Double Proton Decay In H Anti-h Oscillations.

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Double proton decay and H-$\bar{\text{H}}$ oscillations

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Baryon-number-violating ($\Delta B=2$) processes are predicted by a variety of grand (or partially) unified theories of the fundamental interactions. In this paper we explore the double proton decay inside nuclei. We calculate the width Γ for the associated exotic nuclear decay and, by exploiting the experimental data on nuclear stability, we are able to set an upper bound on the coupling constant K of the corresponding effective Lagrangian. This, in turn, can be converted into a lower bound for the time of oscillation of hydrogen into antihydrogen. The latter turns out to be much more stringent than the one inferred from astrophysical observations.

I. INTRODUCTION

Baryon and lepton numbers violating processes are common predictions of the grand (or partially) unified theories of the fundamental interactions. In particular, the occurrence of $\Delta B=2$ and/or $\Delta L=2$ processes is characteristically predicted, at the level of experimental observability, by some models, which furthermore allow the proton to have a lifetime long enough to avoid conflict with the present experimental data.

$\Delta B=2$ and/or $\Delta L=2$ phenomena that have been considered most extensively in the recent literature are the following:

(i) *B-L* conserving processes: $p+p\rightarrow e^{+}+e^{+}$ (double proton decay) and the related process $H\equiv p+e^{-}\leftrightarrow\bar{p}+e^{+}\equiv\bar{H}$ (hydrogen-antihydrogen oscillations).

(ii) *B-L* violating processes: neutron-antineutron oscillations ($\Delta B=2$, $\Delta L=0$), neutrinoless double beta decay ($\Delta B=0$, $\Delta L=2$).

In the frame of the gauge models referred to above, the occurrence of these reactions stems from an enlargement, as compared to the standard (minimal) SU(5), of the Higgs sector, which includes diquark and dilepton scalar bosons; furthermore, interaction terms of higher orders are introduced in the Higgs potential.[1-4]

For instance, in the case of $\Delta B=\Delta L=2$ transitions the relevant diagram is a six-quark—two-lepton graph (Fig. 1) in which three diquark Higgs particles are coupled to a dilepton scalar boson through a quartic term in the Higgs potential.

A thorough analysis of models for the $\Delta B=\Delta L=2$ processes in the context of effective low-energy $SU_C(3)\times SU_L(2)\times U(1)$ interactions generated by partially or grand unified theories has recently been carried out by Nieves and Shanker.[4,5] A typical term of the effective Lagrangian for the $\Delta B=\Delta L=2$ interactions in these models reads[2]

$$\mathscr{L}=G_{H\bar{H}}\,|\,\psi(0)\,|^{4}\,\overline{(\psi_{e,L})^{C}}\psi_{e,L}\overline{(\psi_{p,L})^{C}}\psi_{p,L}+\text{H.c.}\;, \quad (1)$$

where ψ_e and ψ_p are the electron and proton fields, respectively, and $\psi(0)$ is the quark wave function in the nucleon at zero distance; furthermore,

$$G_{H\bar{H}}=\frac{\lambda f_q^3 f_l}{m_{\Delta_q}^6 m_{\Delta_l}^2} \quad (2)$$

for the case considered in Fig. 1 [here λ is the coefficient of the quartic term in the Higgs potential and f_q (f_l) the diquark (dilepton) Higgs-fermion Yukawa coupling].

It should be noticed that in the general scheme of Ref. 4, terms involving right-handed components of the fermion fields are present as well. However as far as the observables dealt with in the present paper are concerned (rate of the double proton decay and H-$\bar{\text{H}}$ oscillation time) these effective $\Delta B=\Delta L=2$ interactions are equivalent to the phenomenological Lagrangian examined by Feinberg, Goldhaber, and Steigman.[7]

