Interplay of cognition and affect in undergraduate math students' careers: insights from recursive partitioning

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Data collected in entrance tests for undergraduate curricula in mathematics at the University of Turin are analysed using the recursive partition method, to obtain classification trees for different "response variables" describing academic achievement or drop-out. The input factors include both math abilities and several affective and motivational factors, the latter having being assessed using internationally validated questionnaires. We argue that classification trees can provide unexpected insight into the interplay of such factors for academic success or failure, specifically for math students.

Students' difficulties related to mathematics in scientific undergraduate curricula have been the subject of studies and surveys in several countries (see e.g. Rylands & Coady, 2009). The present paper deals with the case of undergraduate students in mathematics: we should expect that such students are motivated towards the discipline, or at least do not have negative feelings about it. Nevertheless, a relevant percentage of them drop out (or change curriculum) during or just after the first year. The motivation of our study was to reach a better understanding of the factors affecting mathematics undergraduates' achievement (measured by the number of passed exams, the corresponding marks, and the time needed to graduate) in order to devise effective actions to improve success for students at risk. In analysing the available data relative to students at the University of Turin we initially took into account only cognitive-related variables, such as the diploma type, the final examination marks at high school and the score at the entrance test at university. These turned out to be fairly predictive of success, but not of failure. In other words, students with high scores are most likely to go on and obtain the degree, but the converse could not be said for the students with low scores. Hence, we started to search for other sources of information, taking into account

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Andrà, C. & Magnano, G. (2012). Interplay of cognition and affect in undergraduate math students' careers: insights from recursive partitioning. *Nordic Studies in Mathematics Education*, 17 (3-4), 75–89.

affective-related factors. The research issue has become, thus, twofold: on one hand, one should find out a good tool to assess such affective-related factors (and to decide which one are worthy to be considered); on the other hand, appropriate statistical tools are necessary to investigate the interplay of a large number of factors without oversimplifying assumptions, such as linear (or just monotonic) dependence of the response on each factor separately. In this paper we present an application of a non-linear method of statistical analysis, the recursive-partitioning classification tree, which was introduced around 1980 but only in recent years has been used in the educational domain (Romero & Ventura, 2010).

A theoretical framework on affect and transition

Gueudet (2008) identifies two main stems of research: detecting students' difficulties and planning interventions for making the transition easier. She notes that researchers may adopt different perspectives, consider different aspects of the transition issue and, consequently, draw different conclusions, in terms of didactical actions. Generally, difficulty in the transition is read in terms of a difficulty for students to cope with the new context. Thence the focus of the various studies is mainly on: the different thinking modes that are required at university, as evidenced by all the studies on advanced mathematical thinking (Tall, 1991); the different organization of knowledge and the intrinsic complexity of the new contents to be learnt (see e.g. Robert, 1998); the different processes and activities that are at issue, as discussed e.g. by Moore (1994) for the case of proof; the different didactical contract (Bosch, Fonseca & Gascón, 2004) and, more generally, institutional issues, such as university courses organization (Hoyles, Newman & Noss, 2001).

An emerging perspective regards affect as a lens to scrutinize secondary-tertiary transition, and the subsequent students' decision to pursue and not to drop out. Following McLeod (1992) we consider affective domain as referring "to a wide range of beliefs, feelings, and moods that are generally regarded as going beyond the domain of cognition" (p. 576). The crucial role of affect in mathematics teaching/learning is evidenced by a large amount of studies, some focusing on undergraduates. Some studies explicitly deal with affective factors in the transition issue. For instance, Daskalogianni and Simpson (2001) discuss the concept of "beliefs overhang": some beliefs, developed during schooldays, are carried forward in university, and this fact may cause difficulties. The study points out the crucial role of beliefs (about mathematics) in determining university success or failure.

A hypothesis that informs this study is that not only cognitive, affective and emotional factors are intertwined in shaping the students' path towards success (degree) or dropout, but that such an interplay is culturally shaped, i.e. that there is a cultural and socially-driven way to deal with difficulties. This is in line with Roth and Radford's (2011) understanding of consciousness. Even the role that cognitive-related factors play in reaction to learning difficulties is strongly influenced by the students' perception and interpretation. Emotions, in fact, provide the students with a sense of likelihood of success (see Roth & Radford, 2011), determining their choices. In this sense, it is of crucial interest to collect data concerning their attitudes towards mathematics and towards themselves as professional at the very beginning of undergraduate studies.

