative values of the tachyon correspond to falling down the unbounded side of the tachyon effective potential, which should contribute negatively to the energy. This makes it difficult to understand where the positive energy of the lower dimensional D-brane comes from when $R < 1$. Currently we cannot provide insight into this question, as it may require a (possibly high level) analysis of the solution in level truncation. The first positive energy solution in string field theory was identified in level truncation quite recently [80], in the context of a systematic study of classical solutions describing Ising model boundary conditions. This result provides evidence that positive energy solutions can be understood in a controlled manner in level truncation.[24]

Since we have exact formulas, there are many features of the tachyon profile that can in principle be studied analytically. One particularly interesting property is the fact that the profile is basically fixed for sufficiently large radius. From the perspective of the coefficients t_n this is quite surprising, since they vary substantially with R well past the point where the profile is stable. Since the profile rapidly decays away from the core, one way to understand this phenomenon is that the profile at finite (but sufficiently large) radius is approximately equal to a periodic sum of the profiles at infinite radius. In fact, we claim that this property is exact: the profile at finite radius $t(x, R)$ can be written in terms of the profile at infinite radius $t(x, \infty)$ with the formula

$$t(x, R) = t_0(\infty) + \sum_{n\in\mathbb{Z}} \Big(t(x + 2\pi Rn, \infty) - t_0(\infty) \Big), \tag{7.24}$$

where $t_0(\infty)$ is the zero mode coefficient (7.14) evaluated at $R = \infty$. We give a proof in appendix B. Note that this is a special feature of the solution we have been working with, and does not hold in a more general gauge.

8 Multiple D-brane solutions

In this section we will discuss the construction of backgrounds involving more than one D-brane. We will first describe a solution representing multiple D$(p-1)$-branes in the string field theory of a single Dp-brane. Then we generalize to find a solution describing multiple copies of the perturbative vacuum.

In the last section we found a solution for a single D$(p-1)$-brane on a circle of radius R. This automatically gives a solution for a pair of D$(p-1)$-branes on a circle of radius $2R$, one located at $X^1 = 0$ and the other located at $X^1 = 2\pi R$. Due to the remarkable property (7.24), the "double lump" profile for this two D-brane system is simply the sum of the "single lump" profiles centered at 0 and at $2\pi R$:

$$\begin{aligned} t_{\text{double lump}}(x, 2R) &= t(x, R), \\ &= t(x, 2R) + t(x + 2\pi R, 2R) - t_0(2R). \end{aligned} \tag{8.1}$$

[24]The σ-brane solution constructed in [80] has positive tachyon coefficient $\approx .1454$. This result runs counter to the expectation derived from (3.14), which always produces a negative tachyon coefficient for higher energy solutions. We thank M. Schnabl for sharing this piece of numerical data.

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R	$+t_0$	$-t_1$	$-t_2$	$-t_3$	$-t_4$
$3\sqrt{3}$	0.229663	0.0566884	0.0604255	0.0617708	0.0581978
$2\sqrt{3}$	0.202297	0.0878951	0.0926562	0.0818861	0.0592841
$\sqrt{3}$	0.120199	0.185312	0.118568	0.0381503	0.00775386
$\sqrt{2}$	0.0832971	0.220203	0.0936014	0.0165937	0.00174037
1.2	0.047399	0.242197	0.0643166	0.00615465	0.000323713
1.1	0.025854	0.248766	0.0485287	0.0031664	0.00010647
1	0	0.250000	0.0330126	0.00134221	2.53129×10^{-5}
1/1.1	−0.0284394	0.243616	0.0205228	0.000485494	4.57246×10^{-6}
1/1.2	−0.0568788	0.230414	0.0121757	0.000162893	7.18534×10^{-7}
$1/\sqrt{2}$	−0.1178	0.187203	0.00348075	0.0000121516	8.48188×10^{-9}
$1/\sqrt{3}$	−0.208191	0.114451	0.000404475	1.33399×10^{-7}	3.50603×10^{-12}
$1/2\sqrt{3}$	−0.700776	0.000808949	7.01207×10^{-12}	1.82338×10^{-24}	1.12733×10^{-41}
$1/3\sqrt{3}$	−1.19336	4.00196×10^{-7}	2.73507×10^{-24}	7.5909×10^{-52}	6.925739×10^{-90}

