

# Italian mortgage markets and their dynamics

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## Abstract

This paper deepens previous studies on the analysis of the fixed (FRMs) and adjustable rate mortgages (ARMs) dynamics and the interconnections between FRMs and ARMs markets. In particular, an econometric analysis on the Italian mortgage markets series from 1997 :  $q1$  to 2012 :  $q3$  is set up by involving the VAR estimation technique. Very interesting results are achieved to point out how the effects of the European Central Bank control on the Euribor transmit (*i*) to the behavior of interest rates term structure as well as (*ii*) to interest rates of contracts involved in different technical forms offered in the Italian mortgage markets.

**Keywords:** mortgage market, price fluctuations, market interactions, adjustable and fixed rate mortgage, Italian markets

## 1 Introduction and motivation

The interconnections between the markets of fixed (FRMs) and adjustable rate mortgages (ARMs) are an interesting phenomenon to analyse under different profiles. In particular, with regard to Italy, this phenomenon has been

able to get the effects of the economic crisis triggered in 2008, to deflagrate to the same moment of the Lehman Brothers failure (cfr, for details, Felici et al. (2012)).

If, more generally and for purposes of comparison, the market for housing finance in the industrial countries is taken into account, it is well-known that over the past 25 years this market has changed and developed greatly and the literature concentrates primarily on two countries: the US and UK. An interesting systematic presentation of the state of research and the available literature is traceable in Leece (2004) (see also Bachofner and Lutzkendorf (2005) and, recently, for example, Koijen et al. (2009); Coulibaly and Li (2009), Coles and Hardt (2000); and, in particular, MacDonald and Winson-Geideman (2012) for the choice between FRMs and ARMs in both inflationary and deflationary US environment).

As regards the euro area, the analysis and the comparison of statistics on EU mortgage and housing markets are particularly interesting as well as data and information from several third countries such as the United States; these extensive analysis are available in several publications, for example the reports of the European Mortgage Market Federation (EFM) and the Working Papers of European Central Bank. The reactions of these markets to macroeconomic impulse as changes in monetary policy (see ad example, Calza et al. (2009)) in terms of both prices and quantities (i.e. interest rates level and numbers of new contracts) could have a large impact on the balance sheets of banks, families as well as construction industrial sectors (see European Mortgage Federation (2012), Mori et al. (2010), Coles and Hardt (2000)).

In Casellina et al. (2011) and Uberti et al. (2013) some of the dynamics of the Italian markets of FRMs and ARMs are analysed: the switching mechanism between these two markets is pointed out by historical data on the universe of mortgages in Italy since 1999 (the year of entry into force of EURIBOR rates) to 2011 and an original model is proposed to grasp these peculiar interconnection behaviours.

One of the fundamental assumptions of these studies is that the interest rates for mortgages are also indexed to the EURIBOR rate, among other factors. More precisely, since a mortgage is a form of funding that the financial institutions deliver to customers, part of the funds granted in the mortgages are thought coming from the EURIBOR interbank market. So, from the financial institution's viewpoint, the spread between the charged interest rate

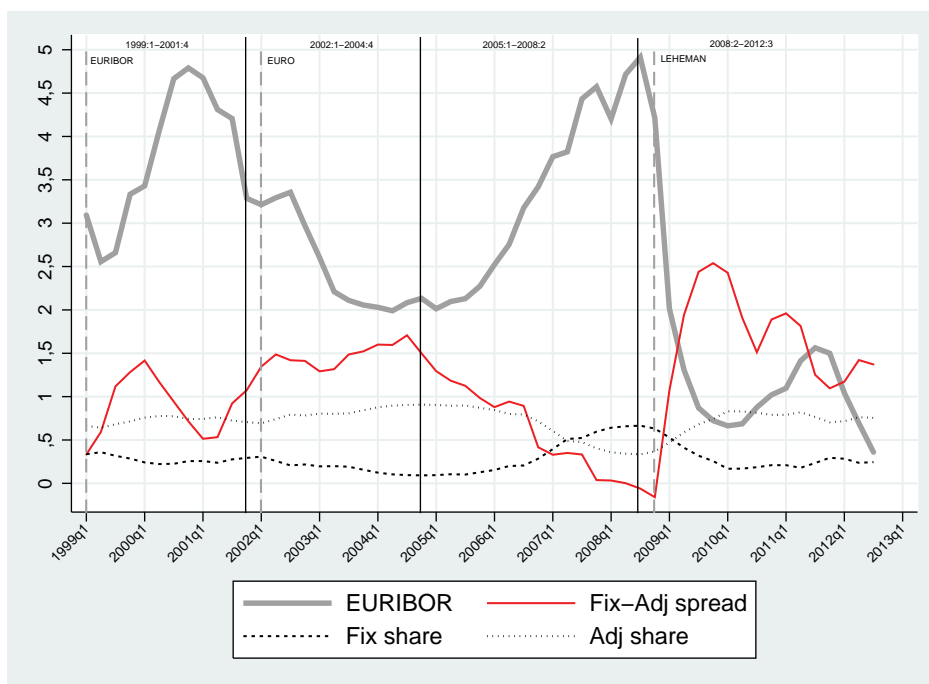


Figure 1: (a) Fixed and adjustable interest rate mortgage contracts shares; (b) EURIBOR interest rate from 1999 :  $q1$  to 2012 :  $q3$ ; (c) fixed and adjustable average market interest rates spread. Source: Bank of Italy.

to the mortgage contract and the interbank interest rate can be considered as an indicator of contract's profitability. The values of the quarterly EURIBOR rates are calculated as the average of the monthly EURIBOR rates at three months for those following its entry into force.

The time series of Italian mortgage markets (see Figure 1; source Bank of Italy) used by Casellina et al. (2011) and Uberti et al. (2013) have become attractive not only, as stressed above, to look back at some phases of the recent past, but also for further aspects: (i) the interconnection between the two markets has become more evident by evaluating the series of the  $k$ -th market values  $V_{k,t} = \rho_{k,t}N_{k,t}$ , where the average rate  $\rho_{k,t}$  of the  $k$ -th type of market is defined as a price and the absolute number of contracts  $N_{k,t}$  evaluates the demanded and, at the same time, the supplied quantities for the  $k$ -th type of contract, (ii) the volatility of market values and, finally, (iii) the effect that could induce a shock to the reference rate for mortgages, the EURIBOR rate  $\iota_t$ .

In order to study the dynamics of market shares, according to cobweb phenomenology, a mathematical model is developed in Casellina et al. (2011) and applied because it turned out able to easily see the switching mechanisms observed between FRMs and ARMs markets. Then, from this initial mathematical model, an econometric model is derived and estimated in Uberti et al. (2013). Although this derived econometric model reveals a good capacity to replicate the time series, it also highlights several questions that remained unanswered, including the effects of monetary policy on mortgage markets. From here, the necessity to resume the historical series and update them to set an econometric analysis aimed to explain the effect of a shock to the EURIBOR on interest rates for FRMs and ARMs.