Therefore for definiteness, in the following we shall consider Eq. (1) as the prototype Lagrangian for the $\Delta B=\Delta L=2$ interaction and introduce a dimensionless coupling constant defined as

$$K=G_{H\bar{H}}\,|\,\psi(0)\,|^{4}m_p^2\;. \quad (3)$$

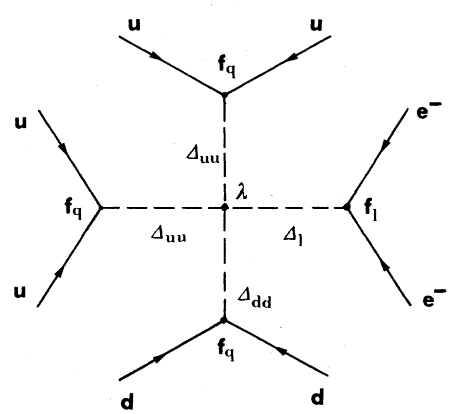

FIG. 1. Quark diagram for double proton decay and H-$\bar{\text{H}}$ oscillations. Δ_{uu} ($Q=-\frac{4}{3}$), Δ_{dd} ($Q=\frac{2}{3}$) are members of the diquark scalar boson Δ_q, whereas Δ_l^{++} is the doubly charged dilepton Higgs boson Δ_l (for other notations, see the text).

In the gauge models we are considering, the $\Delta B=2$, $\Delta L=0$ processes take place with analogous mechanisms. For instance, the n-$\bar{\rm n}$ oscillations can occur either by spontaneous symmetry breaking (see Fig. 2) or by explicit symmetry breaking through cubic terms in the Higgs potential. Neutron-antineutron mixing in nuclei has been analyzed by some of the present authors in the papers of Ref. 8, where data on nuclear stability have been employed to obtain for the n-$\bar{\rm n}$ oscillation time the lower bound $\tau_{n\bar{n}} > (5-7)\times 10^7$ sec. This limit is more stringent than the present experimental bound $\tau_{n\bar{n}} > 10^6$ sec obtained for free neutron oscillations[9] and it substantially agrees, within the theoretical uncertainties, with an independent evaluation given by Dover, Gal, and Richard.[10]

In the present paper we shall be concerned with the $\Delta B=\Delta L=2$ processes originated by the Lagrangian (1) and, in particular, with the double proton decay which leads to a quite peculiar form of nuclear instability. For this to happen, however, the two protons must come very close to each other, a fact likely prevented by the short-range, repulsive nucleon-nucleon correlations; therefore, the latter are obviously going to play a crucial role in this anomalous nuclear decay.

In Sec. II we calculate the decay rate for the double proton decay in nuclei in the frame of the shell model. In Sec. III we develop our approach to deal with proton-proton correlations and in Sec. IV we obtain, from the experimental limits on the nuclear instability, an upper bound on the effective coupling constant K. This limit is, in turn, converted into a lower bound on the oscillation time $\tau_{H\bar{H}}$ for the hydrogen-antihydrogen oscillations. Interestingly, our bound on $\tau_{H\bar{H}}$ turns out to be considerably more stringent than the experimental limit obtained from astrophysical data.

A pioneering analysis of this phenomenon was performed in Ref. 7 in the approximation of constant nuclear density. Recently Vergados[11] has implemented this treatment by evaluating the nuclear effects with phenomenological short-range correlation functions, but allowing only the outer nuclear protons to take part in the process; his calculation has been carried out for some medium weight nuclei and is mainly meant for detectors which involve large quantities of Ni.

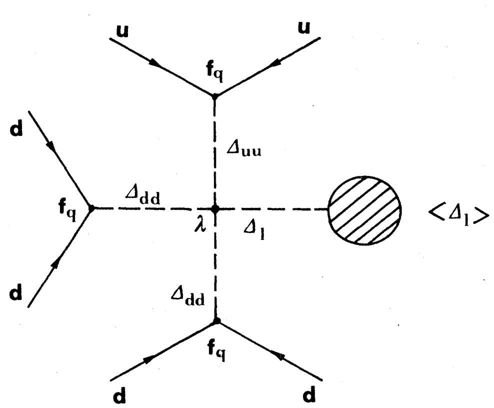

FIG. 2. Quark diagram for n-$\bar{\rm n}$ oscillations through spontaneous breaking of B-L symmetry.

Our present analysis has been prompted partly by the existence of new, more stringent bounds on different modes of matter instability (coming from the big detectors originally designed for proton decay) and partly by the need of treating as accurately as possible the short-range correlations, particularly for the nuclei relevant in the experiments under consideration (^{16}O and ^{56}Fe), allowing all the protons to take part in this exotic nuclear decay.

In Sec. V we finally present the conclusions of our investigation.

