Hidden Insurance in a Moral-Hazard Economy

Giuseppe Bertola* and Winfried Koeniger**

June 2015

Abstract

We analyze the general equilibrium of an economy in which a competitive industry produces non-exclusive insurance services. The equilibrium is inefficient because insurance contracts cannot control moral hazard, and welfare can be improved by policies that reduce insurance by increasing its price above marginal cost. We discuss how insurance production costs that exceed expected claim payments interact with moral hazard in determining the equilibrium's inefficiency, and show that these costs can make insurance premia so actuarially unfair as to validate the standard first-order conditions we exploit in our analysis.

Keywords: Hidden action; Principal agent; First-order approach; Constrained efficiency.

JEL: E21, D81, D82.

*Edhec Business School, CEPR, CESifo, CFS; giuseppe.bertola@edhec.edu. **University of St.Gallen (Swiss Institute for Empirical Economic Research), CEPR, CFS, IZA; winfried.koeniger@unisg.ch. Part of this research was conducted by Giuseppe Bertola at the Università di Torino and by Winfried Koeniger at Queen Mary, University of London. A previous version of this article was circulated as CEPR Discussion Paper No. 9864. This version benefits from helpful comments by the editor and the referees, as well as by colleagues and seminar participants. We thank Adam Pigon and Anil Shamdasani for careful proofreading.

1 Introduction

When a single exclusive insurer of a specific risk does not observe the insured's behavior, but knows how that behavior influences the probability of claims, partial insurance can improve efficiency by giving the insured incentives to exert loss-prevention efforts. If the insured may also obtain coverage from other insurers, however, it can be difficult or impossible to ensure that losses are only partially insured. For example, rental car contracts that do not include collision damage insurance are supposed to encourage safe driving, but do not give such incentives if customers are covered by secondary car insurance contracts.

Competition among non-exclusive insurers implies that coverage is excessive in equilibrium (Pauly, 1974; similar inefficiencies can arise in credit markets where default probabilities depend on the total amount of credit issued by competing lenders, as in e.g. Bizer and DeMarzo, 1992). Insurers can address this problem by sharing information about each individual's coverage, requiring the presentation of original receipts to prevent the submission of claims to other insurers, or refusing to pay claims for risks covered by multiple policies. It can be difficult to discover hidden insurance, however, and the resulting moral-hazard inefficiencies may be particularly relevant for health and income risks, covered by public programs as well as by many privately held contingent assets.

This article revisits such issues, focusing on the effect of moral hazard in a general equilibrium model with costly insurance production. In the model economy, introduced in Section 2, a continuous unobservable effort choice influences the probability of two possible income realizations. A standard competitive-industry cost structure represents "costs of obtaining information and running markets [that] are no less real costs than other forms of production costs" (Greenwald and Stiglitz, 1986, p. 259). We assume that all means to ensure exclusivity are fully exploited, but are not sufficient to rule out residual non-exclusive insurance, which we model in terms of anonymous and competitive trade of infinitely divisible contracts akin to Arrow-Debreu securities. These convenient assumptions make it possible to use standard optimality conditions, and to establish in Section 3 that the equilibrium is constrained inefficient because trade of non-exclusive unit insurance contracts neglects moral-hazard effects. We discuss how the inefficiency depends on the interaction of moral hazard with insurance production costs and show in Section 4 how society can improve welfare. To exploit the incentive effects of partial insurance, optimal public policy reduces equilibrium insurance by increasing insurance premia above production costs.

Although the model's insurance production function is stylized, insurance production costs are realistic: the OECD Insurance Statistics Yearbook records operating expenses amounting to about one third of private non-life insurance claims in advanced countries. These costs influence the size of moral hazard inefficiencies and policy interventions in the model, and make the analysis of competitive insurance markets more straightforward than previous analyses suggest. When competitively priced contingent securities pay off with a probability that depends on hidden actions, standard functional form assumptions are not sufficient to ensure that the expected utility of individual price-takers is concave in effort and insurance, and an equilibrium may fail to exist (Helpman and Laffont, 1975). In the economy we model, the technical results collected in Section 5 establish that the cost of processing claims can make non-exclusive insurance so partial as to ensure that, under interpretable conditions, the objective function of price-taking individuals is locally concave at points where the first-order conditions hold, and that the equilibrium is unique. Section 6 concludes and discusses how the model's convenient but restrictive assumptions could be modified in further work.

2 Model

A continuum, normalized to measure one, of ex-ante identical risk-averse individuals experiences idiosyncratic income shocks.[1] The shocks are observable and verifiable but occur with probabilities that depend on unobservable effort. An individual's resource endowment amounts to y with probability $1-\pi(e)$, but with probability $\pi(e)$ may be reduced to $y-\Delta$, $\Delta > 0$. We make the standard assumptions about the utility derived from consumption: the function $U : \mathbb{R} \to \mathbb{R}$ is continuously differentiable at least twice and strictly increasing, and strictly concave on an interval $\mathcal{C} \subset \mathbb{R}_+$. To ensure interior optima, we also assume $\lim_{x \downarrow \inf \mathcal{C}} U'(x) = \infty$. It will be convenient in what follows to work with the function $u(c) = U(y+c)$, where c denotes the difference between consumption and y.

Effort, denoted e, additively decreases utility and determines the probability $\pi(e)$ of the negative shock. We assume

[1] Our derivations and results are applicable to economies with observably heterogeneous endowments and/or preferences. Pareto-improving policies would then need to include *ex ante* transfers.

[PROB] $0 < \pi(e) < 1;\ \pi'(e) < 0,\ \pi''(e) > 0\ \forall\, e \geq 0;\ \lim_{e \downarrow 0} \pi'(e) = -\infty$:

both realizations have positive probability at all feasible effort levels and the probability of the negative shock is always reduced by higher effort, ensuring that there is moral hazard whenever effort is private information; to obtain an interior optimal solution, $\pi(e)$ is infinitely steep at the zero lower bound of feasible effort.

Production of insurance

The only production is the provision of insurance services. An industry of N firms sells non-exclusive insurance contracts. Realized shocks are verifiable; hence, insurance contracts in terms of the single good can specify payment of a premium p for a unit payoff in the event of a negative income shock. Consumer and insurer rationality dictates neither party suffers a sure loss; hence, $0 \leq p \leq 1$ in any equilibrium. For analytical convenience, we suppose that contracts may be traded in continuously divisible quantities. As usual, competitive behavior can be approximately realistic if N is large and strategic interactions can be disregarded. We further assume that the number of contracts purchased by each individual is unobservable: anonymous competitive trade of contingent securities rules out the optimally nonlinear prices each insurer would charge if it were possible to keep track of each individual's contracts.[2]

In a symmetric competitive equilibrium, let the individuals' common choice of effort be $\bar{e}$ and their common choice of insurance be $\bar{q}$. If there are N insurers, each is issuing $x = \bar{q}/N$ contracts (continuous divisibility implies there is no integer constraint for the number of policies). Taking as given the equilibrium price p of unit contingent claims, insurers maximize profits when that price equals the marginal cost of insurance services, which is only realized when a claim is made (underwriting costs for contracts that do not result in claims would have substantively similar but analytically less elegant implications).