Table 1. List of tachyon coefficients $t_0, \ldots, t_4$ for various values of the compactification radius. Note that for $R < 1$ the coefficients for the nonzero harmonics rapidly become negligibly small, while the zeroth harmonic becomes increasingly negative. Note also that at $R = 1$ we obtain $t_1 = -\frac{1}{4}$, which means that the coefficient of the marginal operator $\cos(X^1/R)$ in the solution is $-1/2$. For solutions describing nonsingular marginal deformations, the coefficient of the marginal operator in the Fock space expansion is equal to the marginal parameter describing the background in BCFT. Thus we consistently find that the solution describes the cosine marginal deformation at the critical value $\lambda = -\frac{1}{2}$ where the boundary condition is Dirichlet.

One might guess that if the D-branes are at positions a and b, we should likewise sum the lump profiles centered at a and b. This immediately suggests that the solution (around the tachyon vacuum) is simply the sum of the solutions creating a D$(p-1)$-brane at position a and a D$(p-1)$-brane at position b:

$$\Phi = -\Sigma_a \Psi_{\rm tv} \overline{\Sigma}_a - \Sigma_b \Psi_{\rm tv} \overline{\Sigma}_b. \tag{8.2}$$

Remarkably, this turns out to be correct. To see this, it is convenient to assemble the bcc operators at a and b into row and column vectors

$$\boldsymbol{\sigma} \equiv \begin{pmatrix} \sigma_a & \sigma_b \end{pmatrix}; \qquad \overline{\boldsymbol{\sigma}} \equiv \begin{pmatrix} \overline{\sigma}_a \\ \overline{\sigma}_b \end{pmatrix}. \tag{8.3}$$

The row and column have the obvious interpretation of "creating" Chan-Paton factors out of a boundary condition where they are absent. Building $\boldsymbol{\Sigma}$ and $\overline{\boldsymbol{\Sigma}}$ from $\boldsymbol{\sigma}$ and $\overline{\boldsymbol{\sigma}}$, we can write the solution as

$$\Phi = -\boldsymbol{\Sigma} \Psi_{\rm tv} \overline{\boldsymbol{\Sigma}}. \tag{8.4}$$

For this to satisfy the equations of motion, $\overline{\boldsymbol{\sigma}}\boldsymbol{\sigma}$ must be equal to the 2×2 identity matrix. Computing we find

$$\overline{\boldsymbol{\sigma}}\boldsymbol{\sigma} = \begin{pmatrix} 1 & \overline{\sigma}_a \sigma_b \\ \overline{\sigma}_b \sigma_a & 1 \end{pmatrix}. \tag{8.5}$$

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The diagonals work correctly because $\overline{\sigma}_a\sigma_a = \overline{\sigma}_b\sigma_b = 1$. To understand what happens with the off-diagonal elements, note that the leading term in the OPE between $\overline{\sigma}_{\mathrm{ND},a}$ and $\sigma_{\mathrm{ND},b}$ must be proportional to a bcc operator which shifts the Dirichlet boundary condition from a to b. If $a \neq b$ (modulo the circumference of the circle), this operator must have positive conformal weight, which means that the leading singularity in the $\overline{\sigma}_{\mathrm{ND},a}$-$\sigma_{\mathrm{ND},b}$ OPE must be *less severe* than $s^{-1/8}$. But the Neumann-Dirichlet twist operators are always accompanied by the timelike Wilson line, and the OPE of these bcc operators vanishes as $s^{1/8}$. Therefore, $\overline{\sigma}_a\sigma_b = 0$ and

$$\overline{\boldsymbol{\sigma}}\boldsymbol{\sigma} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{8.6}$$

Therefore (8.4) is a solution. It is clear that this generalizes to any number of non-coincident D$(p-1)$-branes by simply adding more entries into the row and column vectors.

