On the Validity of the First-Order Approach with Moral Hazard and Hidden Assets

Giuseppe Bertola[*] and Winfried Koeniger[**]

June 2014

Abstract

With moral hazard and anonymous asset trade, first-order conditions need not characterize effort and portfolio choices. A common procedure for establishing validity of the first-order approach in economies with one hidden asset is not fruitful when multiple assets are hidden.

Keywords: Hidden action; Hidden assets; Principal agent; First-order approach.

JEL: E21, D81, D82.

.

---

[*]corresponding author: Edhec Business School, 393/400 Promenade des Anglais - BP3116, 06202 Nice cedex 3 - France, tel.: +33 (0)493189966, fax: +33 (0)493830810; CEPR, CESifo, CFS; giuseppe.bertola@edhec.edu. [**]University of St.Gallen (Swiss Institute for Empirical Economic Research), CEPR, CFS, IZA; winfried.koeniger@unisg.ch. This paper has benefited from particularly helpful comments by an anonymous referee, colleagues and seminar participants. Part of this research has been conducted by Giuseppe Bertola at Università di Torino and by Winfried Koeniger at Queen Mary, University of London.

1 Introduction

Insurance provision is constrained by asymmetric information. Obviously, more insurance decreases incentives to reduce the probability of bad outcomes by exerting unobservable effort. This moral hazard problem becomes more severe when agents not only privately choose effort but also hiddenly own assets that, in anonymous competitive equilibrium, trade at prices that neglect the effect of insurance on effort incentives.

Pauly (1974) uses first-order conditions to characterize this source of inefficiency when insurance is traded against a single event, so that only one asset position is private information. In that setting, Bertola and Koeniger (2013) derive functional form restrictions ensuring validity of that "first-order approach." Ábrahám, Koehne and Pavoni (2011) similarly establish that first-order conditions are necessary and sufficient, under sensible and interpretable functional restrictions, in an economy with exclusive formal insurance and a single hidden non-contingent asset.

In this note we show that the method of these papers is not fruitful if agents can trade multiple hidden assets. This finding is not only a technicality since those and other recent papers build on the classic approach of Rogerson (1985) to establish validity of the first-order approach in moral hazard models. The case of multiple hidden assets is a natural extension of this existing literature, which is motivated by a general concern about asset observability that is not restricted to a single asset.

2 The problem

We assume that privately chosen effort e determines a non-degenerate probability distribution $f(z_i|e)$ for observable income realizations z_i, $i = 1, ..., n$, with $0 < f(z_i|e) < 1$.[1] It will be useful below to index z_i in increasing order, $z_1 < z_2 < ... < z_{n-1} < z_n$. Ex-ante identical individuals derive disutility $-v(e)$ from exerting effort and enjoy expected utility $\mathbf{E}u(c) = \sum_{i=1}^{n} u(c(z_i))f(z_i|e)$ from consumption of $c(z_i)$ upon realization of income z_i. To focus on the interior optima that

---

[1] In this setting a non-exclusive market can be active for trade in securities contingent on idiosyncratic realizations (the sale of securities is non-exclusive since agents can purchase securities from more than one provider). If instead observable outcomes are a deterministic function of effort choices based on privately observed ability realizations, as in the hidden-information economies analyzed by Mirrlees (1971) or Cole and Kocherlakota (2001), private information rules out trade in non-exclusive contingent securities (Golosov and Tsyvinski 2007, appendix A).

may be characterized by first-order conditions, we assume that $v'(0) = 0$ and that consumption c can vary freely along the budget constraint.[2]

Atomistic individuals maximize the objective function

$$V\left(e, \{q(z_i)\}_{i=1}^n\right) = -v(e) + \sum_{i=1}^n u(c(z_i))f(z_i|e) \tag{1}$$

$$\text{for } c(z_i) = z_i + q(z_i) - \sum_{j=1}^n q(z_j)p(z_j),$$

where $q(z_j)$ denotes the (positive or negative) quantity held of a security that pays a unit of consumption upon realization of income z_j, and is traded at a price $p(z_j)$, $j = 1, ..., n$ that is not influenced by each individual's choices.