Methodology

We had the opportunity of analysing careers of all undergraduate math students at the University of Turin along ten years. Among these, only for a restricted group (162 students enrolled in fall 2010) measures of affect-related factors were available; here we shall focus on this group, although the way in which students are categorised, as far as academic achievement is concerned, rests on observations made on the whole population.

Cognitive-related measures come from students' previous career (diploma grades and type); the performance during the non-selective test for the assessment of math prerequisites (TARM) they took when enrolling in the undergraduate course.

TARM is a multiple-choice test which is administered to all students enrolling in scientific-type undergraduate courses. The test is divided in two sets: the first 25 math items are taken from an item pool previously produced by a team of experts within the *Progetto lauree scientifiche*, a national project (connecting all Italian Universities) started in 2004 to foster enrollment in scientific curricula, following the so-called "Lisbon agenda". These items are inspired by the OECD-PISA test in their formulation, but are adapted to the current high-school math curriculum in Italy, and somehow reflect the beliefs shared by Italian university math professors on which specific abilities mostly determine the success or failure in basic math undergraduate courses; this test set is currently used for all scientific (not only math) curricula. The second set of 30 items has been prepared by local teachers, and is curriculum-specific: for math students, they test the comprehension of written texts on mathematics and physics at undergraduate level, as well as the knowledge of English language (one text, and the corresponding questions, are in English). The scores earned in the first and in the second part of the test will be denoted by T1 and T2, respectively, in the sequel.

All items are multiple-choice questions, which drastically limits the capacity of the test to measure some fundamental components of math literacy, and does not match the traditional assessing methods in Italian high schools (by means of oral or open-ended written questions). On the other hand, the purpose of TARM is not to measure the overall math literacy, but rather to reveal some possible sources of learning difficulty.

The actual predictive power of such a test has been the initial motivating question throughout our research. In collaboration with Laura Nota (University of Padua) the 2010/11 TARM was enlarged to include a set of items from the *Career adapt-abilities inventory* (Savickas et al., 2009), from the *Perceived responsibility scale* (Zimmerman & Kitsantas, 2005), from the *Source of school mathematics self-efficacy scale* by Usher and Pajares (2009) and from a questionnaire on generic learning methods, strategies and abilities (Soresi & Nota, 2007).

The affective questionnaire

Savickas' Career adaptability is a multidimensional construct that concerns individual willingness and resources (flexibility, proactivity, conscientiousness, and openness) needed to adopt behaviors appropriate to cope with transitions (Savickas et al., 2009). Adaptability is a psychosocial factor and is distinct from the behaviors that it produces; it includes concern, control, curiosity, confidence, collaboration, and cooperation. The questionnaire assesses 5 distinct apt-adaptability factors: the attitude to *think positively* about one's professional future (adp1), the inclination to *consider oneself responsible* for his own professional future (adp2), the *curiosity* and desire to explore new opportunities in the professional sphere (adp3), the ability of *establishing positive relationships and cooperating with others* (adp4), and *self-confidence* about one's capacity in fostering professional self-realization (adp5). The test set included 11 (5-points) Likert-scale items for each factor.

Zimmerman's and Kitsantas' (2005) Perceived responsibility scale assesses individual's self-efficacy beliefs regarding the use of specific self-regulatory processes in various areas of academic functioning. In the present study, the students were given 18 Likert-scale questions concerning the *attribution of responsibility* of school events to the teacher or to the student (the outcome is a single measure, below denoted by prc). The 24 Likert items of the Usher's and Pajares' scale (2009) aim instead at measuring the influence of four sources on self-efficacy beliefs in mathematics: the *mastery experiences* (sse1), i.e. the perception of one's previous school performance; the *vicarious experiences* (sse2), i.e. the possibility to observe and imitate effective models; the *social persuasion* (sse3), and the *emotional and psychological states* associated to mathematical tasks (sse4). The 18 Likert items by Soresi and Nota (2007) evaluate self-efficacy in five abilities: *identifying learning objectives* (st1), *managing study and leisure time* (st2), *finding help* (st3), *writing* (st4), *identifying key concepts* (st5). The above mentioned measures were complemented with a number of individual data including gender, high school curriculum type, and residency.

