The "Language and argumentation" project: researchers and teachers collaborating in task design

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This contribution illustrates the "Language and argumentation" project, carried out since 2008 by the Mathematics Department of the University of Genoa. The project is aimed at designing, experimenting and refining task sequences for a smooth and meaningful approach to proof in lower secondary school. Two examples illustrate the way of working of the team (the cycles of experimentation and refinement) and some special tasks explicitly aimed at promoting students' reflection on processes and products.

Keywords: language, argumentation, proof, low-secondary school, sequence of tasks, students' processes, cycles of experimentation

Introduction

This contribution presents the "Language and argumentation" Project (lower secondary school strand), carried out by the Mathematics Department of the University of Genoa since 2008. The aim of the paper is to describe the structure of the Project team, the theoretical assumptions underlying the task design activity carried out by the team, the special tasks that were created and the way they were progressively refined throughout cycles of experimentation.

The contribution aims at addressing the following questions, as presented in the ICMI Study 22 Discussion Document (Theme D): *If you identify yourself as a member of a design group that cuts across communities, which ones are they? How did this cross-community come to be formed? When you or your group engages in designing tasks, what are you trying to achieve? What are your primary considerations? Which theoretical, mathematical, pedagogical, technological, cultural, and/or practical aspects are taken into account when designing a task or a task sequence? Are the designed tasks subject to revision in later cycles of the work? If so, what is it that specifically leads to the redesign? On what basis and according to which principles is the redesign carried out?*

In reference to the Discussion Document, we take tasks as the mediating tool between teaching and learning, and we illustrate the way in which, in our project, tasks are used in order to achieve specific educational goals (fostering the approach to argumentation and proof).

Background: proof and task design

Scholars agree on the fact that teachers should set up proper actions so as to arouse students' *need* for proof and proving (Zaslavsky et al., 2012), and also point out that teachers need preparation and support in doing that. Indeed, teachers face many challenges when dealing with proof in the classroom (Lin et al., 2012b): they must establish suitable socio-mathematical norms, choose and manage the good tasks, or even create their own tasks, guide the students towards deductive thinking without turning proving into a “ritual” activity. Also teachers’ beliefs about proof and its role in the teaching and learning of mathematics play a key role in influencing the effectiveness of the teaching of proof (Furinghetti & Morselli, 2011). For instance, many teachers seem to believe that geometry is the most suitable domain to teach proof and that proof is an advanced mathematics issue, to be taught only in secondary school.

In the literature we can found many examples of task sequences that were created with the explicit aim of improving the teaching and learning of mathematical proof (Stylianides, 2007). Lin et al. (2012a) list a series of principles for task design for conjecturing, proving and the transition between conjecture and proof. Concerning conjecturing, we mention the importance of providing an opportunity to engage in observation, construction, and reflection. Concerning proving, the authors point out the importance of promoting the expression of arguments in different modes of argument representation (verbal arguments, symbolic notations etc.), asking the students to create and share their own proofs and to evaluate proofs produced by the teacher (thus “changing the roles”). Finally, concerning the transition from conjecture to proof, the authors suggest that the teacher should establish “*social norms that guide the acceptance or rejection of participants’ mathematical arguments*” (p. 317).

This contribution will illustrate the way the team of the “Language and argumentation” project designed and experimented task sequences aimed at arousing students’ “need for proof” in lower secondary school.

The “Language and argumentation” project

In 2004 the Italian Ministry for Instruction, University and Research (MIUR) founded the national Project “Lauree Scientifiche” (“Scientific degrees”) (PLS in the following), whose aim was fostering the enrolment in university courses with scientific orientation, stimulating young people’s interest in studying sciences and providing a better education in the base sciences. The project had several strands, going from special interventions for “high-achieving students” to pre-university orientation programs. Among them, the so-called PLS Laboratories, that is to say special lessons, performed in the school environment through a collaborative work between university researchers and school teachers.

Within this framework, in 2008 the Mathematics Department of the University of Genoa started the “Language and argumentation “ project, a special case of “PLS Laboratory” aimed at designing and experimenting task sequences with a special focus on argumentation and proof.

Three main features characterize the “Language and argumentation” project: 1) task design is a central part of the collaboration between university and school; 2) argumentation and mathematical proof are the core of the task sequences; 3) teachers of different school levels (and not only higher secondary school) are involved, since the project members share the belief that argumentative competence should be developed in a long-term perspective, starting from the very first years of school and throughout all the school levels.