The notation can accommodate non-contingent assets in a multiple-period economy where consumption and income are indexed by time as well as random realizations. For example, if for some $j = N$ it is the case that $p(z_N) = (1+r)^{-1} r$ and $f(z_N|e) = 1/\beta$, then the return of this asset does not depend on the income realization. The individual can then allocate some resources to first-period consumption (when utility is weighted by an inverse discount factor rather than a probability, and a non-random endowment may be available) rather than to second-period consumption.

The first-order approach is valid if the objective function (1) is concave. If effort decreases welfare at a non-decreasing rate,

[A1] $v'(e) > 0, v''(e) \geq 0 \,\forall\, e > 0$,

and the utility function $u(c)$ is strictly concave in consumption,

[A2] $u'(c) > 0, u''(c) < 0 \,\forall\, c$,

it is straightforward to establish concavity when there is no moral hazard:

Remark 1 *If $f(z_i|e) = \mathbf{f}(z_i)$ for all i, then assumptions [A1] and [A2] suffice to ensure concavity of the objective function (1) in effort e and the quantities $\{q(z_i)\}_{i=1}^n$.*

Concavity, of course, follows immediately from the fact that the objective function is a linear combination of concave transformations of linear functions. It is however useful in what fol-

---

[2]A non-negativity constraint on consumption is not imposed, or is not binding because the marginal utility of consumption diverges to infinity at zero.

lows to refer to a more detailed proof (in the Appendix) that explicitly proves concavity exploiting block-diagonality in effort and security quantities of the objective function's Hessian when probabilities are exogenously given, and showing that the Hessian of a function that maps to the real line a linear combination of variables is negative definite when that function is concave.

3 Establishing concavity under moral hazard

Remark 1's detailed derivation of a fairly obvious result highlights the problems arising when probabilities depend on effort. Even when effort costs are linearly separable in the objective function, if $f(z_i|e)$ depends on e it does not appear possible to replicate the steps of Remark 1's proof and characterize in full generality how the form of the model's primitive functions bears on negative definiteness of the Hessian of (1). Doing so would be exceedingly complicated when not only moral hazard prevents the objective function from being a linear combination of concave functions, but also hidden asset positions appear among the function's arguments. While Jewitt (1988) shows how functional restrictions can establish concavity with respect to effort choices in moral hazard problems without hidden assets, such a direct approach is very cumbersome when derivatives with respect to asset quantities also appear in the Hessian of a multivariate objective function.

Adopting the approach of Rogerson (1985) instead, one may rewrite (1) as

$$
\begin{aligned}
-v(e) + \sum_{i=1}^{n} u(c(z_i))f(z_i|e) &= -v(e) + u(c(z_n)) + \sum_{i=1}^{n-1} f(z_i|e)\left(u(c(z_i)) - u(c(z_n))\right) \\
&= -v(e) + u(c(z_n)) - \sum_{i=1}^{n-1} F(z_i|e)(u(c(z_{i+1})) - u(c(z_i))), \quad (2)
\end{aligned}
$$

where F is the cumulative distribution function. It follows from **[A1]**, **[A2]**, and linearity of $c(z_n)$ in $\{q(z_i)\}_{i=1}^{n}$ that $-v(e) + u(c(z_n))$ is concave in e and $\{q(z_i)\}_{i=1}^{n}$. Thus, it would suffice to establish concavity of the weighted sum of utility differences that appear in (2).

To characterize a summation of products of probability and utility functions, it is potentially useful to make assumptions about the form of these functions. As in Ábrahám, Koehne and Pavoni (2011) and Bertola and Koeniger (2013), a promising restriction for the distribution function is

4

[A3] $F(z_i|e)$ is log-convex in effort.

