MERLO: A NEW TOOL AND A NEW CHALLENGE IN MATHEMATICS TEACHING AND LEARNING

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The core element of this paper is an innovative tool for teaching and learning, based on equivalence of meaning across different kinds of representation and called MERLO (Meaning Equivalence Reusable Learning Objects). After presenting a general MERLO approach, we focus on its application in mathematics education and, in particular, in the Italian institutional context of secondary school. In this paper we describe a first level of experimentation regarding the use of MERLO in teachers' education, as part of an ongoing research. The lens of Meta-Didactical Transposition allows an analysis of the process of teachers' professional development with MERLO.

INTRODUCTION

MERLO (Meaning Equivalence Reusable Learning Objects) is a didactical and methodological tool developed and tested since the 1990s by Uri Shafrir and Masha Etkind at Ontario Institute for Studies in Education (OISE) of University of Toronto, and Ryerson University in Toronto, Canada (Shafrir & Etkind, 2010). It is a very adaptable tool, suitable for several subjects and uses: we can mention, for example, the use of MERLO that Masha Etkind is doing in architecture as an assessment tool to check students' deep understanding of concepts.

The aim of our research is the application of MERLO in mathematics education, linking the MERLO approach with some elements of the Italian institutional context. For this reason, the choice of the research context is that of a Master, held in the University of Turin, for in-service mathematics teachers who will become educators for other teachers.

In the first part of this paper we present the theoretical framework for MERLO approach. Then we focus on our experience concerning the use of MERLO in teachers' education, inside the Master context. A brief description of the Meta-Didactical Transposition model allows us to use it as lens for the analysis of the process of teachers' professional development with MERLO. The paper ends with a final discussion and some proposals for further research that we intend to develop.

MERLO APPROACH: THEORETICAL FRAMEWORK

MERLO (Arzarello, Kenett, Robutti, Shafrir, to be submitted; Etkind, Kenett, Shafrir, 2010) is a database, that is a sorted and organized collection of MERLO activities covering relevant concepts within a discipline, through multi-semiotic representations in multiple sign systems. Each element of the database is a structured MERLO activity, that includes one target statement TS (it encodes different features of an important concept) and four other statements, linked to the target statement by two sorting criteria: shared or not shared *meaning equivalence* with the target statement, shared or not shared *surface similarity* with the target statement.

The term *meaning equivalence* designates a commonality of meaning across several representations. The term *surface similarity* means that representations "look similar": they are similar only in appearance, sharing the same sign system, but not the meaning.

Based on these two sorting criteria it is possible to create four types of statements, called Q1, Q2, Q3, Q4, depending on the fact that they share or not share equivalence of meaning and/or surface similarity with the target statement. The four types of statements are shown in the figure below.

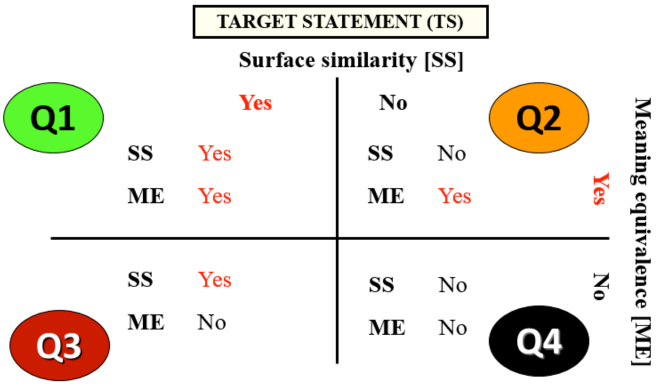


Figure 1: Types of statements, linked to a TS in a MERLO activity

A typical MERLO activity contains five statements: a target statement plus four additional statements of type Q2, Q3 and Q4; they can be in a variable number, provided that at least one Q2 statement is present, in addition to the TS. We avoid the inclusion of Q1 statements, because they make the activity too easy, for their equivalence both in appearance and in meaning with TS.

In the version of MERLO activity for students, obviously, the type of each statement is not revealed. The students are required to recognize the statements in multiple representations that share equivalence of meaning and to write the concept that they had in mind when making the decisions. In this way MERLO activity combines multiple-choice (recognition) and short answer (production). It gives a feedback on students with two main scores: recognition score and production score. This feedback is useful to the teacher for getting information about the level of understanding (the so called "deep understanding") of their students on a particular conceptual knowledge.