II. DOUBLE PROTON DECAY RATE INSIDE A NUCLEUS

From the Lagrangian (1), the decay rate (per proton) of the process

$$(A,Z)\rightarrow(A-2,Z-2)+e^+ + e^+ \tag{4}$$

obtains

$$\frac{\Gamma}{Z}=\frac{1}{\pi}\frac{K^2}{m_p^2}\frac{1}{ZV}\sum_f |\langle f|\hat{\Omega}|i\rangle|^2\,, \tag{5}$$

where V is the nuclear volume $[=(\frac{4}{3})\pi r_0^3 A,\ r_0=1.12\ \text{fm}]$ and

$$\hat{\Omega}=\tfrac{1}{2}\int d\mathbf{x}\,d\mathbf{x}'\,\Omega(\mathbf{x},\mathbf{x}')\hat{\psi}_p(\mathbf{x})\hat{\psi}_p(\mathbf{x}') \tag{6}$$

is the transition operator which annihilates two protons in the ground state $|i\rangle$ of the (A,Z) nucleus.

In (6) $\hat{\psi}_p(\mathbf{x})$ is the annihilation field for the proton and the corresponding first quantized operator reads

$$\Omega=\tfrac{1}{2}\int d\mathbf{x}\,d\mathbf{y}\,\rho_p(\mathbf{x})\delta(\mathbf{x}-\mathbf{y})\rho_p(\mathbf{y})\,, \tag{7}$$

$\rho_p(\mathbf{x})$ being the proton density.

For pointlike particles (7) reduces to

$$\Omega=\tfrac{1}{2}\sum_{\substack{i\neq j\\1}}^{Z}\delta(\mathbf{x}_i-\mathbf{x}_j)\equiv\tfrac{1}{2}\sum_{\substack{i\neq j\\1}}^{Z}\Omega(\mathbf{x}_i,\mathbf{x}_j)\,, \tag{8}$$

whereas, allowing for a finite nucleonic size, i.e., with the proton density

$$\rho_p(\mathbf{x})=\frac{M^3}{8\pi}\sum_{i=1}^{Z}e^{-M|\mathbf{x}-\mathbf{x}_i|} \tag{9}$$

($M\simeq 850$ MeV being the cutoff mass of the dipole nucleon form factor), one gets

$$\Omega=\tfrac{1}{2}\sum_{\substack{i\neq j\\1}}^{Z}\frac{M^3}{64\pi}e^{-M|\mathbf{x}_i-\mathbf{x}_j|}\times\{1+M|\mathbf{x}_i-\mathbf{x}_j|+\tfrac{1}{3}M^2|\mathbf{x}_i-\mathbf{x}_j|^2\}\,. \tag{10}$$

In (5), which can be more conveniently rewritten as

$$\frac{\Gamma}{Z}=5.0\times10^{-4}K^2 m_p\frac{1}{ZA}\sum_f|\langle f|\hat{\Omega}|i\rangle|^2\,, \tag{11}$$

the summation over f is extended to the different states in which the residual $(A-2,\ Z-2)$ nucleus is left.

In the following we shall use the harmonic oscillator (HO) shell model, allowing however for two-body short-range correlations induced by the Reid potential.[12] In this frame the nuclear matrix element for the process we are interested in reads:

$$\langle \Psi^f_{A-2}(\alpha_1^{-1}\alpha_2^{-1}) \,|\, \hat{\Omega} \,|\, \Psi^i_A \rangle = \langle \psi^f_N \,|\, \psi^i_N \rangle \sqrt{(Z-2)^2/Z(Z-1)} \langle 0 \,|\, \Omega \,|\, \alpha_1\alpha_2 \rangle \;, \tag{12}$$

where $|\Psi^f_{A-2}(\alpha_1^{-1}\alpha_2^{-1})\rangle$ is the residual nucleus state, which is obtained by annihilating, in the initial state $|\Psi^i_A\rangle$, two protons in the single particle orbits (α_1,α_2) [α is a shorthand notation for the quantum numbers (nlj) in the HO basis]. Formula (12) also implies that we treat the neutrons as spectators. The Z-dependent factor on the right-hand side of Eq. (12) accounts for the different normalizations of the wave functions of the initial and final nuclei.