Therefore, updating the data reported in Casellina et al. (2011) and Uberti et al. (2013) to the third quarter of 2012 (Figure 1), it turns out that the spread  $\Delta\rho_t = \rho_{X,t} - \rho_{Z,t}$  between the average fixed rate  $\rho_{X,t}$  and the adjustable one  $\rho_{Z,t}$  was inversely related to interest rate EURIBOR  $\iota_t$ : this correlation was  $-0.7882$ . It also emerges that, over time, a mechanism of switching was put in action among customer preferences between FRMs and ARMs.

It is worth highlighting the involved fixed and adjustable interest rates are the average of the corresponding interest rates of financial institutions which, by law, must provide to the Bank of Italy all information concerning all forms of delivered loans. Therefore, the considered interest rates are those

	$\iota_t$	$\Delta\rho_t$	$n_{X,t}$
$\iota_t$	1	—	—
$\Delta\rho_t$	-0.1750	1	—
$n_{X,t}$	-0.7955	-0.3192	1
min	2.5567	0.3330	0.3337
max	4.7900	1.4160	0.3594

Table 1: 1999 :  $q1$  – 2001 :  $q4$  correlations among EURIBOR rate  $\iota_t$ , the spread  $\Delta\rho_t$  between fixed and adjustable rates and the shares  $n_{X,t}$  of FRM contracts; minimum and maximum realisations.

calculated by the Bank of Italy and they are officially published quarterly: these rates are also considered as reference rates for the anti-usury legislation. In addition to this information, the quarterly values of the total number of contracts signed for both FRMs and ARMs by all national financial institutions are used without any distinction between the different branches of Italian mortgages market.

In the following, analyzing the series shown in Figure 1 relative to (a) the shares  $n_{X,t}$  and  $n_{Z,t} = 1 - n_{X,t}$  of FRM and ARM contracts, respectively, (b) the EURIBOR rate  $\iota_t$  series and (c) the spread  $\Delta\rho_t$  between fixed and variable rates, some phases are isolated from 1999 :  $q1$  to 2012 :  $q2$ .

Between 1999 :  $q1$  and 2001 :  $q4$  - see Table 1 - that is between the entry into force of the EURIBOR rate as well as of EURO, the EURIBOR rate has had a cycle among the minimum and maximum values of 1999 :  $q2$  - 2000 :  $q3$ . Up to 2002 :  $q1$ , the spread in rates has always been positive and has peaked in 2000 :  $q1$  and 2002 :  $q1$ : the average fixed interest rate was higher than the average adjustable one for not less than 33 basis points and for not more than 142 basis points. In this triennium, the correlation between the EURIBOR rate and the differential between the rates was  $-0.1750$  and the share of ARMs fluctuated between 64% and 66%: the customers of the financial institutions have evidently had little preference for FRMs because these latter were more expensive and because they were confident in a decline in fixed rates. On the other hand, observing the decreasing trend of the EURIBOR rate from 2000 :  $q3$ , an inversion would have expected with the effect of a reduction of fixed rates, but the opposite has occurred.

	$\iota_t$	$\Delta\rho_t$	$n_{X,t}$
$\iota_t$	1	—	—
$\Delta\rho_t$	-0.5160	1	—
$n_{X,t}$	0.8067	-0.6993	1
min	1.9900	1.2920	0.0923
max	3.3567	1.7070	0.3079

Table 2: 2002 :  $q1$  – 2004 :  $q4$  correlations among EURIBOR rate  $\iota_t$ , the spread  $\Delta\rho_t$  between fixed and adjustable rates and the shares  $n_{X,t}$  of FRM contracts; minimum and maximum realisations.

From 2002 :  $q1$  to 2004 :  $q4$  - see Table 2 - the EURIBOR rate proceeded on the downward trend while the spread between fixed and adjustable rates increased: in this period the correlation between the EURIBOR rate and the spread intensified itself (-0.5106). To a decreasing EURIBOR rate corresponded an increase in the fixed rate respect to the adjustable rate, but with not too large excursions ranging from 129 and 171 basis points. This period was therefore characterized by a further increase in the share of ARMs, which ranged between 69% and 91%, compared to the FRMs share that remained definitely marginal.

From 2005 :  $q1$  to 2008 :  $q2$  - see Table 3 - the EURIBOR rate finally reversed the tendency and it undertook a period almost regular toward growth, except the last year in which underwent some rebound. In any case, from about 2% of 2005 :  $q1$  the EURIBOR rate passed to about 5% of 2008 :  $q2$ , for a variation of 271 basis points in 14 quarters. The most noticeable effect was therefore to reduce the spread between the rates, which begun to decrease since 2004 :  $q2$ , when the EURIBOR rate was almost inert. The spread between the rates decreased by 129 basis points during the same period. In 2008 :  $q2$  the spread became negative and 66% of the mortgage was underwritten at a fixed rate and 34% at a adjustable rate. In this period, demand for ARMs fell from 90% of 2005 :  $q1$  to 34% of 2008 :  $q2$ . But this signal, which was supposed to think of a period of stability, was disturbed by a phenomenon, so to say, *exogenous*.

From 2008 :  $q3$  to 2012 :  $q3$  - see Table 4 - the spread between the rates reached the minimum in 2008 :  $q3$  where, for the first time since the entry into force of the EURIBOR rate, fixed rates was lower than adjustable

	$\iota_t$	$\Delta\rho_t$	$n_{X,t}$
$\iota_t$	1	—	—
$\Delta\rho_t$	-0.9640	1	—
$n_{X,t}$	0.9659	-0.9617	1
min	2.0133	0.0020	0.0952
max	4.7167	1.2940	0.6580

Table 3: 2005 :  $q1$  – 2008 :  $q2$  correlations among EURIBOR rate  $\iota_t$ , the spread  $\Delta\rho_t$  between fixed and adjustable rates and the shares  $n_{X,t}$  of FRM contracts; minimum and maximum realisations.

	$\iota_t$	$\Delta\rho_t$	$n_{X,t}$
$\iota_t$	1	—	—
$\Delta\rho_t$	-0.8361	1	—
$n_{X,t}$	0.8818	-0.7509	1
min	0.3600	-0.1590	0.1688
max	4.9133	2.5390	0.6654

Table 4: 2008 :  $q3$  – 2012 :  $q3$  correlations among EURIBOR rate  $\iota_t$ , the spread  $\Delta\rho_t$  between fixed and adjustable rates and the shares  $n_{X,t}$  of FRM contracts; minimum and maximum realisations.

ones, although of only 59 basis points, but the trend was clear from some time. Looking at the series of shares, from the 2005 :  $q1$  started a process of convergence that in the first two quarters of 2007 showed equal distribution of the mortgages market between the fixed rate and adjustable rate.

