SEMIOTIC AND THEORETIC CONTROL WITHIN AND ACROSS CONCEPTUAL FRAMES

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This paper refers to the concept of semiotic and theoretic control describing resources to conduct decisions in epistemic processes. We consider an argumentation process from a complex problem-solving activity involving different conceptual frames related to parabolas. Using a micro-analytical interpretative lens, we will show that, in order to carry out the argumentation activity, semiotic and theoretic control within conceptual frames (local control) needs to be co-ordinated with control across different conceptual frames (global control).

INTRODUCTION AND THEORETIC BACKGROUND

In the context of argumentation and proof activities, Arzarello & Sabena (2011) show how students' processes are managed and guided according to intertwined modalities of control, namely *semiotic* and *theoretic control*. As introduced by Schoenfeld (1985), control in problem solving activities deals with "global decisions regarding the selection and implementation of resources and strategies" (p. 15). It entails actions such as: planning, monitoring, assessment, decision-making, and conscious meta-cognitive acts. Arzarello & Sabena (2011) speak of *semiotic control* "when the decisions concern mainly the selection and implementation of semiotic resources" (p. 191), and of *theoretic* control

> when the decisions concern mainly the selection and implementation of a more or less explicit theory or parts of it [...]. For example, a semiotic control is necessary to choose a suitable semiotic representation for solving a task (e.g. an algebraic formula vs a Cartesian graph), while a theoretic control intervenes when a subject decides to use a theorem of Calculus or of Euclidean Geometry for supporting an argument. (ibid.)

Although these kinds of control seem relevant for epistemic processes, little is known about how they play together and how they relate to the respective content area.

In this paper we will consider the dialectic between semiotic and theoretic control to give account for students' progresses and standstills during a complex problem-solving activity in geometry context, in which the elaboration of an argument is required. Our analysis will be based on the model of epistemic actions (Bikner-Ahsbahs & Halverscheid, 2014), and on the notion of conceptual frame developed by Arzarello, Bazzini and Chiappini (1995). The former provides a tool to focus on epistemic processes in groups of students working together, the latter allows specifying semiotic and theoretic control by mathematical content.

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The *epistemic actions model* (Bikner-Ahsbahs & Halverscheid, 2014) comprises three collective actions: gathering, connecting and structure-seeing. In order to solve a mathematical problem, students may *gather* mathematical meanings, i.e. collecting similar pieces of knowledge (such as ideas, examples or counter examples). Through *connecting* actions students may link some of these collected mathematics meanings, for instance by checking whether a number of collected coordinates fulfil a specific equation. *Gathering* and *connecting* actions disclose an amount of mathematical meanings that shape the base for *structure-seeing*. Structure-seeing is an epistemic action of perceiving (1) a new entity of relationships built by gathering and connecting actions, condensing a possible infinite number of examples, or (2) a familiar structure in an unfamiliar/new context.

Students' epistemic processes in mathematical activities are usually organized around specific *conceptual frames*, which are related to their knowledge and expectations. For example, the coordinates of a point of a function graph can be framed as a pair of numbers, but also as lengths in the coordinate system. Arzarello, Bazzini, and Chiappini (1995) introduced the conceptual frame in algebraic context, as "an organized set of notions (i.e. mathematical objects, their properties, typical algorithms to use with them, usual arguing strategies in such a field of knowledge, etc.), which suggests them [the students] how to reason, manipulate formulas, anticipate results" (p. 122). The term *frame* is taken from artificial intelligence studies (Minsky, 1975), where it indicates a knowledge structure that contains fixed structural information. In mathematics education, the idea of conceptual frame can be related to the notion of *cadre* (setting) discussed in Douady (1986), for its strong mathematical dimension. It is also akin to *framing* by Krummheuer (1992), i.e. a stabilized and conventionalized way of seeing things based on previous experiences (Krummheuer, 1992).

In the outlined background, our research seeks to answer the following question: What role does semiotic and theoretic control play in the students' epistemic pro-cesses when developing an argumentation? How is it related to different conceptual frames?