However, this misses the important case when the D-branes are coincident. We cannot use the same bcc operators to describe all D-branes in this case, since then $\overline{\boldsymbol{\sigma}}\boldsymbol{\sigma}$ will be a matrix of "ones" rather than the identity matrix. However, there are many choices of $\sigma, \overline{\sigma}$ which implement the same change of boundary condition. In the examples discussed so far, we have chosen $\sigma, \overline{\sigma}$ in such a way that the spacelike factor has the lowest possible conformal weight. But we can also consider "excited" bcc operators. For example, we can build the lump solution using

$$\sigma'(s) = \frac{i}{\sqrt{2}}\partial X^2\,\sigma_{\mathrm{ND}}e^{i\sqrt{17/16}\,X^0}(s); \qquad \overline{\sigma}'(s) = \frac{i}{\sqrt{2}}\partial X^2\,\overline{\sigma}_{\mathrm{ND}}e^{-i\sqrt{17/16}\,X^0}(s), \tag{8.7}$$

where X^2 is a free boson orthogonal to X^1. The computation of observables indicates that the lump solution built from $\sigma', \overline{\sigma}'$ is physically identical to the previous lump solution built from $\sigma, \overline{\sigma}$ in (7.1).[25] In fact, these two sets of bcc operators have vanishing OPE, which means that the row and column vectors

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma & \sigma' \end{pmatrix}; \qquad \overline{\boldsymbol{\sigma}} = \begin{pmatrix} \overline{\sigma} \\ \overline{\sigma}' \end{pmatrix}, \tag{8.8}$$

define a solution for a coincident pair of D$(p-1)$-branes.

By the same mechanism, we can construct a "double brane" solution describing two copies of the perturbative vacuum. Here there is no change of boundary condition, so we simply construct $\boldsymbol{\sigma}, \overline{\boldsymbol{\sigma}}$ using the primaries of BCFT$_0$. For example, if BCFT$_0$ is made from free bosons, we can define two sets of "bcc operators":

$$\begin{aligned} \sigma_1(s) &= \frac{i}{\sqrt{2}}\partial X^1 e^{iX^0}(s); \qquad \overline{\sigma}_1(s) = \frac{i}{\sqrt{2}}\partial X^1 e^{-iX^0}(s); \\ \sigma_2(s) &= \frac{i}{\sqrt{2}}\partial X^2 e^{iX^0}(s); \qquad \overline{\sigma}_2(s) = \frac{i}{\sqrt{2}}\partial X^2 e^{-iX^0}(s). \end{aligned} \tag{8.9}$$

[25] In principle, the lump solutions built from (8.7) and (7.1) should be gauge equivalent. Since they are already in the same gauge, this indicates that the gauge condition does not define the solution uniquely. This phenomenon was already observed in [1], where a 1-parameter family of solutions for the perturbative vacuum was found in Schnabl gauge. We thank M. Schnabl for discussions on this point.

Defining row and column vectors

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_1 & \sigma_2 \end{pmatrix}; \qquad \overline{\boldsymbol{\sigma}} = \begin{pmatrix} \overline{\sigma}_1 \\ \overline{\sigma}_2 \end{pmatrix}, \tag{8.10}$$

we have $\overline{\boldsymbol{\sigma}}\boldsymbol{\sigma} = \mathbb{I}_{2\times 2}$, and the solution creates two copies of the perturbative vacuum. Actually, this is probably the simplest nontrivial solution discussed so far. The n-point functions of $\boldsymbol{\sigma}$ and $\overline{\boldsymbol{\sigma}}$ can be computed by elementary means, and even the Schnabl-gauge solution (6.5) can plausibly be studied in a fairly explicit manner. Note that, contrary to some expectations, this multibrane solution is not formulated within the universal sector. A different approach to multibrane solutions, advanced in [71] and further explored in [81–88], requires only universal states generated by K, B and c. However, the solution is quite singular and an adequate regularization has not been found. Also, it is unclear in this approach how non-abelian gauge bosons emerge in the spectrum of excitations.