If the variable cost of each insurance firm is a differentiable function $\gamma(x)$ of the claims it pays, and a fraction $\pi(\bar{e})$ of contracts issued results in claims, the competitive premium charged

[2]Even when contracts with other insurers are hidden, each insurer does observe the quantity of insurance it sells to each individual. Because customers who purchase more insurance should be charged higher unit prices, and split hidden insurance evenly across insurers. Hence, effort responds to each firm's pricing policy only by a factor of $1/N$. If N is large, nonlinear pricing has a small effect on the firm's profits, can be prevented by bookkeeping costs, and has negligible implications for the economy's equilibrium.

by risk-neutral insurers equals the expected marginal cost of issuing a unit insurance contract:

$$p(\bar{q}, \bar{e}) = \pi(\bar{e})\gamma'(\bar{q}/N)\,. \tag{1}$$

We assume that

[COST] $\gamma'(x) > 0\,\forall x \geq 0, \quad \gamma''(x) > 0\,\forall x \geq 0; \quad \gamma'(0) = 1.$

Because $\gamma(0) = 0$ and $\gamma'(0) = 1$, $\lim_{x\downarrow 0}(\gamma(x)/x) = 1$: the marginal and average variable cost of infinitesimally small amounts of insurance coincide with the claim payment, ensuring that insurance is strictly positive in competitive equilibrium; a zero-insurance corner solution might be optimal if instead $\lim_{x\downarrow 0}(\gamma(x)/x) > 1$ implied that small amounts of insurance are unfairly priced. Because $\gamma''(x) > 0$, marginal cost is increasing at the firm level: there are decreasing returns to the processing or verification activities entailed by the delivery of contingent claim units.

We suppose that organizing risk-sharing operations entails a fixed ϕ for every active insurer. This rules out non-intermediated trade and lets insurers be neutral with respect to idiosyncratic risk, because they each issue a finitely large mass of infinitesimal contracts: profits are negative at $\bar{q}/N = 0$ and increasing in $\bar{q}/N$, so for $\phi > 0$ the free-entry zero-profit condition

$$\pi(\bar{e})\left(\gamma'\left(\frac{\bar{q}}{N}\right)\frac{\bar{q}}{N} - \gamma\left(\frac{\bar{q}}{N}\right)\right) - \phi = 0 \tag{2}$$

implies $\bar{q}/N > 0$. By **[COST]**, strictly positive insurance has marginal cost $\gamma'(\bar{q}/N) > 1$, and it is unfairly priced at $p(\bar{q}, \bar{e}) > \pi(\bar{e})$ by (1).[3]

Demand for insurance

Individuals can only take long positions in contingent securities, and may only buy rather than sell insurance. When they purchase q units of insurance at price p, the difference between their consumption and income is $c_h = -qp$ if the negative shock does not occur, and $c_\ell = -\Delta + (1 - p)q = c_h - \Delta + q$ if it does. Under the assumptions above, the effort and insurance choices that

[3] Under free entry, N adjusts to ensure that each firm's insurance services are produced at the zero-profit efficient scale, and constant returns at the industry level conveniently simplify characterization of the equilibrium. The working paper version also considers the case where a fixed number of competitive firms are owned by the economy's representative individual. The analysis is complicated by the need to account for rents, but delivers a qualitatively identical characterization of the economy's inefficiency and of corrective policies.

maximize

$$\mathcal{U}(q,e) = -e + \Big(1-\pi(e)\Big)u\left(-qp\right) + \pi(e)u\Big(-\Delta + (1-p)q\Big) \tag{3}$$

must satisfy first-order conditions

$$\pi'(e)\Big(u\big(-\Delta + (1-p)q\big) - u(-qp)\Big) = 1 \tag{4}$$

and

$$\Big(1-\pi(e)\Big)u'\left(-qp\right)p = \pi(e)u'\big(-\Delta + (1-p)q\big)(1-p). \tag{5}$$

3 Equilibrium analysis

In competitive equilibrium:

(i) Effort $e^\dagger$ and insurance $q^\dagger$ maximize individual utility (3) at given p;

(ii) Insurance supply q^s by each of N insurers maximizes profits taking as given p and $\pi\left(\bar{e}\right)$, and N is such that profits are zero;

(iii) The insurance market clears ($q^s = q^\dagger = \bar{q}$) and equilibrium effort is individually optimal ($e^\dagger = \bar{e}$).

Points (i) and (iii) imply that equilibrium effort and insurance satisfy conditions (4) and (5), which by our assumptions are necessary for individual optimization and relate the equilibrium insurance premium to insurance and effort through a differentiable function $p\left(\bar{q},\bar{e}\right)$. These conditions are also sufficient if the first-order approach is valid, as we assume now, deferring to Section 5 the discussion of what may ensure that this is the case.

Points (ii) and (iii) imply $p\left(\bar{q},\bar{e}\right) = \pi(\bar{e})\gamma'\left(\bar{q}/N\right)$ as in (1). Hence, in equilibrium the effort first-order condition (4) reads

$$\pi'(\bar{e})\bigg(u\Big(-\Delta + \bar{q} - \pi(\bar{e})\gamma'\left(\bar{q}/N\right)\bar{q}\Big) - u\Big(-\pi(\bar{e})\gamma'\left(\bar{q}/N\right)\bar{q}\Big)\bigg) = 1, \tag{6}$$

and the insurance first-order condition (5) reads

$$\frac{u'\Big(-\Delta + \bar{q} - \pi(\bar{e})\gamma'\left(\bar{q}/N\right)\bar{q}\Big)}{u'\Big(-\pi(\bar{e})\gamma'\left(\bar{q}/N\right)\bar{q}\Big)} = \frac{\gamma'\left(\bar{q}/N\right)\Big(1-\pi(\bar{e})\Big)}{1-\pi(\bar{e})\gamma'\left(\bar{q}/N\right)}. \tag{7}$$

In the Appendix we show formally that, at equilibrium, insurance is positive but actuarially unfair, hence only partial:

Result 1 ***[COST]*** *implies that* $0 < \bar{q} < \Delta$ *in equilibrium.*

The zero-profit condition (2) implies that, if the negative shock is not realized,

$$c_h = -qp\left(\bar{e}, \bar{q}\right) = -\Big(\pi(\bar{e})\gamma\left(\bar{q}/N\right) + \phi\Big)N : \tag{8}$$

consumption is reduced by the total cost

$$k(\bar{q}, \bar{e}) \equiv \pi(\bar{e})\gamma\left(\bar{q}/N\right)N + \phi N = \bar{q}\pi(\bar{e})\gamma'\left(\bar{q}/N\right) \tag{9}$$

of insurance payoffs. We proceed to study how asymmetric information prevents this resource cost from being efficiently traded off against the benefits of consumption stability for the risk-averse representative individual.