METHODOLOGICAL AND METHODICAL CONSIDERATIONS

We investigate this research problem by observing couples of grade 10 students solving the "parabola task". These students are indicated as high achieving by their teachers. The task, adapted from Gilboa, Dreyfus & Kidron (2011), has been designed to investigate epistemic processes with micro lenses of analysis (Krause & Bikner-Ahsbahs, 2012).[1]

Students are firstly given a paper sheet to construct a curve by a folding process (Figure 1a): *(1) Take any point C on the bottom edge of the given sheet of paper. (2) Bend the sheet such that the chosen point touches the given point M. (3) Through point C, draw*

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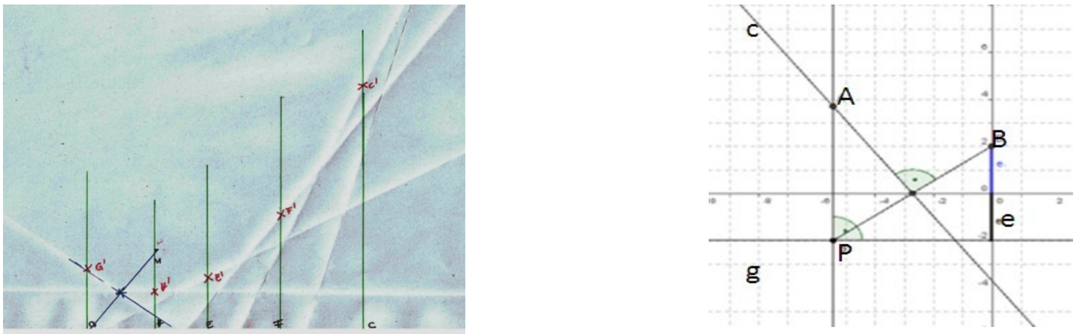
the line perpendicular to the bottom edge. (4) Mark the point of intersection with the folding line. Keep on doing that until you recognize a curve.



(1a) (1b)

Figure 1: Folded sheet and GeoGebra worksheet of the parabola task.

The marked points indicate a parabola and the folds represent the tangents touching the curve at the corresponding marked intersection points, as well as reflecting axes.

The folding process is translated then into the dynamic geometry software GeoGebra (Figure 1b), with *g* representing the edge of the paper sheet, and B the given point (which was M in paper folding). By dragging P (which corresponds to C in fig 1a), a curve is traced. The distance 2*e* between B and the fixed line *g* can be varied by a scroll bar. The task consists now of three steps: *(a) identifying the curve as a parabola, (b) justifying this conjecture (c) providing a definition for the parabola as a locus of points (in analogy with a given definition of a circle)*. During the whole session, an interviewer is sitting at the students' desk: his role concerns organising the process of working on the task and assisting when the students get stuck.

In our view, the parabola task is especially apt to our research question, since it includes complex problem-solving and argumentation activities, fostering the articulation of a variety of conceptual frames (arithmetic, algebra, geometry, etc.).