Regular use of MERLO activities enhances a particular way of thinking, named "conceptual thinking" (Etkind & Shafrir, 2013), requiring that learners explore patterns of equivalence-of-meaning among ideas, relations and underlying issues.

Learners' attention is focused on conceptual contents and on meanings, through comparative analysis of multi-semiotic representations of conceptual situations.

MERLO approach to parsing and analysing concepts is applicable to various subjects for recognizing, representing, organizing, exploring and manipulating knowledge. It is particularly recommended in mathematics, where the ability to shift from one to another representation of the same object and the coordination of multiple representations in more than one semiotic register are fundamental competences, in order to access the underlying meaning and to understand mathematics (Duval, 2006). MERLO approach is in line with national (INVALSI, in Italy) and international (PISA, TIMSS) assessment tests, where the ability of shifting between representations is abundantly evaluated. Moreover, it is also a didactical tool for avoiding or overcoming the so called "duplication obstacle" (Duval, 1983). This kind of obstacle leads students to the consideration of two representations of the same mathematical object as two different mathematical objects, but also, conversely, it may represent students' inability in grasping two different meanings of a mathematical object in only one representation. The duplication obstacle described by Duval is a real source of difficulty in mathematical learning (Fishbein, 1987) and it may be the cause of failure in mathematics at school (examples in Ascari, 2012).

RESEARCH CONTEXT: MASTER FOR IN-SERVICE TEACHERS

The Master for Mathematics Teachers' Educators is an educational program of two years, held in the University of Turin to the Department of Mathematics and directed to Italian in-service secondary school teachers (30 enrolled teachers), who will become teachers' educators.

We think that the choice of this research context is the most appropriate for the application of MERLO in mathematics education and in teaching and learning in Italian secondary school: a fundamental aspect is teachers' education. For this reason, a training process of two years was implemented inside the Master context, in order to make teachers aware of this new didactical tool.

During the first year (2013/2014) teachers were involved in the following training phases:

phase 1. Translation of MERLO activities, produced in other countries (Russia) and solution of them.

phase 2. Construction of new MERLO activities in geometry, inside Italian school context and curriculum.

phase 3. Solution of MERLO activities produced at phase 2.

The experimentation inside the Master context is going on during the second year (2014/2015) with a small group of seven voluntary teachers as teachers-researchers, and with the whole group as learners and practitioners. Teachers in this phase are involved in the design of new MERLO activities linked to INVALSI tests and m@t.abel activities.

INVALSI is a National Institute for Evaluation of Instruction and Education System in Italy and every year it addresses INVALSI tests to monitor the level of Italian students' learning and to compare it with other European realities.

m@t.abel is a national project, started in 2006 and promoted by the Ministry of Education, Universities and Research, the National Agency for School Development and Autonomy (ANSAS - INDIRE), the National Associations for Mathematics and Statistics (UMI - SIS). It points at the renovation and improvement of mathematics teaching and learning and it is aimed at mathematics teachers in secondary school levels. The contents are related to four basic Standards that are part of the curricula of many countries around the world, as well as in OECD-PISA and INVALSI tests: numbers, geometry, relations and functions, data and forecasts.

The choice to link MERLO activities with the national assessment INVALSI and the national project m@t.abel is not accidental, but has the aim to root MERLO approach in the Italian institutional dimension. The institutional dimension is important because the teachers' professional development is contextualised inside and constrained by the institutions, such as national curriculum, the Ministry of Education, policy makers, textbooks, national assessment and so on.

Teachers involved in the experimentation, starting from some INVALSI tests of the previous years and from some m@t.abel activities, produced several MERLO activities. Here we present two examples from INVALSI questions.

The first example (figure 2) is designed for lower secondary school and it is about numbers: aim of the activity is to test students' comprehension of the concept of fraction. With this objective in mind, teachers' design process started from the choice of a graphical representation of a fraction, made of small squares. Using it as target statement, teachers created four other statements: two Q2 statements that share equivalence of meaning with TS but do not share surface similarity, because they are represented in a different semiotic system; a Q3 statement, linked with TS by surface similarity, sharing the same semiotic system, but not the meaning; a Q4 statement that does not share neither equivalence of meaning, nor surface similarity with TS.