Socially optimal insurance

Under the maintained assumption that effort choices are private, the planner cannot control them directly.[4] To characterize the additional inefficiency implied by hidden non-exclusive insurance, we consider the problem that would be solved by a planner who can control insurance. The planner knows that insurance determines individual effort through a function $e(q)$ that is differentiable under our assumptions and that effort satisfies (4), as long as first-order conditions are sufficient as well as necessary. Differentiating either side of the cost expressions in (9) and imposing (2) yields $\partial k(q, e)/\partial q = \pi(e)\gamma'\left(\frac{q}{N}\right)$, and (4) cancels the welfare effect of the variation of effort caused by changes of insurance. Hence, the problem

$$\max_{q}\; -e(q) + \Big(1 - \pi\Big(e(q)\Big)\Big)u\Big(-qp\left(q, e(q)\right)\Big) + \pi\Big(e(q)\Big)u\Big(-\Delta + q - qp\left(q, e(q)\right)\Big)$$

has the first-order condition

$$-\Big(1 - \pi\Big(e(q^*)\Big)\Big)u'\left(c_h^*\right)\frac{dc_h}{dq} = \pi\Big(e(q^*)\Big)u'\left(c_\ell^*\right)\left(1 + \frac{dc_h}{dq}\right), \tag{10}$$

where $c_h^* = -q^* p\left(q^*, e(q^*)\right)$ and $c_\ell^* = c_h^* - \Delta + q^*$ optimize ex-ante utility subject to unobservable effort.

[4] If this were possible, the planner would recognize that effort reduces the probability of the negative shock, and implies a smaller expected utility loss and smaller insurance production costs: differentiating the cost expressions on either side of (9) and imposing (2) yields $\partial k(q, e)/\partial e = \pi'(e)\gamma\left(\frac{q}{N}\right)N < 0$.

Equilibrium inefficiency

Competitive trade of non-exclusive or hidden insurance takes its price as given, but the social optimality condition takes into account the effects of insurance on the equilibrium price. Inserting

$$\frac{dc_h}{dq} = -p\Big(q, e(q)\Big) - q\frac{dp\Big(q, e(q)\Big)}{dq} \tag{11}$$

in (10), we see that the social efficiency condition differs from the individual first-order condition (5) whenever $dp\Big(q, e(q)\Big)/dq \neq 0$.

We can show that, as in Pauly's (1974) partial equilibrium and Helpman and Laffont's (1975) general equilibrium with actuarially fair contracts, insurance is too cheap in the economy we model:

Proposition 1 *The social cost of insurance differs from the competitive general equilibrium price by a positive wedge*

$$\bar{q}\frac{dp\Big(\bar{q}, e(\bar{q})\Big)}{d\bar{q}} = \pi'(\bar{e})\,\frac{d\bar{e}}{d\bar{q}}\,\gamma\left(\frac{\bar{q}}{N}\right)N > 0. \tag{12}$$

The proof, in the Appendix, uses both supply and demand relationships to show that, at the margin, more insurance would reduce effort in equilibrium. Because less effort increases the probability of negative shocks, it also increases the total variable cost of insurance services and the consumption cost (8). Marginal-cost pricing of insurance fails to account for the effects of insurance on effort and on insurance production costs, so the socially optimal ratio of marginal utilities is larger than that implied by price-taking individual optimization, and non-exclusive insurance is excessive.

In our model economy, the wedge (12) is the product of three terms. The first two are $\pi'(\bar{e})$, which determines the intensity of moral hazard, and $d\bar{e}/d\bar{q}$, which represents the negative effect of insurance on loss-prevention efforts. Both of these, and the wedge, would vanish in the absence of moral hazard, and the equilibrium would be efficient regardless of insurance costs. When $\pi'(\bar{e}) < 0$, conversely, insurance production costs interact with moral hazard in determining equilibrium inefficiency: excessive hidden insurance has a larger resource cost when **[COST]** implies that $\gamma\left(\bar{q}/N\right)N > \bar{q}$; i.e., that insurance absorbs more than claim payments. Production costs also influence equilibrium insurance through their effect on equilibrium prices which, as we discuss next, may be influenced by public policy.

4 Policy implications

Unlike atomistic agents, society takes into account the equilibrium effect of individual choices on the price of insurance. A social planner who does not observe individual effort and insurance positions cannot directly control individual consumption, but may improve efficiency with policies that influence individual behavior through observable aggregate variables.

In the model economy the identity of covered individuals is not observable, but the planner may observe the total amount of insurance produced by each firm, or transaction prices. Then, as in Helpman and Laffont (1975), the inefficiency identified by Proposition 1 can be targeted by indirect taxation. In reality, specific taxes are indeed charged for insurance (in the United Kingdom, for example, an Insurance Premium Tax is imposed on common non-life contracts at rates of 6% and 20%).

When effort and insurance are private information, non-concavities generally make it problematic to remove wedges between individual and social optimality with indirect mechanisms that, like taxes, distort the prices facing individual choice problems. In our setting, however, welfare-enhancing taxation is feasible because local concavity is preserved when insurance becomes more partial than required by the sufficient conditions discussed in Section 5 below.

If a unit tax τ is due on insurance contracts, individuals pay a premium

$$p = \pi(\bar{e})\gamma'\left(\bar{q}/N\right) + \tau,$$

where effort and insurance levels are those realized in the equilibrium with taxation, and their resource constraints include rebated tax revenues. We see that a Pigouvian tax amounting to the wedge (12) aligns the marginal utility ratio implied by individual optimization in (5) to that required by social efficiency in (10).

Besides this fully rebated unit tax, other policies can address the economy's inefficiency. If the number of firms N is observable, the planner can reduce it by adding a license fee to the fixed cost in the zero-profit condition (2). At given $\bar{q}$, a smaller N increases the marginal cost $\gamma'\left(\bar{q}/N\right)$ and the competitive insurance premium, so that in equilibrium there is less insurance and more effort. This improves welfare at the laissez faire allocation, where there is a positive wedge between the social and private cost of insurance, and excessive insurance implies inefficiently low effort. As long as the insurance industry remains competitive and license revenues are lump-sum rebated to the economy's representative individual, a smaller N continues to improve welfare until the constrained-efficient allocation is reached, and the equilibrium satisfies the social first-order condition.

In more complex models and in reality, these and other policies may entail additional distortions, such as rent-seeking activities or monopoly power. In our model economy, private insurance services expend resources, and similarly costly policy administration would make it impossible to fully rebate tax or license revenues. From this perspective it is interesting to consider briefly our model's implications for public insurance schemes. In general, private insurance tends to be crowded out by public transfers contingent on the verifiable events it covers. Because in our economy insurance costs influence equilibrium insurance and effort through average consumption levels and local risk aversion, however, public insurance is not fully crowded out in equilibrium if its social production cost differs from that of private insurance.