Having discussed a few explicit examples, let us outline the general construction. Suppose that, starting from BCFT_0, we want to describe a system of N D-branes described by boundary conformal field theories BCFT_i for $i = 1, \ldots, N$. We need N bcc operators

$$\sigma_i(s) = \sigma_{*,i}\, e^{i\sqrt{h_i}X^0}(s); \qquad \overline{\sigma}_i(s) = \overline{\sigma}_{*,i}\, e^{-i\sqrt{h_i}X^0}(s), \tag{8.11}$$

where $\sigma_{*,i}, \overline{\sigma}_{*,i}$ are primaries of weight h_i which act as the identity operator in the time direction and change the boundary condition from BCFT_0 to BCFT_i in the spatial ($c = 25$) directions. If the bcc operators satisfy

$$\lim_{s\to 0} \overline{\sigma}_i(s)\sigma_j(0) = \delta_{ij}, \qquad (s > 0), \tag{8.12}$$

then row and column vectors

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_1 & \ldots & \sigma_N \end{pmatrix}; \qquad \overline{\boldsymbol{\sigma}} = \begin{pmatrix} \overline{\sigma}_1 \\ \vdots \\ \overline{\sigma}_N \end{pmatrix}, \tag{8.13}$$

define a solution for the desired multiple D-brane system. The orthogonality condition (8.12) is nontrivial. In general the $\overline{\sigma}_{*,i}$-$\sigma_{*,j}$ OPE takes the form

$$\overline{\sigma}_{*,i}(s)\sigma_{*,j}(0) = \frac{1}{s^{h_i+h_j-h_{ij}}}\sigma_{*,ij}(0) + \text{less singular}, \qquad (s > 0), \tag{8.14}$$

where the leading term is proportional to a boundary condition changing operator $\sigma_{*,ij}$ relating BCFT_i with BCFT_j with dimension h_{ij}. The orthogonality condition is satisfied provided the conformal weights of the operators in this OPE satisfy the bound

$$|\sqrt{h_i} - \sqrt{h_j}| < \sqrt{h_{ij}}, \qquad (i \neq j). \tag{8.15}$$

We are not certain whether this inequality poses a limitation on the possible multiple D-brane systems that can be constructed by our method. In the case where the $c = 25$ theory is described by free bosons, we have confirmed that it is possible to create an

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arbitrarily large number of copies of the perturbative vacuum by choosing σ_is consistent with this bound.[26]

In noncommutative field theories [89] and vacuum string field theory [41, 90–92], there is a close relation between multiple D-brane systems and higher rank projectors. While the solution (3.14) is not a star algebra projector, there is a natural way to associate a star algebra projector to any classical solution in open string field theory [25, 29]. The construction goes as follows. Consider a "singular" gauge transformation defined by the string field[27]

$$\begin{aligned} U &= Q_\Psi\left(\frac{B}{1+K}\right), \\ &= 1 - \mathbf{\Sigma}\frac{1}{1+K}\overline{\mathbf{\Sigma}}. \end{aligned} \tag{8.16}$$

Formally, U is a gauge parameter defining a (reducible) gauge transformation from the solution to itself:

$$\Psi = U^{-1}(Q+\Psi)U, \qquad \text{(formally)}. \tag{8.17}$$

But in reality this gauge transformation is singular. To see why, note that the definition of U together with the (presumed) existence of U^{-1} implies that the identity string field is trivial in the Q_Ψ cohomology:

$$1 = Q_\Psi\left(U^{-1}\frac{B}{1+K}\right), \tag{8.18}$$

which would mean that the solution supports no open string excitations. Therefore, if the solution is not the tachyon vacuum, we are forced to conclude that U is not invertible. If we think of U as an operator on the space of half string functionals [91, 92], it should have a kernel. The projector onto the kernel is called the *characteristic projector*, and if $U = 1 - X$ we can compute the characteristic projector from the limit

$$X^\infty = \lim_{N\to\infty} X^N. \tag{8.19}$$

Plugging in (8.16) we find

$$\begin{aligned} X^\infty &= \mathbf{\Sigma}\,\Omega^\infty\,\overline{\mathbf{\Sigma}}, \\ &= \Sigma_1\,\Omega^\infty\,\overline{\Sigma}_1 + \ldots + \Sigma_N\,\Omega^\infty\,\overline{\Sigma}_N, \end{aligned} \tag{8.20}$$

where Ω^∞ is the sliver state [40, 93], and in the second step we expanded $\mathbf{\Sigma}, \overline{\mathbf{\Sigma}}$ out into components $\Sigma_i, \overline{\Sigma}_i$ creating the boundary condition of each constituent D-brane. The sliver state factorizes into a wavefunctional on the left and right halves of the open string, and therefore can be interpreted as a rank one projector [91, 92]. Therefore, X^∞ is a