This is a stronger restriction than the convexity assumption that in Rogerson (1985) suffices to prove concavity of the objective function in the absence of hidden assets when **[A2]** holds and the likelihood ratio of $f(z_i|e)$ is monotone.[3] Should restrictions on the form of the utility function that are stronger than **[A2]** ensure log-convexity of $u(c(z_{i+1})) - u(c(z_i))$, when consumption levels are influenced by hidden asset positions, then **[A3]** implies that the terms in the last summation above are convex (because log-convexity is preserved in multiplication and implies convexity), and concave when entering the objective function with a minus sign.

4   A negative result

Log-convexity of $u(c(z_{i+1})) - u(c(z_i))$ requires convexity. Unfortunately, no further restriction on the form of the utility function can ensure convexity of the utility differences across two realizations when the relevant contingent consumption levels depend on holdings of additional non-redundant assets, as is the case in any economy with more than two possible contingencies and securities, and/or multiple periods and non-contingent assets:

Proposition 1 *If $c(z_i)$ and $c(z_j)$ depend on multiple non-redundant assets, $u(c(z_i)) - u(c(z_j))$ cannot be convex in asset quantities when [A2] holds.*

Focusing on the counterpart in this context of the crucial step in Remark 1's proof, the proof of Proposition 1 (in the Appendix) inspects the Hessian of $u(c(z_i)) - u(c(z_j))$. The proof shows formally that if (along with securities contingent on the two realizations z_i and z_j) any other security is traded at a strictly positive price, its quantity can vary contingent consumption levels in positive or negative directions, making it impossible to characterize the curvature of the difference $u(c(z_i)) - u(c(z_j))$ between two concave functions.

For an additional security to have a strictly positive equilibrium price, its payoff should be realized in some other contingency or period $k \neq i, j$. Hence, the negative result applies when the number of possible realizations exceeds two. Should only two income realizations be possible in the period in which income and security payoffs are realized, the portfolio problem would

---

[3]The condition of a monotone likelihood ratio requires that $(\partial f(z_i|e)/\partial e)\,/f(z_i|e)$ be non-decreasing in z_i which has the natural interpretation that more effort increases output on average (see Rogerson, 1985, and his references).

5

feature an additional non-redundant asset (as well as a contingent insurance asset) if hidden savings can non-contingently move resources along the economy's time dimension.

5   The single-hidden-asset case

When the objective function is not concave, the first-order conditions are not sufficient for an optimum, because double deviations in effort and assets can improve welfare. In models with a single hidden asset, it is possible to formulate functional form conditions that, together with **[A3]**, ensure that the first-order conditions are sufficient as well necessary.

If only a non-contingent bond is hidden, deviations in savings move consumption levels in the same direction for all realizations z_i. (Log-)convexity of the utility differences is then ensured by non-increasing absolute risk aversion in the two-period model of Ábrahám, Koehne and Pavoni (2011) who assume exclusive insurance so that positions in contingent assets are controlled by the principal.

In a model with two realizations $z_1 = z_2 - \Delta$, where $\Delta > 0$ is the size of a single negative shock covered by non-exclusive insurance, Bertola and Koeniger (2013) show that the concavity of $u\left(c(z_2)\right)$ is dominated by convexity of $-u\left(c(z_1)\right)$ if absolute risk aversion is decreasing and insurance is sufficiently incomplete, i.e., the asset quantity is in $(0, \Delta)$. The latter condition is satisfied at the optimum if insurance is actuarially unfair enough and implies large enough differences in consumption $c(z_2)$ and $c(z_1)$, and thus also in utility.