A non-linear tool to analyze data

To extract information from the total amount of 24 known variables we used recursive partitioning into a *classification tree*. This method has been used for similar purposes in the past: in Superby, Vandamme and Meskens (2006) it was used, among other methods, as a way to predict academic success (not focusing on mathematics) using both cognitive and self-belief factors, but the resulting correct classification rate was below 50 %. *Decision trees* aimed at supporting students' initial choice among different undergraduate curricula, based on self-evaluation of previous school career and learning efficiency have been implemented in some universities (Vialardi et al., 2009). The main advantages of this tool are that it does not force the researchers to assume a linear correlation between the involved variables – a restriction appearing in most researches in the field of affect (Hannula, 2011) – and that it allows the researcher to take into account all aspects related to longitudinal surveys, allowing to treat categorical variables (as those emerging from interviews and qualitative studies) as well as quantitative data, without loss of complexity.

In the sequel we sketchily recall how the method works, assuming that the reader is not familiar with it and without entering into technicalities: for a thorough and rigorous presentation of the method we refer the reader to (Hastie, Tibshirani & Friedman, 2009).

To build a classification tree, the data involved are: (1) a population; (2) a set of quantitative or categorical factors X_i (sometimes called features), whose values are known for all individuals in the population (in general, a data set might include repeated measures for the same individual, but in our case, for each individual, all features X_i have been measured only once); (3) a categorical response variable Y, which represents the factor which should be predicted; on the given population the response values are known. For simplicity, assume that Y is a binary variable (as it is in our analysis), representing success or failure. The whole population is divided in two groups, the true positives (Y = success) and the true negatives (Y = failure). In the present study, different response variables have been separately considered: as regards the continuation of studies to the second year, the population of 162 students includes 125 successes and 37 failures (dropout).

The first basic concept to understand the method is the *splitting* operation. A splitting criterion consists in the choice of a specific feature X_{i} and in a partition of the possible values of X_{i} in two sets (if X_{i} is a quantitative or ordered factor, this usually amounts to fixing a threshold value). Then, the population is split in two groups, A and B, according to the individual value of X_{i}. Assume, for instance, than the majority of individuals in A are true positives: then, we say that belonging to A *predicts success*.

The second basic concept is the *node impurity* associated to a split. There are different available measures of it: the *misclassification rate*, the *Gini index* or the *cross-entropy*. For simplicity, we refer to the misclassification rate in the sequel, although for technical reasons the other measures are more frequently used. In our example, an individual belonging to A but being a true negative becomes a *false positive*; conversely, if the value of X_{i} predicts failure but the individual is a true positive, we call that individual a *false negative*. The misclassification rate associated to the given split is the total proportion of false positives and false negatives in the population under that particular splitting criterion.

The (computer-aided) construction of a classification tree proceeds as follows. The node impurity for any possible split (i.e. for any possible choice of the feature X_{i} and of the splitting threshold) is computed, and the split with the minimum node impurity (e.g. with the minimum misclassification rate) is selected to produce the first branching. The branching provides two new populations (namely, the groups A and B for that node) on both of which the whole process is iterated, and so on. Notice that at each step all available features are considered, included those that were already used to produce a previous branching: therefore the values of a single feature may result to be segmented into several intervals, corresponding to different nodes in the tree.

The process stops if either (i) all terminal nodes turn out to be pure, i.e. do not produce misclassification, or (ii) further splitting does not yield significant information gain, or (iii) a given maximum number of nodes is reached. The outcome (i), a fully predictive tree, may seem to be optimal, but may be actually unreachable with the available features. As a matter of fact, seeking a fully predictive tree is often not a good strategy, because such a tree could be too complex and might be overfitting: perfectly adapted to the training population, but likely to have a much lower predicting power when applied to a larger population. A tree with less nodes, producing some amount of misclassification, is likely to have a more stable functioning.

It is important to remark that, in spite of being the result of an iterative algorithm, the construction of a classification tree from a given set of data does not lead to a unique result. The outcome depends on the choice of the node impurity measure and on a number of parameters, which have to be assigned, that control at each step the decision to produce a new node. In the tree construction, the researcher is typically led to perform two operations, feature selection and pruning. The first consists in excluding from the process a number of features that are suspected to generate noise; pruning, instead, means deleting some nodes (with all subsequent branches) that seem to be spurious or not significant. Unlike the case of linear methods like multi-linear regression, with classification trees it can happen that restricting the number of independent factors actually improve the accuracy. In order to understand whether a change in the construction rules leads to a better outcome, one should not merely compare the total misclassification rate of the new resulting tree, but also check the stability of the outcome against restricting the population to random subgroups.

