A discrete stochastic model of the transmission cycle of the tick borne encephalitis virus

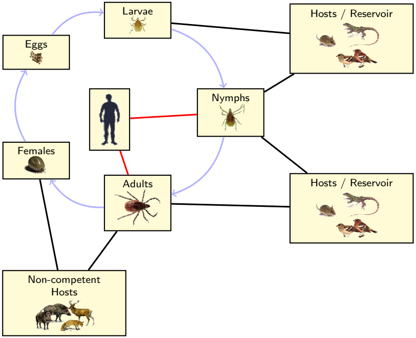
Luca Ferreri Mario Giacobini

Computational Epidemiology Group
Department of Veterinary Sciences

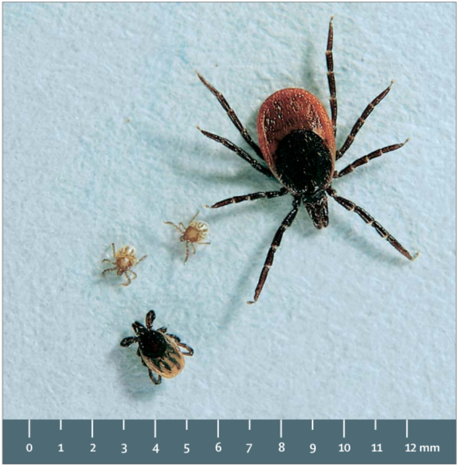
Università degli Studi di Torino
luca.ferreri@unito.it

3 September 2012

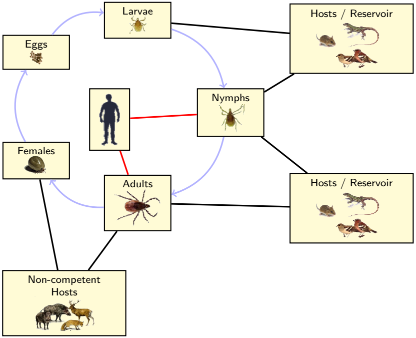
Tick Borne Encephalitis (TBE)



Tick Borne Encephalitis (TBE)



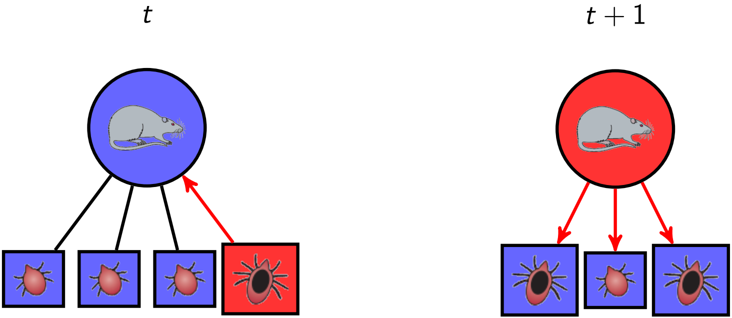
Tick Borne Encephalitis (TBE)



- no consequences for animals
- severe consequences for humans

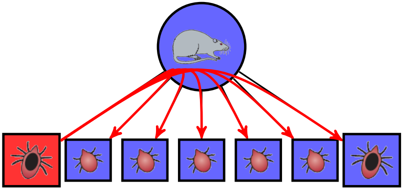
Transmissions

Systemic Transmission:

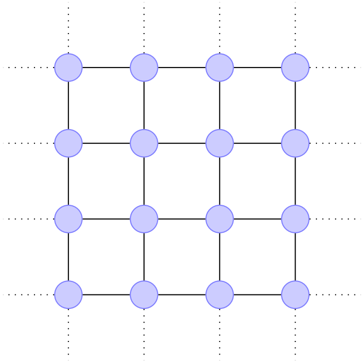


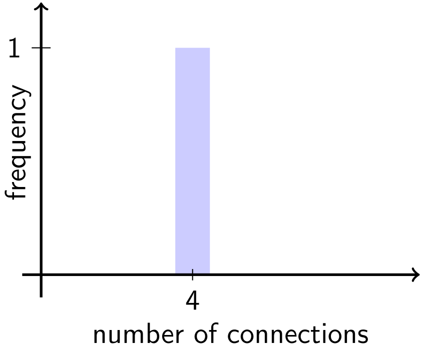
Transmissions

Co-Feeding Transmission:

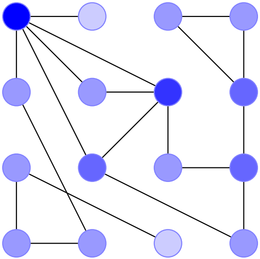


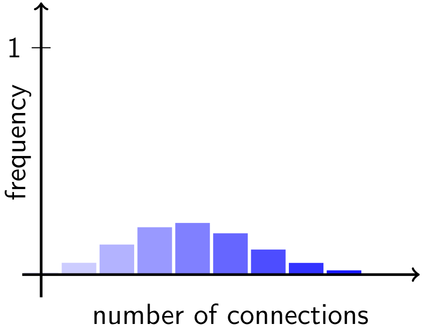
Networks - Lattice



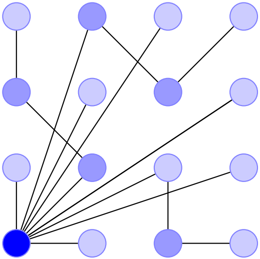


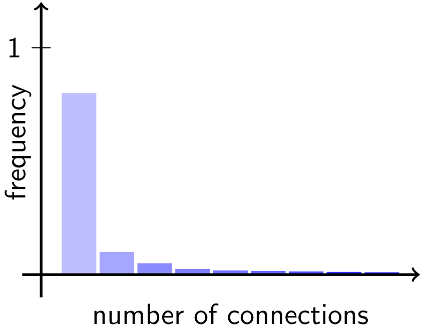
Networks - Random Graph





Networks - Scale Free Network





Compartmental Models on Networks

Susceptible-Infectious-Susceptible (SIS) model:

$$\frac{\mathrm{d}\, i_k}{\mathrm{d}\, t} = -\mu i_k + \beta k \left(1 - i_k\right) \frac{1}{\langle k \rangle} \sum_h h p_h i_h$$

Compartmental Models on Networks

Susceptible-Infectious-Susceptible (SIS) model:

$$\frac{\mathrm{d}\, i_k}{\mathrm{d}\, t} = -\mu i_k + \beta k \left(1 - i_k\right) \frac{1}{\langle k \rangle} \sum_h h p_h i_h$$

the epidemic threshold:

$$\frac{\beta}{\mu} > \frac{\langle k \rangle}{\langle k^2 \rangle}$$

The model

prevalence $i = \frac{\text{number infected}}{\text{total population}}$

The model

$i_{k,y}$ prevalence for larvae that encounter k nymphs at the end of year y

The model

$j_{k,y}$ prevalence for nymphs that encounter k larvae in the year y

The model

a strong assumption(!):

$$i_{k,y} = j_{k,y+1}$$

$$i_{k,y} = 1 - (1 - A_y)^k$$

where A_y is the probability that an edge is transmitting the disease

$$i_{k,y} = 1 - (1 - A_y)^k$$

where A_y is the probability that an edge is transmitting the disease

$$A_y = \frac{\beta}{\langle k \rangle_y^{N \to L}} \sum_h h \cdot p_{h,y}^{N \to L} \cdot j_{h,y}$$

$$i_{k,y} = 1 - (1 - A_y)^k$$

where A_y is the probability that an edge is transmitting the disease

$$A_y = \frac{\beta}{\langle k \rangle_y^{N \to L}} \sum_h h \cdot p_{h,y}^{N \to L} \cdot i_{h,y-1}$$

$$i_{k,y} = 1 - (1 - A_y)^k$$

where A_y is the probability that an edge is transmitting the disease

$$A_y = \frac{\beta}{\langle k \rangle_y^{N\to L}} \sum_h h \cdot p_{h,y}^{N\to L} \cdot i_{h,y-1}$$

therefore

$$A_y = \beta - \frac{\beta}{\langle k \rangle_y^{N\to L}} \sum_h h \cdot p_{h,y}^{N\to L} \cdot (1 - A_{y-1}^h)$$

The Epidemic Threshold

If we can assume that year y and year $y-1$ are ecologically comparable then we check when $A_y = A_{y-1} = x$.