The specific nature of the operator (6) entails that only protons with vanishing total spin (S) and angular momentum (J) can undergo annihilation; in fact, some algebra leads to

$$\langle 0 \,|\, \Omega \,|\, \alpha_1\alpha_2;SJ \rangle = \delta_{S0}\delta_{J0}\delta_{l_1l_2}\delta_{j_1j_2}(-1)^{l_1}\sqrt{(2j_1+1)/2}\,\langle (Z-2)_f;(\alpha_1\alpha_2)_{J=0} \,|\, \}Z_i \rangle \langle 0 \,|\, \Omega \,|\, n_1l_1n_2l_2,0 \rangle \;, \tag{13}$$

where the radial matrix elements, for uncorrelated two-proton states, read

$$\langle 0 \,|\, \Omega \,|\, n_1l_1n_2l_2,0 \rangle = \frac{4\pi}{2l_1+1} \int_0^\infty dr_1\, r_1^2 \int_0^\infty dr_2\, r_2^2 R_{n_1l_1}(r_1) R_{n_2l_2}(r_2) \Omega_{l_1}(r_1,r_2) \;. \tag{14}$$

In the above, $\Omega_l(r,r')$ is the lth multipole of the operator $\Omega(\mathbf{r}-\mathbf{r}')$, according to the standard decomposition

$$\Omega(\mathbf{r}-\mathbf{r}') = \sum_l \frac{4\pi}{2l+1} \Omega_l(r,r') \sum_m Y^*_{lm}(\hat{r}) Y_{lm}(\hat{r}') \;. \tag{15}$$

The analytic expression of the multipoles Ω_l, trivial for the $\delta(\mathbf{r}-\mathbf{r}')$ but quite cumbersome for the operator (10), is given in the Appendix. In formula (13) the probability for two protons in the same orbit $j_1=j_2$ of being coupled to $J=0$ has been included through the two-particle fractional parentage coefficients $\langle (Z-2)_f;(\alpha_1\alpha_2)_{J=0} \,|\, \}Z_i \rangle$.

We remind one that the matrix elements (14) are simply equal to one for the operator (8) and still rather close to unity for the interaction (10). How the dynamical correlations in the two-body wave function will affect these numbers, we shall explore in the next section.

III. CORRELATED TWO-PROTON STATE

The nucleon-nucleon correlations are most conveniently incorporated into the matrix element (13) utilizing the relative and center-of-mass coordinates of the two protons. This is done, in the HO basis, by the method of the Moshinsky transformation, which allows one to rewrite (14) as follows

$$\langle 0 \,|\, \Omega \,|\, n_1l_1n_2l_2,0 \rangle = 4\pi \sum_{n,N} \langle n\,0N0,0 \,|\, n_1l_1n_2l_2,0 \rangle \int_0^\infty dr\, r^2 R_{n0}(r) \Omega(\sqrt{2}r) \int_0^\infty dR\, R^2 R_{N0}(R) \;, \tag{16}$$

the symbols $\langle n\,0N0,0 \,|\, n_1l_1n_2l_2,0 \rangle$ denoting the Moshinsky transformation brackets. The nucleon-nucleon correlations can now be embodied in (16) by replacing the HO wave function for the relative motion, $R_{n0}(r)$, with

$$\psi_{n0}(r) = \mathcal{N}[1-C(r)]R_{n0}(r) \;, \tag{17}$$

$\mathcal{N}$ being a normalization factor and $C(r)$ the two-body correlation function. To obtain the latter we start with the Bethe-Goldstone equation in the operator form

$$G(\mathbf{k},\mathbf{k}_0,K) = V(\mathbf{k},\mathbf{k}_0) - \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{V(\mathbf{k},\mathbf{q})Q(q,K)G(\mathbf{q},\mathbf{k},K)}{E(q,K)-E(k_0,K)} \;, \tag{18}$$

where $Q(q,K)$ is the angle averaged Pauli operator, $\mathbf{k}$ and $\mathbf{k}_0$ the final and initial three-momenta, K the modulus of the center of mass momentum and

$$V(\mathbf{k}_1,\mathbf{k}_2) = \frac{1}{(2\pi)^3} \int d\mathbf{r}\, e^{-i\mathbf{k}_1\cdot\mathbf{r}} V(\mathbf{r}) e^{i\mathbf{k}_2\cdot\mathbf{r}} \tag{19}$$

the two-body potential in momentum space.