For what happened in 2008 :  $q3$ , some inkling was beginning to make itself felt: this was the time of the bankruptcy of Lehman Brothers and the beginning of the financial crisis that still is not exceeded. From 2008 :  $q3$  to 2010 :  $q1$ , that was in 5 quarters, the EURIBOR rate lost more than 324 base points when, under conditions of *normal difficulty*, there took 15 quarters for a growth of approximately 300 base points. The EURIBOR rate decreased exponentially because no one lent more money to someone, there was a climate of enormous distrust because financial institutions did not know if what retained in their portfolios was *toxic* or not and, above all, they were worried about the recovery of claims.

The effect was not long to be seen: the spread between the interest rates grew symmetrically to the decrease of the EURIBOR rate and it reached its

maximum in the 2009 :  $q2$  – 2010 :  $q1$  with an increase of 259 basis points from 2008 :  $q3$ . The fixed rates grew and the adjustable ones decreased: it went back to a market that focused the demand for ARMs between 37% and 82%, as in the mid-2000s, and so it remained until the end of 2012.

From 2010 :  $q1$  on, various events occurred in both the financial setting and the real economy but, above all, in the second half of 2011 a heated debate was generated on the spread between Italian bonds and German Bunds yields. This gap - in some moments definitely worrying - had an impact on domestic interest rates and, therefore, also on the mortgage rates. In the same period, the EURIBOR rate backed to decreasing and this gave the decreasing trend to the fixed-adjustable spread.

The paper is organized as follows. In Section 2, the research question is posed. In Section 3, a phenomenological description of the underlining dynamics is introduced. Section 4 specifies and estimates a suitable Vector-auto-Regression (VAR) model to gather the associate Impulse Response Function (IRF) in order to analyse the effect of a monetary shock to the EURIBOR rate on spreads of both interest rates and shares. Section 5 concludes.

## 2 Research question

Since in the mortgage market there can not be excess-supply, the market is assumed to be demand driven. Then, from the viewpoint of demand, the number of entered contracts  $N_{X,t}$  or  $N_{Z,t}$  for FRMs or ARMs, respectively, can be considered as the volume of the quantities purchased, i.e. the satisfied demand:  $N_t = N_{X,t} + N_{Z,t}$ . However, the fulfilled demand can be less than the amount actually demanded  $\tilde{N}_t$  when, for example, there is credit rationing; in this case, the spread between the actual demand and the fulfilled one generates an excess of demand  $d_t = \tilde{N}_t - N_t \geq 0$ .

In this framework, once a mortgage is subscribed for a given aim, a second one for the same purpose is assumed not to be subscribable, at least not in the short term. Under these assumptions, an almost instantaneous retroactive effect may occur between the actual demand and the supply: what can be observed is only the fulfilled demand and its price, i.e. the volume of entered contracts and the average interest rates  $\rho_{X,t}$  and  $\rho_{Z,t}$ , the fixed rate and the adjustable rate, respectively.



There is no agreement on whether the interest rate value (supply) determines the shares value of FRM and ARM contracts ( $n_{X,t} = N_{X,t}/N_t$  and  $n_{Z,t} = 1 - n_{X,t}$ , respectively) or if the value of fulfilled demand shares is rather to determine the interest rate value because this shares value is an almost exhaustive fraction of the actual demand (see Del Giovane et al. (2011) and literature cited therein). In fact, if the interest rate is the price, it is easy to understand that the actual demand affects the price rather than the satisfied demand. It is also easy to guess that a preference for some type of contract is motivated when the charged interest rate makes the contract more economically profitable and less risky than alternative.

Therefore the interest rates spread  $\Delta\rho_t = \rho_{X,t} - \rho_{Z,t}$  can be assumed to be as a function of the shares spread  $\Delta n_t = n_{X,t} - n_{Z,t}$ . On the other hand, the opposite pattern can also be justified where the shares spread is depending on the interest rates spread. Noting that since  $\Delta n_t = 2n_{X,t} - 1$ , the interest rates spread can be explained in terms of the shares of signed fixed-rate contracts.

Finally, it must also take into account the fact that the interest rates are influenced by the reference rate, the EURIBOR rate  $\iota_t$ , on which they are being indexed and which represents a measure of the cost that a financial institution faces to borrow funds. So, as mentioned above, the spread between the average interest rate charged by a financial institution and the EURIBOR rate can be a measure of its profitability and therefore the EURIBOR rate will have also an effect on shares and not only on rates.

The problem is that the EURIBOR rate is under the control of the European Central Bank (ECB), while the interest rates charged for mortgages are under the control of each Italian financial institution as well as of the Bank of Italy (BI), as a last resort. As it can easily be supposed, there are at least three main sources of interference on interest rates for mortgages considered here: (i) the actual demand, (ii) the real interest rate and (iii) the EURIBOR rate.

The question is then the following: what is and how to explain the possible effect of a monetary policy shock to the EURIBOR rate on the Italian markets of FRMs and ARMs?

To study some possible feedbacks and highlighted interconnections - at least at the level of interactions between the respective interest rates with the

reference rate - and to answer to the formulated research question, it is found appropriate to specify a family of VAR models to assess the reaction to an exogenous shock of monetary policy resulting in the change of the EURIBOR rate.

### 3 A phenomenological description of dynamics

The assumptions underlying the model follow a phenomenologically rational description. Given the demand for mortgages as the supply driver, interest rates (price) on mortgages are directly influenced by the EURIBOR rate, which is an index of the unit cost for a loan from a financial institution, but the reaction speed is necessarily different: the adjustable rate reacts before the fixed one

$$\begin{cases} \rho_{X,t} = a_X + b_X \iota_{t-k_X} + \epsilon_{X,t} \\ \rho_{Z,t} = a_Z + b_Z \iota_{t-k_Z} + \epsilon_{Z,t} \end{cases} : k_X > k_Z \quad (1)$$

Between the two rates some important differences can be highlighted. As regards their determination on the market, there is not only a difference of reaction to EURIBOR rate but also a different level of risk inherent in the contracts. ARMs are contracts with a lower interest rate for the subscriber because the risk is higher whereas for FRMs the subscriber is willing to pay more just because the contract is riskless respect to financial fluctuations.

Furthermore, for FRMs a shock to the EURIBOR rate is completely irrelevant while this last one exerts effects not only in the process of underwriting contracts but also, and above all, on those already signed at floating rates.

The composition of demand  $N_t = N_{X,t} + N_{Z,t}$  can be described by shares  $n_{X,t} = 1 - n_{Z,t}$ , where  $n_{X,t} = N_{X,t}/N_t$ . Therefore, for a complete description of the demand composition, it is sufficient to consider the FRMs share since the ARMs share is the one's complement.