The second example (figure 3) is designed for upper secondary school and requires recognition of relations and functions in different semiotic systems. This MERLO activity is linked to a real life context and concerns two offers for ski-lifts. Teachers' design process started from a natural language description of the two offers, chosen as target statement. Then teachers created four other statements: three Q2 statements that share equivalence of meaning, but do not share surface similarity with TS, representing the same two offers in a different way (Cartesian graph, table and formal language); and a Q4 statement that does not share neither equivalence of meaning, nor surface similarity with TS.

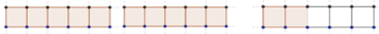
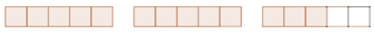
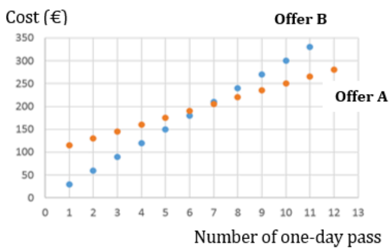
<table>
<tr><td></td><td>TS</td><td>Q2</td></tr>
<tr><td>At least two out of these five statements – but possibly more than two – share equivalence-of-meaning.
1. Mark all statements – but only those – that share equivalence-of-meaning
2. Write down briefly the reasons that guided you in making these decisions</td><td>A[ ]
</td><td>B[ ]
$\frac{12}{5}$</td></tr>
<tr><td>Q2</td><td>Q3</td><td>Q4</td></tr>
<tr><td>C[ ]
$2 + \frac{2}{5}$</td><td>D[ ]
</td><td>E [ ]
$\frac{12+3}{5+3}$</td></tr>
</table>

Figure 2: first example of MERLO activity

<table>
<tr><td colspan="3">Mario is going on holiday at a ski area. To take advantage of ski lifts (chair lifts, cable cars, ...), he can choose between two offers, A and B, both valid for the entire winter season.</td></tr>
<tr><td></td><td>TS</td><td>Q2</td></tr>
<tr><td>At least two out of these five statements – but possibly more than two – share equivalence-of-meaning.
1. Mark all statements – but only those – that share equivalence-of-meaning
2. Write down briefly the reasons that guided you in making these decisions</td><td>A[ ]
Offer A: initial fixed cost 100 € plus 15 € per day (for every day you use ski lifts).

Offer B: 30 € per day, with no initial cost.</td><td>B[ ]

</td></tr>
<tr><td>Q2</td><td>Q2</td><td>Q4</td></tr>
<tr><td>C[ ]
<table>
<tr><td>Number of days in which you use ski lifts</td><td>Offer A (cost in €)</td><td>Offer B (cost in €)</td></tr>
<tr><td>1</td><td>115</td><td>30</td></tr>
<tr><td>2</td><td>130</td><td>60</td></tr>
<tr><td>3</td><td>145</td><td>90</td></tr>
<tr><td>4</td><td>160</td><td>120</td></tr>
<tr><td>5</td><td>175</td><td>150</td></tr>
<tr><td>6</td><td>190</td><td>180</td></tr>
</table></td><td>D[ ]
Offer A: $c = 100 + 15g$

Offer B: $c = 30g$

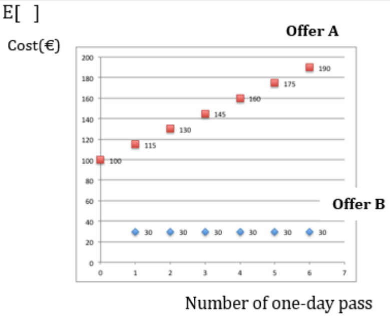
g number of one-day pass
c cost (€)</td><td>E[ ]

</td></tr>
</table>

Figure 3: second example of MERLO activity

The task for students is to recognize the equivalence of meaning across several kinds of representation of the same mathematical object (fraction in the first example, function in the second example). Students have to mark in the first example the three statements in position A, B, C (in the case shown in figure 2) and in the second example the four statements in position A, B, C, D (in the case shown in figure 3) because they share equivalence of meaning. We remind that the position of statements is changeable and their type (Q2, Q3 or Q4) is not revealed to students.