The functional form restrictions proposed in these papers rule out optimality of joint deviations in savings and effort, and insurance and effort, respectively. They do not suffice, however, to rule out optimality of joint deviations in multiple assets, such as deviations in hidden insurance *and* savings. Intuitively, joint deviations in contingent *and* non-contingent hidden assets can remain attractive even when sensible curvature conditions rule out optimality of joint deviations in effort and non-contingent hidden asset quantities. Decreasing risk aversion helps to make such deviations less attractive, but lower (higher) savings and higher (lower) insurance may well increase welfare, to imply that an attractive double deviation cannot be ruled out.

6 Conclusions

In problems with moral hazard and hidden assets, the objective function need not be concave in effort and portfolio choices, and first-order conditions may not be sufficient for optimality. This note has shown that recent progress on this issue relies heavily on a key simplification. The functional form restrictions of Ábrahám, Koehne and Pavoni (2011) establish validity of the first-order approach in a two-period economy with hidden effort and non-contingent savings. Bertola and Koeniger (2013) provide similarly plausible and appealing conditions for an economy with hidden insurance against a single contingency, as in Pauly (1974). In an even mildly more complex and realistic economy, where both non-contingent savings and insurance against a single contingent event are hidden, such restrictions do not suffice to prove validity of the first-order approach building on the approach of Rogerson (1985). Further work may explore, perhaps following the approach of Jewitt (1988), whether other functional restrictions can ensure validity of the first-order approach in economies with multiple contingencies and hidden assets.

Appendix

Proof of Remark 1

If each of the additive terms in (1) is concave, then this is also true of the objective function. Assumption **[A1]** implies that $-v(e)$ is concave. To establish concavity of $\sum_{i=1}^{n} u(c(z_i))\mathbf{f}(z_i)$ we show that its Hessian with respect to the elements of $\{q(z_i)\}_{i=1}^{n}$ is negative definite. The Jacobian of $\sum_{i=1}^{n} u(c(z_i))\mathbf{f}(z_i)$ with respect to the elements of $\{q(z_i)\}_{i=1}^{n}$ is

$$\sum_{i=1}^{n} u(c(z_i))\mathbf{f}(z_i)\left(I_{z_i} - \mathbf{p}\right) \tag{3}$$

where I_{z_i} is the ith column of the identity matrix, and $\mathbf{p}$ is the vector of security prices in terms of the numeraire. The Hessian of $\sum_{i=1}^{n} u(c(z_i))\mathbf{f}(z_i)$ with respect to the elements of $\{q(z_i)\}_{i=1}^{n}$ is the sum of the Hessians of $u(c(z_j))$ with respect to all elements of $\{q(z_i)\}_{i=1}^{n}$,

$$u''(c(z_j))\left(I_{z_j} - \mathbf{p}\right)\left(I_{z_j} - \mathbf{p}\right)^{T}, \tag{4}$$

weighted by the square of the probability weight:

$$\sum_{i=1}^{n} u''(c(z_i))\mathbf{f}(z_i)^2 \left(I_{z_i} - \mathbf{p}\right)\left(I_{z_i} - \mathbf{p}\right)^{T}. \tag{5}$$

By assumption **[A2]** all diagonal terms of (5) are negative, and the leading principal minors of (5)

$$M_j = \left(\prod_{i=1}^{j} u''(c(z_i))\mathbf{f}(z_i)^2\right)\left(1 - \sum_{i=1}^{j} p(z_i)\right)^2$$

are negative for j odd and positive for j even. Hence, $\sum_{i=1}^{n} u(c(z_i))\mathbf{f}(z_i)$ is concave in $\{q(z_i)\}_{i=1}^{n}$.