The preference for one of the two branches of demand depends on the spread between the interest rates in a way not precisely determined because the demand is influenced by several conjunctural factors, by factors specifically related to the signed contract and by the characteristics of the customer

as well as the seller in terms of risk classification and default probabilities:

$$\begin{cases} n_{X,t} = f_X(\Delta\rho_t) + \epsilon_t \\ \Delta\rho_t = \rho_{X,t} - \rho_{Z,t} \end{cases} \quad (2)$$

where  $f_X(\cdot)$  is an unspecified function.

The mechanism of switching in various concentrations of demand between FRMs and ARMs can therefore be reasonably assumed to be explained according to the following phenomenology.

In response to an exogenous positive (negative) shock of monetary policy on EURIBOR rate, the adjustable rate reacts with a delay  $k_Z$  in the same direction of the received pulse. This variation of the adjustable rate approaches the fixed rate, the more inert, leading to a decrease (increase) of the spread between the two rates, for the benefit of a higher concentration of demand in the market of FRMs.

With a longer delay  $k_X > k_Z$ , also the fixed rate increases (decreases) and then the spread between the interest rates goes back up such that the fixed rate moves away from the reached level by the variable rate which curtails little by little the response to the received pulse.

Under normal conditions, i.e. in the absence of conjunctural upheavals or turbulence in the financial markets, when the spread between interest rates reaches the level preceding the shock, albeit on levels now different, there is no more reason to prefer the fixed rate (more expensive but less risky) with respect to adjustable rate (less expensive but more risky).

## 4 The model

The phenomenological description of section 3 constitutes the basic hypothesis to test to explain the observed dynamics on interest rates spread and demand composition in the Italian mortgage market. Due to the lack of a model for (2), estimation of (1) is useless to identify specific delays  $k_X$  and  $k_Z$  for an EURIBOR's shock effect on demand composition. Nevertheless, the hypothesis can be tested by means of a VAR model and by analysing the IRF outcomes of the specified model.

The state variable for the dynamic analysis is the vector

$$x_t = (\iota_t, \Delta\rho_t, n_{X,t})' \in \mathcal{M}_{(3,1)}(\mathbb{R}) \quad (3)$$

where the variables are put in a descendent order of exogeneity as suggested by Hamilton (1994). The VAR(p) structure is

$$x_t - \left( m_0 + \sum_{h=1}^p M_h x_{t-h} \right) = \epsilon_t \rightarrow WN(0, \Sigma) \quad (4)$$

where  $m_0$  is a vector of intercepts with the same dimension of  $x_t$  and  $M_h \in \mathcal{M}_{(3,3)}(\mathbb{R})$  is a matrix of coefficients linking each observable in the  $(t-h)$ -lagged state vector to all the observables in  $x_t$ . The last term  $\epsilon_t$  is a random vector of residuals assumed to follow a White-Noise process, that is with null expected value  $\mathbb{E}[\epsilon_t] = 0$  and a non-singular, constant and positive-definite covariance matrix  $\mathbb{E}[\epsilon_t \epsilon_t'] = \Sigma$ .

## 4.1 Lag-order selection

The auto-correlation functions in Figure 2 suggest that the *a-priori* lag-order to start with the specification of the VAR(p) is  $p = 4$ , that is the relevant data for the present are the same data one year-before on a quarterly time series. With the exception of the interest rate spread, which involves a lag of order 3, the confidence bands show that the auto-correlations become not significantly different from zero for  $p > 4$ . Confidence bands are defined by the Bartlett's approximated formula for the auto-correlation standard error (see Box and Jenkins (1970), formula (6.2.2) p. 177 and formula (2.1.13) p. 35).

Figure 3 is obtained with the procedure `varsoc` of STATA<sup>1</sup>, and it shows that all the selection-order criteria<sup>2</sup> identify  $p = 2$ , see the symbol `'*'`, as the most suitable auto-regression order: that is, a VAR(2) is the most informative as well as parsimonious one. As a consequence, the model to be

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<sup>1</sup>All the data processing have been developed with STATA-Release 10, StataCorp (2007).

<sup>2</sup>These are IC (information criteria) in the Lütkepohl (2005) interpretation, see also StataCorp (2007) on the `varsoc` command.

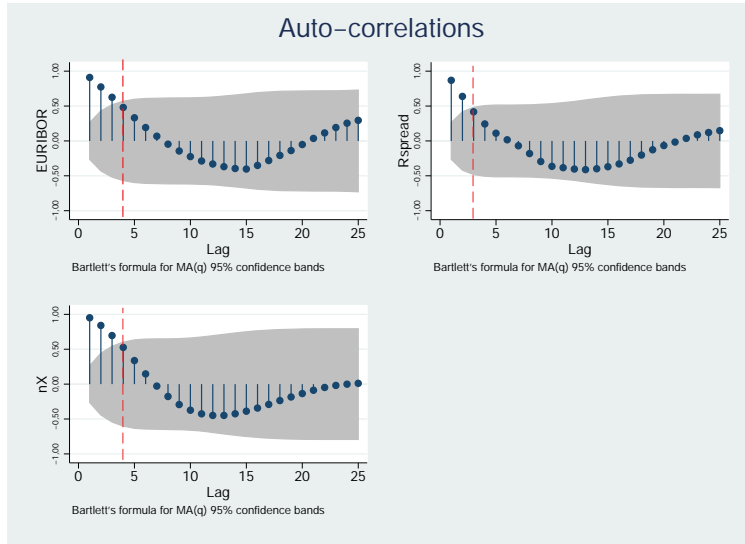


Figure 2: ac: Time-Series Auto-correlation Plot for EURIBOR rate  $\iota_t$ , interest rates spread  $\Delta\rho_t$  and share  $n_{X,t}$  of FRM contracts.

estimated is  $x_t \rightsquigarrow VAR(2)$ . The Likelihood-ratio test LR selects  $VAR(p)$  compared with a  $VAR(p-1)$  under the null hypothesis that all the coefficients of the  $p$ -order are zero StataCorp (2007) while the selection order criteria (Akaike:AIC, Hannan-Quinn:HQIC and Schwatz-Bayesian:SBIC) minimise the prediction error: the selection based on the optimal value of SBIC-HQIC gives a consistent estimate of the  $p$ -th auto-regression order, Lutkepohl (2005); StataCorp (2007).