The Epidemic Threshold

If we can assume that year y and year $y-1$ are ecologically comparable then we check when $A_y = A_{y-1} = x$.
This corresponds to

$$x = F(x) = \beta - \frac{\beta}{\langle k \rangle_y^{N \to L}} \sum h \cdot p_{h,y}^{N \to L} \cdot (1-x)^h$$

The Epidemic Threshold

If we can assume that year y and year $y-1$ are ecologically comparable then we check when $A_y = A_{y-1} = x$.
This corresponds to

$$x = F(x) = \beta - \frac{\beta}{\langle k \rangle_y^{N \to L}} \sum h \cdot p_{h,y}^{N \to L} \cdot (1-x)^h$$

but

$$F'(x) = \frac{\beta}{\langle k \rangle_y^{N \to L}} \sum h^2 p_{hy}^{N \to L} (1-x)^{h-1}$$

The Epidemic Threshold

If we can assume that year y and year $y-1$ are ecologically comparable then we check when $A_y = A_{y-1} = x$.
This corresponds to

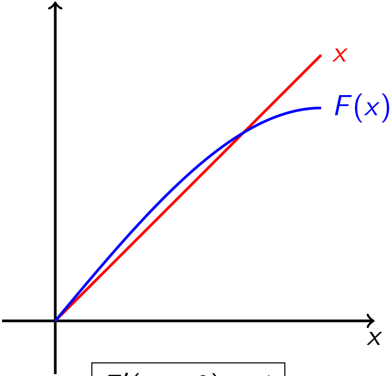
$$x = F(x) = \beta - \frac{\beta}{\langle k \rangle_y^{N \to L}} \sum h \cdot p_{h,y}^{N \to L} \cdot (1-x)^h$$

but

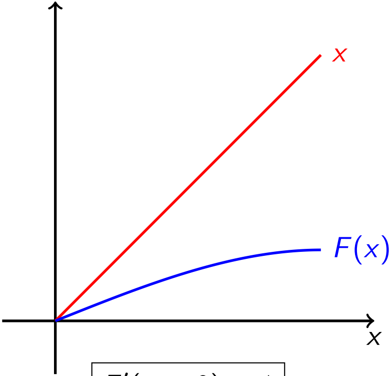
$$F'(x) = \frac{\beta}{\langle k \rangle_y^{N \to L}} \sum h^2 p_{hy}^{N \to L} (1-x)^{h-1}$$

and

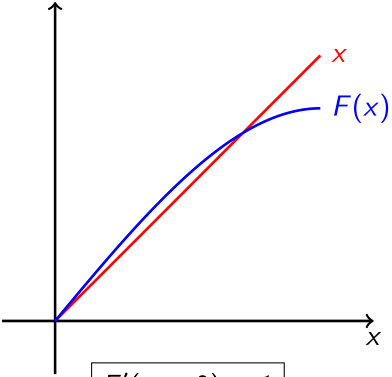
$$F''(x) = -\frac{\beta}{\langle k \rangle_y^{N \to L}} \sum h^2 (h-1) p_{hy}^{N \to L} (1-x)^{h-2}$$



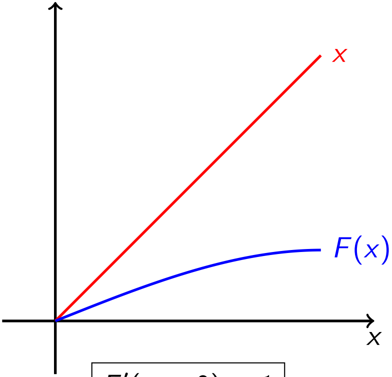
$$\boxed{F'(x=0) > 1}$$



$$\boxed{F'(x=0) < 1}$$



$\boxed{F'(x=0) > 1}$



$\boxed{F'(x=0) < 1}$

$$\beta > \frac{\langle k\rangle_y^{N\to L}}{\langle k^2\rangle_y^{N\to L}}$$

Some recurrent ecological cycles are reported from real world data. Hence, sometimes, instead of the assumption $A_y = A_{y-1}$, it could be more suitable $A_y = A_{y-n}$. In that case the epidemiological threshold would be:

$$\beta^n > \frac{\langle k \rangle_y^{N \to L}}{\langle k^2 \rangle_y^{N \to L}} \cdot \ldots \cdot \frac{\langle k \rangle_{y-n+1}^{N \to L}}{\langle k^2 \rangle_{y-n+1}^{N \to L}}$$