5 Validity of the first-order approach

For the first-order conditions (4) and (5) to be sufficient, the objective function (3) should be concave in its two arguments. This is the case if $\partial^2\mathcal{U}(q,e)/\partial q^2 \leq 0$ and $\partial^2\mathcal{U}(q,e)/\partial e^2 \leq 0$, i.e.

$$u''\left(c_\ell\right)\pi(e)(1-p)^2 + u''(c_h)\Big(1-\pi(e)\Big)p^2 \leq 0, \quad \pi''(e)\Big(u(c_\ell)-u(c_h)\Big) \leq 0, \tag{13}$$

and $\left(\partial^2\mathcal{U}(q,e)/\partial q^2\right)\left(\partial^2\mathcal{U}(q,e)/\partial e^2\right) - \left(\partial^2\mathcal{U}(q,e)/\partial q\partial e\right)^2 \geq 0$, i.e.

$$\frac{u''\left(c_\ell\right)\pi(e)(1-p)^2 + u''(c_h)\Big(1-\pi(e)\Big)p^2}{\pi'(e)\Big(\left(1-p\right)u'(c_\ell) + pu'(c_h)\Big)} \geq \frac{\pi'(e)\Big((1-p)u'(c_\ell)+pu'(c_h)\Big)}{\pi''(e)\Big(u(c_\ell)-u(c_h)\Big)}. \tag{14}$$

Our assumptions imply the inequalities in (13) for all $c_\ell \leq c_h$, but not that in (14): when effort determines the probability distribution of consumption, it is difficult to derive functional form restrictions that ensure concavity of expected utility (3) in both consumption and effort. Because (14) may fail to hold at a consumption and effort combination that satisfies the first-order conditions, those conditions need not identify a competitive equilibrium, which may fail to exist (Helpman and Laffont, 1975).

[Figure 1 here]

To illustrate how non-concavity of individual objective functions may make it inappropriate to rely on the first-order conditions, Figure 1 plots in q, e space the loci that satisfy the insurance and effort first-order conditions. The assumptions we made imply that both loci are continuous and decreasing, but do not prevent them from crossing more than once. In the figure, there is an intersection with low insurance and high effort, where the function is concave and the first-order conditions identify a maximum. The first-order conditions, however, also hold at an allocation with more insurance and less effort, where the objective function is not concave.

Adapting to our single-period economy with non-exclusive insurance an approach similar to that applied by Rogerson (1985) and Ábrahám, Koehne, and Pavoni (2011) to two-period moral-hazard economies with hidden non-contingent savings and exclusive insurance, in this section we show that costly production of insurance services can help ensure the existence of equilibrium for non-exclusive insurance in the presence of moral hazard.

Local concavity

It is convenient to work with a sufficient condition for local concavity:

Result 2 *Given* ***[PROB]****,*

$$\frac{p^2u''(c_h) - (1-p)^2u''(c_\ell)}{\Big(pu'(c_h) + (1-p)u'(c_\ell)\Big)^2}\Big(u(c_h) - u(c_\ell)\Big) \geq \frac{\pi'(e)^2}{\pi''(e)\pi(e)} \tag{15}$$

ensures local concavity of the objective function $\mathcal{U}(q,e)$ *in (3).*

The proof, in the Appendix, shows that (15) implies concavity of the expected utility loss $-\pi(e)\Big(u\left(c_h\right) - u\left(c_\ell\right)\Big)$, which is sufficient to ensure concavity of the welfare function.

Condition (15) is stronger than (14) because its lefthand side numerator omits a positive term, $-p^2u''(c_h)/\pi(e)$, that depends on the shape of both the utility and probability functions and is difficult to bound with interpretable functional form restrictions, as we proceed to do for (15).

The positive term on the righthand side of (15) only depends on the loss probability function, and is bounded below unity if we strengthen **[PROB]:**

[PROB'] Assume **[PROB]** and that $\pi(e)$ is log-convex (which entails $\pi''(e)\pi(e)\pi'(e)^{-2} \geq 1 \; \forall e \geq 0$).

This condition requires the negative shock's probability to decline at a non-increasing proportional rate as effort increases.[5]

[5]The $\pi(e) = \xi \exp(-\beta e^{\alpha})$ function used in drawing Figure 1 satisfies all the conditions in **[PROB']** when $\xi < 1$, $\beta > 0$, $0 < \alpha < 1$. This somewhat awkward functional form maps $x = e^{\alpha}$ into a weakly log-

The lefthand side of (15) only depends on utility terms and on the insurance premium. It is zero when $u(c_h) = u(c_\ell)$. Hence, with full insurance and $\pi'(e)^2 > 0$ the inequality cannot be satisfied. If insurance is only partial and risk aversion is decreasing, then $u''(c_h)p^2 - u''(c_\ell)(1-p)^2$ can be so strongly positive as to satisfy the inequality. Formal analysis of this insight is easier for hyperbolic absolute risk aversion utility,

[HARA] $u(c) = \zeta\left(\eta + \frac{y+c}{\sigma}\right)^{1-\sigma}$, with $(1-\sigma)\zeta > 0$.

For this functional form the lefthand side of the inequality depends only on $u'(c_\ell)/u'(c_h)$, and under decreasing absolute risk aversion it increases above unity as that ratio grows:[6] in the Appendix, we prove

Result 3 *Under [HARA] the lefthand side of (15) is a function*

$$g(\nu) \equiv \frac{\sigma}{\sigma-1}\frac{p^2(\nu)^{-\frac{1+\sigma}{\sigma}} - (1-p)^2}{\left(p\nu^{-1} + (1-p)\right)^2}\left((\nu)^{\frac{1-\sigma}{\sigma}} - 1\right), \tag{16}$$

with $\nu \equiv u'(c_\ell)/u'(c_h)$, *and*

(i) $g'(\nu) > 0$ *for* $\nu > 1$ *and* $g(\nu) > 0$ *if* $\sigma > 0$,

(ii) $g(\nu) < 1$ *for all* $\nu > 0$ *if* $\sigma = -1$ *or* $\sigma \to \infty$,

(iii) $\lim_{\nu\to\infty} g(\nu) > 1$ *if* $0 < \sigma < \infty$.

Concavity at stationary points

Under decreasing risk aversion, insurance can be so partial as to ensure that (15) is satisfied; moreover, in the economy we model insurance production costs can be so large as to imply that first-order conditions only hold where the objective function is locally concave.

convex negative exponential loss probability, $\xi\exp(-\beta x)$. The model's effort variable has linearly negative welfare effects, and $\alpha < 1 \Rightarrow \lim_{e\downarrow 0}\pi'(e) = -\infty$ guarantees an interior solution for e. An equivalent and perhaps more readily interpretable formalization would, as in the working paper version of this article, let the welfare cost of loss-prevention effort x be a convex function, such as $x^{1/\alpha}$, with zero marginal cost at $x = 0$.

[6] For HARA preferences, absolute risk aversion $-u''(c)/u'(c) = \left(\eta + \frac{y+c}{\sigma}\right)^{-1}$ is a decreasing function of c when $0 < \sigma < \infty$; it is constant when $\sigma \to \infty$ yields $u(c) = -\eta\exp((y+c)/\eta)$, and increasing if with $\sigma = -1$ utility is quadratic (see, e.g., Gollier, 2001, pp. 26–27).

At points that satisfy the first-order condition (5) for insurance, the argument of $g(\cdot)$ is

$$\frac{u'(c_\ell)}{u'(c_h)} = \frac{1-\pi(e)}{\pi(e)}\frac{p}{1-p} \equiv \nu(e,p). \tag{17}$$

Under partial insurance, $0 < \pi(e) < p < 1$, so $\nu(e,p)$ exceeds unity and, by Result 3, can be so large as to ensure local concavity wherever the first-order condition for insurance is satisfied. To account for the endogenously determined level of effort, note $\pi'(e) < 0$ implies $\nu(e,p)$ is increasing in e for given p (higher effort decreases the probability of loss, and lets the first-order condition for insurance be satisfied at a larger marginal utility ratio). This makes it possible to rely on the partial-insurance implications of insurance production costs and on interpretable functional restrictions, rather than on global concavity or quasi-concavity of the objective function, to ensure that the first-order conditions identify a unique solution:

Proposition 2 *Under **[PROB']**, **[HARA]**, if absolute risk aversion is decreasing then* p *can exceed* $\pi(0)$ *by a sufficiently large amount to ensure that*

$$g\left(\frac{1-\pi(0)}{\pi(0)}\frac{p}{1-p}\right) \geq 1, \tag{18}$$

and that the objective function (3) is concave at all allocations that satisfy interior first-order conditions.