[26] One of many possible choices is $\sigma_{j,*} = \overline{\sigma}_{j,*} = \frac{i}{\sqrt{2}}\partial X^2 p^{(1)}_{j^2}$ where $p^{(1)}_{j^2}$ are the (properly normalized) zero momentum primaries in the X^1 BCFT of weight j^2.

[27] We can in principle compute a projector given $U = Q_\Psi b$ for any ghost number -1 state b, but the choice (8.16) simplifies the calculation.

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sum of rank one projectors carrying the boundary condition of each constituent D-brane. Moreover, since

$$\overline{\Sigma}_i \Sigma_j = \delta_{ij}, \tag{8.21}$$

the projectors are $*$-orthogonal. Therefore, for a system of N D-branes, the characteristic projector (formally) has rank N. The picture that emerges strongly resembles the boundary conformal field theory construction of D-branes in vacuum string field theory [41], where the equations of motion are solved by adding sliver states with appropriately deformed boundary conditions. There are interesting differences, however. In [41] the projectors are rendered $*$-orthogonal by the nontrivial conformal weight of the boundary condition changing insertions, which under star multiplication produce a vanishing factor due to a singular conformal transformation. In our construction, the matter insertions carry vanishing conformal weight, and the singular multiplication of sliver states is not essential. Rather, the projectors are $*$-orthogonal because the boundary insertions themselves are already $*$-orthogonal.

9 Conclusion

To summarize, the solution takes the form

$$\Psi_{\mathrm{tv}} - \Sigma\Psi_{\mathrm{tv}}\overline{\Sigma}, \tag{9.1}$$

where Ψ_{tv} is the tachyon vacuum (3.1) and Σ and $\overline{\Sigma}$ string fields which change the open string boundary condition between the perturbative vacuum and the D-brane system we wish to describe. The form of the solution is easy to grasp. To find a new background, we first condense to the tachyon vacuum (the first term), then we "reverse" the process of tachyon condensation to create the new D-brane system (the second term). Note, in particular, that $\Sigma\Psi_{\mathrm{tv}}\overline{\Sigma}$ is the tachyon vacuum of the new D-brane system reexpressed (via Σ and $\overline{\Sigma}$) in the variables of the reference boundary conformal field theory. The solution reproduces the physics of the new background in the sense that:

- The action evaluated on the solution describes the difference in tension between the perturbative vacuum and the target D-brane system.
- The solution implies the correct coupling between the new background and closed string states.
- The action expanded around the solution, after a trivial field redefinition, is identical to the string field theory formulated in the new background. In this sense, the background independence of open string field theory is manifest.

These results depend very little on the detailed form of the solution. They follow quite generally from the relations

$$\overline{\Sigma}\Sigma = 1; \qquad Q_{\Psi_{\mathrm{tv}}}\Sigma = Q_{\Psi_{\mathrm{tv}}}\overline{\Sigma} = 0, \tag{9.2}$$

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together with the fact that $\Psi_{\rm tv}$ is a solution for the tachyon vacuum. This suggests that there may be other solutions which share the same basic structure and transcend some limitations in our approach.[28] Our implementation assumes the existence of boundary condition changing operators of vanishing conformal weight, which for time-independent backgrounds we construct by tensoring the background shift with a timelike Wilson line of specific magnitude. This construction does not work for time-dependent backgrounds. Moreover, this construction excites primaries in BCFT_0 which are irrelevant to describing the physics of the new background. This can hide symmetries — such as Lorentz invariance — which we might prefer to be manifest.