## 4.2 Estimation, stability and causality

Once the auto-regression order has been specified, the model  $x_t \rightsquigarrow VAR(2)$  has been estimated by means of the `var` procedure with the option `lags(1/2)` and the Lagrange-multiplier test (LM) has been done on the null hypothesis of non-auto-correlated residuals (`varlmar`). Figure 4 shows that the null hypothesis cannot be rejected, therefore the test gives no hints on the model misspecification: as a consequence, the  $x_t \rightsquigarrow VAR(2)$  model is accepted. Even though residuals have been found not to be serially correlated, as long as the covariance matrix  $\Sigma$  is not diagonal, residuals can be contempora-

selection-order criteria (lutstats)								
Sample: 2000q1 - 2012q3								
Number of obs = 51								
lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	-73.3762				.004012	-5.63613	-5.63613	-5.63613
1	87.8609	322.47	9	0.000	.00001	-11.6062	-11.4759	-11.2653
2	110.816	45.911*	9	0.000	6.0e-06*	-12.1535*	-11.8929*	-11.4717*
3	114.015	6.3967	9	0.700	7.6e-06	-11.926	-11.5352	-10.9032
4	118.337	8.6455	9	0.471	9.3e-06	-11.7426	-11.2215	-10.3789

Endogenous: EURIBOR Rspread nX  
Exogenous: \_cons

Figure 3: varsoc: Lag-order pre-estimation statistics.

Lagrange-multiplier test			
lag	chi2	df	Prob > chi2
1	5.2989	9	0.80751
2	4.8129	9	0.85031

H0: no autocorrelation at lag order

Figure 4: varlmar: Lagrange-multiplier test for residual auto-correlation.

neously correlated. Figure 5 shows the tests on normality of the estimated residuals. As a single observable, only residuals in the equation for EURIBOR fulfill such a hypothesis. If the VAR is considered as a whole its residuals match with the normality character.

More explicitly, according to (4) the model  $x_t \rightsquigarrow VAR(2)$  reads as

$$x_t = m_0 + M_1 x_{t-1} + M_2 x_{t-2} + \epsilon_t \quad (5)$$

and its detailed specification or reduced form is

$$\left\{ \begin{array}{l} \iota_t = m_{0,1} + M_{1,1}^{(1)} \iota_{t-1} + M_{1,2}^{(1)} \Delta \rho_{t-1} + M_{1,3}^{(1)} n_{X,t-1} + \\ \quad M_{1,1}^{(2)} \iota_{t-2} + M_{1,2}^{(2)} \Delta \rho_{t-2} + M_{1,3}^{(2)} n_{X,t-2} + \epsilon_{1,t} \\ \Delta \rho_t = m_{0,2} + M_{2,1}^{(1)} \iota_{t-1} + M_{2,2}^{(1)} \Delta \rho_{t-1} + M_{2,3}^{(1)} n_{X,t-1} + \\ \quad M_{2,1}^{(2)} \iota_{t-2} + M_{2,2}^{(2)} \Delta \rho_{t-2} + M_{2,3}^{(2)} n_{X,t-2} + \epsilon_{2,t} \\ n_{X,t} = m_{0,3} + M_{3,1}^{(1)} \iota_{t-1} + M_{3,2}^{(1)} \Delta \rho_{t-1} + M_{3,3}^{(1)} n_{X,t-1} + \\ \quad M_{3,1}^{(2)} \iota_{t-2} + M_{3,2}^{(2)} \Delta \rho_{t-2} + M_{3,3}^{(2)} n_{X,t-2} + \epsilon_{3,t} \end{array} \right. \quad (6)$$

Model's estimates are given in Figure 6. Column **Coef** of the output table reports parameters values together with their standard errors and sig-

Jarque-Bera test				
Equation	chi2	df	Prob > chi2	
EURIBOR	<b>31,152</b>	<b>2</b>	<b>0,00000</b>	
Rspread	<b>4,120</b>	<b>2</b>	<b>0,12743</b>	
nX	<b>3,271</b>	<b>2</b>	<b>0,19486</b>	
ALL	<b>38,543</b>	<b>6</b>	<b>0,00000</b>	

Skewness test				
Equation	Skewness	chi2	df	Prob > chi2
EURIBOR	<b>-,83388</b>	<b>6,142</b>	<b>1</b>	<b>0,01320</b>
Rspread	<b>,40043</b>	<b>1,416</b>	<b>1</b>	<b>0,23400</b>
nX	<b>,50831</b>	<b>2,282</b>	<b>1</b>	<b>0,13085</b>
ALL	<b>9,841</b>	<b>3</b>	<b>0,01997</b>	

Kurtosis test				
Equation	Kurtosis	chi2	df	Prob > chi2
EURIBOR	<b>6,3653</b>	<b>25,009</b>	<b>1</b>	<b>0,00000</b>
Rspread	<b>4,1066</b>	<b>2,704</b>	<b>1</b>	<b>0,10010</b>
nX	<b>2,3309</b>	<b>0,989</b>	<b>1</b>	<b>0,32010</b>
ALL	<b>28,702</b>	<b>3</b>	<b>0,00000</b>	

Figure 5: varnorm, jbera skewness kurtosis : Test for normally distributed disturbances.

nificance<sup>3</sup>: the first block of the output table refers to equation (7), equations (8) and (9) concern the second and third block respectively.

$$M_{k,1}^{(h)} : \text{effect of } \iota_{t-h} \text{ on } \begin{cases} \iota_t \text{ iff } k = 1 \\ \Delta\rho_t \text{ iff } k = 2 \\ n_{X,t} \text{ iff } k = 3 \end{cases} \quad \forall h = 1, 2 \quad (7)$$

<sup>3</sup>The authors deserve particular care to the significance apparatus, its meaning and its implications: almost contrary to the mood, the authors are convinced that the significance can be used in very a few cases, mainly in natural sciences. Since nobody really knows what is the true underlying model, that is the data generating process is unknown to everybody, the assumption for a White Noise error term is motivated only by practical (i.e. estimation) purposes: if this assumption were true, that is experimentally verified, the significance would play an important role. But this proof is impossible to be handled since a time series is nothing but a sample of a stochastic process, a sequence of an indefinite number of random variables realisations. As a consequence, the significance is to be taken into care if and only if there is no good reason to reject the null hypothesis for the estimated residuals to be drawn from the error terms distribution of the data generating process. Since there is no information about the error term distribution but a hypothesis and, moreover, since there is no evidence or theoretical justification for the stochastic process to be a White Noise but the estimation purpose, the significance has not been commented here.

$$M_{k,2}^{(h)} : \text{effect of } \Delta\rho_{t-h} \text{ on } \begin{cases} \iota_t \text{ iff } k = 1 \\ \Delta\rho_t \text{ iff } k = 2 \\ n_{X,t} \text{ iff } k = 3 \end{cases} \quad \forall h = 1, 2 \quad (8)$$

$$M_{k,3}^{(h)} : \text{effect of } n_{X,t-h} \text{ on } \begin{cases} \iota_t \text{ iff } k = 1 \\ \Delta\rho_t \text{ iff } k = 2 \\ n_{X,t} \text{ iff } k = 3 \end{cases} \quad \forall h = 1, 2 \quad (9)$$

At a first sight it can be seen that the specified model shows a high fitting performance: all the  $R^2$ s are above the 90% of the explained variance, which is almost common in VAR estimation.