For each school level a team (university + school) was created. The different teams met regularly in order to share theoretical references on argumentation and take advantage of the exchanges and discussions. This contribution specifically refers to the work of the lower secondary school team. The next section illustrates the organization of the teamwork and the theoretical tools that were shared within the team and that helped to perform the didactical and methodological choices.

Task design: the team

Structure of the lower-secondary school team

The lower-secondary school team is currently made up of 7 members: the author FM, a researcher in mathematics education, three teachers with at least 10 years of teaching experience (two of them, MT and EZ, have a university degree in mathematics, one, EQ, has a university degree in chemistry), two teachers with less than 10 years of teaching experience (one, EP, has a degree in mathematics, the other one, GA, has a degree in biology), one retired teacher, AS, with a long experience in collaborative research in mathematics education. The teachers entered the project voluntarily and their participation was strongly supported by the school head. All the teachers (except for AS, retired, and GA, who changed the school after the first year but kept the work in the team) work in the same school.

The team was born in 2008-09: this means that the team just finished its fourth year of work. The first two years were dedicated to the development of a common frame (both in terms of theoretical references and of didactical methodologies). Starting form the third year, task design became a crucial activity for the team. This contribution will especially deal with this part of the project. We also point out that the teachers could design and experiment more than one task sequence for the same group of students[1], throughout the school years from 2008-09 to 2011-12. This means that a sort of mini-curriculum with a focus on argumentation and proof was created, and that students could experience more than one task sequence.

The way of working of the team

The author organized all the team meetings (one scheduled meeting per month, starting from November and until June) and acted as an observer during the class sessions. She also made video recordings of the sessions and collected all the students' written productions. Besides the team meetings, she had individual meetings with each teacher, before and after the class experimentation.

The way of working may be synthesized as it follows: during a preliminary meeting, the researcher proposes the theme of the task sequence and sketches a first draft of the core task. The core task is discussed and the teachers, together with the researcher, set up the sequence of tasks, with a special care in the sequencing of tasks. Afterwards, a first experimentation is carried out. Teacher and researcher perform the analysis of the experimentation immediately after the experimentation; the whole team performs an additional analysis during regular meetings. The analysis may lead to the refinement of the task sequence and, thus, to a new experimentation. Two modalities of experimentation were tested: parallel experimentations of the same sequence, and sequential experimentations. In the first modality, two teachers realized the task sequence in their classes, almost in the same period. Regular meetings during and after the experimentation allowed a continuous exchange between the two experiences. In particular, students' processes were compared and the actual development of the task sequence in the two classes was analysed and discussed by

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[1] In Italy, lower secondary school is made up of three years. Usually the teacher teaches the same group of students throughout all the three years.

the whole team. In the second modality, a first experimentation was carried out in one class, afterwards the whole team discussed the way the experimentation was carried out. Possible modifications to the task sequence were discussed, thus leading to a modified sequence to be experimented. In this way, a cycle of planning-experimenting-analyzing-modifying-testing the modified sequence was realized.

In both types of experimentation, the degree of variability left to each teacher is quite high, provided that his/her choices are discussed a priori or analysed a posteriori by the whole team.

In Italy teachers are often involved into research in mathematics education, under the Italian paradigm of the "Research for innovation" (Arzarello & Bartolini Bussi, 1998). Within this paradigm, teachers (who are called "teachers-researchers") collaborate with the researchers in the planning and analysis of the teaching experiments, and theoretical reflections and teaching experiments are performed dialectically, so that the analysis of the teaching experiments may lead to the evolution of the theoretical framework itself. In our case, the teachers were at their first experience of collaboration with researchers. We may say that the "Language and argumentation" project had also the final aim of fostering the professional growth of a new generation of teachers-researchers. Indeed, during the project the teachers did not only receive and implement in their classes the innovative task sequences, rather they were involved in theoretical reflection and a posteriori analysis.

Task design: principles and didactical choices

As regards the level of low secondary school, two educational goals are to be attained: from one side, fostering the development of argumentative and linguistic competences (thus, seeing argumentation as strictly linked to proof, see Durand-Guerrier et al., 2012), from the other side, promoting the first encounter with mathematical proof.