One important question we have not addressed is the behavior of the solution in level truncation. This question poses a technical challenge, both because the solution is somewhat exotic from the perspective of level truncation — due to the excitation of X^0 primaries and the existence of higher energy configurations — and because the gauge condition $\mathcal{B}_{\frac{1}{\sqrt{1+K}},\frac{1}{\sqrt{1+K}}}=0$ produces solutions which are close to being singular from the perspective of the identity string field [66]. Indeed, even the tachyon vacuum (3.1) gives a divergent series for the energy in level truncation, though the series can be resummed to give the expected result within less than a percent [3]. The Schnabl gauge solution (6.5) in theory should be a safer starting point for level truncation studies, but the evaluation of $2n$-point functions of bcc operators presents a substantial technical obstacle.

In this paper we have focused on the bosonic string, but clearly it would be interesting to generalize these results to the superstring. Given the central role of the tachyon vacuum (3.1) in our construction, we expect that the tachyon vacuum of Berkovits superstring field theory, recently found in [4], will likewise play a central role for the superstring. We hope to return to this question soon.

The solution we have found appears to solve several longstanding and fundamental problems in string field theory, and, with remarkable simplicity, demonstrates the power of string field theory to provide a unified description of the multitude of backgrounds of first quantized string theory. We hope to see exciting developments in the near future.

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[28]Note, for example, that the recent solution of [20] can be recast in the form (9.1) due to its formal similarity with the KOS solution. However, the details of [20] are quite different from the solution discussed here.

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A Four point function of twist fields

In this appendix we discuss the 4-point function of Neumann-Dirichlet twist operators for a free boson X^1 compactified on a circle of radius R. Most computations in this paper do not require this correlator, but it implicitly appears (for example) in the computation of the quadratic term in the equations of motion and the kinetic term in the action. (The cubic term implicitly requires the six point function). Our main interest in the 4-point function is as a cross-check on the OPE (7.4), and as an illustration of the algebraic structure of the solution in the context of a correlator which is not completely fixed by conformal invariance.

The complete 4-point function of Neumann-Dirichlet twist fields, including instanton corrections from the compactification, was computed in [76] and takes the form

$$\big\langle I\circ\overline{\sigma}_{\rm ND}(0)\sigma_{\rm ND}(1)\overline{\sigma}_{\rm ND}(s)\sigma_{\rm ND}(0)\big\rangle_{\rm UHP}^{X^1}=\frac{2\pi}{|s(1-s)|^{1/8}}G(s,R), \tag{A.1}$$

where[29]

$$G(s,R)=\frac{1}{\sqrt{\frac{2}{\pi}K\big(\sqrt{s}\,\big)}}\,\vartheta_3\left(0,q\big(\sqrt{s}\,\big)^{R^2}\right). \tag{A.2}$$

Here $\vartheta_3(0,q)$ is the Jacobi theta function, $K(k)$ the complete elliptic integral of the first kind, and $q(k)$ the nome

$$q(k)=e^{-\pi\frac{K(\sqrt{1-k^2})}{K(k)}}. \tag{A.3}$$

Tensoring $\sigma_{\rm ND},\overline{\sigma}_{\rm ND}$ with the timelike Wilson line removes the singular factor $|s(1-s)|^{-1/8}$:

$$\big\langle\overline{\sigma}(\infty)\sigma(1)\overline{\sigma}(s)\sigma(0)\big\rangle_{\rm UHP}^{\rm matter}=2\pi\,G(s,R). \tag{A.4}$$

We plot this for $s\in[0,1]$ in figure 3. Note that the modular property of the theta function,

$$\vartheta_3(0,e^{i\pi(-1/\tau)})=(-i\tau)^{1/2}\vartheta_3(0,e^{i\pi\tau}), \tag{A.5}$$

implies that the 4-point function satisfies

$$G(1-s,R)=\frac{1}{R}G\left(s,\frac{1}{R}\right). \tag{A.6}$$

This is obviously related to T-duality. The correlator at radius R and $1/R$ are related by a switch of Neumann and Dirichlet boundary conditions, which in effect interchanges $\sigma_{\rm ND}$ and $\overline{\sigma}_{\rm ND}$. At the self dual radius, $G(s,R)$ is constant:

$$G(s,1)=1. \tag{A.7}$$

Note that the points $s=0$ and $s=1$ represent a collision between σ and $\overline{\sigma}$, where the 4-point function reduces to a 2-point function. At these points we find

$$G(0,R)=1;\qquad G(1,R)=\frac{1}{R}. \tag{A.8}$$

This is confirms the OPE

$$\lim_{s\to0}\overline{\sigma}(s)\sigma(0)=1;\qquad\lim_{s\to0}\sigma(s)\overline{\sigma}(0)=\frac{1}{R}. \tag{A.9}$$