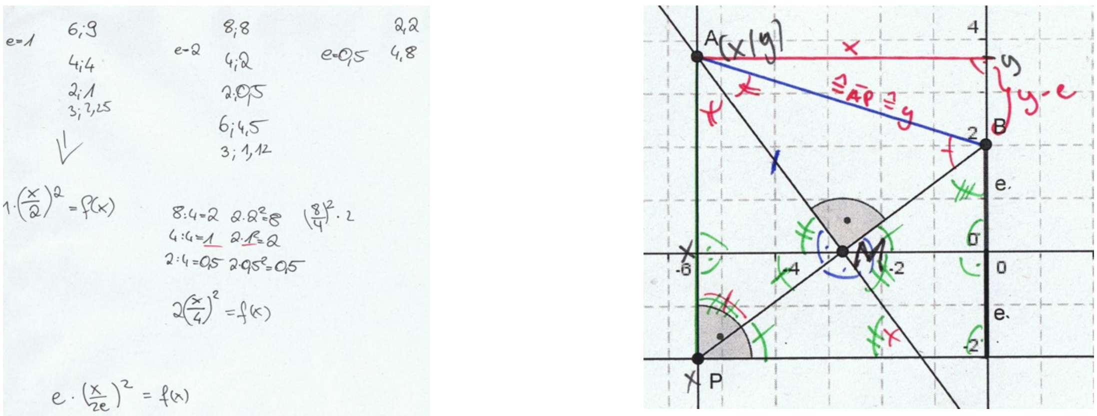
The data collection is organized in order to grasp all the kinds of signs that the students are using (language and gestures, inscriptions, dynamic signs of GeoGebra, etc.). For each couple of students, three video cameras allow us to synchronically observe the gestures, the faces, the computer screen, and the writing processes. Out of this record, a detailed transcript of the language exchanges is obtained, keeping record of verbal and non-verbal modes of expression (Table 1). This allows conducting analyses of social interaction respecting two levels of meaning, the locutionary and the non-locutionary level. Referring to Austin (1975), the former entails what is explicitly expressed, the latter concerns implicit ways of meaning making, for instance through pauses, voice intonation and non-linguistic signs such as gestures.

DATA ANALYSIS

In this paper we consider the epistemic processes of two German students, Rosa and Lisa, who are facing the second part of the parabola task. Rosa and Lisa have done the folding process, clarified how this process has been translated into the dynamic geometry worksheet, and just conjectured that the curve might be a parabola. Now they are asked to justify their conjecture: *How can you convince somebody that it is actually this curve? Use everything you found so far.*

There are several ways to conduct such a justification. The students' knowledge on parabolas is reduced to parabolas as graphs of quadratic equations; thus, a justification of the conjecture must draw on this knowledge. Considering the German curriculum, we expected that the students used the Pythagoras Theorem with subsequent algebraic manipulations. In this argumentation process, it is necessary to consider the geometric conditions and translate them into an algebraic equation: it requires both semiotic and theoretic control encompassing graphical and algebraic frames as well as theoretic knowledge thereof.

We consider the transcript (and video) starting from line 618. In order to prove that the function is a parabola, Rosa and Lisa look for some quadratic equation representing it. The interviewer shows them how to vary *e* with the scroll bar and how to make the coordinates of the point A visible (Figure 1b). The students use the scroll bar several times to create curves, systematically gathering specific coordinates *x* and *y* for *e*=1 and *e*=2 and writing them down in a structured way (Figure 2a, top-left part).



(2a) (2b)

Figure 2: Students' written sheets.

The top part of Figure 2a shows that the girls have structured the writing process, systematically deciding to begin the gathering process with *e*=1, followed by *e*=2. Experiencing that *e*=3 is not provided by the scroll bar, they decide to add some coordinates for *e*=0.5. This phase of gathering signs (lines 618-665) is led by semiotic control of the arithmetic signs, intertwined with theoretic control within a conceptual frame considering parabolas as quadratic (functional) relationships between numbers. From line 666 to 753, the students leave the computer aside and look at the inscriptions gained. The process of interpreting the number pairs is now conducted by the aim of gaining a function equation with squares of *x*. The following transcript shows the last steps of this process:

740 R: (*laughs*) Yeah (*briefly looks up at I, L writes "=f(x)" after "2(x/4)^2"*) (4sec)

741 R: That means (*first points at the brackets in "2(x/4)^2", then at the brackets in "(x/2)^2"*) this here ,so this two can […]

744 /L: yes, yes- (*slides the note sheet to herself, writes "1" in front of the equation for e=1*) times <u>one</u>. (*laughs*)

745 /R: yes that is ,no that is (*points at the just written "1"*) <u>e</u> times'

746 L: Yeah yeah. but here (*points at the "1" in the equation for e=1*) times <u>one</u>' (*points at the "2" in the equation for e=2*) there times <u>two</u>.