Stylianides (2007) proposes the following definition of proof that can be applied in the context of a classroom community at a given time:

> "*Proof* is a *mathematical argument*, a connected sequence of assertions for or against a mathematical claim, with the following characteristics: it uses statements accepted by the classroom community (*set of accepted statements*) that are true and available without further justification; it employs forms of reasoning (*modes of argumentation*) that are valid and known to, or within the conceptual reach of, the classroom community; an it is communicated with forms of expression (*modes of argument representation*) that are appropriate and known to, or within the conceptual reach of, the classroom community". (Stylianides, 2007, p. 291).

Accordingly, the team beliefs that a smooth and meaningful approach to proof requires the students' progressive acquisition of basic content knowledge, but also the ability to manage (from a logical and linguistic point of view) the reasoning steps and their enchaining (modes of argumentation) and the ability to communicate the arguments in an understandable way. It is important to develop a sort of "argumentative attitude", that is to say being aware of the fact that each choice, opinion, affirmation should be justified by means of a discourse that must be understood and accepted by peers. This is also in line with the idea that learning proof is approaching a form of rationality, as expressed by Morselli & Boero (2009), who proposed an adaptation of Habermas' construct of rationality to the special case of proving, showing that the discursive practice of proving may be seen as made up of three interrelated components:

> "- an epistemic aspect, consisting in the conscious validation of statements according to shared premises and legitimate ways of reasoning […];
>
> - a teleological aspect, inherent in the problem solving character of proving, and the conscious choices to be made in order to obtain the aimed product;
>
> - a communicative aspect: the conscious adhering to rules that ensure both the possibility of communicating steps of reasoning, and the conformity of the products (proofs) to standards in a given mathematical culture". (Morselli & Boero, 2009, p. 100)

Starting from the theoretical assumptions that were previously sketched, a list of methodological principles were derived and specific task design principles followed. It is important to underline that argumentation is a major educational goal, but also a means to achieve other educational goals, i.e. a better understanding of specific contents: argumentation is the *goal* and also the *means*. Hence, the task sequences are conceived with argumentation as a pervasive activity. We may distinguish between a core task, setting the problem to be worked on (a property to be discovered and justified), and the further tasks. Core tasks are usually proposed as open-ended questions (*What can you tell about…?*) where, according to the socio-mathematical norms of the class, each answer must be justified. The further tasks are conceived so as to foster students' awareness of the epistemic (*this is true because…*), teleological (*I have this goal…*) and communicative (*how could I communicate it in a proper way*?) requirements inherent in the conjecturing and proving process. More specifically, tasks encompass: formulation of conjectures; comparison between different conjectures; justification of conjectures; comparison between individual processes and between individual final products. Didactical methodologies such as group work and mathematical discussions (Bartolini Bussi, 1996) are widely used. The team also explored the importance of making students to analyse students' written individual solutions, as it is advocated within the theoretical framework of the fields of experience didactics (Boero & Douek, 2008). We point out that the methodological choices are also in line with the principles listed by Lin et al. (2012a).

In this way, two types of argumentation are fostered: argumentation at content level, as a part of the proving process, and argumentation at meta-level, as a means for fostering reflection on the practices of mathematical proof related to the three components of rationality. Within the task design process, a crucial goal was to create occasions for meta-level argumentations aimed at promoting students' awareness of the epistemic, teleological and communicative requirements of proving. To this aim, specific tasks were created. Some examples are illustrated in the subsequent section.

Some examples

Example 1 – Isoperimetric rectangles

The task sequence "*Isoperimetric rectangles*" was conceived for grade 7 (age of the students: 13-14) and encompassed about 20 hours. The core activity is the conjecture and explanation of the fact that, among all the rectangles with fixed perimeter, the square has the maximum area. That is the task sequence that underwent the most evident changes and refinements throughout the years of experimentation. The first version of the task sequence started with an explorative task in paper and pencil ("*Draw four rectangles with a perimeter of 20 cm. What can you tell about the areas of the rectangles?*"), followed by a more direct question concerning the maximum area. The students conjectured that the square is the rectangle with maximum area and in some cases they even tried to provide some numeric justifications. Afterwards, the conjecture was proved by means of algebra, under the guide of the teacher. The starting point of the proof was drawing a generic rectangle and a square (with the same perimeter) and superpose them (see figure 1). Once observed that the rectangle HGDA is part of both the rectangle and the square, the areas of the two figures GFDE and BCGH were expressed algebraically in order to show that the area of the square is greater than that of the rectangle.