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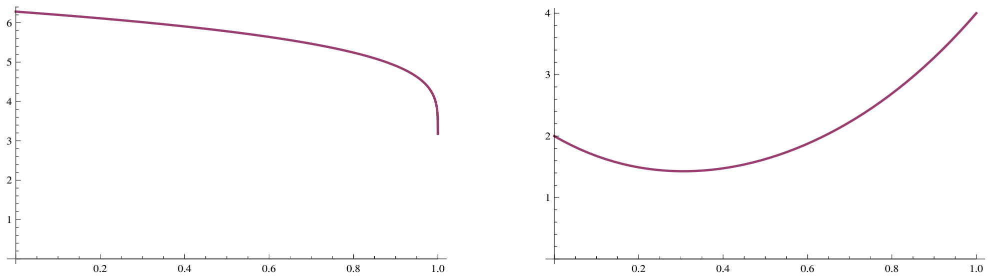

Figure 3. The first plot shows the correlator (A.4) as a function of $s \in [0,1]$ when $R = 2$. Note that the correlator is non-differentiable at $s = 1$, and the value is half that at $s = 0$. For illustrative purposes, in the second plot we show the 4-point function of bcc operators for the 2-brane solution (8.9). Here the value at $x = 1$ is twice that at $x = 0$, which represents the doubling of energy.

Thus the "associativity anomaly" can be seen explicitly in the 4-point function.

Let us take a closer look at the behavior of the 4-point function in the limit $s \to 1$, where we should be able to extract the $\sigma_{\rm ND}$-$\overline{\sigma}_{\rm ND}$ OPE computed in (7.4). To make the expansion somewhat easier, it is convenient to use T-duality (A.6) to map from $s = 1$ to $s = 0$. Then using the power series

$$\vartheta_3(0,q) = 1 + 2q + \mathcal{O}(q^4); \tag{A.10}$$

$$\frac{2}{\pi}K(k) = 1 + \frac{k^2}{4} + \frac{9k^4}{64} + \mathcal{O}(k^6); \tag{A.11}$$

$$q(k) = \frac{k^2}{16} + \frac{k^4}{32} + \mathcal{O}(k^6), \tag{A.12}$$

we obtain

$$\begin{aligned}&\big\langle I\circ\overline{\sigma}_{\rm ND}(0)\sigma_{\rm ND}(1)\overline{\sigma}_{\rm ND}(1-s)\sigma_{\rm ND}(0)\big\rangle^{X^1}_{\rm UHP}\\ &\qquad= \frac{1}{s^{1/8}}\frac{2\pi}{R} + \frac{1}{s^{1/8-1/R^2}}\frac{2\pi\cdot 2^{-4/R^2+1}}{R} + \ldots,\qquad (R>1/\sqrt{2}).\end{aligned} \tag{A.13}$$

The first and second terms represent the contribution from the identity operator and the first cosine harmonic, respectively, in the OPE between $\sigma_{\rm ND}$ and $\overline{\sigma}_{\rm ND}$. The restriction $R > 1/\sqrt{2}$ is assumed otherwise the second term is subleading to terms of the order $s^{15/8}$, which arise from the first Virasoro descendent of the identity. Alternatively, we should be able to compute (A.13) by substituting the OPE (7.4) directly into the correlator:

$$\begin{aligned}&\big\langle I\circ\overline{\sigma}_{\rm ND}(0)\sigma_{\rm ND}(1)\overline{\sigma}_{\rm ND}(1-s)\sigma_{\rm ND}(0)\big\rangle^{X^1}_{\rm UHP}\\ &\quad= \frac{1}{s^{1/8}}\frac{1}{R}\big\langle I\circ\overline{\sigma}_{\rm ND}(0)\sigma_{\rm ND}(0)\big\rangle^{X^1}_{\rm UHP}\\ &\qquad+\frac{1}{s^{1/8-1/R^2}}\frac{2^{-2/R^2+1}}{R}\left\langle I\circ\overline{\sigma}_{\rm ND}(0)\cos\left(\frac{X^1-a}{R}\right)(1)\sigma_{\rm ND}(0)\right\rangle^{X^1}_{\rm UHP} + \ldots,\\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (R>1/\sqrt{2}).\end{aligned} \tag{A.14}$$

[29] Our notation for elliptic functions follows Gradshteyn and Ryzhik [94].