747 /R: that means if we say that more general now that would be-

748 /L: (*writes at the bottom of the note sheet "e·"*) *e* times

749 /R: (*synchronic*) times, *x* divided by, two *e*'

750 /L: (*synchronic*) (writes "(*x*/(2*e*))") two *e*'

751 R: Square

752 /L: (*nearly synchronic*) (*writes "2" as an exponent next to the bracket and then "=f(x)"*) square.

753 R: Thus we would have our' (*R looks up at I, L as well, but only briefly*) (...), our (*lays her hand on the note sheet, briefly looking at it, then looks at I again*) square. (*R looks at the note sheet, L sits leant back and looks towards the note sheet*) (5sec).

I: interviewer *L: Lisa* *R: Rosa*
EXACT: with a loud voice
<u>Exact</u>: emphasized
e-x-a-c-t: prolonged
exact.: dropping the voice
exact´: raising the voice *,exact: with a new onset*
(.),(..)...: 1, 2 ... sec pause
(....): more than 3sec pause
(gets up): nonverbal activity
/S: interrupts the previous speaker
[...] some lines are missing
Table 1: Transcription key

By comparing the structure of the arithmetic terms and keeping the square of *x* as an invariant according to the idea of quadratic functions, Rosa and Lisa gain the equation: $2\left(\frac{x}{4}\right)^2 = f(x)$ (line 741, Figure 2a). By comparing the terms for *e*=1 (744-753, arrow in Figure 2a), they obtain the next equation: $1\cdot\left(\frac{x}{2}\right)^2 = f(x)$. Comparing both equations they finally get: $e\cdot\left(\frac{x}{2e}\right)^2 = f(x)$ (Figure 2a, bottom). In this first phase, the students select and implement theoretic knowledge about arithmetic structures and quadratic functions, which finally are generalized. Indeed, the final equation can be transformed into $(y=)f(x) = \frac{x^2}{4e}$, the correct equation the students will reveal in the end. To carry out this process, the students turn from the numeric to the algebraic system of representation. They start from a gathering process led by semiotic control over the arithmetic-algebraic representation, and then turn into a process of generalisation through connecting actions that lead to structure-seeing. Here, semiotic control intertwines with theoretic control based on arithmetic, algebra, and quadratic functions knowledge.

However, the students have created the equation of the parabola without questioning general validity. A pause of 5 seconds (line 753) appears to the interviewer as an indication that they need some help in this direction:

754	I:	Okay. (..) ,is that universally valid now?
755	R:	No that is a conjecture (..) ,which we found with- (*points at the notes in the upper part of the note sheet, fig. 2a) (.)* eight points or so- (.)
756	I:	Y-e-s- ,so with these two examples (*points at the two lists for e=1 and e=2, fig. 2a*) you have-
757	/R:	yes
758	/L:	(*synchronic*) yes
759	/I:	seen that now already right' (*takes a deep breath*) ,the problem now is that- (..) well- (*points at the screen*) ,here alone you have just seen *e* equals three- ,so you would have to try out all.
760	R:	mhm'
761	I:	It's about general- convincing now
762	R:	(*looks at the note sheets*) Yes. (*looks at the screen*) (…)
763	I:	How can you do that (*both students look at the note sheet*) (9sec), you have now just read out (*points at the screen*), the coordinates right'
764	R:	Mhm' (*nods*)
765	/L:	Mhm- (*I touches some of the sheets on the table*) (...)
766	I:	My proposal would be now that you take another look, at one of these diagrams' (*puts the printed sheet to the top*) (..), and now (*points at A in fig. 2b, slightly shaking his finger*), assume- generally here ,that this- (.) point. A' has the coordinates x y.
767	R:	Yes' (.)
768	I:	(*takes back his hand*) And then proceed from there
		L writes "(x|y)" next to "A" on the printed sheet, then after a short break marks the y-coordinate of A on the y-axis and labels it "y", then labels P "x" (23sec).