The proof, in the Appendix, uses the first-order condition (17) and Result 2 to establish that if the insurance price p exceeds $\pi(0)$ by a sufficiently large amount, then the expected utility loss term $-\pi(e)\Big(u(c_h) - u(c_\ell)\Big)$ would be locally concave should $e = 0$ be the optimal effort choice. Because partial insurance makes it optimal to exert positive effort, at points where first-order conditions are satisfied the marginal utility ratio $\nu(e,p)$ is larger than it would be at $e = 0$, and the properties of $g(\cdot)$ in Result 3 imply that under decreasing risk aversion a larger $\nu(e,p)$ preserves local concavity. Hence, the first-order conditions can only hold at a point where the objective function is concave. There can be only one such point under Proposition 2's conditions: even though the objective function need not be concave where the first-order conditions are not satisfied, our assumptions rule out corner maxima and imply continuity of individual choices. Hence, multiple interior local optima could only exist if, as in Figure 1, the first-order conditions were also satisfied at non-concave local minima or saddle points.

When Proposition 2's assumptions are not satisfied, individual objective functions need not be concave. In Figure 1, $p = \pi(0)$, so full insurance and zero effort also satisfy both first-order conditions and, because the lefthand side of (18) is zero, the inequality in that condition is certainly violated. Even though p is actuarially unfair at positive effort levels (as implied in competitive equilibrium by a marginal cost of claim payment that exceeds unity), it is not so unfair as to guarantee concavity at all of the points where both first-order conditions are satisfied: for the parameterization used in plotting the figure, $g(\cdot)$ exceeds unity only for $q < 0.12$.

For this and any other parameterized example it is possible to establish validity of the first-order conditions by evaluating the exact concavity condition (14). But the interpretable conditions of Proposition 2 offer a comforting general insight: first-order conditions can plausibly hold when insurance is partial, as implied by processing costs, and risk aversion is decreasing.[7]

Equilibrium

To support an equilibrium with positive effort and partial non-exclusive insurance, demand for costly insurance (which depends on risk aversion and on the probability and size of the negative shock) should be so strong as to ensure that the equilibrium marginal cost $\gamma'(\bar{q}/N)$ of insurers exceeds unity by a sufficiently large margin. Proposition 2's condition evaluated at the competitive equilibrium price $p = \pi(\bar{e})\gamma'(\bar{q}/N)$,

$$g\left(\frac{1-\pi(0)}{\pi(0)}\frac{\pi(\bar{e})\gamma'(\bar{q}/N)}{1-\pi(\bar{e})\gamma'(\bar{q}/N)}\right) \geq 1, \tag{19}$$

ensures that first-order conditions are sufficient in equilibrium. Because with positive insurance equilibrium effort $\bar{e}$ is lower than the $\tilde{e}$ that satisfies (4) at zero insurance,

$$\pi(\tilde{e}) < \pi(\bar{e}) \;\Rightarrow\; \frac{\pi(\bar{e})\gamma'(\bar{q}/N)}{1-\pi(\bar{e})\gamma'(\bar{q}/N)} > \frac{\pi(\tilde{e})\gamma'(\bar{q}/N)}{1-\pi(\tilde{e})\gamma'(\bar{q}/N)},$$

[7] It is possible to specify conditions that are less stringent, but also less general and transparent than those of Proposition 2. For example, if absolute risk aversion is constant then $g(\cdot)$ is strictly less than unity, as shown in Result 3. But because $g(\nu) > 0$ when $\nu > 1$ and $p < 0.5$, the sufficient condition (15) can be satisfied if $\pi'(e)^2/(\pi''(e)\pi(e))$ is bounded by additional functional form restrictions: with $\pi(e) = \xi \exp\big(-\beta e^{\alpha}/(1+\mu e^{\alpha})\big)$, for example,

$$\pi'(e)^2/\big(\pi''(e)\pi(e)\big) = \beta\alpha e^{\alpha}/\big((1+\alpha)\,\mu^2 e^{2\alpha} + (2\mu+\alpha\beta)\,e^{\alpha} + 1 - \alpha\big)$$

can be much smaller than unity for all $e > 0$ when $\mu > 0$ is large.

and because $g'(\cdot) > 0$ by Result 3, the inequality in (19) is certainly satisfied when

$$g\left(\frac{1-\pi(0)}{\pi(0)}\frac{\pi(\tilde{e})\gamma'\left(\bar{q}/N\right)}{1-\pi(\tilde{e})\gamma'\left(\bar{q}/N\right)}\right) \geq 1.$$

Like (18), this sufficient condition is more stringent than what would be necessary to validate the first-order approach in a specific parameterized example, but usefully characterizes some relevant structural features. A small $\pi(0)$ helps ensure that price-taking individuals' choice problems are well defined as in Proposition 2, but validity of the first-order approach is more easily established in general equilibrium when $\pi(\tilde{e})$ is large. Hence, $\pi(e)$ should not be too strongly declining in its argument, i.e., the moral hazard problem should not be too severe.

Related literature

In our model, private actions determine a non-degenerate probability distribution for outcomes. Then, as in Pauly (1974) or Ábrahám, Koehne, and Pavoni (2011), a non-exclusive market for trade in securities contingent on idiosyncratic realizations is active unless it is ruled out by assuming exclusive insurance. If instead income realizations were fully determined by effort choices based on privately observed ability, there would be no trade in non-exclusive contingent securities (Golosov and Tsyvinski 2007, appendix A).

In that setting, Ales and Maziero (2009) suppose that exclusive insurance entails a per-individual fixed cost, so that sufficiently rich individuals are perfectly insured by exclusive contracts and the rest of the population is completely uninsured. Our model's cost functions instead imply that partial insurance is available to all of the economy's ex-ante identical individuals.

The characterization results and policy implications of the previous sections are related to those of models that use first-order conditions to characterize interactions between exclusive insurance and public policies (e.g. Golosov and Tsyvinsky, 2007; Chetty and Saez, 2010). The results of this section show that even though that approach may be problematic when there is competitive non-exclusive asset trade under asymmetric information, insurance production costs can provide a structural interpretation for the asset position limits or bid-ask spreads that, in Bisin and Gottardi's (1999) pure-exchange economy, imply partial insurance and can ensure equilibrium existence.