Using the 3-point function (7.2)

$$\big\langle I \circ \overline{\sigma}_{\rm ND}(0) e^{inX^1/R}(1)\sigma(0)\big\rangle_{\rm UHP}^{X^1} = 2\pi 2^{-2n^2/R^2} e^{ina/R}, \tag{A.15}$$

we find agreement between (A.13) and (A.14).

B Additivity of the lump profile

In this appendix, we prove that the lump profile for the solution (3.14) compactified on a circle of radius R is a periodic sum of the uncompactified lump profile. The profile at radius R is given by

$$t(x,R) = t_0(R) + \sum_{n\in\mathbb{Z}-\{0\}} \frac{1}{R} 2^{-2n^2/R^2} g(n^2/R^2) e^{inx/R}, \tag{B.1}$$

where $t_0(R)$ is given by (7.14). In the limit $R\to\infty$, the sum turns into an integral

$$t(x,\infty) = t_0(\infty) + \int_{-\infty}^{\infty} dk\, 2^{-2k^2} g(k^2) e^{ikx}. \tag{B.2}$$

Our goal is to establish

$$t(x,R) = t_0(\infty) + \sum_{n\in\mathbb{Z}} \Big(t(x+2\pi Rn,\infty) - t_0(\infty)\Big). \tag{B.3}$$

Substituting (B.2), we should have

$$t(x,R) = t_0(\infty) + \sum_{n\in\mathbb{Z}} \int_{-\infty}^{\infty} dk\, 2^{-2k^2} g(k^2) e^{ik(x+2\pi Rn)}. \tag{B.4}$$

Performing the sum over Fourier harmonics gives a "Dirac comb" of delta functions:

$$\sum_{n\in\mathbb{Z}} e^{ik(x+2\pi Rn)} = \sum_{n\in\mathbb{Z}} \frac{1}{R}\delta(k-n/R). \tag{B.5}$$

Evaluating the integral then gives

$$t(x,R) = t_0(\infty) + \frac{1}{R}\lim_{h\to 0} g(h) + \sum_{n\in\mathbb{Z}-\{0\}} \frac{1}{R} 2^{-2n^2/R^2} g(n^2/R^2) e^{inx/R}. \tag{B.6}$$

This is almost the expected lump profile. All we have to do is show that the zero mode works correctly. This requires

$$\lim_{h\to 0} g(h) = -t_0(\infty) \approx -.2844. \tag{B.7}$$

From the form of $g(h)$ given in equations (7.19) and (7.20), it is clear that the $h\to 0$ limit is determined by the formula

$$\lim_{h\to 0} h f(s)^{h-1} = \frac{1}{f'(0)}\delta(s), \tag{B.8}$$

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where $f(s)$ is a function that vanishes at $s = 0$ and f' is the first derivative. Plugging into (7.19) and (7.20), the integration over s disappears against the delta function, and the remaining expression turns out to be (minus) the tachyon coefficient of the tachyon vacuum, as required by (B.7). There is a schematic way to understand why this works. The integration variable s in equations (7.19) and (7.20) represents the Schwinger parameter for the factor

$$\partial\sigma\frac{1}{1+K}\overline{\sigma}, \tag{B.9}$$

which appears in the KOS solution. The formula (B.8) effectively says that this factor is replaced by the identity string field in the $h \to 0$ limit. Thus the KOS solution becomes (minus) the tachyon vacuum:

$$-c\partial\sigma\frac{B}{1+K}\overline{\sigma}(1+K)c\frac{1}{1+K} \longrightarrow -c(1+K)Bc\frac{1}{1+K}, \tag{B.10}$$

and the tachyon coefficient is correspondingly that of the tachyon vacuum.

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