For our analysis, we used the *rpart* package within the R software environment (Therneau & Atkinson, 2012). Pruning and feature selection were made "by hand" in order to equate as much as possible the final rates (false negatives)/(negatives) and (false positives)/(positives) – so that accuracy be the same for both previsions – without increasing overall misclassification and tree complexity.

Findings

We present below three classification trees describing freshmen's achievement during their first academic year. In the first one, the predicted variable is CFU, the number of credits earned in the first year (the maximum is 60: from previous studies we observed that 21 is a significant threshold). Thus, we regard the variable CFU as a measure of *academic achievement*. In the second case, we take into account the decision to attend the second year as the target variable, considering the same set of input variables of the first case, plus the variable CFU (the target in the first case) and the average score earned in passed exams (denoted by M). In this case, the target (or response) variable is the *decision to pursue*.

Academic achievement

Figure 1 shows a classification tree which gives a correct prediction rate of 92 %, using only 9 factors (out of 22). The variable yielding the greatest information gain is T2, the score in the second part of the TARM math test. All measures of affective factors have been rescaled so that 50 ± 10 corresponds to the mean ± one standard deviation. The digits 0 and 1 at the bottom of terminal branches mean that the final prediction for that branch is failure (CFU < 21) or success (CFU ≥ 21), respectively. For each terminal group, the number of individuals correctly classified ("T"), or incorrectly classified ("F") is given.

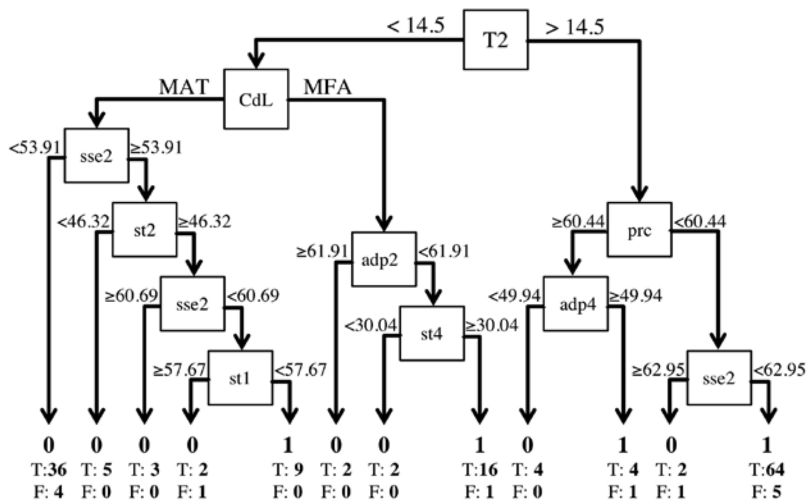

Figure 1. *Classification tree with CFU as predicted variable*

The T2 score ranges from 0 to 30, and the split value determined by the algorithm is 14.

Students on the right branch (scoring 15 or more) are subsequently discriminated by the perceived sense of responsibility: if the latter is not very high (prc < 60.44), then the next split is relative to the factor sse2 (vicarious experiences). If sse2 < 62.95, it predicts "success". If prc is very high (more than one standard deviation above the mean), instead, the variable adp4 (ability of cooperating with others) intervenes.

Going back to the root branching on T2, the variable with the second-greatest information gain on the left branch (i.e., for students who scored 14 or less) is the undergraduate curriculum: MAT (traditional math curriculum) versus MFA (applied math for finance and insurance). Here, the choice among the two curricula is not seen as an achievement factor: however, this datum should be included because reaching 21 CFU may have a different significance in the two curricula. It emerges that the difference affects only the students with a low T2 score. Among these, MFA students are further classified by variables adp2 (inclination to consider oneself as responsible for his own professional future), and st4 (writing ability self-belief). As regards MAT students with low T2, the variable sse2 plays again a remarkable role: unless the possibility to observe and imitate effective model is largely above the mean (< 54 on figure 1), failure is predicted. However, sse2 predicts success if it is high, but not too high (between 54 and 61), and provided st2 (the ability to manage study and leisure time) is not low (> 46), and st1 (the self-belief about the ability to identify learning objectives) is not too high (< 57).

Decision to pursue

In figure 2 the variable CFU is taken as one of the predictors, and the target variable is the decision to pursue in the second year. The predictive power of this tree is 93 %.