The sufficient conditions we derive are for a one-period economy with two possible realizations of uncertainty. Establishing validity of the first-order approach in economies with multiple states and hidden assets is harder, as shown in Bertola and Koeniger (2014). It might be possible to do so with different methods, perhaps adapting to problems with hidden assets the direct approach of Jewitt (1988) rather than that of Rogerson (1985). Otherwise it is necessary to characterize non-concave economies where equilibria may be multiple, or fail to exist. Bisin and Guaitoli (2004), building on Hellwig (1983a), study an exchange economy with two effort levels where non-concavity may let individuals be indifferent between exerting positive effort (and insuring the loss partially) or setting the effort at the lowest possible level (and overinsuring).[8] Partial insurance implies that profits are positive in the absence of production costs, so the positive-effort equilibrium has to be supported by the latent possibility that customers would discontinuously shift to low effort and excessive insurance if any insurer charged a lower premium; such "non-price" equilibria are not particularly plausible (Hellwig, 1983b, p. 4).

6 Conclusion

Insurance is partial for at least two reasons: to limit moral hazard by writing contracts that do not fully cover losses, and because it is costly to write contracts and process claims. Studying interactions between these two reasons in a stylized model of the costs entailed by the verification or delivery of insurance claims, this article has obtained two types of results. At a substantive level, we have shown that, much as one might wish for better coverage, when both effort and insurance are unobservable insurance should be made even more expensive and less complete than is implied by production costs. At a methodological level, we have shown that insurance production costs are technically convenient because partial insurance makes it easier to establish the existence of competitive equilibrium.

In our model economy the information problems that make it difficult to enforce exclusive insurance interact with technological features of insurance production. In reality, both information and technology vary across markets. For specific damages, processing costs vary little with the size of claims, and the resulting contract-level economies of scale push insurance markets towards essentially exclusive arrangements. Increasing returns to scale at the firm level would also favor large, imperfectly competitive companies, which might well supply exclusive insurance that would not be excessive, and might in fact need to be subsidized or nationalized

[8] Loss and Piaser (2013) consider a similar model with a continuous effort choice.

to prevent monopolistic distortions. Our stylized model abstracts from these possibilities to focus on non-exclusive insurance problems, which can be studied using the standard tools of competitive-industry analysis under the assumption that production of insurance services has decreasing returns at the firm level. Comparing the welfare implications for insurance provision across different markets characterized by different technologies and information problems is an interesting direction for future research.

Appendix

Proof of Result 1

As $c_\ell = c_h - \Delta + \bar{q}$ and utility is concave, for any c_h the lefthand side of (7) is a monotonically decreasing function of $\bar{q}$ that ranges from $u'(c_h - \Delta)/u'(c_h) > 1$ at $\bar{q} = 0$, to unity at $\bar{q} = \Delta$. The righthand side of (7) is equal to one at $\bar{q} = 0$ because $\gamma'(0) = 1$ by **[COST]**. It increases in $\bar{q} > 0$, diverging to infinity as $p(\cdot) = \pi(e)\gamma'(q/N) \to 1$, with slope

$$\frac{d}{d(q/N)}\left[\gamma'(q/N)\frac{1-\pi(e)}{1-\pi(e)\gamma'(q/N)}\right] = \gamma''(q/N)\frac{1-\pi(e)}{\Big(1-\pi(e)\gamma'(q/N)\Big)^2} > 0$$

where the inequality follows for any q and e from $\gamma''(x) > 0$ for $x \geq 0$ by **[COST]**. Hence, equality in (7) is obtained for a unique level of equilibrium insurance $\bar{q} \in (0, \Delta)$.

Proof of Proposition 1

In equilibrium, $\bar{q}p(\bar{q}, \bar{e}) = k(\bar{q}, \bar{e})$. Imposing the zero-profit condition (2), differentiation of (9) yields

$$\frac{\partial}{\partial e}k(\bar{q}, \bar{e}) = \pi'(\bar{e})\gamma(\bar{q}/N)N < 0,$$

hence

$$\bar{q}\frac{dp(\bar{q}, \bar{e})}{d\bar{q}} = \frac{\partial}{\partial e}k(\bar{q}, \bar{e})\frac{d\bar{e}}{d\bar{q}} = \pi'(\bar{e})\gamma(\bar{q}/N)N\frac{d\bar{e}}{d\bar{q}},$$

where we again impose (2) to obtain the first equality. As $\pi'(e) < 0$ by **[PROB]** and $\gamma(\bar{q}/N)N > 0$, the Proposition's inequality is proved if $d\bar{e}/d\bar{q} < 0$. Note that individual effort's reaction to changes of $\bar{q}$ maintains equality in the first-order condition (4) and totally differentiate (4) to obtain

$$\frac{d\bar{e}}{d\bar{q}} = -\frac{\pi'(\bar{e})\partial\left[u\big(-\Delta+\bar{q}-k(\bar{q},\bar{e})\big) - u\big(-k(\bar{q},\bar{e})\big)\right]/\partial q}{\pi''(\bar{e})\Big(u\big(-\Delta+\bar{q}-k(\bar{q},\bar{e})\big) - u\big(-k(\bar{q},\bar{e})\big)\Big) + \pi'(\bar{e})\partial\left[u\big(-\Delta+\bar{q}-k(\bar{q},\bar{e})\big) - u\big(-k(\bar{q},\bar{e})\big)\right]/\partial e}.$$

In the numerator, $-\pi'(\bar{e}) > 0$ by **[PROB]**, and

$$\begin{aligned}&\frac{\partial\left[u\big(-\Delta+\bar{q}-k(\bar{q},\bar{e})\big) - u\big(-k(\bar{q},\bar{e})\big)\right]}{\partial q} = \\ = \quad & u'\big(-\Delta+\bar{q}-k(\bar{q},\bar{e})\big)\Big(1-\pi(\bar{e})\gamma'(\bar{q}/N)\Big) + u'\big((-k(\bar{q},\bar{e})\big)\pi(\bar{e})\gamma'(\bar{q}/N) > 0\end{aligned}$$

because $\pi(\bar{e})\gamma'(\bar{q}/N)p < 1$. The denominator is negative because $\pi''(\bar{e})\Big(u(c_\ell) - u(c_h)\Big) < 0$

by the second-order condition for individual effort choice, $\pi'(\bar{e}) < 0$ by **[PROB]**, and

$$\frac{\partial\left[u(c_\ell) - u(c_h)\right]}{\partial e} = \Big(u'(c_\ell) - u'(c_h)\Big)\left(-\frac{\partial}{\partial e}k(\bar{q},\bar{e})\right) > 0$$

because $u'(c_\ell) > u'(c_h)$ when $c_\ell < c_h$ and $\frac{\partial}{\partial e}k(\bar{q},\bar{e}) < 0$.