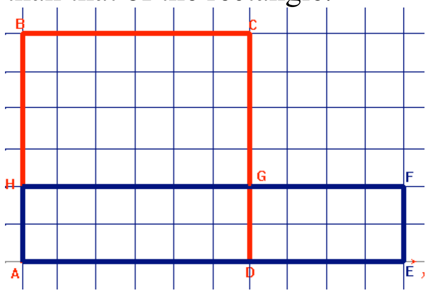

Figure 1 – Isoperimetric rectangles: the starting point for algebraic proof

During the first experimentation, one student, looking at the two superposed figures (see figure 1), raised her hand and proposed to "cut" the rectangle GFDE and place it over the rectangle BCGH, so as to "see" that one surface is bigger than the other one. This fact suggested the teachers of the team that giving students some cardboard to manipulate (and even "cut") could be another step before algebraic proof. A crucial point was the place (and role) of the "cardboard" part with respect to the conjecture about the maximum area: was it to be proposed after the drawing phase (within the conjecturing phase), or after the conjecture of the property of the square (so as to promote the idea of superposing figures and pave the way to the algebraic proof)? Further cycles of experimentation allowed to explore the issue, shedding light on the different roles that the "cardboard phase" may have within the sequence. During the third experimentation, there was also a very rich discussion on the way of drawing rectangles with a given perimeter (*how much are they; how it is possible to create anther rectangle from a given one; is it always possible to draw "couples" of rectangles, that differ only from a rotation of 90 degrees*). This suggested to insert (*in the same experimentation*, after the ingoing analysis of the session) another task of individual reflection, so as to foster the connection between geometrical facts and numerical properties. . The students were asked to write down their reflections about three questions: "*Why is it that adding and subtracting one unit (to the sides) the perimeter doesn't change? Is it the same if we add and subtract two units? Why is it that changing the basis with the height the perimeter doesn't change?*".

At the end of the sequence, there was a "balance" task, conceived in order to foster students' reflection on the value (and limit) of different approaches: drawing on paper, cutting cardboards, using algebra. A key point is that each approach offers a different perspective on the problem, helps to grasp some aspects and contributes to the whole comprehension. The students were asked to answer individually to the following questions:

> Going back to the previous tasks, you may note that we tackled the problem of isoperimetric rectangles by means of different approaches: drawing rectangles in paper & pencil, cutting rectangles on cardboard, using letters.
>
> What may you say of the different approaches?
>
> Did the different approaches allow you to understand the same things?
>
> Were they equally easy to understand?

The "balance" task was aimed at promoting the argumentation at meta-level on the potentialities and limits of each approach (for instance, drawing may only help the conjecturing phase, but it is not a real "proof"), and on the power of algebra as a proving tool. It encouraged the reflection not only on the correctness (epistemic rationality), but also on the comprehensibility (communicative rationality) and usefulness in relation to the final goal of proving (teleological rationality). Furthermore, the task gave the teachers data for an *a posteriori* evaluation of the task sequence.

Example 2: sum of consecutive numbers

The task sequence "*Sum of consecutive numbers*", conceived for grade 7, encompassed exploration, conjecturing and proving in elementary number theory. The whole sequence lasted about 10 hours. The students were proposed a first task ("*What can you tell about the sum of three consecutive numbers?*"). They worked in small groups and shared and compared the group solutions within a mathematical discussion. Afterwards, the students were proposed three connected tasks to be solved individually: "*What can you tell about the sum of two consecutive numbers? What can you tell about the sum of four consecutive numbers? What can you tell about the sum of five consecutive numbers?*". As usual for the norms of the class, each answer was accompanied by a justification. The teacher and the researcher analysed all the individual productions and for each task (sum of 2, 4 and 5 numbers respectively) selected three productions to be compared and commented by the students themselves, according to the following task:

> Read the following answers provided by some of your classmates. Compare them and write your reflections. What about the properties they found? What about the explanations they provided?

A mathematical discussion followed. This task fostered a reflection on two connected issues: the truth and comprehensibility of the conjectures, and the validity and comprehensibility of the related explanations (epistemic and communicative components). Students could reflect on the value of numeric examples (for discovery of the conjecture and communication of the property: teleological and communicative component), but also on their limits for justification (epistemic and teleological component). They could also compare justifications in natural language with justifications in algebraic language. In this ways, an argumentation at meta-level was promoted.