In the energy denominator

$$E(k,K)=\epsilon\left[\frac{\mathbf{K}}{2}+\mathbf{k}\right]+\epsilon\left[\frac{\mathbf{K}}{2}-\mathbf{k}\right] \quad (20)$$

and the standard quadratic form for the single particle energies has been assumed, namely

$$\epsilon(k)=\frac{\hbar^2k^2}{2m^*}+U_0 \quad \text{for } k\le k_F\,,$$
$$\epsilon(k)=\frac{\hbar^2k^2}{2m} \quad \text{for } k>k_F\,, \quad (21)$$

with U_0 and m^* calculated self-consistently, utilizing as a starting point the Reid soft-core potential.

By partial wave decomposition of (18), i.e., with

$$G_l(k,k_0,K)=V_l(k,k_0)-\frac{2}{\pi}\int_0^\infty dq\,q^2\frac{V_l(k,q)Q(q,K)G_l(q,k,K)}{e(k_0,q,K)} \quad (22)$$

and by projecting into configuration space the well-known expression for the correlated wave function

$$|\psi\rangle=|\varphi\rangle-\frac{Q}{e}G\,|\varphi\rangle\,, \quad (23)$$

$|\varphi\rangle$ being the uncorrelated two-body state, we finally arrive at

$$\langle r\,|\,\psi\rangle_l=j_l(k_0r)-\frac{2}{\pi}\int_0^\infty dk\,k^2\frac{Q(k,K)}{e(k,K)}G_l(k,k_0,K)j_l(kr)$$
$$=j_l(k_0r)-C_l(r)j_l(k_0r)\,, \quad (24)$$

which defines the two-body correlation function

$$C_l(r)=+\frac{2}{\pi}\frac{1}{j_l(k_0r)}\int_0^\infty dk\,k^2\frac{Q(k,K)}{e(k,K)}\times G_l(k,k_0,K)j_l(kr)\,. \quad (25)$$

We have evaluated (25) for the $l=0$ case (the only one needed in our problem) calculating the partial wave G-matrix elements $G_l(k,k_0,K)$ along the lines of Ref. 13 and for $k_F=1.2$ fm^{-1} (likely to be a reasonable approximation for ^{16}O and ^{56}Fe). We have also performed the calculation with $k_F=1.3$ fm^{-1} to ascertain the dependence upon the density of $C_0(r)$.

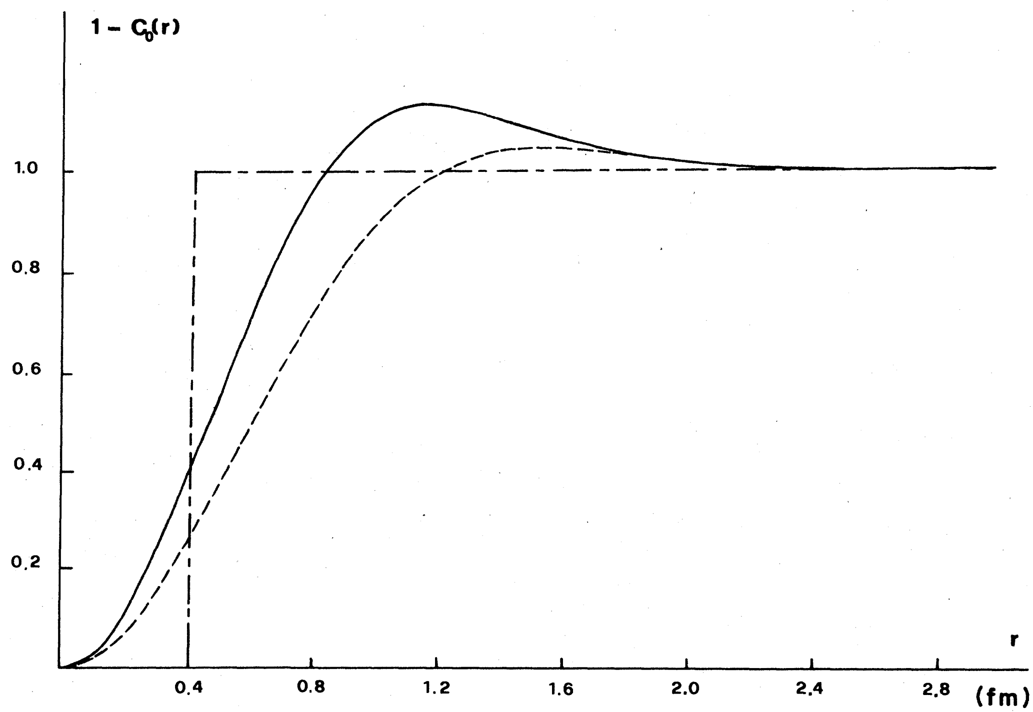

FIG. 3. S-wave two-body correlation function versus the relative distance r in the present calculation (continuous line). Also shown are the correlation functions utilized by Vergados (Ref. 11).