Proof of Result 2

Write $\mathcal{U}(q,e) = -e + u\left(c_h\right) - \pi(e)\Big(u\Big(c_h\Big) - u\left(c_\ell\right)\Big)$. The term $-e + u\left(c_h\right)$ is linear in e and strictly concave in q because $\partial^2 u\left(c_h\right)/\partial q^2 = u''\left(c_h\right)p^2 < 0$. We only need to prove that the conditions imply that $-\pi(e)\Big(u\left(c_h\right) - u\left(c_\ell\right)\Big)$ is concave or, equivalently, $\pi(e)\Big(u\left(c_h\right) - u\left(c_\ell\right)\Big) = \pi(e)\Big(u(-qp) - u(-\Delta + (1-p)q)\Big)$ is convex in q and e. In (15), the righthand side is positive by **[PROB]**, and with $u\left(c_h\right) - u\left(c_\ell\right) > 0$ the inequality implies $u''(c_h)p^2 - u''(c_\ell)(1-p)^2 > 0$. Hence, both diagonal terms of the Hessian

$$H = \begin{bmatrix} \pi(e)\Big(u''(c_h)p^2 - u''(c_\ell)(1-p)^2\Big) & \pi'(e)\Big(-u'(c_h)p - u'(c_\ell)(1-p)\Big) \\ \pi'(e)\Big(-u'(c_h)p - u'(c_\ell)(1-p)\Big) & \pi''(e)\Big(u\left(c_h\right) - u\left(c_\ell\right)\Big) \end{bmatrix},$$

are positive. The determinant

$$|H| = \pi(e)\Big(u''(c_h)p^2 - u''(c_\ell)(1-p)^2\Big)\pi''(e)\Big(u\left(c_h\right) - u\left(c_\ell\right)\Big) - \Big(\pi'(e)\Big(-u'(c_h)p - u'(c_\ell)(1-p)\Big)\Big)^2$$

is also positive if (15) holds.

Proof of Result 3

Rearrange the lefthand side of (15) to

$$\frac{p^2u''(c_h) - (1-p)^2u''(c_\ell)}{\Big(pu'(c_h) + (1-p)u'(c_\ell)\Big)^2}\Big(u(c_h) - u(c_\ell)\Big) = \frac{u''(c_\ell)u(c_\ell)}{u'(c_\ell)^2}\,\frac{p^2\frac{u''(c_h)}{u''(c_\ell)} - (1-p)^2}{\Big(p\frac{u'(c_h)}{u'(c_\ell)} + (1-p)\Big)^2}\left(\frac{u(c_h)}{u(c_\ell)} - 1\right). \quad (20)$$

For $\sigma \neq 1$, **[HARA]** implies

$$\begin{aligned}
\frac{u(c_h)}{u(c_\ell)} &= \left(\frac{\left(\eta + \frac{y+c_h}{\sigma}\right)^{-\sigma}}{\left(\eta + \frac{y+c_\ell}{\sigma}\right)^{-\sigma}}\right)^{-\frac{1-\sigma}{\sigma}} = \left(\frac{u'(c_\ell)}{u'(c_h)}\right)^{\frac{1-\sigma}{\sigma}}, \\
\frac{u''(c_h)}{u''(c_\ell)} &= \left(\frac{u'(c_\ell)}{u'(c_h)}\right)^{-\frac{1+\sigma}{\sigma}}, \\
\frac{u''(c)u(c)}{u'(c)^2} &= \frac{-\zeta\frac{1-\sigma}{\sigma}\left(\eta + \frac{y+c}{\sigma}\right)^{-\sigma-1}\zeta\left(\eta + \frac{y+c}{\sigma}\right)^{1-\sigma}}{\left(\zeta\frac{1-\sigma}{\sigma}\left(\eta + \frac{y+c}{\sigma}\right)^{-\sigma}\right)^2} = \frac{\sigma}{\sigma - 1}.
\end{aligned}$$

Inserting these expressions in (20) and denoting $\nu \equiv u'(c_\ell)/u'(c_h)$ yields (16). For $\sigma = 1$ utility is logarithmic, $u''(c_\ell)u(c_\ell)/u'(c_\ell)^2 = -\ln c_\ell$, and

$$g(\nu) = -\frac{p^2\nu^{-2} - (1-p)^2}{\left(p\nu^{-1} + (1-p)\right)^2} \ln \nu$$

has the same properties as for other positive and finite values of σ.

(i) Write $g(\nu) = \hat{g}(\nu)\frac{\sigma}{\sigma-1}\left(\nu^{\frac{1-\sigma}{\sigma}} - 1\right)$ for $\hat{g}(\nu) = \frac{p^2\nu^{-\frac{1+\sigma}{\sigma}} - (1-p)^2}{(p\nu^{-1}+(1-p))^2}$.
With $\hat{g}'(\nu) = \hat{g}(\nu)\left(-\frac{1+\sigma}{\sigma}\left(\frac{p^2\nu^{-\frac{1}{\sigma}-2}}{p^2\nu^{-\frac{1+\sigma}{\sigma}} - (1-p)^2}\right) + 2\frac{p\nu^{-2}}{(p\nu^{-1}+(1-p))}\right)$,

$$\begin{aligned} g'(\nu) &= \hat{g}'(\nu)\frac{\sigma}{\sigma-1}\left(\nu^{\frac{1-\sigma}{\sigma}} - 1\right) - \hat{g}(\nu)\nu^{\frac{1-\sigma}{\sigma}-1} \\ &= g(\nu)\left(-\frac{1+\sigma}{\sigma}\frac{p^2\nu^{-\left(\frac{1}{\sigma}+2\right)}}{p^2\nu^{-\frac{1+\sigma}{\sigma}} - (1-p)^2} + 2\frac{p\nu^{-2}}{\left(p\nu^{-1} + (1-p)\right)} + \frac{1-\sigma}{\sigma}\frac{\nu^{\frac{1-\sigma}{\sigma}-1}}{\nu^{\frac{1-\sigma}{\sigma}} - 1}\right) \end{aligned}$$

is positive when

$$(\nu > 1, \sigma > 0) \Rightarrow \frac{\sigma}{1-\sigma}\left(\nu^{\frac{1-\sigma}{\sigma}} - 1\right) > 0$$

so that

$$(g(\nu) > 0) \Rightarrow p^2\nu^{-\frac{1+\sigma}{\sigma}} - (1-p)^2 < 0.$$

(ii) For $\sigma = -1$, $g(\nu) = \frac{1}{2}(2p-1)\left(\nu^{-2} - 1\right)\left(\frac{p}{\nu} + (1-p)\right)^{-2}$ has a global maximum of 0.5 at $\nu = (p-1)/p < 0$ and, like

$$\lim_{\sigma\to\infty} g(\nu) = \frac{p^2\nu^{-1} - (1-p)^2}{\left(p\nu^{-1} + (1-p)\right)^2}\left(\nu^{-1} - 1\right) = 1 - \frac{1}{\left(p\nu^{-1} + (1-p)\right)^2\nu} < 1\ \forall \nu > 0,$$

never exceeds unity.

(iii) For $0 < \sigma \leq 1$, $\lim_{\nu\to\infty} g(\nu) = \infty$; for $1 < \sigma < \infty$, $\lim_{\nu\to\infty} g(\nu) = \frac{\sigma}{\sigma-1} > 1$.

Proof of Proposition 2

The righthand side of inequality (15) is bounded below unity by **[PROB']** for all e. Its lefthand side is bounded above unity at $e = 0$ by the first-order condition for insurance choice (17) if (18) holds, where the function $g(\nu)$ is that defined in (16). By Result 3(iii), condition (18) can be satisfied for a finite value of the $g(\cdot)$ function's argument if $0 < \sigma < \infty$ (decreasing risk aversion). For $\sigma > 0$ we know from Result 3(i) that $g'(\cdot) > 0$ when $g(\nu) > 0$ and $\nu > 1$: because the righthand side of (17) increases with e for $\pi'(e) < 0$ as in **[PROB']**,

$$g\left(\frac{1-\pi(e)}{\pi(e)}\frac{p}{1-p}\right) > g\left(\frac{1-\pi(0)}{\pi(0)}\frac{p}{1-p}\right) \geq 1 \text{ for all } e > 0: \quad (21)$$

the inequality in (15) is satisfied, and ensures local concavity by Result 2, at all points where (17) holds. This is sufficient to ensure local concavity at all such points.