Before proceeding further, stationarity of  $x_t$  is to be discussed together with the stability of its VAR(2) representation. The expression (4) with  $p = 2$  gives (5). By using the lag operator  $Lx_t = x_{t-1}$  it then follows that

$$x_t \rightsquigarrow VAR(2) \Rightarrow \begin{cases} G_2(L)x_t = m_0 + \epsilon_t \rightarrow WN(m_0, \Sigma) \\ G_2(L) = I - M_1L - M_2L^2 \end{cases} \quad (10)$$

being  $I$  the identity matrix. The multi-variate stochastic process  $x_t$  is said to be mean-covariance stationary if the absolute value roots of its polynomial  $\det G_2(L) = 0$  are outside the unit circle. Its VAR(2) representation is said to be stable if the matrix  $M$  in the companion matrix representation

$$(I - ML)y_t = \nu_t : \begin{cases} y_{t-h} = (x_{t-h}, x_{t-h-1})' \\ \nu_t = (m_0 + \epsilon_t, 0)' \\ M = \begin{bmatrix} M_1 & M_2 \\ I & 0 \end{bmatrix} \end{cases} \quad (11)$$

is found to be non-singular and that its eigenvalues in absolute value are not outside the unit circle

$$\det(I - M\lambda) \neq 0 : |\lambda| \leq 1 \quad (12)$$

A fundamental result, shown in Hamilton (1994); Lutkepohl (2005) as regarding multivariate time series analysis, is that the stability of the VAR(p) representation is sufficient for the mean-variance stationarity of the multivariate stochastic process  $x_t$ .



Vector autoregression

Sample: 1999q3 - 2012q3  
Log likelihood = 113.8929  
FPE = 6.06e-06  
Det(Sigma\_ml) = 2.73e-06

(1utstats) No. of obs = 53  
AIC = -12.13223  
HQIC = -11.8749  
SBIC = -11.46307

Equation	Parms	RMSE	R-sq	chi2	P>chi2
EURIBOR	7	.380526	0.9273	675.5391	0.0000
Rspread	7	.21081	0.9010	482.391	0.0000
nX	7	.031988	0.9621	1345.242	0.0000

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
<b>EURIBOR</b>						
EURIBOR						
L1.	1.524798	.1375034	11.09	0.000	1.255296	1.794299
L2.	-.5400167	.1514262	-3.57	0.000	-.8368066	-.2432269
Rspread						
L1.	.6547727	.2666727	2.46	0.014	.1321039	1.177442
L2.	-.5069021	.2329851	-2.18	0.030	-.9635446	-.0502597
nX						
L1.	2.873556	1.739194	1.65	0.098	-.5352004	6.282313
L2.	-3.371329	1.590123	-2.12	0.034	-6.487914	-.2547447
_cons	-.0120849	.460028	-0.03	0.979	-.9137232	.8895534
<b>Rspread</b>						
EURIBOR						
L1.	-.3720772	.0761763	-4.88	0.000	-.5213799	-.2227745
L2.	.3116819	.0838894	3.72	0.000	.1472617	.4761022
Rspread						
L1.	.9016081	.1477355	6.10	0.000	.6120519	1.191164
L2.	-.2236451	.1290727	-1.73	0.083	-.476623	.0293329
nX						
L1.	-1.097653	.9635056	-1.14	0.255	-2.98609	.7907831
L2.	1.020514	.8809214	1.16	0.247	-.7060605	2.747088
_cons	.5485332	.2548535	2.15	0.031	.0490295	1.048037
<b>nX</b>						
EURIBOR						
L1.	.0251444	.011559	2.18	0.030	.0024892	.0477997
L2.	-.0287987	.0127294	-2.26	0.024	-.0537479	-.0038495
Rspread						
L1.	-.0463034	.0224174	-2.07	0.039	-.0902408	-.002366
L2.	.0232073	.0195856	1.18	0.236	-.0151796	.0615943
nX						
L1.	1.268595	.1462028	8.68	0.000	.9820428	1.555147
L2.	-.353523	.1336714	-2.64	0.008	-.6155141	-.0915319
_cons	.0602772	.0386716	1.56	0.119	-.0155177	.1360721

Figure 6: Estimation results for  $x_t \rightsquigarrow VAR(2)$ .

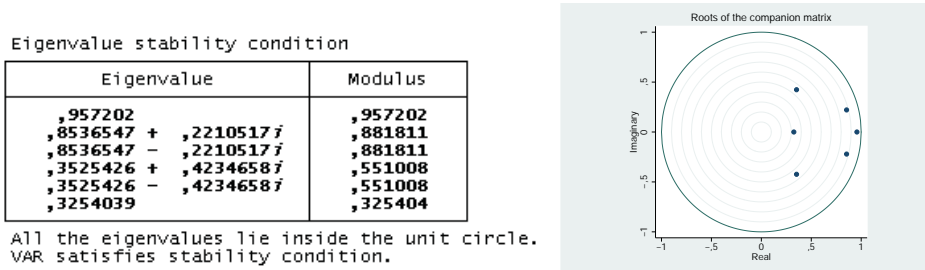


Figure 7: varstable, graph: Check stability condition of VAR estimates.

Therefore, the easiest<sup>4</sup> way to test for stationarity of  $x_t$  is to test for stability of VAR(2) by using the `varstable` procedure. Figure 7 clearly shows that the estimated model is stable, therefore  $x_t$  is found to be stationary; as a consequence, the residuals  $\epsilon_t$  are reasonably found to follow a WN process with null expected value and non-singular constant positive-definite covariance matrix.

According to (6-9) some comments can be made about estimates in Figure 6, keeping in minds that *the goal of VAR analysis is to determine the interrelationships among the variables, 'not' the parameter estimates* Enders (2009). It then turns out that the effects of the Italian demand  $n_{X,t-h}$  on the EURIBOR  $\iota_t$  is greater than the effects of the EURIBOR  $\iota_{t-h}$  on  $n_{X,t}$ , while the contrary would be more reasonable. To be true, there is no good reason for both. Indeed, the EURIBOR contains several European interest rates, which are influenced by either the national mortgage demands and other exogenous facts, such as the real interest rate in the national economies. Perhaps as regarding a different European country, these estimates would be different with respect to the Italian ones, either in magnitude and sign. Indeed, the first equation in (6) concerns the impact of past Italian interest rates spread and demand on the EURIBOR, plus an auto-regressive EURIBOR component.