TABLE I. The matrix elements (16) for (a) ^{16}O and (b) ^{56}Fe. The correlation function has been evaluated with two different values of k_F (in fm^{-1}); ν is the oscillator parameter.

State (n_1l_1 n_2l_2)	Without correlation	With correlation $k_F=1.2$	With correlation $k_F=1.36$
	(a) ^{16}O ($\nu=0.336$ fm^{-2})		
$0s$ $0s$	0.9018	0.7799	0.7798
$0p$ $0p$	0.8428	0.7347	0.7378
	(b) ^{56}Fe ($\nu=0.242$ fm^{-2})		
$0s$ $0s$	0.9274	0.8075	0.8092
$0s$ $1s$	0.0549	0.0558	0.0563
$0p$ $0p$	0.8826	0.7699	0.7732
$0d$ $0d$	0.8405	0.7333	0.7376
$1s$ $1s$	0.8444	0.7374	0.7417
$0f$ $0f$	0.8008	0.6962	0.7008

Although, in principle, $C_0(r)$ is affected by the actual value of the initial relative momentum k_0 of the two interacting particles, this dependence turns out to be mild; typically in (25) we have taken $k_0=1.2$ fm^{-1}. Our result for the function $[1-C_0(r)]$ is displayed in Fig. 3. For the sake of illustration the two phenomenological correlation functions utilized by Vergados[11] are also shown.

The role played by the dynamical proton-proton correlations in the nuclear decay we are considering is illustrated in Table I, where the matrix elements (16) without and with correlations for the two-proton states of relevance for ^{16}O and ^{56}Fe are reported.

A few comments are in order:

(i) The dependence upon k_F of the matrix elements can be safely ignored for the range of k_F pertinent to our problem.

(ii) The "off-diagonal" matrix elements ($n_1 \neq n_2$) are found to be at most 10% of the diagonal ones.

(iii) The reduction associated with the short-range dynamical correlations, although sizable, are not dramatic owing to the finite range of the operator (10). Larger cutoff masses M would sharply change the situation.

This is clearly seen in Table II where the matrix elements for protons artificially reduced in size, by taking $M \simeq 2000$ MeV, are shown in the case of ^{56}Fe. The dramatic change occurring in the correlated matrix elements shows that the interplay between nucleonic size and two-body correlations is critical for the occurrence of the process we are considering here.

IV. RESULTS

A. Double proton decay

According to formula (11), in order to obtain the decay rate per proton we must evaluate

$$P \equiv \frac{1}{ZA} \sum_f |\langle f | \hat{\Omega} | i \rangle|^2 = \frac{1}{2AZ} \sum_{\alpha_1\alpha_2} |\langle \Psi^f_{A-2}(\alpha_1^{-1}\alpha_2^{-1}) | \hat{\Omega} | \Psi^i_A \rangle|^2 , \tag{26}$$

which turns out to be

$$P = 6.7 \times 10^{-3} \text{ for } ^{16}\text{O} , \tag{27a}$$

$$P = 1.6 \times 10^{-3} \text{ for } ^{56}\text{Fe} . \tag{27b}$$

In Table III the partial contributions to (27) arising from each single particle level are reported, with the separated geometrical and dynamical factors. It should be remarked that while the dynamical correlations between protons are treated, in the present framework, quite accurately, the global nuclear wave function is kept within the pure shell model, thus neglecting the residual interaction and the associated configuration mixing. This could change somewhat the "geometrical" probability of two-particle states coupled to $J=0$ as given by the fractional

TABLE II. The same as in Table I(b), with $M=2000$ MeV in the operator (10).

State (n_1l_1 n_2l_2)	Without correlation	With correlation
	^{56}Fe ($\nu=0.242$ fm^{-2}, $M=2000$ MeV)	
$0s$ $0s$	0.9860	0.4265
$0p$ $0p$	0.9769	0.4242
$0d$ $0d$	0.9678	0.4215
$1s$ $1s$	0.9679	0.4216
$0f$ $0f$	0.9588	0.4173

TABLE III. The various terms contributing to Eq. (26). In the second column the fractional parentage coefficients are reported.