References

Ábrahám, Á., Koehne, S. and Pavoni, N. "On the First-Order Approach in Principal-Agent Models with Hidden Borrowing and Lending." *Journal of Economic Theory*, Vol. 146 (2011), pp. 1331–1361.

Ales, L. and Maziero, P. "Non-Exclusive Dynamic Contracts, Competition, and the Limits of Insurance." Tepper School of Business Working Paper, Carnegie Mellon University, 2009.

Bertola, G. and Koeniger, W. "On the Validity of the First-Order Approach with Moral Hazard and Hidden Assets." *Economics Letters*, Vol. 124 (2014), pp. 402–405.

Bisin, A. and Gottardi, P. "Competitive Equilibria with Asymmetric Information." *Journal of Economic Theory*, Vol. 87 (1999), pp. 1–48.

Bisin, A. and Guaitoli, D. "Moral Hazard and Nonexclusive Contracts." *RAND Journal of Economics*, Vol. 35 (2004), pp. 306–328.

Bizer, D. and DeMarzo, P. "Sequential Banking." *Journal of Political Economy*, Vol. 100 (1992), pp. 41–61.

Chetty, R. and Saez, E. "Optimal Taxation and Social Insurance with Endogenous Private Insurance." *American Economic Journal: Economic Policy*, Vol. 2 (2010), pp. 85–114.

Gollier, C. *The Economics of Risk and Time*, Cambridge (MA): MIT Press, 2001.

Golosov, M. and Tsyvinski, A. "Optimal Taxation with Endogenous Insurance Markets." *Quarterly Journal of Economics*, Vol. 122 (2007), pp. 487–534.

Greenwald, B. and Stiglitz, J. "Externalities in Economies with Imperfect Information and Incomplete Markets." *Quarterly Journal of Economics*, Vol. 101 (1986), pp. 229–264.

Hellwig, M. "Moral Hazard and Monopolistically Competitive Insurance Markets." *Geneva Papers on Risk and Insurance*, Vol. 8 (1983a), pp. 44–71.

Hellwig, M. "On Moral Hazard and Non-Price Equilibria in Competitive Insurance Markets." SFB Discussion Paper No. 109, Department of Economics, University of Bonn, 1983b.

Helpman, E. and Laffont, J. "On Moral Hazard in General Equilibrium Theory." *Journal of Economic Theory*, Vol. 10 (1975), pp. 8–23.

Jewitt, I. "Justifying the First-Order Approach to Principal-Agent Problems." *Econometrica*, Vol. 56 (1988), pp. 1177–1190.

Loss, F. and Piaser, G. "Linear Prices Equilibria in a Non-Exclusive Insurance Market." Discussion Paper no. 2013-20, Ecole Polytechnique, Paris, 2013.

Pauly, M. "Overinsurance and Public Provision of Insurance: The Roles of Moral Hazard and Adverse Selection." *Quarterly Journal of Economics*, Vol. 88 (1974), pp. 44–62.

Rogerson, W. "The First-Order Approach to Principal-Agent Problems." *Econometrica*, Vol. 53 (1985), pp. 1357–1367.

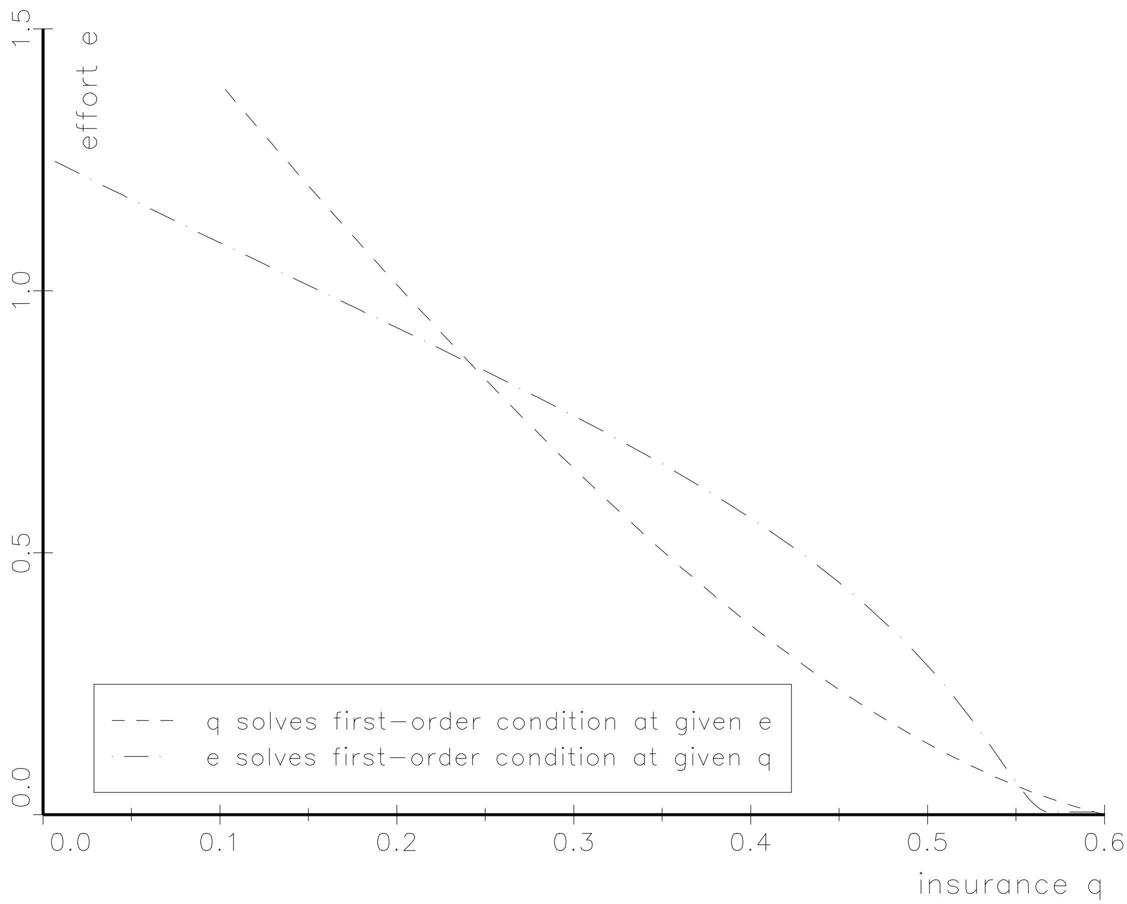

Figure 1: How non-concavity can invalidate the first-order approach. Functional forms $u(c) = ((1+c)^{1-\sigma} - 1)/(1-\sigma)$, $\pi(e) = \xi \exp(-\beta e^{\alpha})$; parameter values $\sigma = 5$, $\xi = 0.7$, $\beta = 2$, $\alpha = 0.9$, $\Delta = 0.6$; insurance price $p = \pi(0) = 0.7$.