There is a second matter of fact: EURIBOR and demand are positively

<sup>4</sup>Mean-variance stationarity could also be tested by solving the polynomial  $\det G_2(L) = 0$  in the variable  $L$ . If all its roots were found to be outside the unit circle, then stationarity would be met. Unfortunately, this methodology does not allow for a closed form solution if the polynomial degree is greater than five. Individual stationarity and unit root tests have been run for the observables in the vector  $x_t = (\iota_t, \Delta\rho_t, n_{X,t})'$ : with the exception of the spread between the interest rates, which could be  $I(1)$ , the variables are not integrated of the first order. Moreover, it has also been tested that there is no co-integration.

Granger causality Wald tests

Equation	Excluded	chi2	df	Prob > chi2
EURIBOR	Rspread	<b>6,463</b>	2	<b>0,039</b>
EURIBOR	nX	<b>6,4536</b>	2	<b>0,040</b>
EURIBOR	ALL	<b>8,7567</b>	4	<b>0,067</b>
Rspread	EURIBOR	<b>25,254</b>	2	<b>0,000</b>
Rspread	nX	<b>1,3507</b>	2	<b>0,509</b>
Rspread	ALL	<b>27,674</b>	4	<b>0,000</b>
nX	EURIBOR	<b>5,2768</b>	2	<b>0,071</b>
nX	Rspread	<b>4,382</b>	2	<b>0,112</b>
nX	ALL	<b>14,966</b>	4	<b>0,005</b>

Figure 8: `vargranger`: Granger causality Wald tests.

related on one period lag but negatively on a two period lag. The effect of the previous period is therefore constructive of the present realisation while the effect of two periods before is destructive for both variables on each other.

As regarding the dynamics of the interest rates differential  $\Delta\rho_{X,t}$  it can be seen that the lagged-demands have almost the same effect in magnitude but with opposite signs: the previous period demand lowers the spread while the demand in the period before the previous one increases the spread. This is not a problem but the fact that, in absolute value, both estimates are about 1 suggests the presence of a possible unit-root trend component for demands on the spread such that the stationary hypothesis might be compromised.

Finally it can be seen that the most influencing components for the present period demand is lagged-demands themselves, more than spreads and EURIBORs.

In Figure 8, Wald tests for Granger causality are reported for the three separate equations in the VAR structure of (6): as a general result it can be said that the null hypothesis that the **Excluded** variables do not Granger-cause the dependent variable can be rejected in all the three equations. Nevertheless, some exception should be made as regarding the demand to seemingly Granger-cause the interest rate spread, and vice-versa. This result is almost expected since demand is actual demand, that is the share of subscribed contracts for which clients have already accepted a supplied interest rate after some bargaining with the credit institute: realisations of demand and interest rate influence each other simultaneously. Due to this aspect, specific Granger-causality cannot be statistically removed and controlled. Therefore, as a final result, the Granger-causality test to be considered is that for **All** the **Excluded** variables not to Granger-cause the dependent

variable jointly.

### 4.3 Impulse-response analysis

If  $x_t \rightsquigarrow VAR(p)$  is found to be stationary-stable, the Wold theorem (see Hamilton (1994); Lutkepohl (2005)) applies to get the following vector-moving-average (VMA) representation

$$x_t - \bar{x} = H(L)\epsilon_t : \epsilon_t \rightarrow WN(0, \Sigma) \quad (13)$$

where  $H(L)$  is a matrix polynomial in  $L$  with an indefinite number of terms and  $\bar{x} = \mathbb{E}[x_t]$ . In real time series analysis, this means that sample  $x_t$  of  $n$  realisations has a representation based on past innovations since the beginning  $\epsilon_{t-n}$  up to the present  $\epsilon_t$ .

Due to this representation of  $x_t$  the so called *innovation accounting* can be developed to answer the following questions: (i) what happens if one exogenous shock or innovation hits a given component of  $x_t$ , let's say the EURIBOR  $\iota_t$ ? (ii) how does the system dynamics change after such a shock realises? (iii) how many periods it takes for the shock diffusion effect to vanish? if no other shock contaminated the innovation  $\epsilon_{j,t-r}$ , what would be the response  $x_{t+s}$ ? This topic is developed in Judge et al. (1998) and it seems either computationally easy and theoretically promising in its interpretation, unfortunately the innovation accounting method does not provide a unique result when estimating the reactions to innovations.

Luckily enough, there is another way to understand how the system reacts to some exogenous shock, it consists in deriving the Impulse-Response-Function (IRF) from the VMA( $\infty$ ) representation in (13). Given a VAR(p) the VMA( $\infty$ ) is

$$x_t = \bar{x} + \sum_{s=0}^{\infty} H_s \epsilon_{t-s} : H_s = H(L^s), H_0 = I \quad (14)$$

from which

$$\frac{\partial x_{t+s}}{\partial \epsilon'_t} = \frac{\partial x_t}{\partial \epsilon'_{t-s}} = H_s \quad (15)$$

is a matrix of coefficients<sup>5</sup>: the  $(i, j)$  entry  $H_s(i, j)$  measures either the response of the  $i$ -th component  $x_{i,t}$  to  $j$ -th impulse component  $\epsilon_{j,t-s}$  received

---

<sup>5</sup>As suggested by Hamilton (1994), this matrix can be obtained by simulations; in the following it is described the methodology to estimate the IRF, for further details see mainly Hamilton (1994) and Judge et al. (1998); Lutkepohl (2005).

$s$  periods before and the response of the  $i$ -th component  $x_{i,t+s}$   $s$  periods ahead given the present period  $j$ -th impulse component  $\epsilon_{j,t}$ .

All in all, the sequence  $IRF_i = \left\{ \frac{\partial x_{i,t+s}}{\partial \epsilon_{j,t}} = H_s(i, j) : \forall j \right\}_{s \leq S}$  estimates the response of the  $i$ -th component  $s$  periods ahead,  $x_{i,t+s}$ , to all the present impulses received through the unanticipated innovations. The sequence  $IRF_{i,j} = \left\{ \frac{\partial x_{i,t+s}}{\partial \epsilon_{j,t}} = H_s(i, j) : s \leq S \right\}$  is the  $i$ -th response to the  $j$ -th impulse through  $S$  periods.

The implicit assumption in these representations of the IRF is that the  $i$ -th response is measured to the  $j$ -th impulse  $\epsilon_{j,t}$  in  $x_{j,t}$  when all the other  $x_{a,\tau}$ , with  $a \neq j$  and  $\tau \leq t$ , are constant. Moreover, it might be considered that if the covariance matrix  $\mathbb{E}[\epsilon_t \epsilon_t'] = \Sigma$  is not diagonal, although constant and positive-definite, the innovations are contemporaneously correlated: as a consequence, the IRF in (15) does not separate the effect of contemporaneously correlated innovations. To overcome this problem it might be considered an orthogonalisation procedure for the IRF<sup>6</sup>.