States	cfp	$\langle 0 \mid \Omega \mid n_1 l_1 n_2 l_2 \rangle$	$\mid \langle \psi^f_{A-2} \mid \Omega \mid \psi^i_A \rangle \mid^2$
$0s_{1/2}\ 0s_{1/2}$	1.000 000	0.779 888	0.391 002
$0p_{1/2}\ 0p_{1/2}$	1.000 000	−0.734 743	0.347 045
$0p_{3/2}\ 0p_{3/2}$	0.408 200	−0.734 743	0.115 654
	$\nu=0.336$ fm^{-2} $P=0.006\,670$		
$0s_{1/2}\ 0s_{1/2}$	1.000 000	0.807 525	0.583 092
$0p_{1/2}\ 0p_{1/2}$	1.000 000	−0.769 940	0.530 077
$0p_{3/2}\ 0p_{3/2}$	0.408 200	−0.769 940	0.176 650
$0d_{3/2}\ 0d_{3/2}$	0.408 200	0.733 288	0.160 232
$0d_{5/2}\ 0d_{5/2}$	0.516 400	0.733 288	0.384 652
$1s_{1/2}\ 1s_{1/2}$	1.000 000	0.737 400	0.486 218
$0f_{7/2}\ 0f_{7/2}$	0.316 200	−0.696 224	0.173 343
	$\nu=0.242$ fm^{-2} $P=0.001\,591$		

parentage coefficient.

Finally, in order to set bounds on the coupling constant K, we rewrite Eq. (11) as follows

$$K=6.8\times10^{-15}\frac{1}{\sqrt{P}}\frac{1}{(\tau/\mathrm{yr})^{1/2}}\ , \tag{28}$$

where $\tau\equiv(\Gamma/Z)^{-1}$ and the definition (26) has been used.

Then, from the NUSEX-collaboration lower limit[14]

$$\tau>3\times10^{31}\ \mathrm{yr}\ (90\%\ \mathrm{C.L.}) \tag{29}$$

and our value of P for ^{56}Fe [Eq. (27b)], the following bound on K follows:

$$K<3\times10^{-29}\ . \tag{30}$$

For ^{16}O a preliminary estimate of the IMB-collaboration lower bound[15]

$$\tau>10^{32}\ \mathrm{yr}\ (90\%\ \mathrm{C.L.}) \tag{31}$$

and the value of Eq. (27a) give

$$K<8\times10^{-30}\ . \tag{32}$$

B. H-$\bar{\mathrm{H}}$ oscillations

As mentioned above, the Lagrangian (1) also gives rise to H-$\bar{\mathrm{H}}$ oscillations with an oscillation time

$$\tau_{\mathrm{H}\bar{\mathrm{H}}}=\left[\frac{K}{m_{\mathrm{p}}^2}\frac{m_{\mathrm{e}}^3\alpha^3}{\pi}\right]^{-1}\ . \tag{33}$$

In Ref. 7 it is shown that an independent limit on K can be set by considering that, if H-$\bar{\mathrm{H}}$ oscillations occur, then a neutral gas of hydrogen atoms, in the absence of external perturbations, can be a source of γ rays through the conversion of a fraction of hydrogen into antihydrogen and the subsequent H-$\bar{\mathrm{H}}$ annihilation. From the data on the γ-rays flux from interstellar regions in our galaxy one obtains the following bound

$$K<6\times10^{-26}\ , \tag{34}$$

or, equivalently,

$$\tau_{\mathrm{H}\bar{\mathrm{H}}}>6\times10^{10}\ \mathrm{yr}\ . \tag{35}$$

By comparing Eqs. (32) and (34) it then follows that our bound on K, derived from the new experimental results on nuclear stability, is much more stringent than the limit deduced from astrophysical data. From the bound of Eq. (32) we obtain for the H-$\bar{\mathrm{H}}$ oscillation time

$$\tau_{\mathrm{H}\bar{\mathrm{H}}}>1\times10^{14}\ \mathrm{yr}\ . \tag{36}$$

Finally our limit on K can be immediately converted into constraints on the parameters of specific gauge models by considering Eq. (3) and expressions of the type (2).