Proof of Proposition 1

The consumption levels contingent on realizations z_i and z_j depend on quantities of securities that pay off contingent on those realizations, as well as of securities that pay off in other realizations z_k, $k \neq i, j$. To show that availability of a contingent payoff for another realization z_k makes it impossible to establish convexity or concavity of $u(c(z_i)) - u(c(z_j))$, we write

$$
\begin{aligned}
c(z_i) &= z_i + q(z_i) - q(z_j)p(z_j) - q(z_k)p(z_k) - \sum_{l \neq j,k} q(z_l)p(z_l), \\
c(z_j) &= z_j + q(z_j) - q(z_j)p(z_j) - q(z_k)p(z_k) - \sum_{l \neq j,k} q(z_l)p(z_l).
\end{aligned}
$$

For positive definiteness, all the leading principal minors of the Hessian of the utility difference $u(c(z_i)) - u(c(z_j))$ with respect to the securities $\{q(z_i)\}_{i=1}^n$ should be positive. To prove Proposition 1 it is thus sufficient to show that the principal minor of the Hessian for a utility difference $u(c(z_i)) - u(c(z_j))$ with respect to $q(z_j)$ and $q(z_k)$ is negative for any utility function as in **[A2]**. The Jacobian is

$$J_{u(c(z_i))-u(c(z_j))} = \begin{bmatrix} -p(z_j)u'(c(z_i)) - (1 - p(z_j))u'(c(z_j)) \\ -p(z_k)u'(c(z_i)) + p(z_k)u'(c(z_j)) \end{bmatrix}$$

and the Hessian is

$$H_{u(c(z_i))-u(c(z_j))}$$
$$= \begin{bmatrix} p(z_j)^2 u''(c(z_i)) - (1 - p(z_j))^2 u''(c(z_j)) & p(z_k)p(z_j)u''(c(z_i)) + p(z_k)(1 - p(z_j))u''(c(z_j)) \\ p(z_k)p(z_j)u''(c(z_i)) + p(z_k)(1 - p(z_j))u''(c(z_j)) & p(z_k)^2 u''(c(z_i)) - p(z_k)^2 u''(c(z_j)) \end{bmatrix}$$
$$= u''(c(z_i)) \begin{bmatrix} p(z_j)^2 & p(z_k)p(z_j) \\ p(z_k)p(z_j) & p(z_k)^2 \end{bmatrix} - u''(c(z_j)) \begin{bmatrix} (1 - p(z_j))^2 & -p(z_k)(1 - p(z_j)) \\ -p(z_k)(1 - p(z_j)) & p(z_k)^2 \end{bmatrix}.$$

The principal minor (the determinant of the Hessian) is

$$-u''(c(z_i)) u''(c(z_j)) p(z_k)^2$$

and cannot be positive, as would be required for convexity of the utility difference $u(c(z_i)) - u(c(z_j))$, when $u''(c(z_i)) < 0$ as in **[A2]** and $p(z_k) > 0$.

References

Ábrahám, Árpád, Sebastian Koehne and Nicola Pavoni (2011) "On the First-Order Approach in Principal-Agent Models with Hidden Borrowing and Lending," Journal of Economic Theory 146, 1331–1361.

Bertola, Giuseppe and Winfried Koeniger (2013) "Hidden Insurance in a Moral Hazard Economy," CEPR Discussion Paper No. 9864, CFS Discussion Paper No. 2013/25, IZA Discussion Paper No. 7806.

Cole, Harold, and Narayana Kocherlakota (2001) "Efficient Allocations with Hidden Income and Hidden Storage," Review of Economic Studies 68, 523–542.

Golosov, Mikhail, and Aleh Tsyvinski (2007) "Optimal Taxation with Endogenous Insurance Markets," Quarterly Journal of Economics 122, 487–534.

Jewitt, Ian (1988) "Justifying the First-Order Approach to Principal-Agent Problems," Econometrica 56, 1177–1190.

Mirrlees, James A. (1971) "An Exploration in the Theory of Optimum Income Taxation," Review of Economic Studies 38, 175–208.

Pauly, Mark V. (1974) "Overinsurance and Public Provision of Insurance: The Roles of Moral Hazard and Adverse Selection," Quarterly Journal of Economics 88, 44–62.

Rogerson, William P. (1985) "The First-Order Approach to Principal-Agent Problems," Econometrica 53, 1357–1367.