Briefly, the covariance matrix  $\Sigma$  is symmetric and positive definite, therefore  $\Sigma = UDU'$  is always a feasible description where  $D$  is diagonal and  $U$  is lower-triangular with 1s on the main-diagonal. Therefore new uncorrelated and orthogonal innovations can be constructed by  $e_t = U^{-1}\epsilon_t$  which can be substituted into (13) to get

$$x_t - \bar{x} = H(L)Ue_t : e_t \rightarrow WN(0, D) \quad (16)$$

A sample estimate of the orthogonalised IRF (OIRF) from the VMA( $\infty$ ) is found to be given by the product of the  $s$ -th matrix of the VMA and the  $j$ -th column vector of  $U$

$$\frac{\partial \mathcal{E}_{j,t}[x_{t+s}]}{\partial x_{j,t}} = H_s U_j \quad (17)$$

where  $s \leq S$  is the time horizon of the sample estimate  $\frac{\partial \mathcal{E}_{j,t}[x_{t+s}]}{\partial x_{j,t}} = \frac{\partial \hat{\mathbb{E}}[x_{t+s} | x_{j,t}, x_{j-1,t}, \dots, x_{1,t}, X_{s-1}]}{\partial x_{j,t}}$  and  $X_{s-1} = (x'_{s-1}, \dots, x'_{s-p})$  according to Hamilton (1994).

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<sup>6</sup>STATA `irf` procedure allows for five kinds of IRF: simple, orthogonalised, cumulative, cumulative orthogonalised and structural. See StataCorp (2007) for technical details.

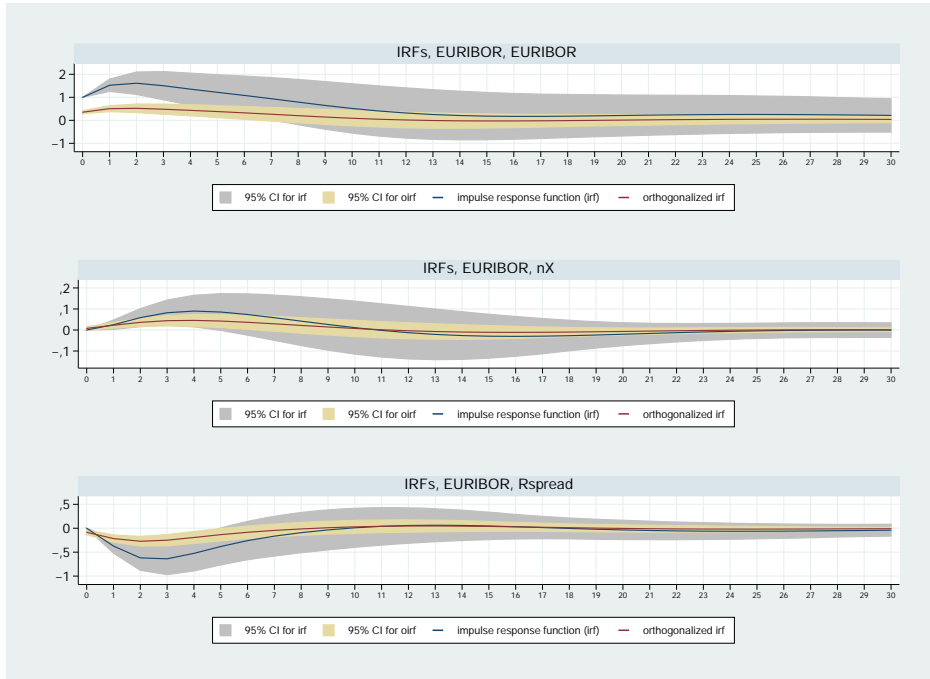


Figure 9: `irf graph irf oirf`: Simple and orthogonalised IRFs with time horizon at  $S = 30$  for  $x_t \rightsquigarrow VAR(2)$ . Impulse=EURIBOR, Response=EURIBOR,  $\Delta\rho_t$ ,  $n_{X,t}$ .

The OIRF with EURIBOR as the more exogenous variable are represented in Figure 9.

Each IRF has been plotted with its confidence bands obtained with asymptotic standard errors estimators<sup>7</sup>. The main information from the IRFs is how long it takes for a given variable to come back to its own regime once it has been hit by an EURIBOR shock: differently put, the IRFs allow for an understanding of the EURIBOR shock persistence. To this end confidence intervals are very important. It does not matter when the IRF lines come back on their zero level, which means that the shock persistence is now completely disappeared, what really matters is when the zero level enters the confidence intervals. Indeed, inside the confidence bands the line can be anywhere, it depends on the sample estimate, but inside the confidence

<sup>7</sup>This option is default with `var` procedure, several other standard errors estimators can be set.

bands there is a 95% confidence that the IRF is not significantly different from zero. Therefore, it might be considered that the EURIBOR shock exerts some effect until the confidence interval includes the zero level.

As it can be seen from the first panel of Figure 9, the EURIBOR shock takes 7 periods before becoming significantly ineffective while, from the second and third panels, it takes 5 periods for demand and interest rate spread to come to their original growth path.

A shock on the EURIBOR gives an impulse to the demand in the same direction while the interest rate spread is hit in the opposite direction: this makes sense since an increase in the EURIBOR implies an increase in the adjustable interest rate first and this shifts the demand on the fixed interest rate market. After three periods the spread keeps on growing since the fixed interest rate is reacting with seemingly two periods of delay: this makes the spread decreasing although both interest rates might have been growing. As a consequence, even though on higher level, the spread decreases because the fixed interest rate has grown enough to converge to the adjustable interest rate. As a consequence, the fixed interest rate becomes no more profitable, even though less risky, if compared to the adjustable one. This makes the demand to switch again towards the adjustable interest rate market. This action-reaction mechanism wears within 5 periods which means one year and a quarter.

## 5 Conclusions

Recently, a meaningful dataset had been arranged by the Bank of Italy on volumes and average interest rates for FRM and ARM on the Italian mortgage market from 1997 :  $q1$  to 2011 :  $q4$ . Together with a brief discussion of the phases of the observed dynamics, Uberti et al. (2013) showed how the original model Casellina et al. (2011) for interdependent markets fitted to data at a good level and, moreover, they pointed out the capability of model to capture the switching mechanism between FRM and ARM markets, that remains significative also in presence of breaks.

The present paper deepens the previous studies of the FRM and ARM dynamics involving also an econometric analysis of real data by means of the VAR estimation technique. This paper shows results in a fairly new research

stream which aims to understand how the effects of the European Central Bank control on the Euribor transmit to different technical form interest rates contracts in the Italian mortgage market, as well as to the behavior of interest rates term structure.

## Acknowledgements

Authors are grateful to two anonymous referees for comments which have improved the paper and led to this final version. This research was partially supported by MIUR (Ministero dell'Istruzione, dell'Università e della Ricerca scientifica), Italy. The views and opinions expressed by the authors are their own and do not involve the responsibility of the institutions to which they belong.

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