In our opinion this task is an application of the principles listed by Lin et al. (2012) concerning making students to produce their own justifications and evaluate justifications presented by others. Furthermore, this task paves the way to the concept of proof as presented by Stylianides (2007) and brings to the fore the importance of three dimensions of rationality, communicative included. We also point out that creating new tasks of reflection starting from students' own productions makes the task sequence very "dynamic", since each implementation must take into account new students' productions.

Discussion and conclusions

The short examples that were described in the previous session show how the model of rationality guided the team in the task design process. The core tasks, aimed at introducing the mathematical content (a property to be discovered and justified), are accompanied by further tasks, aimed at fostering the reflection on the proving process as a rational activity. For instance, the "balance" task (example 1) was aimed at bringing to the fore the role of different methods at epistemic, teleological and communicative level. The "comparison" task (example 2) was aimed at making students to reflect on the fact that the same conjecture and proof may be presented in different ways (communicative component) and that different justifications are possible (epistemic and teleological component).

Example 1 also illustrates the cycles of design, experimentation, analysis and refinement that characterize the teamwork. One key feature is that the task sequences are always under refinement. The team analysis may lead to a change in the task formulation or in the sequencing of the tasks.

Furthermore, as evidenced in example 1, additional tasks, especially tasks fostering reflection, may be inserted. Each teacher may suggest modifications to the task sequence.

Finally, it is worth noting that some tasks are "open" and must be set up during the experimentation. Any task sequence cannot be completely set up a priori, because it depends on the students' processes and products. The teacher, with the cooperation of the team, must be able to evaluate *"on the spot"* the emergence of issues to be deepened (as in example 1), and to analyse students' products and promote students' own reflection on productions (as in example 2).

References

Arzarello, F. & Bartolini Bussi, M.G. (1998). Italian Trends of Research in Mathematics Education: A National Case Study in the International Perspective. In J. Kilpatrick & A. Sierpinska (Eds.), *Mathematics Education as a Research Domain: A Search for Identity*, pp. 243-262. Kluwer, Dordrecht.

Bartolini Bussi, M.G. (1996). Mathematical discussion and perspective drawing in elementary school. *Educational studies in mathematics 31*(1-2), 11-41.

Boero, P. & Douek, N. (2008) La didactique des domaines d'expérience. *Carrefours de l'Education,* 26, 103-119.

Durand-Guerrier, V., Boero, P., Douek, N., Epp, S. S., Tanguay, D. (2012). Argumentation and proof in the Mathematics classroom. In G. Hanna & M. de Villiers (Eds.), *Proof and proving in Mathematics Education, New ICMI Study Series 15* (pp. 349-367). Springer Science + Business media B.V.

Furinghetti, F. & Morselli, F. (2011). Beliefs and beyond: hows and whys in the teaching of proof. ZDM, The international Journal on Mathematics Education, vol. 43, 587-599. ISSN: 1863-9690. Springer Berlin/Heidelberg.

Lin, F.L., Yang, K.L., Lee, K.H., Tabach, M., Stylinanides, G. (2012a). Principles for task design for conjecturing and proving. In G. Hanna & M. de Villiers (Eds.), *Proof and proving in Mathematics Education, New ICMI Study Series 15* (pp. 305-325). Springer Science + Business media B.V.

Lin, F.L., Yang, K.L., Lo, J.J., Tsamir, P., Tirosh, D., Stylianides, G. (2012b). Teachers'professional learning of teaching proof and proving. In G. Hanna & M. de Villiers (Eds.), *Proof and proving in Mathematics Education, New ICMI Study Series 15* (pp. 327-346). Springer Science + Business media B.V.

Morselli, F. & Boero, P. (2011). Using Habermas' theory of rationality to gain insight into students' understanding of algebraic language. In Cai, J. & Knuth, E. (Eds.), *Early algebraization. A global dialogue from multiple perspectives* (pp. 453-481). Springer.

Stylianides, A. J. (2007). Proof and proving in school mathematics. *Journal for Research in Mathematics Education, 38*, 289–321.

Zaslavsky, O., Nickerson, S.D., Stylianides, A.J., Kidron, I., Winicki-Landman, G. (2012). The need for proof and proving: mathematical and pedagogical perspectives. In G. Hanna & M. de Villiers (Eds.), *Proof and proving in Mathematics Education, New ICMI Study Series 15* (pp. 215-229). Springer Science + Business media B.V.