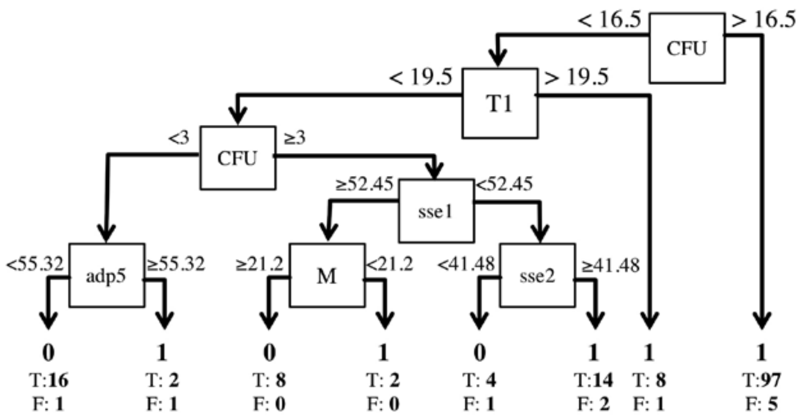

Figure 2. *Classification tree predicting continuation/drop-out on the basis of the same factors in the first case, and of the first year total credits (CFU)*

It is CFU that determines the very first split: when it is greater than 17 (namely, the students have passed at least 2 out of 6 exams), the students are very likely to go on with their career – seemingly without further questioning their choice. If students pass less than 2 exams, but their T1 score is high (> 20/25), then they are likely to pursue. This is a general feature of T1, regardless the affective-related factors: from previous experience we know that a definitely high score in the first part of TARM test is a good predictor of obtaining the degree.

Going back to figure 2, after T1, it is again CFU that determines a further split among students who passed less than 2 exams during the first year. When the number of CFU is very low the students are likely to drop out if adp5 (self-confidence) is below the mean.

It was indeed expected that the number of successful first year exams (CFU) would show up as the major discriminating factor: however, one should be careful in the interpretation of this variable. It provides an objective measure of academic achievement (and as such was used in the previous section), but while regarding it as a factor in the decision to pursue one should rather shift to a subjective viewpoint: in this sense that datum is ambiguous. Most abandoning students take their decision long before the end of the academic year, and stop taking exams as a consequence. Data on *failed* exams would provide useful information, but unfortunately they are not available. Therefore, it is instructive to delete the CFU variable from the input data, and check to which extent dropping out can be foreseen already from the measures taken at the entrance to the university. The result is shown in figure 3.

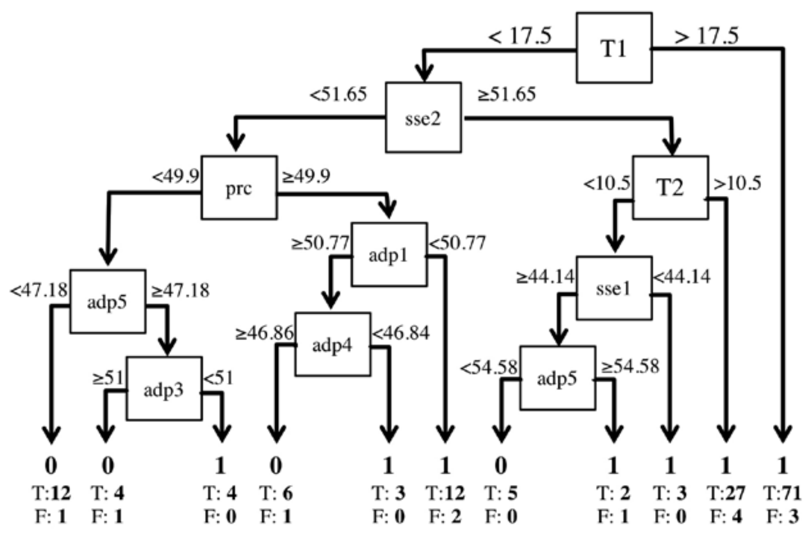

Figure 3. *Classification tree predicting continuation/drop-out on the basis of the factors measured at the beginning of the first year*

The new classification tree has a predictive power of 92 %, only slightly lower than the previous one. A cognitive variable determines the first split: T1 alone predicts success if the score is 17 or more. Only 3 students out of 74 abandon the math curriculum despite a good T1 score.

The situation is much more complex when students with low T1 score are considered. When the score is below 17, vicarious experience (sse2) plays an important role: if it is above the mean (> 52), and if the T2 score is greater than 11/30, then the students are likely to pursue to the second year. If sse2 is below the mean, then low perceived sense of responsibility (prc) and low self-confidence about one's capacity in fostering professional self-realization (adp5) jointly predict drop out. Drop out is also predicted when prc is above the mean, but the attitude to think positively about one's professional future (adp1) is low.