The interviewer's intervention shifts the focus on the general validity of the obtained formula, beyond the specific considered three cases of *e*=1; 2; 0,5 (lines 754-759). On the non-locutionary level, he invites the students to justify their conjecture, i.e. to build an argument proving that their formula is generally valid ("it's about general-convincing", line 761). Under this prompt, the students express awareness about the status of their equation (755: "that is an conjecture"), but they are stuck in the arithmetic-algebraic frame. From voice intonation, pauses and broken language, a certain reluctance is observable. The interviewer reacts by pointing (both verbally and with gesture) to the coordinates of A on the diagram of the work sheet (Figure 2b), and asks explicitly for a change of view ("take another look", line 766). With his speech-gesture intervention, the interviewer is also supporting the students at a semiotic level. In order to shift to a geometric frame he shows that this "other look" must consider the coordinates x and y of the points. Following the interviewer's suggestion, Lisa and Rosa include the coordinates in their considerations while manipulating the graphs on the computer screen. However looking at the value and size of the coordinates, their investigations are still kept in the arithmetic frame (e.g. "the larger e gets the larger y too", line 811): the students are still framing the parabola as a structure of arithmetic relationships. Thus, the interviewer plays the role of a teacher to initiate a change of frame and draws an auxiliary line from A perpendicular to the y-axis (fig. 2b) (naming the intersection point Y):

835 I: I'll do in red then, there's not so much of that in there yet' (4 sec) so I now draw (*draw the red line in fig. 2b*), another- line- here. (..) (*slides the print to the students again*) ,and now- you can- (*circles with his finger around the new triangle AYB*) take a look at this triangle here ,and look what of that is known to you (6 sec) (*R, pointing at the right angle AYB, says: The angle*)

The interviewer first acts at a semiotic level, adding a new line to the figure, and then prompts the students to use their theoretic knowledge ("look what of that is known to you", 834). In this way, he is bringing into the scene a new conceptual frame, the geometric one, and is inviting the students to work in it. Once entered into the new frame, the students suitably connect geometric features with algebraic terms revealing y-e and x for two sides of the triangle AYB. By that, they show a semiotic control on connecting geometric with algebraic signs. In line 857 Rosa sees a structure in the diagram and suggests applying the Pythagoras Theorem on triangle AYB. As a consequence, they get the equation $x^2+(y-e)^2=(y+e)^2$. Finally they transform it into $y=\frac{x^2}{4e}$ and identify its equality to $f(x)=e\cdot\left(\frac{x}{2e}\right)^2$.

CONCLUSION

The analysis of Lisa and Rosa's epistemic processes in the case study shows that *gathering* and *connecting actions* are supported by the *intertwining of semiotic and theoretic control within a given conceptual frame*. The students in fact are successful in gathering and connecting signs first within the arithmetic frame (parabolas as quadratic relationships between numbers, lines 618-665), then within an algebraic frame leading to an equation as a generalization of the arithmetic structures gained (lines 666-753), and finally within a geometric-analytic frame (836 onwards).

However, although the students seem to act according to well-developed semiotic and theoretic control *within* each conceptual frame (we may call it *local* control), they get stuck. Acting as a teacher, the interviewer exploits a synergy between different semiotic resources (written signs, speech and gestures: in particular, lines 766, 835), and supports the students in considering a new frame (geometric), which enriches the previous one (algebraic). With this "new look" at the problem, the students become able to *see a new structure* (Pythagoras Theorem) and to quickly complete the task, exploiting again their semiotic and theoretic control within the new enriched frame.

The interviewer's intervention was not expected by the research plan, but it helped us to seize the importance of theoretic and semiotic control at a global level, encompassing different conceptual frames on the same situation. Our analysis suggests that in order to carry out complex problem-solving or argumentation processes, semiotic/theoretic control within each single conceptual frame (*local control*) needs to be grounded in a higher order kind of control, which allows flexibly looking for other conceptual frames, and suitably connecting them (*global control*).

The next step in our research will consist in validating our results for a wider range of contexts, and in studying what didactical interventions in the mathematics classroom can develop students' semiotic and theoretic control at a global level.

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