V. CONCLUSIONS

Baryon number violating processes ($\Delta B=2$) are predicted by a large variety of grand (or partially) unified theories of the fundamental interactions. Their occurrence gives rise to peculiar forms of nuclear instabilities.

In the present paper we have evaluated the nuclear effects which are relevant to the double proton decay, in the case of the nuclei (^{16}O and ^{56}Fe) involved in the big experimental apparatus originally designed to detect the proton decay. Thus, using the most recent limits on nuclear instability, we have set for the coupling constant K of the effective $\Delta B=\Delta L=2$ interactions an upper bound which is much more stringent than the astrophysical limit on the H-$\bar{\mathrm{H}}$ oscillations.

Our limit, converted into bounds on the parameters (coupling constants and masses) of the grand unified models, implements the constraints obtainable from the analysis of flavor-changing processes and then provides useful information on the possible structure of the unified gauge theories.

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APPENDIX

We evaluate here the partial waves of the operator

$$\Omega(\mathbf{x}-\mathbf{y})=\frac{M^3}{64\pi}e^{-M|\mathbf{x}-\mathbf{y}|}\{1+M\,|\mathbf{x}-\mathbf{y}|+\tfrac{1}{3}M^2|\mathbf{x}-\mathbf{y}|^2\}\ . \tag{A1}$$

It can be directly expanded in terms of Legendre polynomials:

$$\Omega(\mathbf{x}-\mathbf{y})=\sum_l \Omega_l(x,y)P_l(t)\ , \tag{A2}$$

where $t=\hat{x}\cdot\hat{y}$ and

$$\Omega_l(x,y)=\frac{M^3}{64\pi}\frac{2l+1}{2}\int_{-1}^{+1}dt\,P_l(t)\Omega(\sqrt{x^2+y^2-2xyt}\,)\ . \tag{A3}$$

With the change of variable $z=M(x^2+y^2-2xyt)^{1/2}$ and setting $c=Mx$, $d=My$, one gets

$$\Omega_l(x,y)\equiv\Omega_l\left[\frac{c}{M},\frac{d}{M}\right] =\frac{M^3}{64\pi}\frac{2l+1}{2cd}\int_{|c-d|}^{c+d}dz\,e^{-z}\left\{z+z^2+\frac{z^3}{3}\right\} \times P_l\left[\frac{c^2+d^2-z^2}{2cd}\right]. \tag{A4}$$

Finally, inserting in (A4) the following expression for the Legendre polynomials,

$$P_l(a+bz^2)=\frac{1}{2^l}\sum_{K=0}^{[l/2]}(-1)^K\frac{(2l-2K)!}{K!(l-K)!} \times\sum_{m=0}^{l-2K}\frac{a^{l-2K-m}b^m z^{2m}}{m!(l-2K-m)!}, \tag{A5}$$

one ends up with

$$\Omega_l(x,y)=\frac{M^3}{64\pi}\frac{2l+1}{cd}\frac{1}{2^{l+1}} \times\sum_{K=0}^{[l/2]}(-1)^K\frac{(2l-2K)!}{K!(l-K)!} \times\sum_{m=0}^{l-2K}\frac{(-1)^m}{m!(l-2K-m)!}\frac{(c^2+d^2)^{l-2K-m}}{(2cd)^{l-2K}}$$
$$\times\Bigg\{|c-d|^{2m+1}e^{-|c-d|}\Bigg[\tfrac{1}{3}(2m+3)(2m+5)+|c-d|\frac{(2m+6)}{3}+\tfrac{1}{3}(c-d)^2 +\tfrac{1}{3}(2m+3)(2m+5)\sum_{n=1}^{2m+1}\frac{(2m+1)!}{(2m+1-n)!}\frac{1}{|c-d|^n}\Bigg]$$
$$-(c+d)^{2m+1}e^{-(c+d)}\Bigg[\tfrac{1}{3}(2m+3)(2m+5)+\tfrac{1}{3}(2m+6)(c+d) +\tfrac{1}{3}(c+d)^2+\tfrac{1}{3}(2m+3)(2m+5) \times\sum_{n=1}^{2m+1}\frac{(2m+1)!}{(2m+1-n)!}\frac{1}{(c+d)^n}\Bigg]\Bigg\}. \tag{A6}$$

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