What emerges from the classification trees?

The results of our analysis – although based on a small population – already point towards the following general, methodological remarks:

(1) The learning achievement – as measured by the number of passed exams – and the decision to pursue undergraduate studies should be studied as outcomes of two distinct processes. Cognitive and affective factors contribute to both, but in different ways. For math students, math ability indeed determines the first branching in all cases, but the importance of the other factors is demonstrated by the following argument. The obtained rate of above 90 % correct classifications should not be overemphasized, on account of the small size of the population (162 individuals), but can be compared with the fact that if we had partitioned the same students only on the basis of math test scores (T1 and T2) we would have obtained a classification accuracy of 77 %. Now, the actual number of dropouts in that group was 23 %: this means that *blindly predicting success to every student* would yield correct classification in exactly 77 % of cases! In other words, although the scores T1 and T2 play a prominent role, they do not produce by themselves a significant information gain.

(2) Affective factors, motivation and adaptability are relevant for both outcomes, appearing as "second level" factors in the classification trees. Adaptability and perceived responsibility, in this population, are relevant not only for the decision process, as would be expected, but also for learning achievement. This agrees with previous statements on the importance of capability to adapt to a new world and new forms of thinking (Tall, 1991), new organization of knowledge (Robert, 1998), new didactical contract (Bosch, Fonseca & Gascon, 2004), new organization of courses (Hoyles, Newman & Noss, 2001). Such a capability has both cognitive-related and affective-related components, which turn out to be distinguishable but not separable (Roth & Radford, 2011).

(3) Mathematical ability – as measured in the test – is itself the outcome of a previous learning process, to which affective factors already contributed (and will contribute in the sequel, as shown by Furinghetti and

Morselli (2009)): the discriminating power of affective factors displayed in each classification tree should then be regarded as the *additional effect* in the transition from high school to university. On the contrary, gender and other individual factors, such as the high school curriculum, did not show up in any of the classification trees, although included in the set of training data: a very similar observation was reported in (Superby, Vandamme & Meskens, 2006). This should not be misinterpreted (classification tree analysis should not be confused with multiple regression): these factors are indeed likely to have influenced, even strongly, the measured levels of both cognitive and affective factors. What is significant is that they do not seem to play an *independent role* in the transition process.

(4) Concerning mathematical ability (in a broader sense, including analytic reasoning), we observe that measures obtained from differently shaped questionnaires occur at distinct places in the two processes. In this population, "curricular" abilities measured by the T1 score – solving problems in calculus, algebra and geometry – result to be more predictive for the decision to pursue, while comprehension of mathematical reasoning (included in T2) is more predictive of learning achievement. This is an unexpected outcome, which is quite stable against changes in the analysis parameters. A possible (tentative) interpretation is that the T1 score is likely to be in closer accordance with students' previous math ratings in high school, which in turn determine the expectations (and the pressure from the social environment) still affecting the decision after the first year; this hypothesis – *beliefs' overhang*, as described by Daskalogianni and Simpson (2001) – is supported by the observation that the affective factor sse1 (*mastery experience*) has higher correlation with T1 (0.49) than with T2 (0.40). In contrast, learning at university level requires higher and broader understanding skills, which may be better measured by the T2 score.

(5) Nonlinear methods of statistical data analysis, such as classification trees, are apt to uncover effects which could not be detected by traditional multiple regression. This may be particularly useful in developing longitudinal studies in the field of affects, as advocated by Hannula (2011). On the other hand, given the high instability against data and parameter variations – which is an intrinsic characteristic of classification trees – the researcher should be warned against "over-interpretation" of the emerging picture. Misclassification rates tell us to what extent a classification tree is reliable in *predicting an outcome*, not in *describing a process*. Only comparison with a sound theoretical framework, independently validated, would allow to draw conclusions in this sense. Classification trees can however provide useful working hypotheses for further research.

(6) A condition for wider and more systematic studies along these lines would be the definition of more specific affective factors, with appropriate quantitative scales or categorical indicators. The observed relevance of the sources of self-efficacy in mathematics (namely, of the factor sse2 concerning *vicarious experiences*) and of the adaptability factors, suggests that one should not only focus on the individual feelings towards mathematics, but more generally explore the beliefs and attitudes arising in the triangular relationship among mathematical experience, the individual and his/her social environment (Bandura, 1986; Usher & Pajares, 2009).

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