A further request is to explain the reasons for the choice, request that promotes argumentative expertise. The feedback received by teachers is not only that of a closed-answer test, but also an argumentative open-answer. Teachers can decide to use MERLO activities as a tool for final assessment or for formative assessment, in order to support a mathematical discussion in class. MERLO activities are based on the ability to read and interpret several kinds of representation, to see the same mathematical object represented in different sign systems and to be able to recognize it, even if there is a surface similarity but not equivalence of meaning with another object. We think all these aspects are fundamental in mathematics teaching and learning.

META-DIDACTICAL TRANSPOSITION AS LENS FOR ANALYSIS

The Meta-Didactical Transposition model (Arzarello et al., 2014; Clark-Wilson et al., 2014; Aldon et al., 2013) is useful and appropriate as a lens for the analysis of teachers' professional development process, in the research context just described: indeed, this theoretical model has been conceived to take into account the complexity arising from the intertwining of the processes involved during a teacher education program.

The theoretical background for the Meta-Didactical Transposition model is derived from Chevallard's Anthropological Theory of Didactics (Chevallard, 1985, 1992). In particular the model refers to the notions of didactical transposition and praxeology. This is the starting point for an expansion, which focuses mainly on "meta" aspects.

Through the Meta-Didactical Transposition model we can analyse teachers' professional development from a dynamic point of view, highlighting the interactions between the two communities involved in the teachers' education process (the community of researchers and the community of teachers) and observing their initial praxeologies and how they evolve over time, giving birth to new shared praxeologies.

During collaboration between the two communities some components that are internal for researchers become internal also for teachers, like the MERLO theoretical framework. However, there is not only a shift of theoretical knowledge from researchers to teachers, because each community adds something new, with the aim of arriving at a new praxeology shared by both of them, which we could call "MERLO pedagogy". At the moment of our research and experimentation, the shared praxeology is related to the task design of MERLO activities and to their possible use with students. Teachers, during meetings and working together with researchers, arrived to some methodological choices, necessary to have coherence in the design of new MERLO activities. We can mention the choice that all statements must be correct, that is a particular choice in respect to other traditional tests. The sharing of methodological choices in the design process led to the creation of MERLO activities by teachers, such as those presented in the previous section.

FINAL DISCUSSION AND PROPOSALS FOR FURTHER RESEARCH

MERLO activities are rooted on the construct of Meaning Equivalence, that is equivalence of meaning through different kinds of representation: the task for students is to recognize commonality of meaning in several sign systems. The experience with teachers in the research context of the Master highlighted the complexity of Meaning Equivalence construct and the delicacy of some choices during the design process of MERLO activities, because the kind of knowledge that will be tested on students depends from these choices.

The analysis of the examples produced by teachers, the discussions among them during meetings and the next reflection of researchers, led to introduce a sort of "empirical distance" between statements and concepts. For example, if it is simple to recognize a Q2 statement as equivalent with the target statement TS, it is more difficult recognizing what it means that a statement is "closer" to TS than another one, even if both are equivalent to TS.

We think that this sort of "distance" is fundamental in MERLO and so the idea for future research is to explore it in empirical way. In this regard, a possible task is to construct a Boundary of Meaning Map, that is a map where statements have to be placed inside or outside certain boundaries, associated with the meaning of a particular target statement TS, and into three different levels more or less close to the boundary.

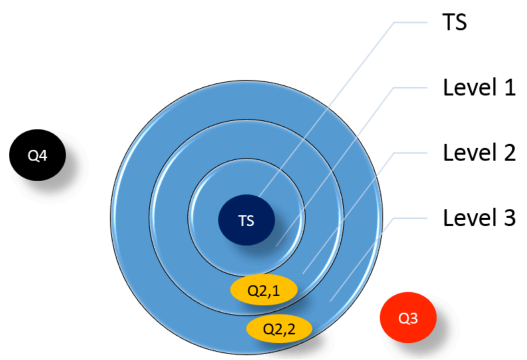


Figure 4: Boundary of Meaning Map

The same task can be addressed to different people, both teachers and students. The analysis of the collected data could give important information to the researchers, because different boundaries of meaning of the same concept might emerge and several gaps between students and teachers could be identified. The awareness of the existence of these gaps is the first step for the development of future research, which aims at didactical and pedagogical interventions to bridge them.

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